```
def derivatives_in_spherical_coordinates():
    Print_Function()
    coords = (r,th,phi) = symbols('r theta phi', real=True)
    (sp3d,er,eth,ephi) = Ga.build('e_r e_theta e_phi',g=[1,r**2,r**2*sin(th)**2],coords=coords)
    grad = sp3d.grad
    f = sp3d.mv('f','scalar',f=True)
    A = sp3d.mv('A','vector',f=True)
    B = sp3d.mv('B','bivector',f=True)
    print 'f =',f
    print 'A =',A
    print 'B =',B
    print 'grad*f =',grad*f
    print 'grad|A =',grad|A
    print 'grad\\times A = -I*(grad^A) =',-sp3d.i*(grad^A)
    print '%\\nabla^{2}f =',grad|(grad*f)
    print 'grad^B =',grad^B
    """
    print '( \\nabla\\W\\nabla )\\bm{e}_{r} =',((grad^grad)*er).trigsimp()
    print '( \\nabla\\W\\nabla )\\bm{e}_{\\theta} =',((grad^grad)*eth).trigsimp()
    print '( \\nabla\\W\\nabla )\\bm{e}_{\\phi} =',((grad^grad)*ephi).trigsimp()
    """
    return
```

Code Output:

$$f = f$$

$$A = A^{r} e_{r} + A^{\theta} e_{\theta} + A^{\phi} e_{\phi}$$

$$B = B^{r\theta} e_{r}\wedge e_{\theta} + B^{r\phi} e_{r}\wedge e_{\phi} + B^{\theta\phi} e_{\theta}\wedge e_{\phi}$$

$$\boldsymbol{\nabla} f = \partial_{r} f e_{r} + \frac{1}{r^{2}} \partial_{\theta} f e_{\theta} + \frac{\partial_{\phi} f}{r^{2} \sin^{2}{\left (\theta \right)}} e_{\phi}$$

$$\boldsymbol{\nabla} \cdot A = \frac{A^{\theta}}{\tan{\left (\theta \right)}} + \partial_{\phi} A^{\phi} + \partial_{r} A^{r} + \partial_{\theta} A^{\theta} + \frac{2 A^{r}}{r}$$

$$\boldsymbol{\nabla} \times A = -I (\boldsymbol{\nabla} \wedge A) = r^{2} \left(\left(- \frac{1}{2} \cos{\left (2 \theta \right)} + \frac{1}{2}\right) \partial_{\theta} A^{\phi} + A^{\phi} \sin{\left (2 \theta \right)} - \partial_{\phi} A^{\theta}\right) e_{r} + \left(\left(- r^{2} \partial_{r} A^{\phi} - 2 r A^{\phi}\right) \sin^{2}{\left (\theta \right)} + \partial_{\phi} A^{r}\right) e_{\theta} + \left(r^{2} \partial_{r} A^{\theta} + 2 r A^{\theta} - \partial_{\theta} A^{r}\right) e_{\phi}$$

$$\nabla^{2} f = \frac{1}{r^{2}} \left(r^{2} \partial^{2}_{r} f + 2 r \partial_{r} f + \partial^{2}_{\theta} f + \frac{\partial_{\theta} f}{\tan{\left (\theta \right)}} + \frac{\partial^{2}_{\phi} f}{\sin^{2}{\left (\theta \right)}}\right)$$

$$\boldsymbol{\nabla} \wedge B = \frac{1}{r^{2}} \left(r^{2} \partial_{r} B^{\theta\phi} + 4 r B^{\theta\phi} - \frac{2 B^{r\phi}}{\tan{\left (\theta \right)}} - \partial_{\theta} B^{r\phi} + \frac{\partial_{\phi} B^{r\theta}}{\sin^{2}{\left (\theta \right)}}\right) e_{r}\wedge e_{\theta}\wedge e_{\phi}$$

```
def derivatives_in_paraboloidal_coordinates():
    Print_Function()
    coords = (u,v,phi) = symbols('u v phi', real=True)
    (par3d,er,eth,ephi) = Ga.build('e_u e_v e_phi',X=[u*v*cos(phi),u*v*sin(phi),(u**2-v**2)/2],coords=coords,norm=True)
    grad = par3d.grad
    f = par3d.mv('f','scalar',f=True)
    A = par3d.mv('A','vector',f=True)
    B = par3d.mv('B','bivector',f=True)
    print '#Derivatives in Paraboloidal Coordinates'
    print 'f =',f
    print 'A =',A
    print 'B =',B
    print 'grad*f =',grad*f
    print 'grad|A =',grad|A
    (-par3d.i*(grad^A)).Fmt(3,'grad\\times A = -I*(grad^A)')
    print 'grad^B =',grad^B
    return
```

Code Output: Derivatives in Paraboloidal Coordinates

$$f = f$$

$$A = A^{u} e_{u} + A^{v} e_{v} + A^{\phi} e_{\phi}$$

$$B = B^{uv} e_{u} \wedge e_{v} + B^{u\phi} e_{u} \wedge e_{\phi} + B^{v\phi} e_{v} \wedge e_{\phi}$$

$$\boldsymbol{\nabla} f = \frac{\partial_{u} f}{\sqrt{u^{2} + v^{2}}} e_{u} + \frac{\partial_{v} f}{\sqrt{u^{2} + v^{2}}} e_{v} + \frac{\partial_{\phi} f}{uv} e_{\phi}$$

$$\boldsymbol{\nabla} \cdot A = \left(\frac{u}{\left(u^{2} + v^{2}\right)^{\frac{3}{2}}} + \frac{1}{u\sqrt{u^{2} + v^{2}}} \right) A^{u} + \left(\frac{v}{\left(u^{2} + v^{2}\right)^{\frac{3}{2}}} + \frac{1}{v\sqrt{u^{2} + v^{2}}} \right) A^{v} + \frac{\partial_{u} A^{u}}{\sqrt{u^{2} + v^{2}}} + \frac{\partial_{v} A^{v}}{\sqrt{u^{2} + v^{2}}} + \frac{\partial_{\phi} A^{\phi}}{uv}$$

$$\begin{aligned} \boldsymbol{\nabla} \times A = -I(\boldsymbol{\nabla} \wedge A) = & \frac{1}{uv\left(u^{2} + v^{2}\right)} \left(uv\sqrt{u^{2} + v^{2}} \partial_{v} A^{\phi} + u\sqrt{u^{2} + v^{2}} A^{\phi} + \left(-u^{2} - v^{2}\right) \partial_{\phi} A^{v} \right) e_{u} \\ & + \frac{1}{uv\left(u^{2} + v^{2}\right)} \left(-uv\sqrt{u^{2} + v^{2}} \partial_{u} A^{\phi} - v\sqrt{u^{2} + v^{2}} A^{\phi} + \left(u^{2} + v^{2}\right) \partial_{\phi} A^{u} \right) e_{v} \\ & + \frac{1}{\left(u^{2} + v^{2}\right)^{\frac{3}{2}}} \left(uA^{v} - vA^{u} + \left(u^{2} + v^{2}\right) \left(-\partial_{v} A^{u} + \partial_{u} A^{v}\right) \right) e_{\phi} \end{aligned}$$

$$\boldsymbol{\nabla} \wedge B = \left(\left(\frac{u}{\left(u^{2} + v^{2}\right)^{\frac{3}{2}}} + \frac{1}{u\sqrt{u^{2} + v^{2}}} \right) B^{v\phi} + \left(-\frac{v}{\left(u^{2} + v^{2}\right)^{\frac{3}{2}}} - \frac{1}{v\sqrt{u^{2} + v^{2}}} \right) B^{u\phi} - \frac{\partial_{v} B^{u\phi}}{\sqrt{u^{2} + v^{2}}} + \frac{\partial_{u} B^{v\phi}}{\sqrt{u^{2} + v^{2}}} + \frac{\partial_{\phi} B^{uv}}{uv} \right) e_{u} \wedge e_{v} \wedge e_{\phi}$$

```
def derivatives_in_elliptic_cylindrical_coordinates():
    Print_Function()
    a = symbols('a', real=True)
    coords = (u,v,z) = symbols('u v z', real=True)
    (elip3d,er,eth,ephi) = Ga.build('e_u e_v e_z',X=[a*cosh(u)*cos(v),a*sinh(u)*sin(v),z],coords=coords,norm=True)
    grad = elip3d.grad
    f = elip3d.mv('f','scalar',f=True)
    A = elip3d.mv('A','vector',f=True)
    B = elip3d.mv('B','bivector',f=True)
    print 'f =',f
    print 'A =',A
    print 'B =',B
    print 'grad*f =',grad*f
    print 'grad|A =',grad|A
    print '-I*(grad^A) =',-elip3d.i*(grad^A)
    print 'grad^B =',grad^B
    return
```

Code Output:

$$f = f$$

$$A = A^{u} e_{u} + A^{v} e_{v} + A^{z} e_{z}$$

$$B = B^{uv} e_{u} \wedge e_{v} + B^{uz} e_{u} \wedge e_{z} + B^{vz} e_{v} \wedge e_{z}$$

$$\boldsymbol{\nabla} f = \frac{\partial_{u} f}{\sqrt{\sin^{2}(v)\cosh^{2}(u) + \cos^{2}(v)\sinh^{2}(u)}\,|a|} e_{u} + \frac{\partial_{v} f}{\sqrt{\sin^{2}(v)\cosh^{2}(u) + \cos^{2}(v)\sinh^{2}(u)}\,|a|} e_{v} + \partial_{z} f e_{z}$$

$$\boldsymbol{\nabla}\cdot A = \frac{|a|}{2\left(\left(\cos^{2}(v)\cos(2v) - \cos^{2}(v)\cosh(2u)\right)\sinh^{2}(u) + \left(\cos(2v)\cosh^{2}(u) - \cosh^{2}(u)\cosh(2u)\right)\sin^{2}(v)\right)\left(\sin^{2}(v)\cosh^{2}(u) + \cos^{2}(v)\sinh^{2}(u)\right)^{\frac{5}{2}}} \Big(\left(A^{u}\sinh(2u) + A^{v}\sin(2v)\right)\left(\sin^{2}(v)\cosh^{2}(u) + \cos^{2}(v)\sinh^{2}(u)\right)\big(\sin^{2}(v)\cos(2v)\cosh^{2}(u) - \sin^{2}(v)\cosh^{2}(u)\cosh(2u) + \cos^{2}(v)\cos(2v)\sinh^{2}(u) - \cos^{2}(v)\sinh^{2}(u)\cosh(2$$

$$-I(\boldsymbol{\nabla}\wedge A) = \left(-\partial_{z} A^{v} + \frac{\partial_{v} A^{z}}{\sqrt{\sin^{2}(v) + \sinh^{2}(u)}\,|a|}\right) e_{u} + \left(\partial_{z} A^{u} - \frac{\partial_{u} A^{z}}{\sqrt{\sin^{2}(v) + \sinh^{2}(u)}\,|a|}\right) e_{v} - \frac{\sqrt{2}\,|a|}{2\left(-\cos(2v) + \cosh(2u)\right)^{\frac{5}{2}}} \Big(\left(-2\left(\cosh(2u) - 1\right)^{2} + 2\right)\partial_{u} A^{v} + \left(2\left(\cosh(2u) - 1\right)^{2} - 2\right)\partial_{v} A^{u} + \left(2\partial_{v} A^{u} - 2\partial_{u} A^{v}\right)\cos^{2}(2v) + \left(2A^{u}\sin(2v) - 4\cos(2v)\partial_{v} A^{u} + 4\cos(2v)\partial_{u} A^{v} + 4\partial_{v} A^{u} - 4\partial_{u} A^{v}\right)\cosh(2u) - A^{u}\sin(4v) + 2A^{v}\cos(2v$$

$$\boldsymbol{\nabla}\wedge B = -\frac{|a|}{2\left(\left(\cos^{2}(v)\cos(2v) - \cos^{2}(v)\cosh(2u)\right)\sinh^{2}(u) + \left(\cos(2v)\cosh^{2}(u) - \cosh^{2}(u)\cosh(2u)\right)\sin^{2}(v)\right)\left(\sin^{2}(v)\cosh^{2}(u) + \cos^{2}(v)\sinh^{2}(u)\right)^{\frac{5}{2}}} \Big(\left(B^{uz}\sin(2v) - B^{vz}\sinh(2u)\right)\left(\sin^{2}(v)\cosh^{2}(u) + \cos^{2}(v)\sinh^{2}(u)\right)\big(\sin^{2}(v)\cos(2v)\cosh^{2}(u) - \sin^{2}(v)\cosh^{2}(u)\cosh(2u) + \cos^{2}(v)\cos(2v)\sinh^{2}(u) - \cos^{2}(v)\sinh^{2}(u)\,\mathrm{co}$$

```
def derivatives_in_prolate_spheroidal_coordinates():
    Print_Function()
    a = symbols('a', real=True)
    coords = (xi,eta,phi) = symbols('xi eta phi', real=True)
    (ps3d,er,eth,ephi) = Ga.build('e_xi e_eta e_phi',X=[a*sinh(xi)*sin(eta)*cos(phi),a*sinh(xi)*sin(eta)*sin(phi),
                                                        a*cosh(xi)*cos(eta)],coords=coords,norm=True)
    grad = ps3d.grad
    f = ps3d.mv('f','scalar',f=True)
    A = ps3d.mv('A','vector',f=True)
    B = ps3d.mv('B','bivector',f=True)
    print 'f =',f
    print 'A =',A
    print 'B =',B
    print 'grad*f =',grad*f
    print 'grad|A =',grad|A
    (-ps3d.i*(grad^A)).Fmt(3,'-I*(grad^A)')
    (grad^B).Fmt(3,'grad^B')
    return
```

Code Output:

$$f = f$$

$$A = A^{\xi}e_{\xi} + A^{\eta}e_{\eta} + A^{\phi}e_{\phi}$$

$$B = B^{\xi\xi}e_{\xi}\wedge e_{\eta} + B^{\xi\xi}e_{\xi}\wedge e_{\phi} + B^{\phi\phi}e_{\eta}\wedge e_{\phi}$$

$$\boldsymbol{\nabla} f = \frac{\partial_{\xi} f}{\sqrt{\sin^{2}{\left (\eta \right)} + \sinh^{2}{\left (\xi \right)}} \left|{a}\right|} e_{\xi} + \frac{\partial_{\eta} f}{\sqrt{\sin^{2}{\left (\eta \right)} + \sinh^{2}{\left (\xi \right)}} \left|{a}\right|} e_{\eta} + \frac{\partial_{\phi} f}{a \sin{\left (\eta \right)} \sinh{\left (\xi \right)}} e_{\phi}$$

$$\boldsymbol{\nabla} \cdot A = \frac{1}{a^{2} \left(\sin^{2}{\left (\eta \right)} + \sinh^{2}{\left (\xi \right)}\right)^{3} \sin{\left (\eta \right)} \sinh{\left (\xi \right)}} \left(a \left(\sin^{2}{\left (\eta \right)} + \sinh^{2}{\left (\xi \right)}\right)^{3} \partial_{\phi} A^{\phi} + \left(\sin^{2}{\left (\eta \right)} + \sinh^{2}{\left (\xi \right)}\right)^{\frac{5}{2}} A^{\eta} \cos{\left (\eta \right)} \sinh{\left (\xi \right)} \left|{a}\right| + \left(\frac{1}{2} \left(A^{\eta} \sin{\left (2 \eta \right)} + A^{\xi} \sinh{\left (2 \xi \right)}\right) \left(\sin^{2}{\left (\eta \right)} + \sinh^{2}{\left (\xi \right)}\right)^{\frac{3}{2}} \sin{\left (\eta \right)} \sinh{\left (\xi \right)} + \left(\sin^{2}{\left (\eta \right)} + \sinh^{2}{\left (\xi \right)}\right)^{\frac{5}{2}} \left(\partial_{\eta} A^{\eta} + \partial_{\xi} A^{\xi}\right) \sin{\left (\eta \right)} \sinh{\left (\xi \right)} + \left(\sin^{2}{\left (\eta \right)} + \sinh^{2}{\left (\xi \right)}\right)^{\frac{5}{2}} A^{\xi} \sin{\left (\eta \right)} \cosh{\left (\xi \right)}\right) \left|{a}\right|\right)$$

$$\begin{aligned} -I (\boldsymbol{\nabla} \wedge A) = & \frac{1}{a^{2} \sin{\left (\eta \right)}} \left(- \frac{a \partial_{\phi} A^{\eta}}{\sinh{\left (\xi \right)}} + \left(\frac{A^{\phi} \cos{\left (\eta \right)}}{\sqrt{\sin^{2}{\left (\eta \right)} + \sinh^{2}{\left (\xi \right)}}} + \frac{\sin{\left (\eta \right)} \partial_{\eta} A^{\phi}}{\sqrt{\sin^{2}{\left (\eta \right)} + \sinh^{2}{\left (\xi \right)}}}\right) \left|{a}\right|\right) e_{\xi} \\ & - \frac{1}{a^{2} \sinh{\left (\xi \right)}} \left(- \frac{a \partial_{\phi} A^{\xi}}{\sin{\left (\eta \right)}} + \left(\frac{A^{\phi} \cosh{\left (\xi \right)}}{\sqrt{\sin^{2}{\left (\eta \right)} + \sinh^{2}{\left (\xi \right)}}} + \frac{\sinh{\left (\xi \right)} \partial_{\xi} A^{\phi}}{\sqrt{\sin^{2}{\left (\eta \right)} + \sinh^{2}{\left (\xi \right)}}}\right) \left|{a}\right|\right) e_{\eta} \\ & + \frac{1}{2 \left(\sin^{2}{\left (\eta \right)} + \sinh^{2}{\left (\xi \right)}\right)^{\frac{3}{2}} \left|{a}\right|} \left(\left(\sin^{2}{\left (\eta \right)} + \sinh^{2}{\left (\xi \right)}\right) \left(2 \partial_{\xi} A^{\eta} - 2 \partial_{\eta} A^{\xi}\right) + A^{\eta} \sinh{\left (2 \xi \right)} - A^{\xi} \sin{\left (2 \eta \right)}\right) e_{\phi} \end{aligned}$$

$$\boldsymbol{\nabla} \wedge B = \frac{1}{a^{2} \left(\sin^{2}{\left (\eta \right)} + \sinh^{2}{\left (\xi \right)}\right)^{3} \sin{\left (\eta \right)} \sinh{\left (\xi \right)}} \left(a \left(\sin^{2}{\left (\eta \right)} + \sinh^{2}{\left (\xi \right)}\right)^{3} \partial_{\phi} B^{\xi\xi} + \left(\frac{1}{2} \left(B^{\phi\phi} \sinh{\left (2 \xi \right)} - B^{\xi\xi} \sin{\left (2 \eta \right)}\right) \left(\sin^{2}{\left (\eta \right)} + \sinh^{2}{\left (\xi \right)}\right)^{\frac{3}{2}} \sin{\left (\eta \right)} \sinh{\left (\xi \right)} + \left(\sin^{2}{\left (\eta \right)} + \sinh^{2}{\left (\xi \right)}\right)^{\frac{5}{2}} \left(\partial_{\xi} B^{\phi\phi} - \partial_{\eta} B^{\xi\xi}\right) \sin{\left (\eta \right)} \sinh{\left (\xi \right)} + \left(\sin^{2}{\left (\eta \right)} + \sinh^{2}{\left (\xi \right)}\right)^{\frac{5}{2}} B^{\phi\phi} \sin{\left (\eta \right)} \cosh{\left (\xi \right)} - \left(\sin^{2}{\left (\eta \right)} + \sinh^{2}{\left (\xi \right)}\right)^{\frac{5}{2}} B^{\xi\xi} \cos{\left (\eta \right)} \sinh{\left (\xi \right)}\right) \left|{a}\right|\right) e_{\xi}\wedge e_{\eta}\wedge e_{\phi}$$