

3d orthogonal (A, B are linear transformations)

$$A = \left\{ \begin{array}{l} L(\mathbf{e}_x) = A_{xx}\mathbf{e}_x + A_{yx}\mathbf{e}_y + A_{zx}\mathbf{e}_z \\ L(\mathbf{e}_y) = A_{xy}\mathbf{e}_x + A_{yy}\mathbf{e}_y + A_{zy}\mathbf{e}_z \\ L(\mathbf{e}_z) = A_{xz}\mathbf{e}_x + A_{yz}\mathbf{e}_y + A_{zz}\mathbf{e}_z \end{array} \right\}$$

$$\text{mat}(A) = \begin{bmatrix} A_{xx} & A_{xy} & A_{xz} \\ A_{yx} & A_{yy} & A_{yz} \\ A_{zx} & A_{zy} & A_{zz} \end{bmatrix}$$

$$\det(A) = A_{xz}(A_{yx}A_{zy} - A_{yy}A_{zx}) - A_{yz}(A_{xx}A_{zy} - A_{xy}A_{zx}) + A_{zz}(A_{xx}A_{yy} - A_{xy}A_{yx})$$

$$\overline{A} = \left\{ \begin{array}{l} L(\mathbf{e}_x) = A_{xx}\mathbf{e}_x + A_{xy}\mathbf{e}_y + A_{xz}\mathbf{e}_z \\ L(\mathbf{e}_y) = A_{yx}\mathbf{e}_x + A_{yy}\mathbf{e}_y + A_{yz}\mathbf{e}_z \\ L(\mathbf{e}_z) = A_{zx}\mathbf{e}_x + A_{zy}\mathbf{e}_y + A_{zz}\mathbf{e}_z \end{array} \right\}$$

$$\text{Tr}(A) = A_{xx} + A_{yy} + A_{zz}$$

$$A(\mathbf{e}_x \wedge \mathbf{e}_y) = (A_{xx}A_{yy} - A_{xy}A_{yx})\mathbf{e}_x \wedge \mathbf{e}_y + (A_{xx}A_{zy} - A_{xy}A_{zx})\mathbf{e}_x \wedge \mathbf{e}_z + (A_{yx}A_{zy} - A_{yy}A_{zx})\mathbf{e}_y \wedge \mathbf{e}_z$$

$$A(\mathbf{e}_x) \wedge A(\mathbf{e}_y) = (A_{xx}A_{yy} - A_{xy}A_{yx})\mathbf{e}_x \wedge \mathbf{e}_y + (A_{xx}A_{zy} - A_{xy}A_{zx})\mathbf{e}_x \wedge \mathbf{e}_z + (A_{yx}A_{zy} - A_{yy}A_{zx})\mathbf{e}_y \wedge \mathbf{e}_z$$

$$A + B = \left\{ \begin{array}{l} L(\mathbf{e}_x) = (A_{xx} + B_{xx})\mathbf{e}_x + (A_{yx} + B_{yx})\mathbf{e}_y + (A_{zx} + B_{zx})\mathbf{e}_z \\ L(\mathbf{e}_y) = (A_{xy} + B_{xy})\mathbf{e}_x + (A_{yy} + B_{yy})\mathbf{e}_y + (A_{zy} + B_{zy})\mathbf{e}_z \\ L(\mathbf{e}_z) = (A_{xz} + B_{xz})\mathbf{e}_x + (A_{yz} + B_{yz})\mathbf{e}_y + (A_{zz} + B_{zz})\mathbf{e}_z \end{array} \right\}$$

$$AB = \left\{ \begin{array}{l} L(\mathbf{e}_x) = (A_{xx}B_{xx} + A_{xy}B_{yx} + A_{xz}B_{zx})\mathbf{e}_x + (A_{yx}B_{xx} + A_{yy}B_{yx} + A_{yz}B_{zx})\mathbf{e}_y + (A_{zx}B_{xx} + A_{zy}B_{yx} + A_{zz}B_{zx})\mathbf{e}_z \\ L(\mathbf{e}_y) = (A_{xx}B_{xy} + A_{xy}B_{yy} + A_{xz}B_{zy})\mathbf{e}_x + (A_{yx}B_{xy} + A_{yy}B_{yy} + A_{yz}B_{zy})\mathbf{e}_y + (A_{zx}B_{xy} + A_{zy}B_{yy} + A_{zz}B_{zy})\mathbf{e}_z \\ L(\mathbf{e}_z) = (A_{xx}B_{xz} + A_{xy}B_{yz} + A_{xz}B_{zz})\mathbf{e}_x + (A_{yx}B_{xz} + A_{yy}B_{yz} + A_{yz}B_{zz})\mathbf{e}_y + (A_{zx}B_{xz} + A_{zy}B_{yz} + A_{zz}B_{zz})\mathbf{e}_z \end{array} \right\}$$

$$A - B = \left\{ \begin{array}{l} L(\mathbf{e}_x) = (A_{xx} - B_{xx})\mathbf{e}_x + (A_{yx} - B_{yx})\mathbf{e}_y + (A_{zx} - B_{zx})\mathbf{e}_z \\ L(\mathbf{e}_y) = (A_{xy} - B_{xy})\mathbf{e}_x + (A_{yy} - B_{yy})\mathbf{e}_y + (A_{zy} - B_{zy})\mathbf{e}_z \\ L(\mathbf{e}_z) = (A_{xz} - B_{xz})\mathbf{e}_x + (A_{yz} - B_{yz})\mathbf{e}_y + (A_{zz} - B_{zz})\mathbf{e}_z \end{array} \right\}$$

2d general (A, B are linear transformations)

$$A = \left\{ \begin{array}{l} L(\mathbf{e}_u) = A_{uu}\mathbf{e}_u + A_{vu}\mathbf{e}_v \\ L(\mathbf{e}_v) = A_{uv}\mathbf{e}_u + A_{vv}\mathbf{e}_v \end{array} \right\}$$

$$\det(A) = A_{uu}A_{vv} - A_{uv}A_{vu}$$

$$\text{Tr}(A) = -\frac{(e_u \cdot e_u)(e_v \cdot e_v)A_{uu}}{-(e_u \cdot e_u)(e_v \cdot e_v) + (e_u \cdot e_v)^2} - \frac{(e_u \cdot e_u)(e_v \cdot e_v)A_{vv}}{-(e_u \cdot e_u)(e_v \cdot e_v) + (e_u \cdot e_v)^2} + \frac{(e_u \cdot e_v)^2 A_{uu}}{-(e_u \cdot e_u)(e_v \cdot e_v) + (e_u \cdot e_v)^2} + \frac{(e_u \cdot e_v)^2 A_{vv}}{-(e_u \cdot e_u)(e_v \cdot e_v) + (e_u \cdot e_v)^2}$$

$$A(\mathbf{e}_u \wedge \mathbf{e}_v) = (A_{uu}A_{vv} - A_{uv}A_{vu})\mathbf{e}_u \wedge \mathbf{e}_v$$

$$A(\mathbf{e}_u) \wedge A(\mathbf{e}_v) = (A_{uu}A_{vv} - A_{uv}A_{vu})\mathbf{e}_u \wedge \mathbf{e}_v$$

$$B = \left\{ \begin{array}{l} L(\mathbf{e}_u) = B_{uu}\mathbf{e}_u + B_{vu}\mathbf{e}_v \\ L(\mathbf{e}_v) = B_{uv}\mathbf{e}_u + B_{vv}\mathbf{e}_v \end{array} \right\}$$

$$A + B = \left\{ \begin{array}{l} L(\mathbf{e}_u) = (A_{uu} + B_{uu})\mathbf{e}_u + (A_{vu} + B_{vu})\mathbf{e}_v \\ L(\mathbf{e}_v) = (A_{uv} + B_{uv})\mathbf{e}_u + (A_{vv} + B_{vv})\mathbf{e}_v \end{array} \right\}$$

$$AB = \left\{ \begin{array}{l} L(\mathbf{e}_u) = (A_{uu}B_{uu} + A_{uv}B_{vu})\mathbf{e}_u + (A_{vu}B_{uu} + A_{vv}B_{vu})\mathbf{e}_v \\ L(\mathbf{e}_v) = (A_{uu}B_{uv} + A_{uv}B_{vv})\mathbf{e}_u + (A_{vu}B_{uv} + A_{vv}B_{vv})\mathbf{e}_v \end{array} \right\}$$

$$A - B = \left\{ \begin{array}{l} L(\mathbf{e}_u) = (A_{uu} - B_{uu})\mathbf{e}_u + (A_{vu} - B_{vu})\mathbf{e}_v \\ L(\mathbf{e}_v) = (A_{uv} - B_{uv})\mathbf{e}_u + (A_{vv} - B_{vv})\mathbf{e}_v \end{array} \right\}$$

$$a \cdot \overline{A}(b) - b \cdot \underline{A}(a) = 0$$

4d Minkowski space (Space Time)

$$g = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$\underline{T} = \left\{ \begin{array}{l} L(\mathbf{e}_t) = T_{tt}\mathbf{e}_t + T_{xt}\mathbf{e}_x + T_{yt}\mathbf{e}_y + T_{zt}\mathbf{e}_z \\ L(\mathbf{e}_x) = T_{tx}\mathbf{e}_t + T_{xx}\mathbf{e}_x + T_{yx}\mathbf{e}_y + T_{zx}\mathbf{e}_z \\ L(\mathbf{e}_y) = T_{ty}\mathbf{e}_t + T_{xy}\mathbf{e}_x + T_{yy}\mathbf{e}_y + T_{zy}\mathbf{e}_z \\ L(\mathbf{e}_z) = T_{tz}\mathbf{e}_t + T_{xz}\mathbf{e}_x + T_{yz}\mathbf{e}_y + T_{zz}\mathbf{e}_z \end{array} \right\}$$

$$\overline{T} = \left\{ \begin{array}{l} L(\mathbf{e}_t) = T_{tt}\mathbf{e}_t - T_{tx}\mathbf{e}_x - T_{ty}\mathbf{e}_y - T_{tz}\mathbf{e}_z \\ L(\mathbf{e}_x) = -T_{xt}\mathbf{e}_t + T_{xx}\mathbf{e}_x + T_{xy}\mathbf{e}_y + T_{xz}\mathbf{e}_z \\ L(\mathbf{e}_y) = -T_{yt}\mathbf{e}_t + T_{yx}\mathbf{e}_x + T_{yy}\mathbf{e}_y + T_{yz}\mathbf{e}_z \\ L(\mathbf{e}_z) = -T_{zt}\mathbf{e}_t + T_{zx}\mathbf{e}_x + T_{zy}\mathbf{e}_y + T_{zz}\mathbf{e}_z \end{array} \right\}$$

$$\text{tr}(\underline{T}) = T_{tt} + T_{xx} + T_{yy} + T_{zz}$$

$$a \cdot \overline{T}(b) - b \cdot \underline{T}(a) = 0$$