

```
def Maxwells_Equations_in_Geom_Calculus():
    Print_Function()
    X = symbols('t x y z',real=True)
    (st4d,g0,g1,g2,g3) = Ga.build('gamma*t|x|y|z',g=[1,-1,-1,-1],coords=X)
    I = st4d.i
    B = st4d.mv('B','vector',f=True)
    E = st4d.mv('E','vector',f=True)
    B.set_coef(1,0,0)
    E.set_coef(1,0,0)
    B *= g0
    E *= g0
    J = st4d.mv('J','vector',f=True)
    F = E+I*B
    print r'\text{Pseudo Scalar \;\;} I =',I
    print '\\text{Magnetic Field Bi-Vector \;\;\;\;} B = \\bm{B\\gamma_{t}} =',B
    print '\\text{Electric Field Bi-Vector \;\;\;\;} E = \\bm{E\\gamma_{t}} =',E
    print '\\text{Electromagnetic Field Bi-Vector \;\;\;\;} F = E+IB =',F
    print '%\\text{Four Current Density \;\;\;\;} J =',J
    gradF = st4d.grad*F
    print '#Geom Derivative of Electomagnetic Field Bi-Vector'
    gradF.Fmt(3,'grad*F')
    print '#Maxwell Equations'
    print 'grad*F = J'
    print '#Div $E$ and Curl $H$ Equations'
    (gradF.get_grade(1)-J).Fmt(3,'%\\grade{\\nabla F}_{1} -J = 0')
    print '#Curl $E$ and Div $B$ equations'
    (gradF.get_grade(3)).Fmt(3,'%\\grade{\\nabla F}_{3} = 0')
    return
```

Code Output:

Pseudo Scalar $I = \boldsymbol{\gamma}_t \wedge \boldsymbol{\gamma}_x \wedge \boldsymbol{\gamma}_y \wedge \boldsymbol{\gamma}_z$

Magnetic Field Bi-Vector $B = \boldsymbol{B}\boldsymbol{\gamma}_t = -B^x\boldsymbol{\gamma}_t \wedge \boldsymbol{\gamma}_x - B^y\boldsymbol{\gamma}_t \wedge \boldsymbol{\gamma}_y - B^z\boldsymbol{\gamma}_t \wedge \boldsymbol{\gamma}_z$

Electric Field Bi-Vector $E = \boldsymbol{E}\boldsymbol{\gamma}_t = -E^x\boldsymbol{\gamma}_t \wedge \boldsymbol{\gamma}_x - E^y\boldsymbol{\gamma}_t \wedge \boldsymbol{\gamma}_y - E^z\boldsymbol{\gamma}_t \wedge \boldsymbol{\gamma}_z$

Electromagnetic Field Bi-Vector $F = E + IB = -E^x\boldsymbol{\gamma}_t \wedge \boldsymbol{\gamma}_x - E^y\boldsymbol{\gamma}_t \wedge \boldsymbol{\gamma}_y - E^z\boldsymbol{\gamma}_t \wedge \boldsymbol{\gamma}_z - B^z\boldsymbol{\gamma}_x \wedge \boldsymbol{\gamma}_y + B^y\boldsymbol{\gamma}_x \wedge \boldsymbol{\gamma}_z - B^x\boldsymbol{\gamma}_y \wedge \boldsymbol{\gamma}_z$

Four Current Density $J = J^t\boldsymbol{\gamma}_t + J^x\boldsymbol{\gamma}_x + J^y\boldsymbol{\gamma}_y + J^z\boldsymbol{\gamma}_z$

Geom Derivative of Electomagnetic Field Bi-Vector

$$\begin{aligned} \nabla F = & (\partial_x E^x + \partial_y E^y + \partial_z E^z) \boldsymbol{\gamma}_t \\ & + (-\partial_z B^y + \partial_y B^z - \partial_t E^x) \boldsymbol{\gamma}_x \\ & + (\partial_z B^x - \partial_x B^z - \partial_t E^y) \boldsymbol{\gamma}_y \\ & + (-\partial_y B^x + \partial_x B^y - \partial_t E^z) \boldsymbol{\gamma}_z \\ & + (-\partial_t B^z + \partial_y E^x - \partial_x E^y) \boldsymbol{\gamma}_t \wedge \boldsymbol{\gamma}_x \wedge \boldsymbol{\gamma}_y \\ & + (\partial_t B^y + \partial_z E^x - \partial_x E^z) \boldsymbol{\gamma}_t \wedge \boldsymbol{\gamma}_x \wedge \boldsymbol{\gamma}_z \\ & + (-\partial_t B^x + \partial_z E^y - \partial_y E^z) \boldsymbol{\gamma}_t \wedge \boldsymbol{\gamma}_y \wedge \boldsymbol{\gamma}_z \\ & + (\partial_x B^x + \partial_y B^y + \partial_z B^z) \boldsymbol{\gamma}_x \wedge \boldsymbol{\gamma}_y \wedge \boldsymbol{\gamma}_z \end{aligned}$$

Maxwell Equations

$$\nabla F = J$$

Div E and Curl H Equations

$$\begin{aligned} \langle \nabla F \rangle_1 - J = 0 = & (-J^t + \partial_x E^x + \partial_y E^y + \partial_z E^z) \boldsymbol{\gamma}_t \\ & + (-J^x - \partial_z B^y + \partial_y B^z - \partial_t E^x) \boldsymbol{\gamma}_x \\ & + (-J^y + \partial_z B^x - \partial_x B^z - \partial_t E^y) \boldsymbol{\gamma}_y \\ & + (-J^z - \partial_y B^x + \partial_x B^y - \partial_t E^z) \boldsymbol{\gamma}_z \end{aligned}$$

Curl E and Div B equations

$$\begin{aligned}\langle \nabla F \rangle_3 = 0 = & (-\partial_t B^z + \partial_y E^x - \partial_x E^y) \gamma_t \wedge \gamma_x \wedge \gamma_y \\ & + (\partial_t B^y + \partial_z E^x - \partial_x E^z) \gamma_t \wedge \gamma_x \wedge \gamma_z \\ & + (-\partial_t B^x + \partial_z E^y - \partial_y E^z) \gamma_t \wedge \gamma_y \wedge \gamma_z \\ & + (\partial_x B^x + \partial_y B^y + \partial_z B^z) \gamma_x \wedge \gamma_y \wedge \gamma_z\end{aligned}$$

```
def Dirac_Equation_in_Geog_Calculus():
    Print_Function()
    coords = symbols('t x y z',real=True)
    (st4d,g0,g1,g2,g3) = Ga.build('gamma*t|x|y|z',g=[1,-1,-1,-1],coords=coords)
    I = st4d.i
    (m,e) = symbols('m e')
    psi = st4d.mv('psi','spinor',f=True)
    A = st4d.mv('A','vector',f=True)
    sig_z = g3*g0
    print '\\text{4-Vector Potential\\;\\;}\\bm{A} =',A
    print '\\text{8-component real spinor\\;\\;}\\bm{\\psi} =',psi
    dirac_eq = (st4d.grad*psi)*I*sig_z-e*A*psi-m*psi*g0
    dirac_eq = dirac_eq.simplify()
    dirac_eq.Fmt(3,r'%\\text{Dirac Equation\\;\\;}\\nabla \\bm{\\psi} I \\sigma_{z}-e\\bm{A}\\bm{\\psi}-m\\bm{\\psi}\\gamma_{t} = 0')
    return
```

Code Output:

4-Vector Potential $\mathbf{A} = A^t \gamma_t + A^x \gamma_x + A^y \gamma_y + A^z \gamma_z$

8-component real spinor $\psi = \psi + \psi^{tx} \gamma_t \wedge \gamma_x + \psi^{ty} \gamma_t \wedge \gamma_y + \psi^{tz} \gamma_t \wedge \gamma_z + \psi^{xy} \gamma_x \wedge \gamma_y + \psi^{xz} \gamma_x \wedge \gamma_z + \psi^{yz} \gamma_y \wedge \gamma_z + \psi^{txyz} \gamma_t \wedge \gamma_x \wedge \gamma_y \wedge \gamma_z$

Dirac Equation $\nabla \psi I \sigma_z - e \mathbf{A} \psi - m \psi \gamma_t = 0 =$

$$\begin{aligned} & (-eA^t \psi - eA^x \psi^{tx} - eA^y \psi^{ty} - eA^z \psi^{tz} - m\psi - \partial_y \psi^{tx} - \partial_z \psi^{txyz} + \partial_x \psi^{ty} + \partial_t \psi^{xy}) \gamma_t \\ & + (-eA^t \psi^{tx} - eA^x \psi - eA^y \psi^{xy} - eA^z \psi^{xz} + m\psi^{tx} + \partial_y \psi - \partial_t \psi^{ty} - \partial_x \psi^{xy} + \partial_z \psi^{yz}) \gamma_x \\ & + (eA^x \psi^{xy} - eA^y \psi - eA^z \psi^{yz} + (-eA^t + m) \psi^{ty} - \partial_x \psi + \partial_t \psi^{tx} - \partial_y \psi^{xy} - \partial_z \psi^{xz}) \gamma_y \\ & + (eA^x \psi^{xz} + eA^y \psi^{yz} - eA^z \psi + (-eA^t + m) \psi^{tz} + \partial_t \psi^{txyz} - \partial_z \psi^{xy} + \partial_y \psi^{xz} - \partial_x \psi^{yz}) \gamma_z \\ & + (eA^x \psi^{ty} - eA^y \psi^{tx} - eA^z \psi^{tyz} + (-eA^t - m) \psi^{xy} - \partial_t \psi + \partial_x \psi^{tx} + \partial_y \psi^{ty} + \partial_z \psi^{tz}) \gamma_t \wedge \gamma_x \wedge \gamma_y \\ & + (-eA^t \psi^{xz} + eA^x \psi^{tz} + eA^y \psi^{txyz} - eA^z \psi^{tx} - m\psi^{xz} + \partial_x \psi^{txyz} + \partial_z \psi^{ty} - \partial_y \psi^{tz} - \partial_t \psi^{yz}) \gamma_t \wedge \gamma_x \wedge \gamma_z \\ & + (-eA^t \psi^{yz} - eA^x \psi^{txyz} + eA^y \psi^{tz} - eA^z \psi^{ty} - m\psi^{yz} - \partial_z \psi^{tx} + \partial_y \psi^{txyz} + \partial_x \psi^{tz} + \partial_t \psi^{xz}) \gamma_t \wedge \gamma_y \wedge \gamma_z \\ & + (-eA^t \psi^{txyz} - eA^x \psi^{yz} + eA^y \psi^{xz} - eA^z \psi^{xy} + m\psi^{txyz} + \partial_z \psi - \partial_t \psi^{tz} - \partial_x \psi^{xz} - \partial_y \psi^{yz}) \gamma_x \wedge \gamma_y \wedge \gamma_z \end{aligned}$$

```
def Lorentz_Tranformation_in_Geog_Algebra():
    Print_Function()
    (alpha,beta,gamma) = symbols('alpha beta gamma')
    (x,t,xp,tp) = symbols("x t x' t'",real=True)
    (st2d,g0,g1) = Ga.build('gamma*t|x',g=[1,-1])
    from sympy import sinh,cosh
    R = cosh(alpha/2)+sinh(alpha/2)*(g0^g1)
    X = t*g0+x*g1
    Xp = tp*g0+xp*g1
    print 'R =',R
    print r"%t\\bm{\\gamma_{t}}+x\\bm{\\gamma_{x}} = t'\\bm{\\gamma_{t'}}+x'\\bm{\\gamma_{x'}} = R\\lp t'\\bm{\\gamma_{t}}+x'\\bm{\\gamma_{x}}\\rp R^{\\dagger}"
    Xpp = R*Xp*R.rev()
    Xpp = Xpp.collect()
    Xpp = Xpp.trigsimp()
    print r"%t\\bm{\\gamma_{t}}+x\\bm{\\gamma_{x}} ="',Xpp
    Xpp = Xpp.subs({sinh(alpha):gamma*beta,cosh(alpha):gamma})
    print r'%f{\\sinh}{\\alpha} = \\gamma\\beta'
    print r'%f{\\cosh}{\\alpha} = \\gamma'
    print r"%t\\bm{\\gamma_{t}}+x\\bm{\\gamma_{x}} ="',Xpp.collect()
    return
```

Code Output:

$$R = \cosh\left(\frac{\alpha}{2}\right) + \sinh\left(\frac{\alpha}{2}\right)\gamma_t \wedge \gamma_x$$

$$t\gamma_t + x\gamma_x = t'\gamma_t' + x'\gamma_x' = R\left(t'\gamma_t + x'\gamma_x\right)R^\dagger$$

$$t\gamma_t + x\gamma_x = \left(t'\cosh(\alpha) - x'\sinh(\alpha)\right)\gamma_t + \left(-t'\sinh(\alpha) + x'\cosh(\alpha)\right)\gamma_x$$

$$\sinh(\alpha) = \gamma\beta$$

$$\cosh(\alpha) = \gamma$$

$$t\gamma_t + x\gamma_x = \gamma\left(-\beta x' + t'\right)\gamma_t + \gamma\left(-\beta t' + x'\right)\gamma_x$$