

Normalized Spherical Coordinates:

$$g = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\nabla = e_r \frac{\partial}{\partial r} + e_\theta \frac{1}{r} \frac{\partial}{\partial \theta} + e_\phi \frac{1}{r \sin(\theta)} \frac{\partial}{\partial \phi}$$

$$\nabla^2 = \frac{2}{r} \frac{\partial}{\partial r} + \frac{\cos(\theta)}{r^2 \sin(\theta)} \frac{\partial}{\partial \theta} + \frac{\partial^2}{\partial r^2} + r^{-2} \frac{\partial^2}{\partial \theta^2} + \frac{1}{r^2 \sin^2(\theta)} \frac{\partial^2}{\partial \phi^2}$$

$$(\nabla^2) f = \frac{1}{r^2} \left( r^2 \partial_r^2 f + 2r \partial_r f + \partial_\theta^2 f + \frac{\partial_\theta f}{\tan(\theta)} + \frac{\partial_\phi^2 f}{\sin^2(\theta)} \right)$$

$$\nabla \cdot (\nabla f) = \frac{1}{r^2} \left( r^2 \partial_r^2 f + 2r \partial_r f + \partial_\theta^2 f + \frac{\partial_\theta f}{\tan(\theta)} + \frac{\partial_\phi^2 f}{\sin^2(\theta)} \right)$$

Unnormalized Spherical Coordinates:

$$g = \begin{bmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & r^2 \sin^2(\theta) \end{bmatrix}$$

$$\nabla = e_r \frac{\partial}{\partial r} + e_\theta r^{-2} \frac{\partial}{\partial \theta} + e_\phi \frac{1}{r^2 \sin^2(\theta)} \frac{\partial}{\partial \phi}$$

$$\nabla^2 = \frac{2}{r} \frac{\partial}{\partial r} + \frac{1}{r^2 \tan(\theta)} \frac{\partial}{\partial \theta} + \frac{\partial^2}{\partial r^2} + r^{-2} \frac{\partial^2}{\partial \theta^2} + \frac{1}{r^2 \sin^2(\theta)} \frac{\partial^2}{\partial \phi^2} + e_\theta \wedge e_\phi \frac{\sin(2\theta)}{2r^4 \sin^4(\theta)} + \frac{1}{r^4 \sin^2(\theta) \tan(\theta)} - \frac{2 \cos(\theta)}{r^4 \sin^3(\theta)} \frac{\partial}{\partial \phi}$$

$$(\nabla^2) f = \frac{1}{r^2} \left( r^2 \partial_r^2 f + 2r \partial_r f + \partial_\theta^2 f + \frac{\partial_\theta f}{\tan(\theta)} + \frac{\partial_\phi^2 f}{\sin^2(\theta)} \right) + \frac{(\sin(2\theta) \tan(\theta) + \cos(2\theta) - 1) \partial_\phi f}{2r^4 \sin^4(\theta) \tan(\theta)} e_\theta \wedge e_\phi$$

$$\nabla \cdot (\nabla f) = \frac{1}{r^2} \left( r^2 \partial_r^2 f + 2r \partial_r f + \partial_\theta^2 f + \frac{\partial_\theta f}{\tan(\theta)} + \frac{\partial_\phi^2 f}{\sin^2(\theta)} \right)$$