```
def derivatives_in_spherical_coordinates():
    Print_Function()
    coords = (r,th,phi) = symbols('r theta phi', real=True)
    (sp3d,er,eth,ephi) = Ga.build('e_r e_theta e_phi',g=[1,r**2,r**2*sin(th)**2],coords=coords,norm=True)
    grad = sp3d.grad
    f = sp3d.mv('f','scalar',f=True)
    A = sp3d.mv('A','vector',f=True)
    B = sp3d.mv('B','bivector',f=True)
    print 'f =',f
    print 'A =',A
    print 'B =',B
    print 'grad*f =',grad*f
    print 'grad|A =',grad|A
    print 'grad\\times A = -I*(grad^A) =',-sp3d.i*(grad^A)
    print '%\\nabla^{2}f =',grad|(grad*f)
    print 'grad^B =',grad^B
    """
    print '( \\nabla\\W\\nabla )\\bm{e}_{r} =',((grad^grad)*er).trigsimp()
    print '( \\nabla\\W\\nabla )\\bm{e}_{\\theta} =',((grad^grad)*eth).trigsimp()
    print '( \\nabla\\W\\nabla )\\bm{e}_{\\phi} =',((grad^grad)*ephi).trigsimp()
    """
    return
```

Code Output:

$$f = f$$

$$A = A^{r} e_{r} + A^{\theta} e_{\theta} + A^{\phi} e_{\phi}$$

$$B = B^{r\theta} e_{r}\wedge e_{\theta} + B^{r\phi} e_{r}\wedge e_{\phi} + B^{\phi\phi} e_{\theta}\wedge e_{\phi}$$

$$\boldsymbol{\nabla} f = \partial_{r} f e_{r} + \frac{1}{r}\partial_{\theta} f e_{\theta} + \frac{\partial_{\phi} f}{r\sin{(\theta)}} e_{\phi}$$

$$\boldsymbol{\nabla}\cdot A = \frac{1}{r}\left(r\partial_{r} A^{r} + 2A^{r} + \frac{A^{\theta}}{\tan{(\theta)}} + \partial_{\theta} A^{\theta} + \frac{\partial_{\phi} A^{\phi}}{\sin{(\theta)}}\right)$$

$$\boldsymbol{\nabla}\times A = -I(\boldsymbol{\nabla}\wedge A) = \frac{1}{r}\left(\frac{A^{\phi}}{\tan{(\theta)}} + \partial_{\theta} A^{\phi} - \frac{\partial_{\phi} A^{\theta}}{\sin{(\theta)}}\right) e_{r} + \frac{1}{r}\left(-r\partial_{r} A^{\phi} - A^{\phi} + \frac{\partial_{\phi} A^{r}}{\sin{(\theta)}}\right) e_{\theta} + \frac{1}{r}\left(r\partial_{r} A^{\theta} + A^{\theta} - \partial_{\theta} A^{r}\right) e_{\phi}$$

$$\nabla^{2} f = \frac{1}{r^{2}}\left(r^{2}\partial^{2}_{r} f + 2r\partial_{r} f + \partial^{2}_{\theta} f + \frac{\partial_{\theta} f}{\tan{(\theta)}} + \frac{\partial^{2}_{\phi} f}{\sin^{2}{(\theta)}}\right)$$

$$\boldsymbol{\nabla}\wedge B = \frac{1}{r}\left(r\partial_{r} B^{\phi\phi} - \frac{B^{r\phi}}{\tan{(\theta)}} + 2B^{\phi\phi} - \partial_{\theta} B^{r\phi} + \frac{\partial_{\phi} B^{r\theta}}{\sin{(\theta)}}\right) e_{r}\wedge e_{\theta}\wedge e_{\phi}$$

```
def derivatives_in_paraboloidal_coordinates():
    Print_Function()
    coords = (u,v,phi) = symbols('u v phi', real=True)
    (par3d,er,eth,ephi) = Ga.build('e_u e_v e_phi',X=[u*v*cos(phi),u*v*sin(phi),(u**2-v**2)/2],coords=coords,norm=True)
    grad = par3d.grad
    f = par3d.mv('f','scalar',f=True)
    A = par3d.mv('A','vector',f=True)
    B = par3d.mv('B','bivector',f=True)
    print '#Derivatives in Paraboloidal Coordinates'
    print 'f =',f
    print 'A =',A
    print 'B =',B
    print 'grad*f =',grad*f
    print 'grad|A =',grad|A
    (-par3d.i*(grad^A)).Fmt(3,'grad\\times A = -I*(grad^A)')
    print 'grad^B =',grad^B
    return
```

Code Output: Derivatives in Paraboloidal Coordinates

$$f = f$$

$$A = A^{u} e_{u} + A^{v} e_{v} + A^{\phi} e_{\phi}$$

$$B = B^{uv} e_{u}\wedge e_{v} + B^{u\phi} e_{u}\wedge e_{\phi} + B^{v\phi} e_{v}\wedge e_{\phi}$$

$$\boldsymbol{\nabla} f = \frac{\partial_{u} f}{\sqrt{u^{2} + v^{2}}} e_{u} + \frac{\partial_{v} f}{\sqrt{u^{2} + v^{2}}} e_{v} + \frac{\partial_{\phi} f}{uv} e_{\phi}$$

$$\boldsymbol{\nabla} \cdot A = \left (\frac{u}{\left(u^{2} + v^{2}\right)^{\frac{3}{2}}} + \frac{1}{u \sqrt{u^{2} + v^{2}}}\right) A^{u} + \left (\frac{v}{\left(u^{2} + v^{2}\right)^{\frac{3}{2}}} + \frac{1}{v \sqrt{u^{2} + v^{2}}}\right) A^{v} + \frac{\partial_{u} A^{u}}{\sqrt{u^{2} + v^{2}}} + \frac{\partial_{v} A^{v}}{\sqrt{u^{2} + v^{2}}} + \frac{\partial_{\phi} A^{\phi}}{uv}$$

$$\begin{aligned} \boldsymbol{\nabla} \times A = -I (\boldsymbol{\nabla} \wedge A) = & \frac{1}{uv \left(u^{2} + v^{2}\right)} \left (uv \sqrt{u^{2} + v^{2}} \partial_{v} A^{\phi} + u \sqrt{u^{2} + v^{2}} A^{\phi} + \left(- u^{2} - v^{2}\right) \partial_{\phi} A^{v}\right) e_{u} \\ & + \frac{1}{uv \left(u^{2} + v^{2}\right)} \left (- uv \sqrt{u^{2} + v^{2}} \partial_{u} A^{\phi} - v \sqrt{u^{2} + v^{2}} A^{\phi} + \left(u^{2} + v^{2}\right) \partial_{\phi} A^{u}\right) e_{v} \\ & + \frac{1}{\left(u^{2} + v^{2}\right)^{\frac{3}{2}}} \left (u A^{v} - v A^{u} + \left(u^{2} + v^{2}\right) \left(- \partial_{v} A^{u} + \partial_{u} A^{v}\right)\right) e_{\phi} \end{aligned}$$

$$\boldsymbol{\nabla} \wedge B = \left (\left (\frac{u}{\left(u^{2} + v^{2}\right)^{\frac{3}{2}}} + \frac{1}{u \sqrt{u^{2} + v^{2}}}\right) B^{v\phi} + \left (- \frac{v}{\left(u^{2} + v^{2}\right)^{\frac{3}{2}}} - \frac{1}{v \sqrt{u^{2} + v^{2}}}\right) B^{u\phi} - \frac{\partial_{v} B^{u\phi}}{\sqrt{u^{2} + v^{2}}} + \frac{\partial_{u} B^{v\phi}}{\sqrt{u^{2} + v^{2}}} + \frac{\partial_{\phi} B^{uv}}{uv}\right) e_{u}\wedge e_{v}\wedge e_{\phi}$$

```
def derivatives_in_elliptic_cylindrical_coordinates():
    Print_Function()
    a = symbols('a', real=True)
    coords = (u,v,z) = symbols('u v z', real=True)
    (elip3d,er,eth,ephi) = Ga.build('e_u e_v e_z',X=[a*cosh(u)*cos(v),a*sinh(u)*sin(v),z],coords=coords,norm=True)
    grad = elip3d.grad
    f = elip3d.mv('f','scalar',f=True)
    A = elip3d.mv('A','vector',f=True)
    B = elip3d.mv('B','bivector',f=True)
    print 'f =',f
    print 'A =',A
    print 'B =',B
    print 'grad*f =',grad*f
    print 'grad|A =',grad|A
    print '-I*(grad^A) =',-elip3d.i*(grad^A)
    print 'grad^B =',grad^B
    return
```

Code Output:

$$f = f$$

$$A = A^{u} e_{u} + A^{v} e_{v} + A^{z} e_{z}$$

$$B = B^{uv} e_{u}\wedge e_{v} + B^{uz} e_{u}\wedge e_{z} + B^{vz} e_{v}\wedge e_{z}$$

$$\boldsymbol{\nabla} f = \frac{\partial_{u} f}{\sqrt{\sin^{2}{\left (v \right)} \cosh^{2}{\left (u \right)} + \cos^{2}{\left (v \right)} \sinh^{2}{\left (u \right)}} \left|{a}\right|} e_{u} + \frac{\partial_{v} f}{\sqrt{\sin^{2}{\left (v \right)} \cosh^{2}{\left (u \right)} + \cos^{2}{\left (v \right)} \sinh^{2}{\left (u \right)}} \left|{a}\right|} e_{v} + \partial_{z} f e_{z}$$

$$\boldsymbol{\nabla} \cdot A = \frac{\left|{a}\right|}{2 \left(\left(\cos^{2}{\left (v \right)} \cos{\left (2 v \right)} - \cos^{2}{\left (v \right)} \cosh{\left (2 u \right)}\right) \sinh^{2}{\left (u \right)} + \left(\cos{\left (2 v \right)} \cosh^{2}{\left (u \right)} - \cosh^{2}{\left (u \right)} \cosh{\left (2 u \right)}\right) \sin^{2}{\left (v \right)}\right) \left(\sin^{2}{\left (v \right)} \cosh^{2}{\left (u \right)} + \cos^{2}{\left (v \right)} \sinh^{2}{\left (u \right)}\right)^{\frac{5}{2}}} \Bigg(\left(A^{u} \sinh{\left (2 u \right)} + A^{v} \sin{\left (2 v \right)}\right) \left(\sin^{2}{\left (v \right)} \cosh^{2}{\left (u \right)} + \cos^{2}{\left (v \right)} \sinh^{2}{\left (u \right)}\right) \big(\sin^{2}{\left (v \right)} \cos{\left (2 v \right)} \cosh^{2}{\left (u \right)} - \sin^{2}{\left (v \right)} \cosh^{2}{\left (u \right)} \cosh{\left (2 u \right)} + \cos^{2}{\left (v \right)} \cos{\left (2 v \right)} \sinh^{2}{\left (u \right)} - \cos^{2}{\left (v \right)} \sinh^{2}{\left (u \right)} \cosh (2$$

$$-I (\boldsymbol{\nabla} \wedge A) = \left (- \partial_{z} A^{v} + \frac{\partial_{v} A^{z}}{\sqrt{\sin^{2}{\left (v \right)} + \sinh^{2}{\left (u \right)}} \left|{a}\right|}\right) e_{u} + \left (\partial_{z} A^{u} - \frac{\partial_{u} A^{z}}{\sqrt{\sin^{2}{\left (v \right)} + \sinh^{2}{\left (u \right)}} \left|{a}\right|}\right) e_{v} - \frac{\sqrt{2} \left|{a}\right|}{2 \left(- \cos{\left (2 v \right)} + \cosh{\left (2 u \right)}\right)^{\frac{5}{2}}} \Big(\left(- 2 \left(\cosh{\left (2 u \right)} - 1\right)^{2} + 2\right) \partial_{u} A^{v} + \left(2 \left(\cosh{\left (2 u \right)} - 1\right)^{2} - 2\right) \partial_{v} A^{u} + \left(2 \partial_{v} A^{u} - 2 \partial_{u} A^{v}\right) \cos^{2}{\left (2 v \right)} + \left(2 A^{u} \sin{\left (2 v \right)} - 4 \cos{\left (2 v \right)} \partial_{v} A^{u} + 4 \cos{\left (2 v \right)} \partial_{u} A^{v} + 4 \partial_{v} A^{u} - 4 \partial_{u} A^{v}\right) \cosh{\left (2 u \right)} - A^{u} \sin{\left (4 v \right)} + 2 A^{v} \cos (2v$$

$$\boldsymbol{\nabla} \wedge B = - \frac{\left|{a}\right|}{2 \left(\left(\cos^{2}{\left (v \right)} \cos{\left (2 v \right)} - \cos^{2}{\left (v \right)} \cosh{\left (2 u \right)}\right) \sinh^{2}{\left (u \right)} + \left(\cos{\left (2 v \right)} \cosh^{2}{\left (u \right)} - \cosh^{2}{\left (u \right)} \cosh{\left (2 u \right)}\right) \sin^{2}{\left (v \right)}\right) \left(\sin^{2}{\left (v \right)} \cosh^{2}{\left (u \right)} + \cos^{2}{\left (v \right)} \sinh^{2}{\left (u \right)}\right)^{\frac{5}{2}}} \Bigg(\left(B^{uz} \sin{\left (2 v \right)} - B^{vz} \sinh{\left (2 u \right)}\right) \left(\sin^{2}{\left (v \right)} \cosh^{2}{\left (u \right)} + \cos^{2}{\left (v \right)} \sinh^{2}{\left (u \right)}\right) \big(\sin^{2}{\left (v \right)} \cos{\left (2 v \right)} \cosh^{2}{\left (u \right)} - \sin^{2}{\left (v \right)} \cosh^{2}{\left (u \right)} \cosh{\left (2 u \right)} + \cos^{2}{\left (v \right)} \cos{\left (2 v \right)} \sinh^{2}{\left (u \right)} - \cos^{2}{\left (v \right)} \sinh^{2}{\left (u \right)} \mathrm{co}$$

```
def derivatives_in_prolate_spheroidal_coordinates():
    Print_Function()
    a = symbols('a', real=True)
    coords = (xi,eta,phi) = symbols('xi eta phi', real=True)
    (ps3d,er,eth,ephi) = Ga.build('e_xi e_eta e_phi',X=[a*sinh(xi)*sin(eta)*cos(phi),a*sinh(xi)*sin(eta)*sin(phi),
                                                    a*cosh(xi)*cos(eta)],coords=coords,norm=True)
    grad = ps3d.grad
    f = ps3d.mv('f','scalar',f=True)
    A = ps3d.mv('A','vector',f=True)
    B = ps3d.mv('B','bivector',f=True)
    print 'f =',f
    print 'A =',A
    print 'B =',B
    print 'grad*f =',grad*f
    print 'grad|A =',grad|A
    (-ps3d.i*(grad^A)).Fmt(3,'-I*(grad^A)')
    (grad^B).Fmt(3,'grad^B')
    return
```

Code Output:

$$f = f$$

$$A = A^{\xi} e_{\xi} + A^{\eta} e_{\eta} + A^{\phi} e_{\phi}$$

$$B = B^{\xi\xi} e_{\xi} \wedge e_{\eta} + B^{\xi\xi} e_{\xi} \wedge e_{\phi} + B^{\phi\phi} e_{\eta} \wedge e_{\phi}$$

$$\boldsymbol{\nabla} f = \frac{\partial_{\xi} f}{\sqrt{\sin^{2}(\eta) + \sinh^{2}(\xi)}\,|a|} e_{\xi} + \frac{\partial_{\eta} f}{\sqrt{\sin^{2}(\eta) + \sinh^{2}(\xi)}\,|a|} e_{\eta} + \frac{\partial_{\phi} f}{a \sin(\eta)\sinh(\xi)} e_{\phi}$$

$$\boldsymbol{\nabla} \cdot A = \frac{1}{a^{2}\left(\sin^{2}(\eta) + \sinh^{2}(\xi)\right)^{3} \sin(\eta)\sinh(\xi)} \left(a\left(\sin^{2}(\eta) + \sinh^{2}(\xi)\right)^{3} \partial_{\phi} A^{\phi} + \left(\sin^{2}(\eta) + \sinh^{2}(\xi)\right)^{\frac{5}{2}} A^{\eta}\cos(\eta)\sinh(\xi)\,|a| + \left(\frac{1}{2}\left(A^{\eta}\sin(2\eta) + A^{\xi}\sinh(2\xi)\right)\left(\sin^{2}(\eta) + \sinh^{2}(\xi)\right)^{\frac{3}{2}}\sin(\eta)\sinh(\xi) + \left(\sin^{2}(\eta) + \sinh^{2}(\xi)\right)^{\frac{5}{2}}\left(\partial_{\eta} A^{\eta} + \partial_{\xi} A^{\xi}\right)\sin(\eta)\sinh(\xi) + \left(\sin^{2}(\eta) + \sinh^{2}(\xi)\right)^{\frac{5}{2}} A^{\xi}\sin(\eta)\cosh(\xi)\right)|a|\right)$$

$$\begin{aligned} -I(\boldsymbol{\nabla} \wedge A) = & \frac{1}{a^{2}\sin(\eta)}\left(-\frac{a\partial_{\phi} A^{\eta}}{\sinh(\xi)} + \left(\frac{A^{\phi}\cos(\eta)}{\sqrt{\sin^{2}(\eta) + \sinh^{2}(\xi)}} + \frac{\sin(\eta)\partial_{\eta} A^{\phi}}{\sqrt{\sin^{2}(\eta) + \sinh^{2}(\xi)}}\right)|a|\right) e_{\xi} \\ & - \frac{1}{a^{2}\sinh(\xi)}\left(-\frac{a\partial_{\phi} A^{\xi}}{\sin(\eta)} + \left(\frac{A^{\phi}\cosh(\xi)}{\sqrt{\sin^{2}(\eta) + \sinh^{2}(\xi)}} + \frac{\sinh(\xi)\partial_{\xi} A^{\phi}}{\sqrt{\sin^{2}(\eta) + \sinh^{2}(\xi)}}\right)|a|\right) e_{\eta} \\ & + \frac{1}{2\left(\sin^{2}(\eta) + \sinh^{2}(\xi)\right)^{\frac{3}{2}}|a|}\left(\left(\sin^{2}(\eta) + \sinh^{2}(\xi)\right)\left(2\partial_{\xi} A^{\eta} - 2\partial_{\eta} A^{\xi}\right) + A^{\eta}\sinh(2\xi) - A^{\xi}\sin(2\eta)\right) e_{\phi} \end{aligned}$$

$$\boldsymbol{\nabla} \wedge B = \frac{1}{a^{2}\left(\sin^{2}(\eta) + \sinh^{2}(\xi)\right)^{3}\sin(\eta)\sinh(\xi)}\left(a\left(\sin^{2}(\eta) + \sinh^{2}(\xi)\right)^{3}\partial_{\phi} B^{\xi\xi} + \left(\frac{1}{2}\left(B^{\phi\phi}\sinh(2\xi) - B^{\xi\xi}\sin(2\eta)\right)\left(\sin^{2}(\eta) + \sinh^{2}(\xi)\right)^{\frac{3}{2}}\sin(\eta)\sinh(\xi) + \left(\sin^{2}(\eta) + \sinh^{2}(\xi)\right)^{\frac{5}{2}}\left(\partial_{\xi} B^{\phi\phi} - \partial_{\eta} B^{\xi\xi}\right)\sin(\eta)\sinh(\xi) + \left(\sin^{2}(\eta) + \sinh^{2}(\xi)\right)^{\frac{5}{2}} B^{\phi\phi}\sin(\eta)\cosh(\xi) - \left(\sin^{2}(\eta) + \sinh^{2}(\xi)\right)^{\frac{5}{2}} B^{\xi\xi}\cos(\eta)\sinh(\xi)\right)|a|\right) e_{\xi}\wedge e_{\eta}\wedge e_{\phi}$$