$$\boldsymbol{A} = A + A^{x}\boldsymbol{e_x} + A^{y}\boldsymbol{e_y} + A^{z}\boldsymbol{e_z} + A^{xy}\boldsymbol{e_x}\wedge\boldsymbol{e_y} + A^{xz}\boldsymbol{e_x}\wedge\boldsymbol{e_z} + A^{yz}\boldsymbol{e_y}\wedge\boldsymbol{e_z} + A^{xyz}\boldsymbol{e_x}\wedge\boldsymbol{e_y}\wedge\boldsymbol{e_z}$$

$$\begin{aligned}\boldsymbol{A} = &A \\ &+ A^{x}\boldsymbol{e_x} + A^{y}\boldsymbol{e_y} + A^{z}\boldsymbol{e_z} \\ &+ A^{xy}\boldsymbol{e_x}\wedge\boldsymbol{e_y} + A^{xz}\boldsymbol{e_x}\wedge\boldsymbol{e_z} + A^{yz}\boldsymbol{e_y}\wedge\boldsymbol{e_z} \\ &+ A^{xyz}\boldsymbol{e_x}\wedge\boldsymbol{e_y}\wedge\boldsymbol{e_z}\end{aligned}$$

$$\begin{aligned}\boldsymbol{A} = &A \\ &+ A^{x}\boldsymbol{e_x} \\ &+ A^{y}\boldsymbol{e_y} \\ &+ A^{z}\boldsymbol{e_z} \\ &+ A^{xy}\boldsymbol{e_x}\wedge\boldsymbol{e_y} \\ &+ A^{xz}\boldsymbol{e_x}\wedge\boldsymbol{e_z} \\ &+ A^{yz}\boldsymbol{e_y}\wedge\boldsymbol{e_z} \\ &+ A^{xyz}\boldsymbol{e_x}\wedge\boldsymbol{e_y}\wedge\boldsymbol{e_z}\end{aligned}$$

$$\boldsymbol{A} = A^{x}\boldsymbol{e_x} + A^{y}\boldsymbol{e_y} + A^{z}\boldsymbol{e_z}$$

$$\boldsymbol{B} = B^{xy}\boldsymbol{e_x}\wedge\boldsymbol{e_y} + B^{xz}\boldsymbol{e_x}\wedge\boldsymbol{e_z} + B^{yz}\boldsymbol{e_y}\wedge\boldsymbol{e_z}$$

$$\boldsymbol{\nabla} f = \partial_{x} f\boldsymbol{e_x} + \partial_{y} f\boldsymbol{e_y} + \partial_{z} f\boldsymbol{e_z}$$

$$\boldsymbol{\nabla}\cdot\boldsymbol{A} = \partial_{x} A^{x} + \partial_{y} A^{y} + \partial_{z} A^{z}$$

$$\boldsymbol{\nabla}\boldsymbol{A} = \left(\partial_{x} A^{x} + \partial_{y} A^{y} + \partial_{z} A^{z}\right) + \left(-\partial_{y} A^{x} + \partial_{x} A^{y}\right)\boldsymbol{e_x}\wedge\boldsymbol{e_y} + \left(-\partial_{z} A^{x} + \partial_{x} A^{z}\right)\boldsymbol{e_x}\wedge\boldsymbol{e_z} + \left(-\partial_{z} A^{y} + \partial_{y} A^{z}\right)\boldsymbol{e_y}\wedge\boldsymbol{e_z}$$

$$-I(\boldsymbol{\nabla}\wedge\boldsymbol{A}) = \left(-\partial_{z} A^{y} + \partial_{y} A^{z}\right)\boldsymbol{e_x} + \left(\partial_{z} A^{x} - \partial_{x} A^{z}\right)\boldsymbol{e_y} + \left(-\partial_{y} A^{x} + \partial_{x} A^{y}\right)\boldsymbol{e_z}$$

$$\boldsymbol{\nabla}\boldsymbol{B} = \left(-\partial_{y} B^{xy} - \partial_{z} B^{xz}\right)\boldsymbol{e_x} + \left(\partial_{x} B^{xy} - \partial_{z} B^{yz}\right)\boldsymbol{e_y} + \left(\partial_{x} B^{xz} + \partial_{y} B^{yz}\right)\boldsymbol{e_z} + \left(\partial_{z} B^{xy} - \partial_{y} B^{xz} + \partial_{x} B^{yz}\right)\boldsymbol{e_x}\wedge\boldsymbol{e_y}\wedge\boldsymbol{e_z}$$

$$\boldsymbol{\nabla}\wedge\boldsymbol{B} = \left(\partial_{z} B^{xy} - \partial_{y} B^{xz} + \partial_{x} B^{yz}\right)\boldsymbol{e_x}\wedge\boldsymbol{e_y}\wedge\boldsymbol{e_z}$$

$$\boldsymbol{\nabla}\cdot\boldsymbol{B} = \left(-\partial_{y} B^{xy} - \partial_{z} B^{xz}\right)\boldsymbol{e_x} + \left(\partial_{x} B^{xy} - \partial_{z} B^{yz}\right)\boldsymbol{e_y} + \left(\partial_{x} B^{xz} + \partial_{y} B^{yz}\right)\boldsymbol{e_z}$$

$$g_{ij} = \left[\begin{array}{cccc}(a\cdot a) & (a\cdot b) & (a\cdot c) & (a\cdot d)\\ (a\cdot b) & (b\cdot b) & (b\cdot c) & (b\cdot d)\\ (a\cdot c) & (b\cdot c) & (c\cdot c) & (c\cdot d)\\ (a\cdot d) & (b\cdot d) & (c\cdot d) & (d\cdot d)\end{array}\right]$$

$$\boldsymbol{a\cdot (bc)} = -\left(a\cdot c\right)\boldsymbol{b} + \left(a\cdot b\right)\boldsymbol{c}$$

$$\boldsymbol{a\cdot (b\wedge c)} = -\left(a\cdot c\right)\boldsymbol{b} + \left(a\cdot b\right)\boldsymbol{c}$$

$$\boldsymbol{a\cdot (b\wedge c\wedge d)} = \left(a\cdot d\right)\boldsymbol{b}\wedge\boldsymbol{c} - \left(a\cdot c\right)\boldsymbol{b}\wedge\boldsymbol{d} + \left(a\cdot b\right)\boldsymbol{c}\wedge\boldsymbol{d}$$

$$\boldsymbol{a\cdot (b\wedge c)+c\cdot (a\wedge b)+b\cdot (c\wedge a)} = 0$$

$$\boldsymbol{a(b\wedge c)-b(a\wedge c)+c(a\wedge b)} = 3\boldsymbol{a}\wedge\boldsymbol{b}\wedge\boldsymbol{c}$$

$$\boldsymbol{a(b\wedge c\wedge d)-b(a\wedge c\wedge d)+c(a\wedge b\wedge d)-d(a\wedge b\wedge c)} = 4\boldsymbol{a}\wedge\boldsymbol{b}\wedge\boldsymbol{c}\wedge\boldsymbol{d}$$

$$\boldsymbol{(a\wedge b)\cdot (c\wedge d)} = -\left(a\cdot c\right)\left(b\cdot d\right) + \left(a\cdot d\right)\left(b\cdot c\right)$$

$$\boldsymbol{((a\wedge b)\cdot c)\cdot d} = -\left(a\cdot c\right)\left(b\cdot d\right) + \left(a\cdot d\right)\left(b\cdot c\right)$$

$$\boldsymbol{(a\wedge b)\times (c\wedge d)} = -\left(b\cdot d\right)\boldsymbol{a}\wedge\boldsymbol{c} + \left(b\cdot c\right)\boldsymbol{a}\wedge\boldsymbol{d} + \left(a\cdot d\right)\boldsymbol{b}\wedge\boldsymbol{c} - \left(a\cdot c\right)\boldsymbol{b}\wedge\boldsymbol{d}$$

$$E = \boldsymbol{e_1}\wedge\boldsymbol{e_2}\wedge\boldsymbol{e_3}$$

$$E^{2} = \left(e_{1}\cdot e_{2}\right)^{2} - 2\left(e_{1}\cdot e_{2}\right)\left(e_{1}\cdot e_{3}\right)\left(e_{2}\cdot e_{3}\right) + \left(e_{1}\cdot e_{3}\right)^{2} + \left(e_{2}\cdot e_{3}\right)^{2} - 1$$

$$E1 = (e2\wedge e3)E = \left(\left(e_{2}\cdot e_{3}\right)^{2} - 1\right)\boldsymbol{e_1} + \left(\left(e_{1}\cdot e_{2}\right) - \left(e_{1}\cdot e_{3}\right)\left(e_{2}\cdot e_{3}\right)\right)\boldsymbol{e_2} + \left(-\left(e_{1}\cdot e_{2}\right)\left(e_{2}\cdot e_{3}\right) + \left(e_{1}\cdot e_{3}\right)\right)\boldsymbol{e_3}$$

$$E2 = -(e1\wedge e3)E = \left(\left(e_{1}\cdot e_{2}\right) - \left(e_{1}\cdot e_{3}\right)\left(e_{2}\cdot e_{3}\right)\right)\boldsymbol{e_1} + \left(\left(e_{1}\cdot e_{3}\right)^{2} - 1\right)\boldsymbol{e_2} + \left(-\left(e_{1}\cdot e_{2}\right)\left(e_{1}\cdot e_{3}\right) + \left(e_{2}\cdot e_{3}\right)\right)\boldsymbol{e_3}$$

$$E3 = (e1\wedge e2)E = \left(-\left(e_{1}\cdot e_{2}\right)\left(e_{2}\cdot e_{3}\right) + \left(e_{1}\cdot e_{3}\right)\right)\boldsymbol{e_1} + \left(-\left(e_{1}\cdot e_{2}\right)\left(e_{1}\cdot e_{3}\right) + \left(e_{2}\cdot e_{3}\right)\right)\boldsymbol{e_2} + \left(\left(e_{1}\cdot e_{2}\right)^{2} - 1\right)\boldsymbol{e_3}$$

$E1 \cdot e2 = 0$

$E1 \cdot e3 = 0$

$E2 \cdot e1 = 0$

$E2 \cdot e3 = 0$

$E3 \cdot e1 = 0$

$E3 \cdot e2 = 0$

$(E1 \cdot e1)/E^2 = 1$

$(E2 \cdot e2)/E^2 = 1$

$(E3 \cdot e3)/E^2 = 1$

$A = A^r \boldsymbol{e_r} + A^\theta \boldsymbol{e_\theta} + A^\phi \boldsymbol{e_\phi}$

$B = B^{r\theta} \boldsymbol{e_r} \wedge \boldsymbol{e_\theta} + B^{r\phi} \boldsymbol{e_r} \wedge \boldsymbol{e_\phi} + B^{\phi\phi} \boldsymbol{e_\theta} \wedge \boldsymbol{e_\phi}$

$\boldsymbol{\nabla} f = \partial_r f \boldsymbol{e_r} + \frac{1}{r} \partial_\theta f \boldsymbol{e_\theta} + \frac{\partial_\phi f}{r \sin(\theta)} \boldsymbol{e_\phi}$

$\boldsymbol{\nabla} \cdot A = \frac{1}{r}\left(r\partial_r A^r + 2A^r + \frac{A^\theta}{\tan(\theta)} + \partial_\theta A^\theta + \frac{\partial_\phi A^\phi}{\sin(\theta)}\right)$

$-I(\boldsymbol{\nabla} \wedge A) = \frac{1}{r}\left(\frac{A^\phi}{\tan(\theta)} + \partial_\theta A^\phi - \frac{\partial_\phi A^\theta}{\sin(\theta)}\right)\boldsymbol{e_r} + \frac{1}{r}\left(-r\partial_r A^\phi - A^\phi + \frac{\partial_\phi A^r}{\sin(\theta)}\right)\boldsymbol{e_\theta} + \frac{1}{r}\left(r\partial_r A^\theta + A^\theta - \partial_\theta A^r\right)\boldsymbol{e_\phi}$

$\boldsymbol{\nabla} \wedge B = \frac{1}{r}\left(r\partial_r B^{\phi\phi} - \frac{B^{r\phi}}{\tan(\theta)} + 2B^{\phi\phi} - \partial_\theta B^{r\phi} + \frac{\partial_\phi B^{r\theta}}{\sin(\theta)}\right)\boldsymbol{e_r} \wedge \boldsymbol{e_\theta} \wedge \boldsymbol{e_\phi}$

$B = \boldsymbol{B}\boldsymbol{\gamma_t} = -B^x \boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_x} - B^y \boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_y} - B^z \boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_z}$

$E = \boldsymbol{E}\boldsymbol{\gamma_t} = -E^x \boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_x} - E^y \boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_y} - E^z \boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_z}$

$F = E + IB = -E^x \boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_x} - E^y \boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_y} - E^z \boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_z} - B^z \boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_y} + B^y \boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_z} - B^x \boldsymbol{\gamma_y} \wedge \boldsymbol{\gamma_z}$

$J = J^t \boldsymbol{\gamma_t} + J^x \boldsymbol{\gamma_x} + J^y \boldsymbol{\gamma_y} + J^z \boldsymbol{\gamma_z}$

$$\begin{aligned}
\boldsymbol{\nabla} F = & \left(\partial_x E^x + \partial_y E^y + \partial_z E^z\right)\boldsymbol{\gamma_t} \\
& + \left(-\partial_z B^y + \partial_y B^z - \partial_t E^x\right)\boldsymbol{\gamma_x} \\
& + \left(\partial_z B^x - \partial_x B^z - \partial_t E^y\right)\boldsymbol{\gamma_y} \\
& + \left(-\partial_y B^x + \partial_x B^y - \partial_t E^z\right)\boldsymbol{\gamma_z} \\
& + \left(-\partial_t B^z + \partial_y E^x - \partial_x E^y\right)\boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_y} \\
& + \left(\partial_t B^y + \partial_z E^x - \partial_x E^z\right)\boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_z} \\
& + \left(-\partial_t B^x + \partial_z E^y - \partial_y E^z\right)\boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_y} \wedge \boldsymbol{\gamma_z} \\
& + \left(\partial_x B^x + \partial_y B^y + \partial_z B^z\right)\boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_y} \wedge \boldsymbol{\gamma_z}
\end{aligned}$$

$\boldsymbol{\nabla} F = J$

$$\begin{aligned}
\langle \nabla F \rangle_1 - J = 0 = & \left(-J^t + \partial_x E^x + \partial_y E^y + \partial_z E^z\right)\boldsymbol{\gamma_t} \\
& + \left(-J^x - \partial_z B^y + \partial_y B^z - \partial_t E^x\right)\boldsymbol{\gamma_x} \\
& + \left(-J^y + \partial_z B^x - \partial_x B^z - \partial_t E^y\right)\boldsymbol{\gamma_y} \\
& + \left(-J^z - \partial_y B^x + \partial_x B^y - \partial_t E^z\right)\boldsymbol{\gamma_z}
\end{aligned}$$

$$\begin{aligned}
\langle \nabla F \rangle_3 = 0 = & \left(-\partial_t B^z + \partial_y E^x - \partial_x E^y\right)\boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_y} \\
& + \left(\partial_t B^y + \partial_z E^x - \partial_x E^z\right)\boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_z} \\
& + \left(-\partial_t B^x + \partial_z E^y - \partial_y E^z\right)\boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_y} \wedge \boldsymbol{\gamma_z} \\
& + \left(\partial_x B^x + \partial_y B^y + \partial_z B^z\right)\boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_y} \wedge \boldsymbol{\gamma_z}
\end{aligned}$$

$R = \cosh\left(\frac{\alpha}{2}\right) + \sinh\left(\frac{\alpha}{2}\right)\boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_x}$

$$t\boldsymbol{\gamma_t} + x\boldsymbol{\gamma_x} = t'\boldsymbol{\gamma_t'} + x'\boldsymbol{\gamma_x'} = R\left(t'\boldsymbol{\gamma_t} + x'\boldsymbol{\gamma_x}\right)R^{\dagger}$$

$$t\boldsymbol{\gamma_t} + x\boldsymbol{\gamma_x} = \left(t'\cosh\left(\alpha\right) - x'\sinh\left(\alpha\right)\right)\boldsymbol{\gamma_t} + \left(-t'\sinh\left(\alpha\right) + x'\cosh\left(\alpha\right)\right)\boldsymbol{\gamma_x}$$

$$\sinh\left(\alpha\right) = \gamma\beta$$

$$\cosh\left(\alpha\right) = \gamma$$

$$t\boldsymbol{\gamma_t} + x\boldsymbol{\gamma_x} = \gamma\left(-\beta x' + t'\right)\boldsymbol{\gamma_t} + \gamma\left(-\beta t' + x'\right)\boldsymbol{\gamma_x}$$

$$\boldsymbol{A} = A^t\boldsymbol{\gamma_t} + A^x\boldsymbol{\gamma_x} + A^y\boldsymbol{\gamma_y} + A^z\boldsymbol{\gamma_z}$$

$$\boldsymbol{\psi} = \psi + \psi^{tx}\boldsymbol{\gamma_t}\wedge\boldsymbol{\gamma_x} + \psi^{ty}\boldsymbol{\gamma_t}\wedge\boldsymbol{\gamma_y} + \psi^{tz}\boldsymbol{\gamma_t}\wedge\boldsymbol{\gamma_z} + \psi^{xy}\boldsymbol{\gamma_x}\wedge\boldsymbol{\gamma_y} + \psi^{xz}\boldsymbol{\gamma_x}\wedge\boldsymbol{\gamma_z} + \psi^{yz}\boldsymbol{\gamma_y}\wedge\boldsymbol{\gamma_z} + \psi^{txyz}\boldsymbol{\gamma_t}\wedge\boldsymbol{\gamma_x}\wedge\boldsymbol{\gamma_y}\wedge\boldsymbol{\gamma_z}$$

$$\begin{aligned}
\nabla\boldsymbol{\psi}I\sigma_z - e\boldsymbol{A}\boldsymbol{\psi} - m\boldsymbol{\psi}\gamma_t = 0 = &\left(-eA^x\psi^{tx} - eA^y\psi^{ty} - eA^z\psi^{tz} + \left(-eA^t - m\right)\psi - \partial_y\psi^{tx} - \partial_z\psi^{txyz} + \partial_x\psi^{ty} + \partial_t\psi^{xy}\right)\boldsymbol{\gamma_t} \\
&+ \left(-eA^x\psi - eA^y\psi^{xy} - eA^z\psi^{xz} + \left(-eA^t + m\right)\psi^{tx} + \partial_y\psi - \partial_t\psi^{ty} - \partial_x\psi^{xy} + \partial_z\psi^{yz}\right)\boldsymbol{\gamma_x} \\
&+ \left(-eA^t\psi^{ty} + eA^x\psi^{xy} - eA^y\psi - eA^z\psi^{yz} + m\psi^{ty} - \partial_x\psi + \partial_t\psi^{tx} - \partial_y\psi^{xy} - \partial_z\psi^{xz}\right)\boldsymbol{\gamma_y} \\
&+ \left(-eA^t\psi^{tz} + eA^x\psi^{xz} + eA^y\psi^{yz} - eA^z\psi + m\psi^{tz} + \partial_t\psi^{txyz} - \partial_z\psi^{xy} + \partial_y\psi^{xz} - \partial_x\psi^{yz}\right)\boldsymbol{\gamma_z} \\
&+ \left(eA^x\psi^{ty} - eA^y\psi^{tx} - eA^z\psi^{txyz} + \left(-eA^t - m\right)\psi^{xy} - \partial_t\psi + \partial_x\psi^{tx} + \partial_y\psi^{ty} + \partial_z\psi^{tz}\right)\boldsymbol{\gamma_t}\wedge\boldsymbol{\gamma_x}\wedge\boldsymbol{\gamma_y} \\
&+ \left(eA^x\psi^{tz} + eA^y\psi^{txyz} - eA^z\psi^{tx} + \left(-eA^t - m\right)\psi^{xz} + \partial_x\psi^{txyz} + \partial_z\psi^{ty} - \partial_y\psi^{tz} - \partial_t\psi^{yz}\right)\boldsymbol{\gamma_t}\wedge\boldsymbol{\gamma_x}\wedge\boldsymbol{\gamma_z} \\
&+ \left(-eA^t\psi^{yz} - eA^x\psi^{txyz} + eA^y\psi^{tz} - eA^z\psi^{ty} - m\psi^{yz} - \partial_z\psi^{tx} + \partial_y\psi^{txyz} + \partial_x\psi^{tz} + \partial_t\psi^{xz}\right)\boldsymbol{\gamma_t}\wedge\boldsymbol{\gamma_y}\wedge\boldsymbol{\gamma_z} \\
&+ \left(-eA^t\psi^{txyz} - eA^x\psi^{yz} + eA^y\psi^{xz} - eA^z\psi^{xy} + m\psi^{txyz} + \partial_z\psi - \partial_t\psi^{tz} - \partial_x\psi^{xz} - \partial_y\psi^{yz}\right)\boldsymbol{\gamma_x}\wedge\boldsymbol{\gamma_y}\wedge\boldsymbol{\gamma_z}
\end{aligned}$$