

Program:

```
from sympy import expand, simplify
from sympy.galgebra.printer import Format, xpdf
from sympy.galgebra.ga import Ga
```

```
g = '1 # #, '+ \
    '# 1 #, '+ \
    '# # 1'
```

```
Format()
```

```
ng3d = Ga('e1 e2 e3', g=g)
(e1, e2, e3) = ng3d.mv()
print 'g_{ij} =', ng3d.g
E = e1^e2^e3
Esq = (E*E).scalar()
print 'E =', E
print '%E^{2} =', Esq
Esq_inv = 1/Esq
E1 = (e2^e3)*E
E2 = (-1)*(e1^e3)*E
E3 = (e1^e2)*E
print 'E1 = (e2^e3)*E =', E1
print 'E2 = -(e1^e3)*E =', E2
print 'E3 = (e1^e2)*E =', E3
w = (E1|e2)
w = w.expand()
print 'E1|e2 =', w
w = (E1|e3)
w = w.expand()
print 'E1|e3 =', w
w = (E2|e1)
w = w.expand()
print 'E2|e1 =', w
w = (E2|e3)
w = w.expand()
print 'E2|e3 =', w
w = (E3|e1)
w = w.expand()
print 'E3|e1 =', w
w = (E3|e2)
w = w.expand()
print 'E3|e2 =', w
w = (E1|e1)
w = (w.expand()).scalar()
```

```

Esq = expand(Esq)
print '%(E1\\cdot e1)/E^{2} =', simplify(w/Esq)
w = (E2|e2)
w = (w.expand()).scalar()
print '%(E2\\cdot e2)/E^{2} =', simplify(w/Esq)
w = (E3|e3)
w = (w.expand()).scalar()
print '%(E3\\cdot e3)/E^{2} =', simplify(w/Esq)
xpdf(paper='letter',prog=True)

```

Code Output:

$$g_{ij} = \begin{bmatrix} 1 & (e_1 \cdot e_2) & (e_1 \cdot e_3) \\ (e_1 \cdot e_2) & 1 & (e_2 \cdot e_3) \\ (e_1 \cdot e_3) & (e_2 \cdot e_3) & 1 \end{bmatrix}$$

$$E = \mathbf{e}_1 \wedge \mathbf{e}_2 \wedge \mathbf{e}_3$$

$$E^2 = (e_1 \cdot e_2)^2 - 2(e_1 \cdot e_2)(e_1 \cdot e_3)(e_2 \cdot e_3) + (e_1 \cdot e_3)^2 + (e_2 \cdot e_3)^2 - 1$$

$$E1 = (e2 \wedge e3)E = \left((e_2 \cdot e_3)^2 - 1\right) \mathbf{e}_1 + ((e_1 \cdot e_2) - (e_1 \cdot e_3)(e_2 \cdot e_3)) \mathbf{e}_2 + (- (e_1 \cdot e_2)(e_2 \cdot e_3) + (e_1 \cdot e_3)) \mathbf{e}_3$$

$$E2 = -(e1 \wedge e3)E = ((e_1 \cdot e_2) - (e_1 \cdot e_3)(e_2 \cdot e_3)) \mathbf{e}_1 + \left((e_1 \cdot e_3)^2 - 1\right) \mathbf{e}_2 + (- (e_1 \cdot e_2)(e_1 \cdot e_3) + (e_2 \cdot e_3)) \mathbf{e}_3$$

$$E3 = (e1 \wedge e2)E = (- (e_1 \cdot e_2)(e_2 \cdot e_3) + (e_1 \cdot e_3)) \mathbf{e}_1 + (- (e_1 \cdot e_2)(e_1 \cdot e_3) + (e_2 \cdot e_3)) \mathbf{e}_2 + \left((e_1 \cdot e_2)^2 - 1\right) \mathbf{e}_3$$

$$E1 \cdot e2 = 0$$

$$E1 \cdot e3 = 0$$

$$E2 \cdot e1 = 0$$

$$E2 \cdot e3 = 0$$

$$E3 \cdot e1 = 0$$

$$E3 \cdot e2 = 0$$

$$(E1 \cdot e1)/E^2 = 1$$

$$(E2 \cdot e2)/E^2 = 1$$

$$(E3 \cdot e3)/E^2 = 1$$