

$$\boldsymbol{A} = A + A^xe_x + A^ye_y + A^ze_z + A^{xy}e_x \wedge e_y + A^{xz}e_x \wedge e_z + A^{yz}e_y \wedge e_z + A^{xyz}e_x \wedge e_y \wedge e_z$$

$$\begin{aligned}\boldsymbol{A} = & A \\ & + A^xe_x + A^ye_y + A^ze_z \\ & + A^{xy}e_x \wedge e_y + A^{xz}e_x \wedge e_z + A^{yz}e_y \wedge e_z \\ & + A^{xyz}e_x \wedge e_y \wedge e_z\end{aligned}$$

$$\begin{aligned}\boldsymbol{A} = & A \\ & + A^xe_x \\ & + A^ye_y \\ & + A^ze_z \\ & + A^{xy}e_x \wedge e_y \\ & + A^{xz}e_x \wedge e_z \\ & + A^{yz}e_y \wedge e_z \\ & + A^{xyz}e_x \wedge e_y \wedge e_z\end{aligned}$$

$$\boldsymbol{A} = A^xe_x + A^ye_y + A^ze_z$$

$$\boldsymbol{B} = B^{xy}e_x \wedge e_y + B^{xz}e_x \wedge e_z + B^{yz}e_y \wedge e_z$$

$$\boldsymbol{\nabla} f = \partial_x f e_x + \partial_y f e_y + \partial_z f e_z$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{A} = \partial_x A^x + \partial_y A^y + \partial_z A^z$$

$$\boldsymbol{\nabla} \boldsymbol{A} = (\partial_x A^x + \partial_y A^y + \partial_z A^z) + (-\partial_y A^x + \partial_x A^y) e_x \wedge e_y + (-\partial_z A^x + \partial_x A^z) e_x \wedge e_z + (-\partial_z A^y + \partial_y A^z) e_y \wedge e_z$$

$$-I(\boldsymbol{\nabla} \wedge \boldsymbol{A}) = (-\partial_z A^y + \partial_y A^z) e_x + (\partial_z A^x - \partial_x A^z) e_y + (-\partial_y A^x + \partial_x A^y) e_z$$

$$\boldsymbol{\nabla} \boldsymbol{B} = (-\partial_y B^{xy} - \partial_z B^{xz}) e_x + (\partial_x B^{xy} - \partial_z B^{yz}) e_y + (\partial_x B^{xz} + \partial_y B^{yz}) e_z + (\partial_z B^{xy} - \partial_y B^{xz} + \partial_x B^{yz}) e_x \wedge e_y \wedge e_z$$

$$\boldsymbol{\nabla} \wedge \boldsymbol{B} = (\partial_z B^{xy} - \partial_y B^{xz} + \partial_x B^{yz}) e_x \wedge e_y \wedge e_z$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{B} = (-\partial_y B^{xy} - \partial_z B^{xz}) e_x + (\partial_x B^{xy} - \partial_z B^{yz}) e_y + (\partial_x B^{xz} + \partial_y B^{yz}) e_z$$

$$g_{ij} = \begin{bmatrix} (a \cdot a) & (a \cdot b) & (a \cdot c) & (a \cdot d) \\ (a \cdot b) & (b \cdot b) & (b \cdot c) & (b \cdot d) \\ (a \cdot c) & (b \cdot c) & (c \cdot c) & (c \cdot d) \\ (a \cdot d) & (b \cdot d) & (c \cdot d) & (d \cdot d) \end{bmatrix}$$

$$\boldsymbol{a} \cdot (\boldsymbol{bc}) = -(a \cdot c) b + (a \cdot b) c$$

$$\boldsymbol{a} \cdot (\boldsymbol{b} \wedge \boldsymbol{c}) = -(a \cdot c) b + (a \cdot b) c$$

$$\boldsymbol{a} \cdot (\boldsymbol{b} \wedge \boldsymbol{c} \wedge \boldsymbol{d}) = (a \cdot d) b \wedge c - (a \cdot c) b \wedge d + (a \cdot b) c \wedge d$$

$$\boldsymbol{a} \cdot (\boldsymbol{b} \wedge \boldsymbol{c}) + \boldsymbol{c} \cdot (\boldsymbol{a} \wedge \boldsymbol{b}) + \boldsymbol{b} \cdot (\boldsymbol{c} \wedge \boldsymbol{a}) = 0$$

$$\boldsymbol{a}(\boldsymbol{b} \wedge \boldsymbol{c}) - \boldsymbol{b}(\boldsymbol{a} \wedge \boldsymbol{c}) + \boldsymbol{c}(\boldsymbol{a} \wedge \boldsymbol{b}) = 3\boldsymbol{a} \wedge \boldsymbol{b} \wedge \boldsymbol{c}$$

$$\boldsymbol{a}(\boldsymbol{b} \wedge \boldsymbol{c} \wedge \boldsymbol{d}) - \boldsymbol{b}(\boldsymbol{a} \wedge \boldsymbol{c} \wedge \boldsymbol{d}) + \boldsymbol{c}(\boldsymbol{a} \wedge \boldsymbol{b} \wedge \boldsymbol{d}) - \boldsymbol{d}(\boldsymbol{a} \wedge \boldsymbol{b} \wedge \boldsymbol{c}) = 4\boldsymbol{a} \wedge \boldsymbol{b} \wedge \boldsymbol{c} \wedge \boldsymbol{d}$$

$$(\boldsymbol{a} \wedge \boldsymbol{b}) \cdot (\boldsymbol{c} \wedge \boldsymbol{d}) = -(a \cdot c) (b \cdot d) + (a \cdot d) (b \cdot c)$$

$$((\boldsymbol{a} \wedge \boldsymbol{b}) \cdot \boldsymbol{c}) \cdot \boldsymbol{d} = -(a \cdot c) (b \cdot d) + (a \cdot d) (b \cdot c)$$

$$(\boldsymbol{a} \wedge \boldsymbol{b}) \times (\boldsymbol{c} \wedge \boldsymbol{d}) = -(b \cdot d) a \wedge c + (b \cdot c) a \wedge d + (a \cdot d) b \wedge c - (a \cdot c) b \wedge d$$

$$E = e_1 \wedge e_2 \wedge e_3$$

$$E^2 = (e_1 \cdot e_2)^2 - 2(e_1 \cdot e_2)(e_1 \cdot e_3)(e_2 \cdot e_3) + (e_1 \cdot e_3)^2 + (e_2 \cdot e_3)^2 - 1$$

$$E1 = (e2 \wedge e3)E = \left( (e_2 \cdot e_3)^2 - 1 \right) e_1 + ((e_1 \cdot e_2) - (e_1 \cdot e_3)(e_2 \cdot e_3)) e_2 + (-(e_1 \cdot e_2)(e_2 \cdot e_3) + (e_1 \cdot e_3)) e_3$$

$$E2 = -(e1 \wedge e3)E = ((e_1 \cdot e_2) - (e_1 \cdot e_3)(e_2 \cdot e_3)) e_1 + \left( (e_1 \cdot e_3)^2 - 1 \right) e_2 + (-(e_1 \cdot e_2)(e_1 \cdot e_3) + (e_2 \cdot e_3)) e_3$$

$$E3 = (e1 \wedge e2)E = (-(e_1 \cdot e_2)(e_2 \cdot e_3) + (e_1 \cdot e_3)) e_1 + (-(e_1 \cdot e_2)(e_1 \cdot e_3) + (e_2 \cdot e_3)) e_2 + \left( (e_1 \cdot e_2)^2 - 1 \right) e_3$$

$$E1 \cdot e2 = 0$$

$$E1 \cdot e3 = 0$$

$$E2 \cdot e1 = 0$$

$$E2 \cdot e3 = 0$$

$$E3 \cdot e1 = 0$$

$$E3 \cdot e2 = 0$$

$$(E1 \cdot e1)/E^2 = 1$$

$$(E2 \cdot e2)/E^2 = 1$$

$$(E3 \cdot e3)/E^2 = 1$$

$$A = A^r e_r + A^\theta e_\theta + A^\phi e_\phi$$

$$B = B^{r\theta} e_r \wedge e_\theta + B^{r\phi} e_r \wedge e_\phi + B^{\phi\phi} e_\theta \wedge e_\phi$$

$$\nabla f = \partial_r f e_r + \frac{1}{r} \partial_\theta f e_\theta + \frac{\partial_\phi f}{r \sin(\theta)} e_\phi$$

$$\nabla \cdot A = \frac{1}{r} \left( r \partial_r A^r + 2A^r + \frac{A^\theta}{\tan(\theta)} + \partial_\theta A^\theta + \frac{\partial_\phi A^\phi}{\sin(\theta)} \right)$$

$$-I(\nabla \wedge A) = \frac{1}{r} \left( \frac{A^\phi}{\tan(\theta)} + \partial_\theta A^\phi - \frac{\partial_\phi A^\theta}{\sin(\theta)} \right) e_r + \frac{1}{r} \left( -r \partial_r A^\phi - A^\phi + \frac{\partial_\phi A^r}{\sin(\theta)} \right) e_\theta + \frac{1}{r} (r \partial_r A^\theta + A^\theta - \partial_\theta A^r) e_\phi$$

$$\nabla \wedge B = \frac{1}{r} \left( r \partial_r B^{\phi\phi} - \frac{B^{r\phi}}{\tan(\theta)} + 2B^{\phi\phi} - \partial_\theta B^{r\phi} + \frac{\partial_\phi B^{r\theta}}{\sin(\theta)} \right) e_r \wedge e_\theta \wedge e_\phi$$

$$B = \boldsymbol{B}\boldsymbol{\gamma}_t = -B^x \gamma_t \wedge \gamma_x - B^y \gamma_t \wedge \gamma_y - B^z \gamma_t \wedge \gamma_z$$

$$E = \boldsymbol{E}\boldsymbol{\gamma}_t = -E^x \gamma_t \wedge \gamma_x - E^y \gamma_t \wedge \gamma_y - E^z \gamma_t \wedge \gamma_z$$

$$F = E + IB = -E^x \gamma_t \wedge \gamma_x - E^y \gamma_t \wedge \gamma_y - E^z \gamma_t \wedge \gamma_z - B^z \gamma_x \wedge \gamma_y + B^y \gamma_x \wedge \gamma_z - B^x \gamma_y \wedge \gamma_z$$

$$J = J^t \gamma_t + J^x \gamma_x + J^y \gamma_y + J^z \gamma_z$$

$$\begin{aligned} \nabla F &= (\partial_x E^x + \partial_y E^y + \partial_z E^z) \gamma_t \\ &\quad + (-\partial_z B^y + \partial_y B^z - \partial_t E^x) \gamma_x \\ &\quad + (\partial_z B^x - \partial_x B^z - \partial_t E^y) \gamma_y \\ &\quad + (-\partial_y B^x + \partial_x B^y - \partial_t E^z) \gamma_z \\ &\quad + (-\partial_t B^z + \partial_y E^x - \partial_x E^y) \gamma_t \wedge \gamma_x \wedge \gamma_y \\ &\quad + (\partial_t B^y + \partial_z E^x - \partial_x E^z) \gamma_t \wedge \gamma_x \wedge \gamma_z \\ &\quad + (-\partial_t B^x + \partial_z E^y - \partial_y E^z) \gamma_t \wedge \gamma_y \wedge \gamma_z \\ &\quad + (\partial_x B^x + \partial_y B^y + \partial_z B^z) \gamma_x \wedge \gamma_y \wedge \gamma_z \end{aligned}$$

$$\nabla F = J$$

$$\begin{aligned} \langle \nabla F \rangle_1 - J = 0 &= (-J^t + \partial_x E^x + \partial_y E^y + \partial_z E^z) \gamma_t \\ &\quad + (-J^x - \partial_z B^y + \partial_y B^z - \partial_t E^x) \gamma_x \\ &\quad + (-J^y + \partial_z B^x - \partial_x B^z - \partial_t E^y) \gamma_y \\ &\quad + (-J^z - \partial_y B^x + \partial_x B^y - \partial_t E^z) \gamma_z \end{aligned}$$

$$\begin{aligned} \langle \nabla F \rangle_3 = 0 &= (-\partial_t B^z + \partial_y E^x - \partial_x E^y) \gamma_t \wedge \gamma_x \wedge \gamma_y \\ &\quad + (\partial_t B^y + \partial_z E^x - \partial_x E^z) \gamma_t \wedge \gamma_x \wedge \gamma_z \\ &\quad + (-\partial_t B^x + \partial_z E^y - \partial_y E^z) \gamma_t \wedge \gamma_y \wedge \gamma_z \\ &\quad + (\partial_x B^x + \partial_y B^y + \partial_z B^z) \gamma_x \wedge \gamma_y \wedge \gamma_z \end{aligned}$$

$$R = \cosh\left(\frac{\alpha}{2}\right) + \sinh\left(\frac{\alpha}{2}\right) \gamma_t \wedge \gamma_x$$

$$t\gamma_t + x\gamma_x = t'\gamma'_t + x'\gamma'_x = R(t'\gamma_t + x'\gamma_x)R^\dagger$$

$$t\gamma_t + x\gamma_x = (t'\cosh(\alpha) - x'\sinh(\alpha))\gamma_t + (-t'\sinh(\alpha) + x'\cosh(\alpha))\gamma_x$$

$$\sinh(\alpha) = \gamma\beta$$

$$\cosh(\alpha) = \gamma$$

$$t\gamma_t + x\gamma_x = \gamma(-\beta x' + t')\gamma_t + \gamma(-\beta t' + x')\gamma_x$$

$$\boldsymbol{A} = A^t\gamma_t + A^x\gamma_x + A^y\gamma_y + A^z\gamma_z$$

$$\boldsymbol{\psi} = \psi + \psi^{tx}\gamma_t \wedge \gamma_x + \psi^{ty}\gamma_t \wedge \gamma_y + \psi^{tz}\gamma_t \wedge \gamma_z + \psi^{xy}\gamma_x \wedge \gamma_y + \psi^{xz}\gamma_x \wedge \gamma_z + \psi^{yz}\gamma_y \wedge \gamma_z + \psi^{txyz}\gamma_t \wedge \gamma_x \wedge \gamma_y \wedge \gamma_z$$

$$\begin{aligned} \nabla\boldsymbol{\psi}I\sigma_z - e\boldsymbol{A}\boldsymbol{\psi} - m\boldsymbol{\psi}\gamma_t = 0 = & \left(-eA^x\psi^{tx} - eA^y\psi^{ty} - eA^z\psi^{tz} + (-eA^t - m)\psi - \partial_y\psi^{tx} - \partial_z\psi^{txyz} + \partial_x\psi^{ty} + \partial_t\psi^{xy}\right)\gamma_t \\ & + \left(-eA^x\psi - eA^y\psi^{xy} - eA^z\psi^{xz} + (-eA^t + m)\psi^{tx} + \partial_y\psi - \partial_t\psi^{ty} - \partial_x\psi^{xy} + \partial_z\psi^{yz}\right)\gamma_x \\ & + \left(-eA^t\psi^{ty} + eA^x\psi^{xy} - eA^y\psi - eA^z\psi^{yz} + m\psi^{ty} - \partial_x\psi + \partial_t\psi^{tx} - \partial_y\psi^{xy} - \partial_z\psi^{xz}\right)\gamma_y \\ & + \left(-eA^t\psi^{tz} + eA^x\psi^{xz} + eA^y\psi^{yz} - eA^z\psi + m\psi^{tz} + \partial_t\psi^{txyz} - \partial_z\psi^{xy} + \partial_y\psi^{xz} - \partial_x\psi^{yz}\right)\gamma_z \\ & + \left(eA^x\psi^{ty} - eA^y\psi^{tx} - eA^z\psi^{txyz} + (-eA^t - m)\psi^{xy} - \partial_t\psi + \partial_x\psi^{tx} + \partial_y\psi^{ty} + \partial_z\psi^{tz}\right)\gamma_t \wedge \gamma_x \wedge \gamma_y \\ & + \left(eA^x\psi^{tz} + eA^y\psi^{txyz} - eA^z\psi^{tx} + (-eA^t - m)\psi^{xz} + \partial_x\psi^{txyz} + \partial_z\psi^{ty} - \partial_y\psi^{tz} - \partial_t\psi^{yz}\right)\gamma_t \wedge \gamma_x \wedge \gamma_z \\ & + \left(-eA^t\psi^{yz} - eA^x\psi^{txyz} + eA^y\psi^{tz} - eA^z\psi^{ty} - m\psi^{yz} - \partial_z\psi^{tx} + \partial_y\psi^{txyz} + \partial_x\psi^{tz} + \partial_t\psi^{xz}\right)\gamma_t \wedge \gamma_y \wedge \gamma_z \\ & + \left(-eA^t\psi^{txyz} - eA^x\psi^{yz} + eA^y\psi^{xz} - eA^z\psi^{xy} + m\psi^{txyz} + \partial_z\psi - \partial_t\psi^{tz} - \partial_x\psi^{xz} - \partial_y\psi^{yz}\right)\gamma_x \wedge \gamma_y \wedge \gamma_z \end{aligned}$$