

Program:

```
import sys
from sympy import symbols, exp, I, Matrix, solve, simplify
from sympy.galgebra.printer import Format, xpdf, Get_Program, Print_Function
from sympy.galgebra.ga import Ga
from sympy.galgebra.metric import linear_expand

Format()
X = (t, x, y, z) = symbols('t x y z', real=True)
(st4d, g0, g1, g2, g3) = Ga.build('gamma*t|x|y|z', g=[1, -1, -1, -1], coords=X)

i = st4d.i

B = st4d.mv('B', 'vector')
E = st4d.mv('E', 'vector')
B.set_coef(1, 0, 0)
E.set_coef(1, 0, 0)
B *= g0
E *= g0
F = E + i * B

kx, ky, kz, w = symbols('k_x k_y k_z omega', real=True)
kv = kx*g1 + ky*g2 + kz*g3
xv = x*g1 + y*g2 + z*g3
KX = ((w*g0 + kv) | (t*g0 + xv)).scalar()

Ixyz = g1*g2*g3

F = F*exp(I*KX)

print r'\text{Pseudo Scalar\\;} I =', i
print r'%I_{xyz} =', Ixyz
F.Fmt(3, '\\text{Electromagnetic Field Bi-Vector\\;\\;} F')
gradF = st4d.grad*F

print '#Geom Derivative of Electomagnetic Field Bi-Vector'
gradF.Fmt(3, 'grad*F = 0')

gradF = gradF / (I * exp(I*KX))
gradF.Fmt(3, r'%\lp\bm{\nabla}F\rp /\lp i e^{\iK\cdot X}\rp = 0')

g = '1 # 0 0, # 1 0 0, 0 0 1 0, 0 0 0 -1'
X = (xE, xB, xk, t) = symbols('x_E x_B x_k t', real=True)
(EBkst, eE, eB, ek, et) = Ga.build('e_E e_B e_k t', g=g, coords=X)
```

```

i = EBkst.i

E,B,k,w = symbols('E B k omega',real=True)

F = E*eE*et+i*B*eB*et
kv = k*ek+w*et
xv = xE*eE+xB*eB+xk*ek+t*et
KX = (kv|xv).scalar()
F = F*exp(I*KX)

print r'%\mbox{ set } e_{E}\cdot e_{k} = e_{B}\cdot e_{k} = 0'+\
      r'\mbox{ and } e_{E}\cdot e_{E} = e_{B}\cdot e_{B} = '+\
      r'e_{k}\cdot e_{k} = -e_{t}\cdot e_{t} = 1'

print 'g =', EBkst.g

print 'K|X =',KX
print 'F =',F
(EBkst.grad*F).Fmt(3,'grad*F = 0')

gradF_reduced = (EBkst.grad*F)/(I*exp(I*KX))

gradF_reduced.Fmt(3,r'%\lp\bm{\nabla}F\rp/\lp ie^{iK\cdot X} \rp = 0')

print r'%\mbox{ Previous equation requires that: }e_{E}\cdot e_{B} = 0'+\
      r'\mbox{ if }B\ne 0\mbox{ and }k\ne 0'

gradF_reduced = gradF_reduced.subs({EBkst.g[0,1]:0})
gradF_reduced.Fmt(3,r'%\lp\bm{\nabla}F\rp/\lp ie^{iK\cdot X} \rp = 0')

(coefs,bases) = linear_expand(gradF_reduced.obj)

eq1 = coefs[0]
eq2 = coefs[1]

B1 = solve(eq1,B)[0]
B2 = solve(eq2,B)[0]

print r'\mbox{eq1: }B =',B1
print r'\mbox{eq2: }B =',B2

eq3 = B1-B2

print r'\mbox{eq3 = eq1-eq2: }0 =',eq3
eq3 = simplify(eq3 / E)

```

```

print r '\mbox{eq3 = (eq1-eq2)/E: }0 =' ,eq3
print '#Solutions for $k$ and $B$ in terms of $\omega$ and $E$: '
print 'k =',Matrix(solve(eq3,k))
print 'B =',Matrix([B1.subs(w,k),B1.subs(-w,k)])
xpdf(paper='landscape',prog=True)

```

Code Output:

Pseudo Scalar $I = \gamma_t \wedge \gamma_x \wedge \gamma_y \wedge \gamma_z$

$$I_{xyz} = \gamma_x \wedge \gamma_y \wedge \gamma_z$$

$$\begin{aligned}
 \text{Electromagnetic Field Bi-Vector } F = & -E^x e^{-i(-\omega t + k_x x + k_y y + k_z z)} \gamma_t \wedge \gamma_x \\
 & -E^y e^{-i(-\omega t + k_x x + k_y y + k_z z)} \gamma_t \wedge \gamma_y \\
 & -E^z e^{-i(-\omega t + k_x x + k_y y + k_z z)} \gamma_t \wedge \gamma_z \\
 & -B^z e^{-i(-\omega t + k_x x + k_y y + k_z z)} \gamma_x \wedge \gamma_y \\
 & +B^y e^{i(\omega t - k_x x - k_y y - k_z z)} \gamma_x \wedge \gamma_z \\
 & -B^x e^{-i(-\omega t + k_x x + k_y y + k_z z)} \gamma_y \wedge \gamma_z
 \end{aligned}$$

Geom Derivative of Electomagnetic Field Bi-Vector

$$\begin{aligned}
 \nabla F = 0 = & -i(E^x k_x + E^y k_y + E^z k_z) e^{-i(-\omega t + k_x x + k_y y + k_z z)} \gamma_t \\
 & +i(B^y k_z - B^z k_y - E^x \omega) e^{i(\omega t - k_x x - k_y y - k_z z)} \gamma_x \\
 & +i(-B^x k_z + B^z k_x - E^y \omega) e^{i(\omega t - k_x x - k_y y - k_z z)} \gamma_y \\
 & +i(B^x k_y - B^y k_x - E^z \omega) e^{i(\omega t - k_x x - k_y y - k_z z)} \gamma_z \\
 & +i(-B^z \omega - E^x k_y + E^y k_x) e^{i(\omega t - k_x x - k_y y - k_z z)} \gamma_t \wedge \gamma_x \wedge \gamma_y \\
 & +i(B^y \omega - E^x k_z + E^z k_x) e^{i(\omega t - k_x x - k_y y - k_z z)} \gamma_t \wedge \gamma_x \wedge \gamma_z \\
 & +i(-B^x \omega - E^y k_z + E^z k_y) e^{i(\omega t - k_x x - k_y y - k_z z)} \gamma_t \wedge \gamma_y \wedge \gamma_z \\
 & -i(B^x k_x + B^y k_y + B^z k_z) e^{-i(-\omega t + k_x x + k_y y + k_z z)} \gamma_x \wedge \gamma_y \wedge \gamma_z
 \end{aligned}$$

$$\begin{aligned}
 (\nabla F) / (ie^{iK \cdot X}) = 0 = & (-E^x k_x - E^y k_y - E^z k_z) \gamma_t \\
 & + (B^y k_z - B^z k_y - E^x \omega) \gamma_x \\
 & + (-B^x k_z + B^z k_x - E^y \omega) \gamma_y \\
 & + (B^x k_y - B^y k_x - E^z \omega) \gamma_z \\
 & + (-B^z \omega - E^x k_y + E^y k_x) \gamma_t \wedge \gamma_x \wedge \gamma_y \\
 & + (B^y \omega - E^x k_z + E^z k_x) \gamma_t \wedge \gamma_x \wedge \gamma_z \\
 & + (-B^x \omega - E^y k_z + E^z k_y) \gamma_t \wedge \gamma_y \wedge \gamma_z \\
 & + (-B^x k_x - B^y k_y - B^z k_z) \gamma_x \wedge \gamma_y \wedge \gamma_z
 \end{aligned}$$

set $e_E \cdot e_k = e_B \cdot e_k = 0$ and $e_E \cdot e_E = e_B \cdot e_B = e_k \cdot e_k = -e_t \cdot e_t = 1$

$$g = \begin{bmatrix} 1 & (e_E \cdot e_B) & 0 & 0 \\ (e_E \cdot e_B) & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$K \cdot X = -\omega t + kx_k$$

$$F = -Be^{-i(\omega t - kx_k)} \mathbf{e}_E \wedge \mathbf{e}_k + Ee^{i(-\omega t + kx_k)} \mathbf{e}_E \wedge \mathbf{t} + (e_E \cdot e_B) Be^{i(-\omega t + kx_k)} \mathbf{e}_B \wedge \mathbf{e}_k$$

$$\begin{aligned} \nabla F = 0 = & i(Bk + E\omega) e^{i(-\omega t + kx_k)} \mathbf{e}_E \\ & - i(e_E \cdot e_B) Bk e^{-i(\omega t - kx_k)} \mathbf{e}_B \\ & - i(B\omega + Ek) e^{-i(\omega t - kx_k)} \mathbf{e}_E \wedge \mathbf{e}_k \wedge \mathbf{t} \\ & + i(e_E \cdot e_B) B\omega e^{i(-\omega t + kx_k)} \mathbf{e}_B \wedge \mathbf{e}_k \wedge \mathbf{t} \end{aligned}$$

$$\begin{aligned} (\nabla F) / (ie^{iK \cdot X}) = 0 = & (Bk + E\omega) \mathbf{e}_E \\ & - (e_E \cdot e_B) Bk \mathbf{e}_B \\ & + (-B\omega - Ek) \mathbf{e}_E \wedge \mathbf{e}_k \wedge \mathbf{t} \\ & + (e_E \cdot e_B) B\omega \mathbf{e}_B \wedge \mathbf{e}_k \wedge \mathbf{t} \end{aligned}$$

Previous equation requires that: $e_E \cdot e_B = 0$ if $B \neq 0$ and $k \neq 0$

$$\begin{aligned} (\nabla F) / (ie^{iK \cdot X}) = 0 = & (Bk + E\omega) \mathbf{e}_E \\ & + (-B\omega - Ek) \mathbf{e}_E \wedge \mathbf{e}_k \wedge \mathbf{t} \end{aligned}$$

$$\text{eq1: } B = -\frac{E\omega}{k}$$

$$\text{eq2: } B = -\frac{Ek}{\omega}$$

$$\text{eq3} = \text{eq1} - \text{eq2: } 0 = -\frac{E\omega}{k} + \frac{Ek}{\omega}$$

$$\text{eq3} = (\text{eq1} - \text{eq2})/E: 0 = -\frac{\omega}{k} + \frac{k}{\omega}$$

Solutions for k and B in terms of ω and E :

$$k = \begin{bmatrix} -\omega \\ \omega \end{bmatrix}$$

$$B = \begin{bmatrix} -E \\ E \end{bmatrix}$$