3D Orthogonal Metric
Multvectors:

$$s = s$$

$$v = v^{x}\boldsymbol{e_{x}} + v^{y}\boldsymbol{e_{y}} + v^{z}\boldsymbol{e_{z}}$$

$$b = b^{xy}\boldsymbol{e_{x}}\wedge \boldsymbol{e_{y}} + b^{xz}\boldsymbol{e_{x}}\wedge \boldsymbol{e_{z}} + b^{yz}\boldsymbol{e_{y}}\wedge \boldsymbol{e_{z}}$$

Products:

$$ss = s^{2}$$

$$s\wedge s = s^{2}$$

$$s\rfloor s = s^{2}$$

$$s\lfloor s = s^{2}$$

$$sv = sv^{x}\boldsymbol{e_{x}} + sv^{y}\boldsymbol{e_{y}} + sv^{z}\boldsymbol{e_{z}}$$

$$s\wedge v = sv^{x}\boldsymbol{e_{x}} + sv^{y}\boldsymbol{e_{y}} + sv^{z}\boldsymbol{e_{z}}$$

$$s\rfloor v = sv^{x}\boldsymbol{e_{x}} + sv^{y}\boldsymbol{e_{y}} + sv^{z}\boldsymbol{e_{z}}$$

$$s\lfloor v = sv^{x}\boldsymbol{e_{x}} + sv^{y}\boldsymbol{e_{y}} + sv^{z}\boldsymbol{e_{z}}$$

$$sb = b^{xy}s\boldsymbol{e_{x}}\wedge \boldsymbol{e_{y}} + b^{xz}s\boldsymbol{e_{x}}\wedge \boldsymbol{e_{z}} + b^{yz}s\boldsymbol{e_{y}}\wedge \boldsymbol{e_{z}}$$

$$s\wedge b = b^{xy}s\boldsymbol{e_{x}}\wedge \boldsymbol{e_{y}} + b^{xz}s\boldsymbol{e_{x}}\wedge \boldsymbol{e_{z}} + b^{yz}s\boldsymbol{e_{y}}\wedge \boldsymbol{e_{z}}$$

$$s\rfloor b = b^{xy}s\boldsymbol{e_{x}}\wedge \boldsymbol{e_{y}} + b^{xz}s\boldsymbol{e_{x}}\wedge \boldsymbol{e_{z}} + b^{yz}s\boldsymbol{e_{y}}\wedge \boldsymbol{e_{z}}$$

$$s\lfloor b = b^{xy}s\boldsymbol{e_{x}}\wedge \boldsymbol{e_{y}} + b^{xz}s\boldsymbol{e_{x}}\wedge \boldsymbol{e_{z}} + b^{yz}s\boldsymbol{e_{y}}\wedge \boldsymbol{e_{z}}$$

$$vs = sv^{x}\boldsymbol{e_{x}} + sv^{y}\boldsymbol{e_{y}} + sv^{z}\boldsymbol{e_{z}}$$

$$v\wedge s = sv^{x}\boldsymbol{e_{x}} + sv^{y}\boldsymbol{e_{y}} + sv^{z}\boldsymbol{e_{z}}$$

$$v\rfloor s = sv^{x}\boldsymbol{e_{x}} + sv^{y}\boldsymbol{e_{y}} + sv^{z}\boldsymbol{e_{z}}$$

$$v\lfloor s = sv^{x}\boldsymbol{e_{x}} + sv^{y}\boldsymbol{e_{y}} + sv^{z}\boldsymbol{e_{z}}$$

$$vv = \left(v^{x}\right)^{2} + \left(v^{y}\right)^{2} + \left(v^{z}\right)^{2}$$

$$v\wedge v = 0$$

$$v\cdot v = \left(v^{x}\right)^{2} + \left(v^{y}\right)^{2} + \left(v^{z}\right)^{2}$$

$$v\rfloor v = \left(v^{x}\right)^{2} + \left(v^{y}\right)^{2} + \left(v^{z}\right)^{2}$$

$$v\lfloor v = \left(v^{x}\right)^{2} + \left(v^{y}\right)^{2} + \left(v^{z}\right)^{2}$$

$$vb = \left(- b^{xy}v^{y} - b^{xz}v^{z}\right)\boldsymbol{e_{x}} + \left(b^{xy}v^{x} - b^{yz}v^{z}\right)\boldsymbol{e_{y}} + \left(b^{xz}v^{x} + b^{yz}v^{y}\right)\boldsymbol{e_{z}} + \left(b^{xy}v^{z} - b^{xz}v^{y} + b^{yz}v^{x}\right)\boldsymbol{e_{x}}\wedge \boldsymbol{e_{y}}\wedge \boldsymbol{e_{z}}$$

$$v\wedge b = \left(b^{xy}v^{z} - b^{xz}v^{y} + b^{yz}v^{x}\right)\boldsymbol{e_{x}}\wedge \boldsymbol{e_{y}}\wedge \boldsymbol{e_{z}}$$

$$v\cdot b = \left(- b^{xy}v^{y} - b^{xz}v^{z}\right)\boldsymbol{e_{x}} + \left(b^{xy}v^{x} - b^{yz}v^{z}\right)\boldsymbol{e_{y}} + \left(b^{xz}v^{x} + b^{yz}v^{y}\right)\boldsymbol{e_{z}}$$

$$v\rfloor b = \left(-b^{xy}v^{y} - b^{xz}v^{z}\right)\boldsymbol{e_{x}} + \left(b^{xy}v^{x} - b^{yz}v^{z}\right)\boldsymbol{e_{y}} + \left(b^{xz}v^{x} + b^{yz}v^{y}\right)\boldsymbol{e_{z}}$$

$$v\lfloor b = 0$$

$$bs = b^{xy}s\boldsymbol{e_{x}}\wedge\boldsymbol{e_{y}} + b^{xz}s\boldsymbol{e_{x}}\wedge\boldsymbol{e_{z}} + b^{yz}s\boldsymbol{e_{y}}\wedge\boldsymbol{e_{z}}$$

$$b\wedge s = b^{xy}s\boldsymbol{e_{x}}\wedge\boldsymbol{e_{y}} + b^{xz}s\boldsymbol{e_{x}}\wedge\boldsymbol{e_{z}} + b^{yz}s\boldsymbol{e_{y}}\wedge\boldsymbol{e_{z}}$$

$$b\rfloor s = b^{xy}s\boldsymbol{e_{x}}\wedge\boldsymbol{e_{y}} + b^{xz}s\boldsymbol{e_{x}}\wedge\boldsymbol{e_{z}} + b^{yz}s\boldsymbol{e_{y}}\wedge\boldsymbol{e_{z}}$$

$$b\lfloor s = b^{xy}s\boldsymbol{e_{x}}\wedge\boldsymbol{e_{y}} + b^{xz}s\boldsymbol{e_{x}}\wedge\boldsymbol{e_{z}} + b^{yz}s\boldsymbol{e_{y}}\wedge\boldsymbol{e_{z}}$$

$$bv = \left(b^{xy}v^{y} + b^{xz}v^{z}\right)\boldsymbol{e_{x}} + \left(-b^{xy}v^{x} + b^{yz}v^{z}\right)\boldsymbol{e_{y}} + \left(-b^{xz}v^{x} - b^{yz}v^{y}\right)\boldsymbol{e_{z}} + \left(b^{xy}v^{z} - b^{xz}v^{y} + b^{yz}v^{x}\right)\boldsymbol{e_{x}}\wedge\boldsymbol{e_{y}}\wedge\boldsymbol{e_{z}}$$

$$b\wedge v = \left(b^{xy}v^{z} - b^{xz}v^{y} + b^{yz}v^{x}\right)\boldsymbol{e_{x}}\wedge\boldsymbol{e_{y}}\wedge\boldsymbol{e_{z}}$$

$$b\cdot v = \left(b^{xy}v^{y} + b^{xz}v^{z}\right)\boldsymbol{e_{x}} + \left(-b^{xy}v^{x} + b^{yz}v^{z}\right)\boldsymbol{e_{y}} + \left(-b^{xz}v^{x} - b^{yz}v^{y}\right)\boldsymbol{e_{z}}$$

$$b\rfloor v = 0$$

$$b\lfloor v = \left(b^{xy}v^{y} + b^{xz}v^{z}\right)\boldsymbol{e_{x}} + \left(-b^{xy}v^{x} + b^{yz}v^{z}\right)\boldsymbol{e_{y}} + \left(-b^{xz}v^{x} - b^{yz}v^{y}\right)\boldsymbol{e_{z}}$$

$$bb = -\left(b^{xy}\right)^{2} - \left(b^{xz}\right)^{2} - \left(b^{yz}\right)^{2}$$

$$b\wedge b = 0$$

$$b\cdot b = -\left(b^{xy}\right)^{2} - \left(b^{xz}\right)^{2} - \left(b^{yz}\right)^{2}$$

$$b\rfloor b = -\left(b^{xy}\right)^{2} - \left(b^{xz}\right)^{2} - \left(b^{yz}\right)^{2}$$

$$b\lfloor b = -\left(b^{xy}\right)^{2} - \left(b^{xz}\right)^{2} - \left(b^{yz}\right)^{2}$$

Multivector Functions:

$$s(X) = s$$

$$v(X) = v^{x}\boldsymbol{e_{x}} + v^{y}\boldsymbol{e_{y}} + v^{z}\boldsymbol{e_{z}}$$

$$b(X) = b^{xy}\boldsymbol{e_{x}}\wedge\boldsymbol{e_{y}} + b^{xz}\boldsymbol{e_{x}}\wedge\boldsymbol{e_{z}} + b^{yz}\boldsymbol{e_{y}}\wedge\boldsymbol{e_{z}}$$

Products:

$$\boldsymbol{\nabla}s = \partial_{x}s\boldsymbol{e_{x}} + \partial_{y}s\boldsymbol{e_{y}} + \partial_{z}s\boldsymbol{e_{z}}$$

$$\boldsymbol{\nabla}\wedge s = \partial_{x}s\boldsymbol{e_{x}} + \partial_{y}s\boldsymbol{e_{y}} + \partial_{z}s\boldsymbol{e_{z}}$$

$$\boldsymbol{\nabla}\rfloor s = \partial_{x}s\boldsymbol{e_{x}} + \partial_{y}s\boldsymbol{e_{y}} + \partial_{z}s\boldsymbol{e_{z}}$$

$$\boldsymbol{\nabla}\lfloor s = \partial_{x}s\boldsymbol{e_{x}} + \partial_{y}s\boldsymbol{e_{y}} + \partial_{z}s\boldsymbol{e_{z}}$$

$$\boldsymbol{\nabla}v = \left(\partial_{x}v^{x} + \partial_{y}v^{y} + \partial_{z}v^{z}\right) + \left(-\partial_{y}v^{x} + \partial_{x}v^{y}\right)\boldsymbol{e_{x}}\wedge\boldsymbol{e_{y}} + \left(-\partial_{z}v^{x} + \partial_{x}v^{z}\right)\boldsymbol{e_{x}}\wedge\boldsymbol{e_{z}} + \left(-\partial_{z}v^{y} + \partial_{y}v^{z}\right)\boldsymbol{e_{y}}\wedge\boldsymbol{e_{z}}$$

$$\boldsymbol{\nabla}\wedge v = \left(-\partial_{y}v^{x} + \partial_{x}v^{y}\right)\boldsymbol{e_{x}}\wedge\boldsymbol{e_{y}} + \left(-\partial_{z}v^{x} + \partial_{x}v^{z}\right)\boldsymbol{e_{x}}\wedge\boldsymbol{e_{z}} + \left(-\partial_{z}v^{y} + \partial_{y}v^{z}\right)\boldsymbol{e_{y}}\wedge\boldsymbol{e_{z}}$$

$$\boldsymbol{\nabla}\cdot v = \partial_{x}v^{x} + \partial_{y}v^{y} + \partial_{z}v^{z}$$

$$\boldsymbol{\nabla}\rfloor v = \partial_x v^x + \partial_y v^y + \partial_z v^z$$

$$\boldsymbol{\nabla}\lfloor v = \partial_x v^x + \partial_y v^y + \partial_z v^z$$

$$\boldsymbol{\nabla} b = \left(-\partial_y b^{xy} - \partial_z b^{xz}\right)\boldsymbol{e_x} + \left(\partial_x b^{xy} - \partial_z b^{yz}\right)\boldsymbol{e_y} + \left(\partial_x b^{xz} + \partial_y b^{yz}\right)\boldsymbol{e_z} + \left(\partial_z b^{xy} - \partial_y b^{xz} + \partial_x b^{yz}\right)\boldsymbol{e_x}\wedge\boldsymbol{e_y}\wedge\boldsymbol{e_z}$$

$$\boldsymbol{\nabla}\wedge b = \left(\partial_z b^{xy} - \partial_y b^{xz} + \partial_x b^{yz}\right)\boldsymbol{e_x}\wedge\boldsymbol{e_y}\wedge\boldsymbol{e_z}$$

$$\boldsymbol{\nabla}\cdot b = \left(-\partial_y b^{xy} - \partial_z b^{xz}\right)\boldsymbol{e_x} + \left(\partial_x b^{xy} - \partial_z b^{yz}\right)\boldsymbol{e_y} + \left(\partial_x b^{xz} + \partial_y b^{yz}\right)\boldsymbol{e_z}$$

$$\boldsymbol{\nabla}\rfloor b = \left(-\partial_y b^{xy} - \partial_z b^{xz}\right)\boldsymbol{e_x} + \left(\partial_x b^{xy} - \partial_z b^{yz}\right)\boldsymbol{e_y} + \left(\partial_x b^{xz} + \partial_y b^{yz}\right)\boldsymbol{e_z}$$

$$\boldsymbol{\nabla}\lfloor b = 0$$

$$s\boldsymbol{\nabla} = s\boldsymbol{e_x}\frac{\partial}{\partial x} + s\boldsymbol{e_y}\frac{\partial}{\partial y} + s\boldsymbol{e_z}\frac{\partial}{\partial z}$$

$$s\wedge\boldsymbol{\nabla} = s\boldsymbol{e_x}\frac{\partial}{\partial x} + s\boldsymbol{e_y}\frac{\partial}{\partial y} + s\boldsymbol{e_z}\frac{\partial}{\partial z}$$

$$s\rfloor\boldsymbol{\nabla} = s\boldsymbol{e_x}\frac{\partial}{\partial x} + s\boldsymbol{e_y}\frac{\partial}{\partial y} + s\boldsymbol{e_z}\frac{\partial}{\partial z}$$

$$s\lfloor\boldsymbol{\nabla} = s\boldsymbol{e_x}\frac{\partial}{\partial x} + s\boldsymbol{e_y}\frac{\partial}{\partial y} + s\boldsymbol{e_z}\frac{\partial}{\partial z}$$

$$ss = s^2$$

$$s\wedge s = s^2$$

$$s\rfloor s = s^2$$

$$s\lfloor s = s^2$$

$$sv = sv^x\boldsymbol{e_x} + sv^y\boldsymbol{e_y} + sv^z\boldsymbol{e_z}$$

$$s\wedge v = sv^x\boldsymbol{e_x} + sv^y\boldsymbol{e_y} + sv^z\boldsymbol{e_z}$$

$$s\rfloor v = sv^x\boldsymbol{e_x} + sv^y\boldsymbol{e_y} + sv^z\boldsymbol{e_z}$$

$$s\lfloor v = sv^x\boldsymbol{e_x} + sv^y\boldsymbol{e_y} + sv^z\boldsymbol{e_z}$$

$$sb = b^{xy}s\boldsymbol{e_x}\wedge\boldsymbol{e_y} + b^{xz}s\boldsymbol{e_x}\wedge\boldsymbol{e_z} + b^{yz}s\boldsymbol{e_y}\wedge\boldsymbol{e_z}$$

$$s\wedge b = b^{xy}s\boldsymbol{e_x}\wedge\boldsymbol{e_y} + b^{xz}s\boldsymbol{e_x}\wedge\boldsymbol{e_z} + b^{yz}s\boldsymbol{e_y}\wedge\boldsymbol{e_z}$$

$$s\rfloor b = b^{xy}s\boldsymbol{e_x}\wedge\boldsymbol{e_y} + b^{xz}s\boldsymbol{e_x}\wedge\boldsymbol{e_z} + b^{yz}s\boldsymbol{e_y}\wedge\boldsymbol{e_z}$$

$$s\lfloor b = b^{xy}s\boldsymbol{e_x}\wedge\boldsymbol{e_y} + b^{xz}s\boldsymbol{e_x}\wedge\boldsymbol{e_z} + b^{yz}s\boldsymbol{e_y}\wedge\boldsymbol{e_z}$$

$$v\boldsymbol{\nabla} = \left(v^x - v^y\boldsymbol{e_x}\wedge\boldsymbol{e_y} - v^z\boldsymbol{e_x}\wedge\boldsymbol{e_z}\right)\frac{\partial}{\partial x} + \left(v^y + v^x\boldsymbol{e_x}\wedge\boldsymbol{e_y} - v^z\boldsymbol{e_y}\wedge\boldsymbol{e_z}\right)\frac{\partial}{\partial y} + \left(v^z + v^x\boldsymbol{e_x}\wedge\boldsymbol{e_z} + v^y\boldsymbol{e_y}\wedge\boldsymbol{e_z}\right)\frac{\partial}{\partial z}$$

$$v\wedge\boldsymbol{\nabla} = \left(-v^y\boldsymbol{e_x}\wedge\boldsymbol{e_y} - v^z\boldsymbol{e_x}\wedge\boldsymbol{e_z}\right)\frac{\partial}{\partial x} + \left(v^x\boldsymbol{e_x}\wedge\boldsymbol{e_y} - v^z\boldsymbol{e_y}\wedge\boldsymbol{e_z}\right)\frac{\partial}{\partial y} + \left(v^x\boldsymbol{e_x}\wedge\boldsymbol{e_z} + v^y\boldsymbol{e_y}\wedge\boldsymbol{e_z}\right)\frac{\partial}{\partial z}$$

$$v \cdot \boldsymbol{\nabla} = v^{x} \frac{\partial}{\partial x} + v^{y} \frac{\partial}{\partial y} + v^{z} \frac{\partial}{\partial z}$$

$$v \rfloor \boldsymbol{\nabla} = v^{x} \frac{\partial}{\partial x} + v^{y} \frac{\partial}{\partial y} + v^{z} \frac{\partial}{\partial z}$$

$$v \lfloor \boldsymbol{\nabla} = v^{x} \frac{\partial}{\partial x} + v^{y} \frac{\partial}{\partial y} + v^{z} \frac{\partial}{\partial z}$$

$$vs = sv^{x}\boldsymbol{e_{x}} + sv^{y}\boldsymbol{e_{y}} + sv^{z}\boldsymbol{e_{z}}$$

$$v \wedge s = sv^{x}\boldsymbol{e_{x}} + sv^{y}\boldsymbol{e_{y}} + sv^{z}\boldsymbol{e_{z}}$$

$$v \rfloor s = sv^{x}\boldsymbol{e_{x}} + sv^{y}\boldsymbol{e_{y}} + sv^{z}\boldsymbol{e_{z}}$$

$$v \lfloor s = sv^{x}\boldsymbol{e_{x}} + sv^{y}\boldsymbol{e_{y}} + sv^{z}\boldsymbol{e_{z}}$$

$$vv = (v^{x})^{2} + (v^{y})^{2} + (v^{z})^{2}$$

$$v \wedge v = 0$$

$$v \cdot v = (v^{x})^{2} + (v^{y})^{2} + (v^{z})^{2}$$

$$v \rfloor v = (v^{x})^{2} + (v^{y})^{2} + (v^{z})^{2}$$

$$v \lfloor v = (v^{x})^{2} + (v^{y})^{2} + (v^{z})^{2}$$

$$vb = \left(-b^{xy}v^{y} - b^{xz}v^{z}\right)\boldsymbol{e_{x}} + \left(b^{xy}v^{x} - b^{yz}v^{z}\right)\boldsymbol{e_{y}} + \left(b^{xz}v^{x} + b^{yz}v^{y}\right)\boldsymbol{e_{z}} + \left(b^{xy}v^{z} - b^{xz}v^{y} + b^{yz}v^{x}\right)\boldsymbol{e_{x}} \wedge \boldsymbol{e_{y}} \wedge \boldsymbol{e_{z}}$$

$$v \wedge b = \left(b^{xy}v^{z} - b^{xz}v^{y} + b^{yz}v^{x}\right)\boldsymbol{e_{x}} \wedge \boldsymbol{e_{y}} \wedge \boldsymbol{e_{z}}$$

$$v \cdot b = \left(-b^{xy}v^{y} - b^{xz}v^{z}\right)\boldsymbol{e_{x}} + \left(b^{xy}v^{x} - b^{yz}v^{z}\right)\boldsymbol{e_{y}} + \left(b^{xz}v^{x} + b^{yz}v^{y}\right)\boldsymbol{e_{z}}$$

$$v \rfloor b = \left(-b^{xy}v^{y} - b^{xz}v^{z}\right)\boldsymbol{e_{x}} + \left(b^{xy}v^{x} - b^{yz}v^{z}\right)\boldsymbol{e_{y}} + \left(b^{xz}v^{x} + b^{yz}v^{y}\right)\boldsymbol{e_{z}}$$

$$v \lfloor b = 0$$

$$b\boldsymbol{\nabla} = \left(-b^{xy}\boldsymbol{e_{y}} - b^{xz}\boldsymbol{e_{z}} + b^{yz}\boldsymbol{e_{x}} \wedge \boldsymbol{e_{y}} \wedge \boldsymbol{e_{z}}\right)\frac{\partial}{\partial x} + \left(b^{xy}\boldsymbol{e_{x}} - b^{yz}\boldsymbol{e_{z}} - b^{xz}\boldsymbol{e_{x}} \wedge \boldsymbol{e_{y}} \wedge \boldsymbol{e_{z}}\right)\frac{\partial}{\partial y} + \left(b^{xz}\boldsymbol{e_{x}} + b^{yz}\boldsymbol{e_{y}} + b^{xy}\boldsymbol{e_{x}} \wedge \boldsymbol{e_{y}} \wedge \boldsymbol{e_{z}}\right)\frac{\partial}{\partial z}$$

$$b \wedge \boldsymbol{\nabla} = b^{yz}\boldsymbol{e_{x}} \wedge \boldsymbol{e_{y}} \wedge \boldsymbol{e_{z}}\frac{\partial}{\partial x} - b^{xz}\boldsymbol{e_{x}} \wedge \boldsymbol{e_{y}} \wedge \boldsymbol{e_{z}}\frac{\partial}{\partial y} + b^{xy}\boldsymbol{e_{x}} \wedge \boldsymbol{e_{y}} \wedge \boldsymbol{e_{z}}\frac{\partial}{\partial z}$$

$$b \cdot \boldsymbol{\nabla} = \left(-b^{xy}\boldsymbol{e_{y}} - b^{xz}\boldsymbol{e_{z}}\right)\frac{\partial}{\partial x} + \left(b^{xy}\boldsymbol{e_{x}} - b^{yz}\boldsymbol{e_{z}}\right)\frac{\partial}{\partial y} + \left(b^{xz}\boldsymbol{e_{x}} + b^{yz}\boldsymbol{e_{y}}\right)\frac{\partial}{\partial z}$$

$$b \rfloor \boldsymbol{\nabla} = 0$$

$$b \lfloor \boldsymbol{\nabla} = \left(-b^{xy}\boldsymbol{e_{y}} - b^{xz}\boldsymbol{e_{z}}\right)\frac{\partial}{\partial x} + \left(b^{xy}\boldsymbol{e_{x}} - b^{yz}\boldsymbol{e_{z}}\right)\frac{\partial}{\partial y} + \left(b^{xz}\boldsymbol{e_{x}} + b^{yz}\boldsymbol{e_{y}}\right)\frac{\partial}{\partial z}$$

$$bs = b^{xy}s\boldsymbol{e_{x}} \wedge \boldsymbol{e_{y}} + b^{xz}s\boldsymbol{e_{x}} \wedge \boldsymbol{e_{z}} + b^{yz}s\boldsymbol{e_{y}} \wedge \boldsymbol{e_{z}}$$

$$b \wedge s = b^{xy}s\boldsymbol{e_{x}} \wedge \boldsymbol{e_{y}} + b^{xz}s\boldsymbol{e_{x}} \wedge \boldsymbol{e_{z}} + b^{yz}s\boldsymbol{e_{y}} \wedge \boldsymbol{e_{z}}$$

$$b \rfloor s = b^{xy}s\boldsymbol{e_{x}} \wedge \boldsymbol{e_{y}} + b^{xz}s\boldsymbol{e_{x}} \wedge \boldsymbol{e_{z}} + b^{yz}s\boldsymbol{e_{y}} \wedge \boldsymbol{e_{z}}$$

$$b\lfloor s = b^{xy} s \boldsymbol{e_x} \wedge \boldsymbol{e_y} + b^{xz} s \boldsymbol{e_x} \wedge \boldsymbol{e_z} + b^{yz} s \boldsymbol{e_y} \wedge \boldsymbol{e_z}$$

$$bv = \left(b^{xy} v^{y} + b^{xz} v^{z}\right) \boldsymbol{e_x} + \left(- b^{xy} v^{x} + b^{yz} v^{z}\right) \boldsymbol{e_y} + \left(- b^{xz} v^{x} - b^{yz} v^{y}\right) \boldsymbol{e_z} + \left(b^{xy} v^{z} - b^{xz} v^{y} + b^{yz} v^{x}\right) \boldsymbol{e_x} \wedge \boldsymbol{e_y} \wedge \boldsymbol{e_z}$$

$$b \wedge v = \left(b^{xy} v^{z} - b^{xz} v^{y} + b^{yz} v^{x}\right) \boldsymbol{e_x} \wedge \boldsymbol{e_y} \wedge \boldsymbol{e_z}$$

$$b \cdot v = \left(b^{xy} v^{y} + b^{xz} v^{z}\right) \boldsymbol{e_x} + \left(- b^{xy} v^{x} + b^{yz} v^{z}\right) \boldsymbol{e_y} + \left(- b^{xz} v^{x} - b^{yz} v^{y}\right) \boldsymbol{e_z}$$

$$b\rfloor v = 0$$

$$b\lfloor v = \left(b^{xy} v^{y} + b^{xz} v^{z}\right) \boldsymbol{e_x} + \left(- b^{xy} v^{x} + b^{yz} v^{z}\right) \boldsymbol{e_y} + \left(- b^{xz} v^{x} - b^{yz} v^{y}\right) \boldsymbol{e_z}$$

$$bb = - \left(b^{xy}\right)^{2} - \left(b^{xz}\right)^{2} - \left(b^{yz}\right)^{2}$$

$$b \wedge b = 0$$

$$b \cdot b = - \left(b^{xy}\right)^{2} - \left(b^{xz}\right)^{2} - \left(b^{yz}\right)^{2}$$

$$b\rfloor b = - \left(b^{xy}\right)^{2} - \left(b^{xz}\right)^{2} - \left(b^{yz}\right)^{2}$$

$$b\lfloor b = - \left(b^{xy}\right)^{2} - \left(b^{xz}\right)^{2} - \left(b^{yz}\right)^{2}$$

General 2D Metric
Multivector Functions:

$$s(X) = s$$

$$v(X) = v^{x} \boldsymbol{e_x} + v^{y} \boldsymbol{e_y}$$

$$b(X) = v^{xy} \boldsymbol{e_x} \wedge \boldsymbol{e_y}$$

Products:

$$\boldsymbol{\nabla} s = \frac{- \left(e_{x} \cdot e_{y}\right) \partial_{y} s + \left(e_{y} \cdot e_{y}\right) \partial_{x} s}{\left(e_{x} \cdot e_{x}\right) \left(e_{y} \cdot e_{y}\right) - \left(e_{x} \cdot e_{y}\right)^{2}} \boldsymbol{e_x} + \frac{\left(e_{x} \cdot e_{x}\right) \partial_{y} s - \left(e_{x} \cdot e_{y}\right) \partial_{x} s}{\left(e_{x} \cdot e_{x}\right) \left(e_{y} \cdot e_{y}\right) - \left(e_{x} \cdot e_{y}\right)^{2}} \boldsymbol{e_y}$$

$$\boldsymbol{\nabla} \wedge s = \frac{- \left(e_{x} \cdot e_{y}\right) \partial_{y} s + \left(e_{y} \cdot e_{y}\right) \partial_{x} s}{\left(e_{x} \cdot e_{x}\right) \left(e_{y} \cdot e_{y}\right) - \left(e_{x} \cdot e_{y}\right)^{2}} \boldsymbol{e_x} + \frac{\left(e_{x} \cdot e_{x}\right) \partial_{y} s - \left(e_{x} \cdot e_{y}\right) \partial_{x} s}{\left(e_{x} \cdot e_{x}\right) \left(e_{y} \cdot e_{y}\right) - \left(e_{x} \cdot e_{y}\right)^{2}} \boldsymbol{e_y}$$

$$\boldsymbol{\nabla}\rfloor s = \frac{- \left(e_{x} \cdot e_{y}\right) \partial_{y} s + \left(e_{y} \cdot e_{y}\right) \partial_{x} s}{\left(e_{x} \cdot e_{x}\right) \left(e_{y} \cdot e_{y}\right) - \left(e_{x} \cdot e_{y}\right)^{2}} \boldsymbol{e_x} + \frac{\left(e_{x} \cdot e_{x}\right) \partial_{y} s - \left(e_{x} \cdot e_{y}\right) \partial_{x} s}{\left(e_{x} \cdot e_{x}\right) \left(e_{y} \cdot e_{y}\right) - \left(e_{x} \cdot e_{y}\right)^{2}} \boldsymbol{e_y}$$

$$\boldsymbol{\nabla}\lfloor s = \frac{- \left(e_{x} \cdot e_{y}\right) \partial_{y} s + \left(e_{y} \cdot e_{y}\right) \partial_{x} s}{\left(e_{x} \cdot e_{x}\right) \left(e_{y} \cdot e_{y}\right) - \left(e_{x} \cdot e_{y}\right)^{2}} \boldsymbol{e_x} + \frac{\left(e_{x} \cdot e_{x}\right) \partial_{y} s - \left(e_{x} \cdot e_{y}\right) \partial_{x} s}{\left(e_{x} \cdot e_{x}\right) \left(e_{y} \cdot e_{y}\right) - \left(e_{x} \cdot e_{y}\right)^{2}} \boldsymbol{e_y}$$

$$\boldsymbol{\nabla} v = \left(\partial_{x} v^{x} + \partial_{y} v^{y}\right) + \frac{- \left(e_{x} \cdot e_{x}\right) \partial_{y} v^{x} + \left(e_{x} \cdot e_{y}\right) \partial_{x} v^{x} - \left(e_{x} \cdot e_{y}\right) \partial_{y} v^{y} + \left(e_{y} \cdot e_{y}\right) \partial_{x} v^{y}}{\left(e_{x} \cdot e_{x}\right) \left(e_{y} \cdot e_{y}\right) - \left(e_{x} \cdot e_{y}\right)^{2}} \boldsymbol{e_x} \wedge \boldsymbol{e_y}$$

$$\boldsymbol{\nabla} \wedge v = \frac{- \left(e_{x} \cdot e_{x}\right) \partial_{y} v^{x} + \left(e_{x} \cdot e_{y}\right) \partial_{x} v^{x} - \left(e_{x} \cdot e_{y}\right) \partial_{y} v^{y} + \left(e_{y} \cdot e_{y}\right) \partial_{x} v^{y}}{\left(e_{x} \cdot e_{x}\right) \left(e_{y} \cdot e_{y}\right) - \left(e_{x} \cdot e_{y}\right)^{2}} \boldsymbol{e_x} \wedge \boldsymbol{e_y}$$

$$\boldsymbol{\nabla} \cdot v = \partial_{x} v^{x} + \partial_{y} v^{y}$$

$$\boldsymbol{\nabla}\rfloor v = \partial_{x} v^{x} + \partial_{y} v^{y}$$

$$\boldsymbol{\nabla}\lfloor v = \partial_x v^x + \partial_y v^y$$

$$s\boldsymbol{\nabla} = \left(\frac{(e_y\cdot e_y)\, s}{(e_x\cdot e_x)\,(e_y\cdot e_y) - (e_x\cdot e_y)^2}\boldsymbol{e_x} - \frac{(e_x\cdot e_y)\, s}{(e_x\cdot e_x)\,(e_y\cdot e_y) - (e_x\cdot e_y)^2}\boldsymbol{e_y} \right)\frac{\partial}{\partial x} + \left(-\frac{(e_x\cdot e_y)\, s}{(e_x\cdot e_x)\,(e_y\cdot e_y) - (e_x\cdot e_y)^2}\boldsymbol{e_x} + \frac{(e_x\cdot e_x)\, s}{(e_x\cdot e_x)\,(e_y\cdot e_y) - (e_x\cdot e_y)^2}\boldsymbol{e_y} \right)\frac{\partial}{\partial y}$$

$$s\wedge\boldsymbol{\nabla} = \left(\frac{(e_y\cdot e_y)\, s}{(e_x\cdot e_x)\,(e_y\cdot e_y) - (e_x\cdot e_y)^2}\boldsymbol{e_x} - \frac{(e_x\cdot e_y)\, s}{(e_x\cdot e_x)\,(e_y\cdot e_y) - (e_x\cdot e_y)^2}\boldsymbol{e_y} \right)\frac{\partial}{\partial x} + \left(-\frac{(e_x\cdot e_y)\, s}{(e_x\cdot e_x)\,(e_y\cdot e_y) - (e_x\cdot e_y)^2}\boldsymbol{e_x} + \frac{(e_x\cdot e_x)\, s}{(e_x\cdot e_x)\,(e_y\cdot e_y) - (e_x\cdot e_y)^2}\boldsymbol{e_y} \right)\frac{\partial}{\partial y}$$

$$s\rfloor\boldsymbol{\nabla} = \left(\frac{(e_y\cdot e_y)\, s}{(e_x\cdot e_x)\,(e_y\cdot e_y) - (e_x\cdot e_y)^2}\boldsymbol{e_x} - \frac{(e_x\cdot e_y)\, s}{(e_x\cdot e_x)\,(e_y\cdot e_y) - (e_x\cdot e_y)^2}\boldsymbol{e_y} \right)\frac{\partial}{\partial x} + \left(-\frac{(e_x\cdot e_y)\, s}{(e_x\cdot e_x)\,(e_y\cdot e_y) - (e_x\cdot e_y)^2}\boldsymbol{e_x} + \frac{(e_x\cdot e_x)\, s}{(e_x\cdot e_x)\,(e_y\cdot e_y) - (e_x\cdot e_y)^2}\boldsymbol{e_y} \right)\frac{\partial}{\partial y}$$

$$s\lfloor\boldsymbol{\nabla} = \left(\frac{(e_y\cdot e_y)\, s}{(e_x\cdot e_x)\,(e_y\cdot e_y) - (e_x\cdot e_y)^2}\boldsymbol{e_x} - \frac{(e_x\cdot e_y)\, s}{(e_x\cdot e_x)\,(e_y\cdot e_y) - (e_x\cdot e_y)^2}\boldsymbol{e_y} \right)\frac{\partial}{\partial x} + \left(-\frac{(e_x\cdot e_y)\, s}{(e_x\cdot e_x)\,(e_y\cdot e_y) - (e_x\cdot e_y)^2}\boldsymbol{e_x} + \frac{(e_x\cdot e_x)\, s}{(e_x\cdot e_x)\,(e_y\cdot e_y) - (e_x\cdot e_y)^2}\boldsymbol{e_y} \right)\frac{\partial}{\partial y}$$

$$ss = s^2$$

$$s\wedge s = s^2$$

$$s\rfloor s = s^2$$

$$s\lfloor s = s^2$$

$$sv = sv^x\boldsymbol{e_x} + sv^y\boldsymbol{e_y}$$

$$s\wedge v = sv^x\boldsymbol{e_x} + sv^y\boldsymbol{e_y}$$

$$s\rfloor v = sv^x\boldsymbol{e_x} + sv^y\boldsymbol{e_y}$$

$$s\lfloor v = sv^x\boldsymbol{e_x} + sv^y\boldsymbol{e_y}$$

$$v\boldsymbol{\nabla} = \left(v^x - \frac{(e_x\cdot e_y)\, v^x + (e_y\cdot e_y)\, v^y}{(e_x\cdot e_x)\,(e_y\cdot e_y) - (e_x\cdot e_y)^2}\boldsymbol{e_x}\wedge\boldsymbol{e_y} \right)\frac{\partial}{\partial x} + \left(v^y + \frac{(e_x\cdot e_x)\, v^x + (e_x\cdot e_y)\, v^y}{(e_x\cdot e_x)\,(e_y\cdot e_y) - (e_x\cdot e_y)^2}\boldsymbol{e_x}\wedge\boldsymbol{e_y} \right)\frac{\partial}{\partial y}$$

$$v\wedge\boldsymbol{\nabla} = -\frac{(e_x\cdot e_y)\, v^x + (e_y\cdot e_y)\, v^y}{(e_x\cdot e_x)\,(e_y\cdot e_y) - (e_x\cdot e_y)^2}\boldsymbol{e_x}\wedge\boldsymbol{e_y}\frac{\partial}{\partial x} + \frac{(e_x\cdot e_x)\, v^x + (e_x\cdot e_y)\, v^y}{(e_x\cdot e_x)\,(e_y\cdot e_y) - (e_x\cdot e_y)^2}\boldsymbol{e_x}\wedge\boldsymbol{e_y}\frac{\partial}{\partial y}$$

$$v\cdot\boldsymbol{\nabla} = v^x\frac{\partial}{\partial x} + v^y\frac{\partial}{\partial y}$$

$$v\rfloor\boldsymbol{\nabla} = v^x\frac{\partial}{\partial x} + v^y\frac{\partial}{\partial y}$$

$$v\lfloor\boldsymbol{\nabla} = v^x\frac{\partial}{\partial x} + v^y\frac{\partial}{\partial y}$$

$$vs = sv^x\boldsymbol{e_x} + sv^y\boldsymbol{e_y}$$

$$v\wedge s = sv^x\boldsymbol{e_x} + sv^y\boldsymbol{e_y}$$

$$v\rfloor s = sv^x\boldsymbol{e_x} + sv^y\boldsymbol{e_y}$$

$$v\lfloor s = sv^x\boldsymbol{e_x} + sv^y\boldsymbol{e_y}$$

$$vv = (e_x\cdot e_x)\,(v^x)^2 + 2\,(e_x\cdot e_y)\,v^xv^y + (e_y\cdot e_y)\,(v^y)^2$$

$$v \wedge v = 0$$

$$v \cdot v = (e_x \cdot e_x)\,(v^x)^2 + 2\,(e_x \cdot e_y)\,v^x v^y + (e_y \cdot e_y)\,(v^y)^2$$

$$v \rfloor v = (e_x \cdot e_x)\,(v^x)^2 + 2\,(e_x \cdot e_y)\,v^x v^y + (e_y \cdot e_y)\,(v^y)^2$$

$$v \lfloor v = (e_x \cdot e_x)\,(v^x)^2 + 2\,(e_x \cdot e_y)\,v^x v^y + (e_y \cdot e_y)\,(v^y)^2$$