```python
def derivatives_in_spherical_coordinates():
    Print_Function()
    coords = (r,th,phi) = symbols('r theta phi', real=True)
    (sp3d,er,eth,ephi) = Ga.build('e_r e_theta e_phi',g=[1,r**2,r**2*sin(th)**2],coords=coords)
    grad = sp3d.grad
    f = sp3d.mv('f','scalar',f=True)
    A = sp3d.mv('A','vector',f=True)
    B = sp3d.mv('B','bivector',f=True)
    print 'f =',f
    print 'A =',A
    print 'B =',B
    print 'grad*f =',grad*f
    print 'grad|A =',grad|A
    print 'grad\\times A = -I*(grad^A) =',-sp3d.i*(grad^A)
    print '%\\nabla^{2}f =',grad|(grad*f)
    print 'grad^B =',grad^B
    """
    print '( \\nabla\\W\\nabla )\\bm{e}_{r} =',((grad^grad)*er).trigsimp()
    print '( \\nabla\\W\\nabla )\\bm{e}_{\\theta} =',((grad^grad)*eth).trigsimp()
    print '( \\nabla\\W\\nabla )\\bm{e}_{\\phi} =',((grad^grad)*ephi).trigsimp()
    """
    return
```

Code Output:

$$f = f$$

$$A = A^{r}\boldsymbol{e_{r}} + A^{\theta}\boldsymbol{e_{\theta}} + A^{\phi}\boldsymbol{e_{\phi}}$$

$$B = B^{r\theta}\boldsymbol{e_{r}}\wedge\boldsymbol{e_{\theta}} + B^{r\phi}\boldsymbol{e_{r}}\wedge\boldsymbol{e_{\phi}} + B^{\phi\phi}\boldsymbol{e_{\theta}}\wedge\boldsymbol{e_{\phi}}$$

$$\boldsymbol{\nabla} f = \partial_{r} f\boldsymbol{e_{r}} + \frac{1}{r^{2}}\partial_{\theta} f\boldsymbol{e_{\theta}} + \frac{\partial_{\phi} f}{r^{2}\sin^{2}{(\theta)}}\boldsymbol{e_{\phi}}$$

$$\boldsymbol{\nabla}\cdot A = \frac{A^{\theta}}{\tan{(\theta)}} + \partial_{\phi} A^{\phi} + \partial_{r} A^{r} + \partial_{\theta} A^{\theta} + \frac{2A^{r}}{r}$$

$$\boldsymbol{\nabla}\times A = -I(\boldsymbol{\nabla}\wedge A) = r^{2}\left(\left(-\frac{1}{2}\cos{(2\theta)} + \frac{1}{2}\right)\partial_{\theta} A^{\phi} + A^{\phi}\sin{(2\theta)} - \partial_{\phi} A^{\theta}\right)\boldsymbol{e_{r}} + \left(\left(-r^{2}\partial_{r} A^{\phi} - 2rA^{\phi}\right)\sin^{2}{(\theta)} + \partial_{\phi} A^{r}\right)\boldsymbol{e_{\theta}} + \left(r^{2}\partial_{r} A^{\theta} + 2rA^{\theta} - \partial_{\theta} A^{r}\right)\boldsymbol{e_{\phi}}$$

$$\nabla^{2} f = \frac{1}{r^{2}}\left(r^{2}\partial^{2}_{r} f + 2r\partial_{r} f + \partial^{2}_{\theta} f + \frac{\partial_{\theta} f}{\tan{(\theta)}} + \frac{\partial^{2}_{\phi} f}{\sin^{2}{(\theta)}}\right)$$

$$\boldsymbol{\nabla}\wedge B = \frac{1}{r^{2}}\left(r^{2}\partial_{r} B^{\phi\phi} + 4rB^{\phi\phi} - \frac{2B^{r\phi}}{\tan{(\theta)}} - \partial_{\theta} B^{r\phi} + \frac{\partial_{\phi} B^{r\theta}}{\sin^{2}{(\theta)}}\right)\boldsymbol{e_{r}}\wedge\boldsymbol{e_{\theta}}\wedge\boldsymbol{e_{\phi}}$$

```python
def derivatives_in_paraboloidal_coordinates():
    Print_Function()
    coords = (u,v,phi) = symbols('u v phi', real=True)
    (par3d,er,eth,ephi) = Ga.build('e_u e_v e_phi',X=[u*v*cos(phi),u*v*sin(phi),(u**2-v**2)/2],coords=coords,norm=True)
    grad = par3d.grad
    f = par3d.mv('f','scalar',f=True)
    A = par3d.mv('A','vector',f=True)
    B = par3d.mv('B','bivector',f=True)
    print '#Derivatives in Paraboloidal Coordinates'
    print 'f =',f
    print 'A =',A
    print 'B =',B
    print 'grad*f =',grad*f
    print 'grad|A =',grad|A
    (-par3d.i*(grad^A)).Fmt(3,'grad\\times A = -I*(grad^A)')
    print 'grad^B =',grad^B
    return
```

Code Output: Derivatives in Paraboloidal Coordinates

$$f = f$$

$$A = A^{u}\boldsymbol{e_{u}} + A^{v}\boldsymbol{e_{v}} + A^{\phi}\boldsymbol{e_{\phi}}$$

$$B = B^{uv}\boldsymbol{e_{u}}\wedge\boldsymbol{e_{v}} + B^{u\phi}\boldsymbol{e_{u}}\wedge\boldsymbol{e_{\phi}} + B^{v\phi}\boldsymbol{e_{v}}\wedge\boldsymbol{e_{\phi}}$$

$$\boldsymbol{\nabla} f = \frac{\partial_{u} f}{\sqrt{u^{2} + v^{2}}}\boldsymbol{e_{u}} + \frac{\partial_{v} f}{\sqrt{u^{2} + v^{2}}}\boldsymbol{e_{v}} + \frac{\partial_{\phi} f}{uv}\boldsymbol{e_{\phi}}$$

$$\boldsymbol{\nabla}\cdot A = \left(\frac{u}{\left(u^{2} + v^{2}\right)^{\frac{3}{2}}} + \frac{1}{u\sqrt{u^{2} + v^{2}}}\right)A^{u} + \left(\frac{v}{\left(u^{2} + v^{2}\right)^{\frac{3}{2}}} + \frac{1}{v\sqrt{u^{2} + v^{2}}}\right)A^{v} + \frac{\partial_{u} A^{u}}{\sqrt{u^{2} + v^{2}}} + \frac{\partial_{v} A^{v}}{\sqrt{u^{2} + v^{2}}} + \frac{\partial_{\phi} A^{\phi}}{uv}$$

$$\begin{aligned}\boldsymbol{\nabla}\times A = -I(\boldsymbol{\nabla}\wedge A) = & \frac{1}{uv\left(u^{2} + v^{2}\right)}\left(uv\sqrt{u^{2} + v^{2}}\partial_{v} A^{\phi} + u\sqrt{u^{2} + v^{2}}A^{\phi} + \left(-u^{2} - v^{2}\right)\partial_{\phi} A^{v}\right)\boldsymbol{e_{u}} \\ & + \frac{1}{uv\left(u^{2} + v^{2}\right)}\left(-uv\sqrt{u^{2} + v^{2}}\partial_{u} A^{\phi} - v\sqrt{u^{2} + v^{2}}A^{\phi} + \left(u^{2} + v^{2}\right)\partial_{\phi} A^{u}\right)\boldsymbol{e_{v}} \\ & + \frac{1}{\left(u^{2} + v^{2}\right)^{\frac{3}{2}}}\left(uA^{v} - vA^{u} + \left(u^{2} + v^{2}\right)\left(-\partial_{v} A^{u} + \partial_{u} A^{v}\right)\right)\boldsymbol{e_{\phi}}\end{aligned}$$

$$\boldsymbol{\nabla}\wedge B = \left(\left(\frac{u}{\left(u^{2} + v^{2}\right)^{\frac{3}{2}}} + \frac{1}{u\sqrt{u^{2} + v^{2}}}\right)B^{v\phi} + \left(-\frac{v}{\left(u^{2} + v^{2}\right)^{\frac{3}{2}}} - \frac{1}{v\sqrt{u^{2} + v^{2}}}\right)B^{u\phi} - \frac{\partial_{v} B^{u\phi}}{\sqrt{u^{2} + v^{2}}} + \frac{\partial_{u} B^{v\phi}}{\sqrt{u^{2} + v^{2}}} + \frac{\partial_{\phi} B^{uv}}{uv}\right)\boldsymbol{e_{u}}\wedge\boldsymbol{e_{v}}\wedge\boldsymbol{e_{\phi}}$$

```python
def derivatives_in_elliptic_cylindrical_coordinates():
    Print_Function()
    a = symbols('a', real=True)
    coords = (u,v,z) = symbols('u v z', real=True)
    (elip3d,er,eth,ephi) = Ga.build('e_u e_v e_z',X=[a*cosh(u)*cos(v),a*sinh(u)*sin(v),z],coords=coords,norm=True)
    grad = elip3d.grad
    f = elip3d.mv('f','scalar',f=True)
    A = elip3d.mv('A','vector',f=True)
    B = elip3d.mv('B','bivector',f=True)
    print 'f =',f
    print 'A =',A
    print 'B =',B
    print 'grad*f =',grad*f
    print 'grad|A =',grad|A
    print '-I*(grad^A) =',-elip3d.i*(grad^A)
    print 'grad^B =',grad^B
    return
```

Code Output:

$$f = f$$

$$A = A^{u}\boldsymbol{e_{u}} + A^{v}\boldsymbol{e_{v}} + A^{z}\boldsymbol{e_{z}}$$

$$B = B^{uv}\boldsymbol{e_{u}}\wedge\boldsymbol{e_{v}} + B^{uz}\boldsymbol{e_{u}}\wedge\boldsymbol{e_{z}} + B^{vz}\boldsymbol{e_{v}}\wedge\boldsymbol{e_{z}}$$

$$\boldsymbol{\nabla} f = \frac{\partial_{u} f}{\sqrt{\sin^{2}(v)\cosh^{2}(u) + \cos^{2}(v)\sinh^{2}(u)}\,|a|}\boldsymbol{e_{u}} + \frac{\partial_{v} f}{\sqrt{\sin^{2}(v)\cosh^{2}(u) + \cos^{2}(v)\sinh^{2}(u)}\,|a|}\boldsymbol{e_{v}} + \partial_{z} f\boldsymbol{e_{z}}$$

$$\boldsymbol{\nabla}\cdot A = \frac{|a|}{2\left(\left(\cos^{2}(v)\cos(2v) - \cos^{2}(v)\cosh(2u)\right)\sinh^{2}(u) + \left(\cos(2v)\cosh^{2}(u) - \cosh^{2}(u)\cosh(2u)\right)\sin^{2}(v)\right)\left(\sin^{2}(v)\cosh^{2}(u) + \cos^{2}(v)\sinh^{2}(u)\right)^{\frac{5}{2}}}\Big(\left(A^{u}\sinh(2u) + A^{v}\sin(2v)\right)\left(\sin^{2}(v)\cosh^{2}(u) + \cos^{2}(v)\sinh^{2}(u)\right)\left(\sin^{2}(v)\cos(2v)\cosh^{2}(u) - \sin^{2}(v)\cosh^{2}(u)\cosh(2u) + \cos^{2}(v)\cos(2v)\sinh^{2}(u) - \cos^{2}(v)\sinh^{2}(u)\cosh(2$$

$$-I(\boldsymbol{\nabla}\wedge A) = \left(-\partial_{z} A^{v} + \frac{\partial_{v} A^{z}}{\sqrt{\sin^{2}(v) + \sinh^{2}(u)}\,|a|}\right)\boldsymbol{e_{u}} + \left(\partial_{z} A^{u} - \frac{\partial_{u} A^{z}}{\sqrt{\sin^{2}(v) + \sinh^{2}(u)}\,|a|}\right)\boldsymbol{e_{v}} - \frac{\sqrt{2}\,|a|}{2\left(-\cos(2v) + \cosh(2u)\right)^{\frac{5}{2}}}\Big(\left(-2\left(\cosh(2u) - 1\right)^{2} + 2\right)\partial_{u} A^{v} + \left(2\left(\cosh(2u) - 1\right)^{2} - 2\right)\partial_{v} A^{u} + \left(2\partial_{v} A^{u} - 2\partial_{u} A^{v}\right)\cos^{2}(2v) + \left(2A^{u}\sin(2v) - 4\cos(2v)\partial_{v} A^{u} + 4\cos(2v)\partial_{u} A^{v} + 4\partial_{v} A^{u} - 4\partial_{u} A^{v}\right)\cosh(2u) - A^{u}\sin(4v) + 2A^{v}\cos(2$$

$$\boldsymbol{\nabla}\wedge B = -\frac{|a|}{2\left(\left(\cos^{2}(v)\cos(2v) - \cos^{2}(v)\cosh(2u)\right)\sinh^{2}(u) + \left(\cos(2v)\cosh^{2}(u) - \cosh^{2}(u)\cosh(2u)\right)\sin^{2}(v)\right)\left(\sin^{2}(v)\cosh^{2}(u) + \cos^{2}(v)\sinh^{2}(u)\right)^{\frac{5}{2}}}\Big(\left(B^{uz}\sin(2v) - B^{vz}\sinh(2u)\right)\left(\sin^{2}(v)\cosh^{2}(u) + \cos^{2}(v)\sinh^{2}(u)\right)\left(\sin^{2}(v)\cos(2v)\cosh^{2}(u) - \sin^{2}(v)\cosh^{2}(u)\cosh(2u) + \cos^{2}(v)\cos(2v)\sinh^{2}(u) - \cos^{2}(v)\sinh^{2}(u)\cosh$$

```python
def derivatives_in_prolate_spheroidal_coordinates():
    Print_Function()
    a = symbols('a', real=True)
    coords = (xi,eta,phi) = symbols('xi eta phi', real=True)
    (ps3d,er,eth,ephi) = Ga.build('e_xi e_eta e_phi',X=[a*sinh(xi)*sin(eta)*cos(phi),a*sinh(xi)*sin(eta)*sin(phi),
                                                         a*cosh(xi)*cos(eta)],coords=coords,norm=True)
    grad = ps3d.grad
    f = ps3d.mv('f','scalar',f=True)
    A = ps3d.mv('A','vector',f=True)
    B = ps3d.mv('B','bivector',f=True)
    print 'f =',f
    print 'A =',A
    print 'B =',B
    print 'grad*f =',grad*f
    print 'grad|A =',grad|A
    (-ps3d.i*(grad^A)).Fmt(3,'-I*(grad^A)')
    (grad^B).Fmt(3,'grad^B')
    return
```

Code Output:

$$f = f$$

$$A = A^{\xi}\boldsymbol{e_{\xi}} + A^{\eta}\boldsymbol{e_{\eta}} + A^{\phi}\boldsymbol{e_{\phi}}$$

$$B = B^{\xi\xi}\boldsymbol{e_{\xi}}\wedge\boldsymbol{e_{\eta}} + B^{\xi\xi}\boldsymbol{e_{\xi}}\wedge\boldsymbol{e_{\phi}} + B^{\phi\phi}\boldsymbol{e_{\eta}}\wedge\boldsymbol{e_{\phi}}$$

$$\boldsymbol{\nabla} f = \frac{\partial_{\xi} f}{\sqrt{\sin^{2}(\eta) + \sinh^{2}(\xi)}\,|a|}\boldsymbol{e_{\xi}} + \frac{\partial_{\eta} f}{\sqrt{\sin^{2}(\eta) + \sinh^{2}(\xi)}\,|a|}\boldsymbol{e_{\eta}} + \frac{\partial_{\phi} f}{a\sin(\eta)\sinh(\xi)}\boldsymbol{e_{\phi}}$$

$$\boldsymbol{\nabla}\cdot A = \frac{1}{a^{2}\left(\sin^{2}(\eta) + \sinh^{2}(\xi)\right)^{3}\sin(\eta)\sinh(\xi)}\left(a\left(\sin^{2}(\eta) + \sinh^{2}(\xi)\right)^{3}\partial_{\phi}A^{\phi} + \left(\sin^{2}(\eta) + \sinh^{2}(\xi)\right)^{\frac{5}{2}}A^{\eta}\cos(\eta)\sinh(\xi)\,|a| + \left(\frac{1}{2}\left(A^{\eta}\sin(2\eta) + A^{\xi}\sinh(2\xi)\right)\left(\sin^{2}(\eta) + \sinh^{2}(\xi)\right)^{\frac{3}{2}}\sin(\eta)\sinh(\xi) + \left(\sin^{2}(\eta) + \sinh^{2}(\xi)\right)^{\frac{5}{2}}\left(\partial_{\eta}A^{\eta} + \partial_{\xi}A^{\xi}\right)\sin(\eta)\sinh(\xi) + \left(\sin^{2}(\eta) + \sinh^{2}(\xi)\right)^{\frac{5}{2}}A^{\xi}\sin(\eta)\cosh(\xi)\right)|a|\right)$$

$$\begin{aligned}
-I(\boldsymbol{\nabla}\wedge A) = & \frac{1}{a^{2}\sin(\eta)}\left(-\frac{a\partial_{\phi}A^{\eta}}{\sinh(\xi)} + \left(\frac{A^{\phi}\cos(\eta)}{\sqrt{\sin^{2}(\eta) + \sinh^{2}(\xi)}} + \frac{\sin(\eta)\partial_{\eta}A^{\phi}}{\sqrt{\sin^{2}(\eta) + \sinh^{2}(\xi)}}\right)|a|\right)\boldsymbol{e_{\xi}} \\
& - \frac{1}{a^{2}\sinh(\xi)}\left(-\frac{a\partial_{\phi}A^{\xi}}{\sin(\eta)} + \left(\frac{A^{\phi}\cosh(\xi)}{\sqrt{\sin^{2}(\eta) + \sinh^{2}(\xi)}} + \frac{\sinh(\xi)\partial_{\xi}A^{\phi}}{\sqrt{\sin^{2}(\eta) + \sinh^{2}(\xi)}}\right)|a|\right)\boldsymbol{e_{\eta}} \\
& + \frac{1}{2\left(\sin^{2}(\eta) + \sinh^{2}(\xi)\right)^{\frac{3}{2}}|a|}\left(\left(\sin^{2}(\eta) + \sinh^{2}(\xi)\right)\left(2\partial_{\xi}A^{\eta} - 2\partial_{\eta}A^{\xi}\right) + A^{\eta}\sinh(2\xi) - A^{\xi}\sin(2\eta)\right)\boldsymbol{e_{\phi}}
\end{aligned}$$

$$\boldsymbol{\nabla}\wedge B = \frac{1}{a^{2}\left(\sin^{2}(\eta) + \sinh^{2}(\xi)\right)^{3}\sin(\eta)\sinh(\xi)}\left(a\left(\sin^{2}(\eta) + \sinh^{2}(\xi)\right)^{3}\partial_{\phi}B^{\xi\xi} + \left(\frac{1}{2}\left(B^{\phi\phi}\sinh(2\xi) - B^{\xi\xi}\sin(2\eta)\right)\left(\sin^{2}(\eta) + \sinh^{2}(\xi)\right)^{\frac{3}{2}}\sin(\eta)\sinh(\xi) + \left(\sin^{2}(\eta) + \sinh^{2}(\xi)\right)^{\frac{5}{2}}\left(\partial_{\xi}B^{\phi\phi} - \partial_{\eta}B^{\xi\xi}\right)\sin(\eta)\sinh(\xi) + \left(\sin^{2}(\eta) + \sinh^{2}(\xi)\right)^{\frac{5}{2}}B^{\phi\phi}\sin(\eta)\cosh(\xi) - \left(\sin^{2}(\eta) + \sinh^{2}(\xi)\right)^{\frac{5}{2}}B^{\xi\xi}\cos(\eta)\sinh(\xi)\right)|a|\right)\boldsymbol{e_{\xi}}\wedge\boldsymbol{e_{\eta}}\wedge\boldsymbol{e_{\phi}}$$