

Pseudo Scalar $I = \gamma_t \wedge \gamma_x \wedge \gamma_y \wedge \gamma_z$

$$I_{xyz} = \gamma_x \wedge \gamma_y \wedge \gamma_z$$

Electromagnetic Field Bi-Vector $F = -E^x e^{-i(-\omega t + k_x x + k_y y + k_z z)} \gamma_t \wedge \gamma_x$
 $-E^y e^{-i(-\omega t + k_x x + k_y y + k_z z)} \gamma_t \wedge \gamma_y$
 $-E^z e^{-i(-\omega t + k_x x + k_y y + k_z z)} \gamma_t \wedge \gamma_z$
 $-B^z e^{-i(-\omega t + k_x x + k_y y + k_z z)} \gamma_x \wedge \gamma_y$
 $+B^y e^{i(\omega t - k_x x - k_y y - k_z z)} \gamma_x \wedge \gamma_z$
 $-B^x e^{-i(-\omega t + k_x x + k_y y + k_z z)} \gamma_y \wedge \gamma_z$

Geom Derivative of Electromagnetic Field Bi-Vector

$$\begin{aligned} \nabla F = 0 = & -i(E^x k_x + E^y k_y + E^z k_z) e^{-i(-\omega t + k_x x + k_y y + k_z z)} \gamma_t \\ & + i(B^y k_z - B^z k_y - E^x \omega) e^{i(\omega t - k_x x - k_y y - k_z z)} \gamma_x \\ & + i(-B^x k_z + B^z k_x - E^y \omega) e^{i(\omega t - k_x x - k_y y - k_z z)} \gamma_y \\ & + i(B^x k_y - B^y k_x - E^z \omega) e^{i(\omega t - k_x x - k_y y - k_z z)} \gamma_z \\ & + i(-B^z \omega - E^x k_y + E^y k_x) e^{i(\omega t - k_x x - k_y y - k_z z)} \gamma_t \wedge \gamma_x \wedge \gamma_y \\ & + i(B^y \omega - E^x k_z + E^z k_x) e^{i(\omega t - k_x x - k_y y - k_z z)} \gamma_t \wedge \gamma_x \wedge \gamma_z \\ & + i(-B^x \omega - E^y k_z + E^z k_y) e^{i(\omega t - k_x x - k_y y - k_z z)} \gamma_t \wedge \gamma_y \wedge \gamma_z \\ & - i(B^x k_x + B^y k_y + B^z k_z) e^{-i(-\omega t + k_x x + k_y y + k_z z)} \gamma_x \wedge \gamma_y \wedge \gamma_z \end{aligned}$$

$$\begin{aligned} (\nabla F) / (ie^{iK \cdot X}) = 0 = & (-E^x k_x - E^y k_y - E^z k_z) \gamma_t \\ & + (B^y k_z - B^z k_y - E^x \omega) \gamma_x \\ & + (-B^x k_z + B^z k_x - E^y \omega) \gamma_y \\ & + (B^x k_y - B^y k_x - E^z \omega) \gamma_z \\ & + (-B^z \omega - E^x k_y + E^y k_x) \gamma_t \wedge \gamma_x \wedge \gamma_y \\ & + (B^y \omega - E^x k_z + E^z k_x) \gamma_t \wedge \gamma_x \wedge \gamma_z \\ & + (-B^x \omega - E^y k_z + E^z k_y) \gamma_t \wedge \gamma_y \wedge \gamma_z \\ & + (-B^x k_x - B^y k_y - B^z k_z) \gamma_x \wedge \gamma_y \wedge \gamma_z \end{aligned}$$

set $e_E \cdot e_k = e_B \cdot e_k = 0$ and $e_E \cdot e_E = e_B \cdot e_B = e_k \cdot e_k = -e_t \cdot e_t = 1$

$$g = \begin{bmatrix} 1 & (e_E \cdot e_B) & 0 & 0 \\ (e_E \cdot e_B) & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$K \cdot X = -\omega t + kx_k$$

$$F = -B e^{-i(\omega t - kx_k)} e_E \wedge e_k + E e^{i(-\omega t + kx_k)} e_E \wedge t + (e_E \cdot e_B) B e^{i(-\omega t + kx_k)} e_B \wedge e_k$$

$$\begin{aligned} \nabla F = 0 = & i(Bk + E\omega) e^{i(-\omega t + kx_k)} e_E \\ & - i(e_E \cdot e_B) B k e^{-i(\omega t - kx_k)} e_B \\ & - i(B\omega + Ek) e^{-i(\omega t - kx_k)} e_E \wedge e_k \wedge t \\ & + i(e_E \cdot e_B) B \omega e^{i(-\omega t + kx_k)} e_B \wedge e_k \wedge t \end{aligned}$$

$$\begin{aligned} (\nabla F) / (ie^{iK \cdot X}) = 0 = & (Bk + E\omega) e_E \\ & - (e_E \cdot e_B) B k e_B \\ & + (-B\omega - Ek) e_E \wedge e_k \wedge t \\ & + (e_E \cdot e_B) B \omega e_B \wedge e_k \wedge t \end{aligned}$$

Previous equation requires that: $e_E \cdot e_B = 0$ if $B \neq 0$ and $k \neq 0$

$$\begin{aligned} (\nabla F) / \left(i e^{i K \cdot X} \right) = 0 = & (B k + E \omega) e_E \\ & + (-B \omega - E k) e_E \wedge e_k \wedge t \end{aligned}$$

$$\text{eq1: } B = -\frac{E \omega}{k}$$

$$\text{eq2: } B = -\frac{E k}{\omega}$$

$$\text{eq3 = eq1-eq2: } 0 = -\frac{E \omega}{k} + \frac{E k}{\omega}$$

$$\text{eq3 = (eq1-eq2)/E: } 0 = -\frac{\omega}{k} + \frac{k}{\omega}$$

$$k = \begin{bmatrix} -\omega \\ \omega \end{bmatrix}$$

$$B = \begin{bmatrix} -E \\ E \end{bmatrix}$$