Corona Virus Math: For the Cynics and Sceptics

-Sounder Rajendran | 22/03/2020 | sounderajendran@gmail.com

"The unexamined life is not worth living."
― **Socrates**

Greetings, fellow cynics and sceptics; according to Socrates, the only true wisdom is acknowledging that we know nothing of anything. And thus here we are, digging deep into why we are asked to wash our hands and #selfQuarantine ourselves at the wake of a mass extinction crisis.

We will (hopefully) try to understand the math, science and the logic behind what we're doing whatever we're trying to do and why we're doing them the way that we're trying to.

To make some sense out of the math and leave out the hard stuff, let's assume some boundaries on our theory. We'll consider four types of people on the general;

- S(t) : The susceptible
- I(t) : The infected
- R(t) : The recovered
- D(t) : The succumbed

Now, we'll define what these terms mean.

- Total number of previously unaffected people that are exposed to the COVID-19 virus at a given time 't' is given by S(t)
- Total number of people infected by the COVID-19 (subset of previously susceptible) at the given time 't' is given by I(t)
- Total number of people recovered from COVID-19 (subset of previously infected) at time 't' is given by R(t)
- Total number of people dead due to COVID-19 at the given time 't' is given by D(t)

The flow of numbers from S(t) to other stages can be represented as follows:

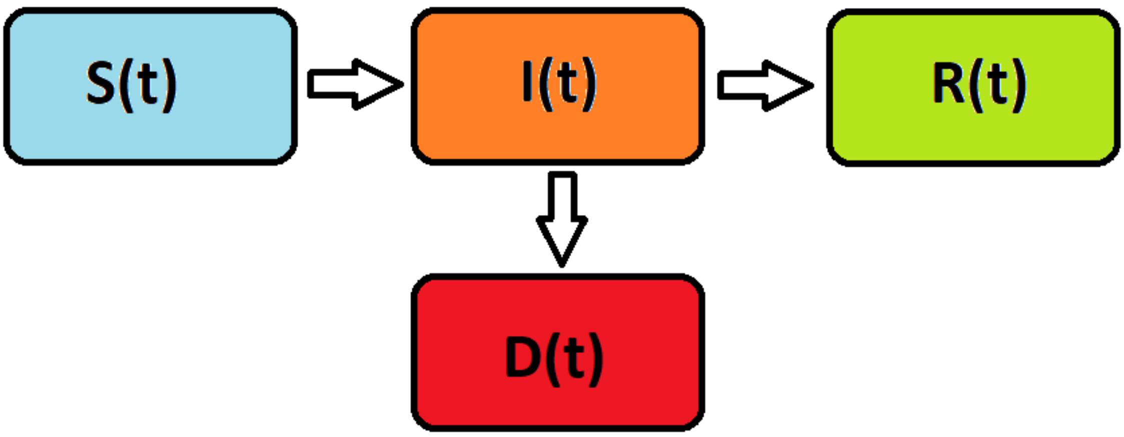

Figure 1: Flow of cases between stages

Credits to "*Learning Scientific Programming with Python*" with Christian Hill for SIR model reference

By definition we find that S(t) is the superset of the other three terms' cases. Let's now introduce a population N as the superset of S(t) at any given time (and is equal to S(t) during time t=0)

Thus:

S(t) + I(t) + R(t) + D(t) = N (eventually)

Let's look at a time at and near the initial outbreak t=0.

$$S(0) = S_0 = N \quad | \quad I(0) = I_0 = 0 \quad | \quad R(0) = R_0 = 0 \quad | \quad D(0) = D_0 = 0$$

(This means that the entire population is susceptible to the virus at t=0, and there are no infected people, no recovered people and no deaths due to COVID-19 at the initial time of outbreak. S(0) = N and S(t) ≠ 0 (at all times) as people get immunity once they are infected and recover)

For now, let's concentrate on how to bring all people to R(t) and remove focus from D(t).

Thus,

$$S'(t) = -a(SI) \quad | \quad I'(t) = a\,(SI) - bI \quad | \quad R'(t) = bI$$

$$\text{as } (\, dS/dt = S'(t) \quad | \quad dI/dt = I'(t) \quad | \quad dR/dt = R'(t)\,)$$

(This, by definition means that rate of change of the Susceptibles equals to a negative of constant 'a' times the product of S and I (**a = rate of transmission**); Thus when the infected come in contact with the susceptibles they become infected depending on factor 'a'.

Rate of change of recovered equals the product of constant 'b' and I (**b = rate of recovery**)

Thus we can put together the rate of change of Infected as the difference between those who were infected –S(t) and those who have recovered R(t))

We shall build a model from the above equations that gives graphs as follows:

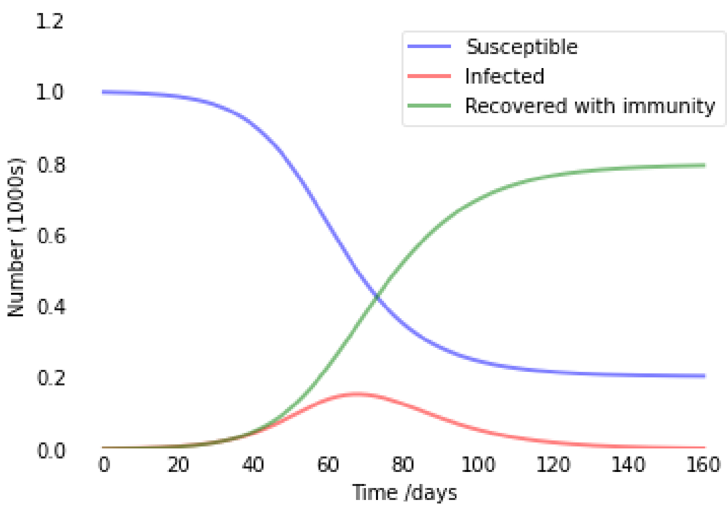

Figure 2: Model output

Feel free to skim: https://github.com/RSounder/PythonForFun/blob/master/SIR_Model.ipynb

Credits to "*Learning Scientific Programming with Python*" with Christian Hill for SIR model reference

Understanding the graph: S(t) represents an inverse sigmoid that decreases exponentially initially before flattening. R(t) represents a sigmoid that increases exponentially before flattening out. I(t) depends on both S(t) and R(t) and thus increases during decrease of S(t) and decreases during increase of R(t).

Now let's see how we can decrease the number of cases of I(t). Let's take dI/dt at time t=0.

$$dI/dt \text{ at } t=0 \text{ is given as: } aS_0I_0 - bI_0$$

Now we already know that I =0 at t=0. Thus we can safely assume, dI/dt = 0 at t=0. But during the rate of change at $t \neq 0$, dI/dt can vary from positive to negative. To decrease the number of infections as per the graph, we take dI/dt <0.

$$aS_0I_0 - bI_0 < 0 \rightarrow (aS_0 - b)I_0 < 0$$

$\therefore$ For infected cases to start decreasing, $\boxed{\mathbf{(aS_0 / b) < 1}}$ ------> eq1

By equation eq1, we find that the way to decrease rate of infections is to:

1. Decrease 'a' (Rate of transmission)
2. Decrease S_0 (Number of people exposed to the virus)
3. Increase 'b' (Rate of recovery)

Human intuition always has a hard time understanding exponential outburst. We are deep rooted to linear trends that we sometimes even linearize exponential curves in logarithmic scales just because it is easier to look at.

$$dI/dt = I\,(aS_0 - b), \text{ when 't' is near to } t=0 \ (t-t_0 = \text{low})$$

$$\boxed{\therefore\ \mathbf{I(t) = e^{(aS_0 - b)t}}} \quad \text{------> eq2}$$

This equation eq2 clearly shows that the infection I(t) is exponential.

When S_0 becomes 0 the curve starts to head in the opposite direction.

Growth Factor: It is the ratio of change in number of cases today to the change in number of cases yesterday on a constant step basis (step = 1 for daily basis).

Growth Factor: $\triangle \mathbf{I_d} / \triangle \mathbf{I_{d+1}}$

The infection curve starts to decrease while the number of new cases on any given day is less than the number of old cases the previous day. This can be achieved only when the three factors from eq1 are adherently decreased or increased. Thus to conclude,

1. To decrease 'a' (Rate of transmission), **Wash your hands properly and frequently; plus don't touch your face!**
2. To decrease S_0 (Number of people exposed to the virus), **don't go outdoors unless it is absolutely necessary.**
3. To Increase 'b' (Rate of recovery), we must hope for vaccines/treatments to come around sooner.

Credits to "*Learning Scientific Programming with Python*" with Christian Hill for SIR model reference