AEM 5253—Computational Fluid Mechanics

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Final Project

Suryanarayan(Surya) Ramachandran

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A MATLAB code is written to solve the compressible Euler equations using Steger-Warming flux-splitting methodology in curvilinear (elliptic) coordinates. The code consists 7 subroutines:

- 1. Euler_Finite_Diff.m-the main solver routine
- 2. F_plus.m-computes the F^+ and G^+ flux locally.
- 3. F_minus.m-computes the F^- and G^- flux locally
- 4. $compute_dt.m$ -computes the minimum stable dt possible for advance
- 5. lambda_max.m-helper function for compute_dt.m to retrieve max local eigen value
- 6. post_process.m-to generate contours
- 7. compute_drag.m-to compute drag coefficient

The contents of each of these routines are presented below:

1 Main solver:

Euler_Finite_Diff.m

```
1 %-----%
2 % Name: Suryanarayan Ramachandran
3 % Course: AEM5253 Computational Fluid Mechanics
  % Date: Dec 26th, 2022
  % E-Mail: ramac106@umn.edu
  % Course instructor: Dr.Pramod K Subbareddy
  %-----%
8
  clear all; close all; clc; %#ok<CLALL>
9
10
  %-----%
11
  %# of points along xi/eta
13 N=100;
14
  %generate xi, eta
15
16 xi = linspace(0, 2*pi, N);
eta = linspace(atanh(0.25),6.2,N);
18
  %meshgrid to get structured mesh
  [eta, xi] = meshgrid(eta, xi);
20
21
 %get physical space coords
22
x = \cosh(eta) \cdot *\cos(xi);
y = \sinh(eta) \cdot *\sin(xi);
25
  %-----%
26
  %get Delta(xi), Delta(eta)
 dxi = xi(2,1)-xi(1,1);
 deta = eta(1,2) - eta(1,1);
30
31 %Declare arrays for mesh-metrics and Jacobians
32 \text{ xi}_{-}x = zeros(N,N);
xi_y = zeros(N,N);
34 \text{ eta}_x = zeros(N,N);
35 \text{ eta_y} = \text{zeros}(N, N);
  J = zeros(N,N);
37
  %Compute mesh-metrics and Jacobians
  for i=1:N
39
40
     for j=1:N
         if(i \ge 2 \&\& i \le N-1)
41
             x_x = (x(i+1,j)-x(i-1,j))/2/dxi;
42
             y_x = (y(i+1,j)-y(i-1,j))/2/dxi;
43
         elseif(i==1)
44
45
             x_x = (x(i+1,j)-x(i,j))/dxi;
46
             y_xi = (y(i+1,j)-y(i,j))/dxi;
         elseif(i==N)
47
             x_x = (x(i,j)-x(i-1,j))/dxi;
48
             y_x = (y(i,j)-y(i-1,j))/dxi;
49
50
51
         end
52
53
         if (j \ge 2 \& \& j \le N-1)
54
             x_{eta} = (x(i, j+1) - x(i, j-1))/2/deta;
```

```
y_{eta} = (y(i, j+1) - y(i, j-1))/2/deta;
56
          elseif(j==1)
57
              x_{eta} = (x(i,j+1)-x(i,j))/deta;
58
59
              y_{eta} = (y(i,j+1)-y(i,j))/deta;
          elseif(j==N)
60
              x_{eta} = (x(i,j)-x(i,j-1))/deta;
61
              y_{eta} = (y(i,j)-y(i,j-1))/deta;
62
          end
63
64
65
          J(i,j) = 1/(x_xi*y_eta - x_eta*y_xi);
          xi_x(i,j) = y_eta*J(i,j);
66
          xi_y(i,j) = -y_xi*J(i,j);
67
          eta_x(i,j) = -x_eta*J(i,j);
68
          eta_y(i,j) = x_xi*J(i,j);
69
70
      end
   end
71
72
   %-----%
73
  %Time-level: n arrays
75 rho = zeros(N, N);
  u = zeros(N,N);
v = zeros(N,N);
78 p = zeros(N,N);
79 %Time-level:n+1 arrays
so rho_n = zeros(N,N);
u_n = zeros(N,N);
v_n = zeros(N,N);
  p_n = zeros(N, N);
83
84
85 %matrices for storing norms
86 u_norm = [];
v_norm = [];
88 rho_norm = [];
89 p_norm = [];
  time = [];
90
91
  %-----%
92
93 %freestream conditions
94 M = 6;
95 \text{ gamma} = 1.4;
96 p_inflow = 1/gamma;
97 rho_inflow=1;
  u_inflow = M*sqrt(gamma*p_inflow/rho_inflow);
  cfl=0.5;
99
100
101 %Initialize
102 for i=1:N
      for j=1:N
103
104
          u(i,j) = u_inflow;
105
          p(i,j) = p_i nflow;
          rho(i, j) = rho_inflow;
106
107
          v(i, j) = 0;
108
      end
109 end
110
  %-----%
111
112 %set time:
113 t=0;
  %set max time of simulation:
115 t_max=10;
```

```
116
117 %Solve:
   %Index notation followed: i for xi direction, j for eta direction
118
   while(t<t_max)</pre>
120
         %compute minimum dt:
         dt_min = cfl*compute_dt(u,v,p,rho, xi_x, xi_y, eta_x, eta_y, dxi, deta);
121
122
         %advance i=1 and i=N, in the interior j's
123
         % For i=1, i+1=2, i-1 = N-1
124
125
         for j=2:N-1
                  %Compute RHS
126
                  R_{ij} = (F_{plus}(u(1,j),...)
127
                                   v(1,j),...
128
                                   p(1, j), ...
129
130
                                   rho(1,j),...
131
                                   xi_x(1,j),...
                                   xi_y(1,j),...
132
133
                                   J(1, j))...
                         -F_{\text{plus}}(u(N-1,j),...
134
135
                                   v(N-1, j), ...
136
                                   p(N-1, j), ...
137
                                   rho(N-1,j),...
138
                                   xi_x(N-1,j),...
                                   xi_y(N-1,j),...
139
140
                                   J(N-1, j)))/dxi...
141
                         +(F_{\min us}(u(2,j),...
                                     v(2,j),...
142
                                     p(2,j),...
143
                                      rho(2,j),...
144
                                      xi_x(2,j),...
145
146
                                     xi_{-y}(2,j),...
                                      J(2,j))...
147
                            -F_{\text{minus}}(u(1,j),...
148
                                     v(1,j),...
149
150
                                     p(1, j), ...
                                      rho(1, j), ...
151
                                      xi_x(1,j),...
152
153
                                      xi_-y(1,j),...
154
                                      J(1,j)))/dxi...
                            +(F_plus(u(1,j),
155
                                     v(1,j),...
156
157
                                     p(1, j), ...
                                     rho(1,j),...
158
                                      eta_x(1, j), ...
159
                                      eta_y(1,j),...
160
                                      J(1,j))...
161
                            -F_plus(u(1,j-1),...
162
                                     v(1, j-1), ...
163
                                     p(1, j-1), ...
164
165
                                     rho(1, j-1), ...
                                      eta_x(1, j-1), ...
166
                                      eta_y(1, j-1), ...
167
                                      J(1, j-1))/deta...
168
                            +(F_{minus}(u(1,j+1),...
169
170
                                     v(1,j+1),...
171
                                     p(1, j+1), ...
172
                                      rho(1, j+1), ...
173
                                      eta_x(1, j+1), ...
                                      eta_y (1, j+1),...
174
175
                                      J(1, j+1))...
```

```
-F_{minus}(u(1,j),...
176
                                      v(1,j),...
177
                                      p(1,j),...
178
179
                                      rho(1, j), ...
180
                                      eta_x(1,j),...
181
                                       eta_y(1,j),...
                                      J(1,j)))/deta;
182
183
                  %Locally construct conserved state vector
184
185
                  U_{\text{old}} = [\text{rho}(1,j); \dots]
                             rho(1, j) *u(1, j); ...
186
                             rho(1, j) *v(1, j); ...
187
                             rho(1, j) * (0.5*u(1, j)^2+0.5*v(1, j)^2)+p(1, j)/(gamma-1)];
188
189
190
                  %advance
191
                  U_{\text{new}} = U_{\text{old}} - dt_{\text{min}} * J(1, j) * R_{\text{ij}};
192
                  %get back new primitive vars
193
                  rho_new = U_new(1);
194
195
                  u_new = U_new(2)/rho_new;
196
                  v_{new} = U_{new}(3)/rho_{new};
                  q_new = 0.5*(u_new^2+v_new^2);
197
                  p_new = (gamma-1) * (U_new (4) - rho_new * q_new);
198
199
200
                  %assign to new arrays
                  % since xi-direction is periodic, we note i=1 and i=N are the same
201
                  rho_n(1,j) = rho_new;
202
                  rho_n(N,j) = rho_new;
203
                  u_n(1,j) = u_new;
204
                  u_n(N,j) = u_new;
205
206
                  v_n(1,j) = v_new;
                  v_n(N,j) = v_new;
207
208
                  p_n(1,j) = p_new;
                  p_n(N,j) = p_new;
209
        end
210
211
        %advance all other interior j's and i's i.e. 2:N-1
212
213
         for j=2:N-1 %iterate through the interior ellipses-outer loop, we will fill ...
            j=1, N later
             for i=2:N-1 %iterate along the interior ellipses-inner loop, we will fill ...
214
                 i=1,N later
                       %compute RHS
215
                       R_{ij} = (F_{plus}(u(i,j),
216
                                     v(i,j),...
217
                                     p(i,j),...
218
                                     rho(i,j),...
219
                                     xi_x(i,j),...
220
                                     xi_y(i,j),...
221
222
                                     J(i,j))...
                              -F_plus(u(i-1,j),...
223
                                       v(i-1, j), ...
224
225
                                       p(i-1,j),...
                                       rho(i-1,j),...
226
227
                                        xi_x(i-1, j), ...
228
                                        xi_{y}(i-1, j), ...
229
                                        J(i-1, j)))/dxi...
                              +(F_{\min}(u(i+1,j),...)
230
                                          v(i+1,j),...
231
232
                                          p(i+1, j), ...
                                          rho(i+1,j),...
233
```

```
xi_x(i+1,j),...
234
                                          xi_-y(i+1, j), ...
235
                                          J(i+1, j))...
236
                               -F_{\text{minus}}(u(i,j),...
237
238
                                          v(i,j),...
239
                                          p(i,j),...
                                          rho(i,j),...
240
                                          xi_x(i,j),...
241
                                          xi_y(i,j),...
242
243
                                          J(i,j)))/dxi...
                              +(F_plus(u(i,j),...
244
                                         v(i,j),...
245
                                         p(i,j),...
246
                                         rho(i,j),...
247
248
                                         eta_x(i,j),...
249
                                         eta_y(i,j),...
                                         J(i,j))...
250
                               -F_plus(u(i,j-1),...
251
                                         v(i, j-1), ...
252
253
                                         p(i, j-1), ...
254
                                         rho(i, j-1), ...
255
                                         eta_x(i,j-1),...
                                         eta_y(i, j-1), ...
256
                                         J(i, j-1))/deta...
257
                              +(F_{\min}(u(i,j+1),...)
258
259
                                          v(i, j+1), ...
                                          p(i, j+1), ...
260
                                          rho(i, j+1), ...
261
                                          eta_x(i,j+1),...
262
                                          eta_y(i,j+1),...
263
264
                                          J(i, j+1))...
                               -F_{\text{minus}}(u(i,j),...
265
266
                                          v(i,j),...
                                          p(i,j),...
267
                                          rho(i,j),...
268
269
                                          eta_x(i,j),...
270
                                          eta_y(i,j),...
271
                                          J(i,j))/deta;
272
                       %construct U_old
273
                       U_{\text{old}} = [\text{rho}(i, j); ...]
274
275
                                 rho(i, j) *u(i, j); ...
                                 rho(i,j)*v(i,j);...
276
277
                                  0.5*rho(i,j)*(u(i,j)^2+v(i,j)^2)+p(i,j)/(gamma-1)];
278
                       %advance
279
                       U_new = U_old - dt_min*J(i, j)*R_ij;
280
281
                       %extract fields from solution
282
283
                       rho_new = U_new(1);
                       u_new = U_new(2)/rho_new;
284
                       v_new = U_new(3)/rho_new;
285
                       q_{new} = 0.5*(u_{new}^2+v_{new}^2);
286
                       p_new = (gamma-1)*(U_new(4)-rho_new*q_new);
287
288
289
                       %assign to new arrays:
290
                       rho_n(i,j) = rho_new;
                       u_n(i,j) = u_new;
291
292
                       v_n(i,j) = v_new;
                       p_n(i,j) = p_new;
293
```

```
294
             end
295
        end
296
        %advance j=1 and j=N
297
        for i=1:N
298
            %j=1 i.e. airfoil surface
299
            M = [xi_x(i,1) xi_y(i,1); eta_x(i,1) eta_y(i,1)];
300
            D = [xi_x(i,2) *u_n(i,2) + xi_y(i,2) *v_n(i,2); 0];
301
302
            A=M\setminus D;
303
            %get velocities
            u_n(i,1) = A(1); v_n(i,1) = A(2);
304
            %get density
305
            rho_n(i,1) = rho_n(i,2);
306
            %get pressure
307
308
            q_1 = 0.5*(u_n(i,1)^2 + v_n(i,1)^2);
309
            q_2 = 0.5*(u_n(i,2)^2 + v_n(i,2)^2);
            p_n(i,1) = p_n(i,2) + (gamma-1)*(rho_n(i,2)*q_2-rho_n(i,1)*q_1);
310
311
            %j=N, i.e. outer boundary
312
             if(i>N/2 && i \le 3*N/2)
313
                  %one half of the domain is an inlet
314
                 u_n(i,N) = u_inflow;
315
316
                  v_n(i,N)=0;
                  p_n(i,N) = p_inflow;
317
318
                  rho_n(i,N)=rho_inflow;
319
             else
                  %other half is an outlet
320
                  u_n(i,N)=u_n(i,N-1);
321
                  v_n(i, N) = v_n(i, N-1);
322
                  p_n(i,N) = p_n(i,N-1);
323
324
                  rho_n(i,N) = rho_n(i,N-1);
             end
325
        end
326
327
        %impose periodicity along i once again
328
329
        u_n(N, :) = u_n(1, :);
330
        v_n(N, :) = v_n(1, :);
        p_n(N,:) = p_n(1,:);
331
332
        rho_n(N, :) = rho_n(1, :);
333
        %compute norms
334
335
        L2_u = norm(u-u_n, 2);
        L2_v = norm(v-v_n, 2);
336
337
        L2_p = norm(p-p_n, 2);
        L2-rho = norm(rho-rho_n,2);
338
339
        %update norm vectors
340
        u_norm = [L2_u, u_norm];
341
        v_norm = [L2_v, v_norm];
342
343
        p_norm = [L2_p, p_norm];
        rho_norm = [L2_rho, rho_norm];
344
345
        %update solution arrays
346
        u = u_n;
347
348
        v = v_n;
349
        p = p_n;
350
        rho = rho_n;
351
        %advance time
352
        t = t + dt_min;
353
```

```
354
        %display time
355
        %store time
356
357
        time = [t, time];
   end
358
359
   %plot residuals of primitive variables vs time
360
   loglog(time, p_norm,...
362
             time, rho_norm, ...
363
             time, u_norm, ...
            time, v_norm);
364
   xlabel("time");legend("pressure", "rho", "u", "v")
365
```

2 F^+, G^+

```
1
       function fp = F_plus(u, v, p, rho, xi_x, xi_y, J)
2
       %returns the 3 column F^+ vector for the advance step
       %all inputs are scalars hence values at a point and not vectors
3
       r_xi = sqrt(xi_x^2+xi_y^2);
4
       gamma = 1.4;
5
6
       n_x = xi_x/r_xi_i
7
       n_y = xi_y/r_xi;
       u_d = u * n_x + v * n_y;
8
       q = (u^2+v^2)/2;
9
       a = sqrt(gamma*p/rho);
10
11
       h0 = q + a^2/(gamma-1);
       e = 1e-6;
12
       %construct lambda_plus matrix
13
       L_p = zeros(4,4);
14
       L_p(1,1) = r_xi*0.5*(u_d-a+sqrt((u_d-a)^2+e^2));
15
       L_p(2,2) = r_xi*0.5*(u_d+sqrt(u_d^2+e^2));
16
17
       L_p(3,3) = r_xi*0.5*(u_d+a+sqrt((u_d+a)^2+e^2));
       L_p(4,4) = r_xi*0.5*(u_d+sqrt(u_d^2+e^2));
18
19
       %construct left eigen vector
20
       R = zeros(4,4);
^{21}
       R(1,1) = 1;
22
       R(1,2) = 1;
23
       R(1,3) = 1;
24
25
       R(2,1) = u-a*n_x;
26
       R(2,2) = u;
27
       R(2,3) = u+a*n_x;
28
       R(2,4) = n_-y;
29
30
31
       R(3,1) = v-a*n_y;
       R(3,2) = v;
32
33
       R(3,3) = v+a*n_y;
34
       R(3,4) = -n_x;
35
36
       R(4,1) = h0 - a*u_d;
       R(4,2) = q;
37
38
       R(4,3) = h0 + a*u_d;
       R(4,4) = u*n_y - v*n_x;
39
40
       %construct left-egien vector
41
```

```
R_{inv} = zeros(4,4);
42
43
       R_{inv}(1,1) = ((gamma-1)*q+a*u_d)/(2*a*a);
44
45
       R_{inv}(1,2) = ((1-gamma)*u-a*n_x)/(2*a*a);
       R_{inv}(1,3) = ((1-gamma)*v-a*n_y)/(2*a*a);
46
       R_{inv}(1,4) = ((gamma-1))/(2*a*a);
^{47}
48
       R_{inv}(2,1) = (a^2 - (gamma - 1) * q) / (a * a);
49
       R_{inv}(2,2) = ((gamma-1)*u)/(a*a);
50
51
        R_{inv}(2,3) = ((gamma-1)*v)/(a*a);
       R_{inv}(2,4) = ((1-gamma))/(a*a);
52
53
       R_{inv}(3,1) = ((gamma-1)*q-a*u_d)/(2*a*a);
54
       R_{inv}(3,2) = ((1-qamma)*u+a*n_x)/(2*a*a);
55
56
       R_{inv}(3,3) = ((1-gamma)*v+a*n_y)/(2*a*a);
       R_{inv}(3,4) = ((gamma-1))/(2*a*a);
57
58
       R_{inv}(4,1) = v*n_x-u*n_y;
59
60
       R_{inv}(4,2) = n_{y};
61
       R_{inv}(4,3) = -n_{x};
62
       R_{inv}(4,4) = 0;
63
       %construct state vector:
64
       U = [rho; rho*u; rho*v; rho*q+p/(gamma-1)];
65
66
       U_Tilde = U/J;
67
       A_p = R*L_p*R_inv;
68
        fp = A_p * U_Tilde;
69
70
  end
```

3 F^-, G^-

```
function fm = F_minus(u, v, p, rho, xi_x, xi_y, J)
1
2
       %returns the 3 column F^+ vector for the advance step
       %all inputs are scalars hence values at a point and not vectors
3
       r_xi = sqrt(xi_x^2+xi_y^2);
4
       gamma = 1.4;
5
       n_x = xi_x/r_xi_i
6
       n_y = xi_y/r_xi;
7
       u_d = u * n_x + v * n_y;
8
       q = (u^2+v^2)/2;
9
10
       a = sqrt(qamma*p/rho);
       h0 = q + a^2/(qamma-1);
11
       e = 1e-6;
12
13
       %construct lambda_plus matrix
       L_m = zeros(4,4);
14
       L_m(1,1) = r_xi*0.5*(u_d-a-sqrt((u_d-a)^2+e^2));
15
       L_m(2,2) = r_xi*0.5*(u_d-sqrt(u_d^2+e^2));
16
17
       L_m(3,3) = r_xi*0.5*(u_d+a-sqrt((u_d+a)^2+e^2));
       L_m(4,4) = r_xi*0.5*(u_d-sqrt(u_d^2+e^2));
18
19
       %construct left eigen vector
20
21
       R = zeros(4,4);
       R(1,1) = 1;
22
       R(1,2) = 1;
23
       R(1,3) = 1;
24
```

```
R(1,4) = 0;
25
26
       R(2,1) = u-a*n_x;
27
       R(2,2) = u;
28
29
       R(2,3) = u+a*n_x;
30
       R(2,4) = n_{-y};
31
       R(3,1) = v-a*n_y;
32
       R(3,2) = v;
33
34
       R(3,3) = v+a*n_y;
       R(3,4) = -n_x;
35
36
       R(4,1) = h0 - a*u_d;
37
       R(4,2) = q;
38
39
       R(4,3) = h0 + a*u_d;
       R(4,4) = u * n_y - v * n_x;
40
41
       %construct left-egien vector
42
       R_{inv} = zeros(4,4);
43
44
45
       R_{inv}(1,1) = ((gamma-1)*q+a*u_d)/(2*a*a);
       R_{inv}(1,2) = ((1-gamma)*u-a*n_x)/(2*a*a);
46
47
       R_{inv}(1,3) = ((1-gamma)*v-a*n_y)/(2*a*a);
       R_{inv}(1,4) = ((gamma-1))/(2*a*a);
48
49
       R_{inv}(2,1) = (a^2 - (gamma - 1) * q) / (a*a);
50
       R_{inv}(2,2) = ((gamma-1)*u)/(a*a);
51
       R_{inv}(2,3) = ((gamma-1)*v)/(a*a);
52
       R_{inv}(2,4) = ((1-gamma))/(a*a);
53
54
55
       R_{inv}(3,1) = ((gamma-1)*g-a*u_d)/(2*a*a);
       R_{inv}(3,2) = ((1-gamma)*u+a*n_x)/(2*a*a);
56
       R_{inv}(3,3) = ((1-gamma)*v+a*n_y)/(2*a*a);
57
       R_{inv}(3,4) = ((gamma-1))/(2*a*a);
58
59
60
       R_{inv}(4,1) = v*n_x-u*n_y;
       R_{-inv}(4,2) = n_{-y};
61
62
       R_{inv}(4,3) = -n_x;
       R_{inv}(4,4) = 0;
63
       %construct state vector:
65
       U = [rho; rho*u; rho*v; rho*q+p/(qamma-1)];
66
       U_Tilde = U/J;
67
68
       A_m = R*L_m*R_inv;
69
       fm = A_m * U_Tilde;
70
  end
71
```

4 Compute stable dt

```
function dt = compute_dt(u,v,rho,p,xi_x, xi_y, eta_x, eta_y, dxi, deta)
   N = length(u);
2
3
    dt = 1e2;
    for i = 1:N
4
           for j=1:N
5
                lmax = lambda_max(u(i,j),...
6
                                     v(i,j),...
7
                                     rho(i,j),...
8
                                     p(i,j),...
9
                                     xi_x(i,j),...
10
                                     xi_y(i,j),...
11
                                     eta_x(i,j),...
12
                                     eta_y(i,j));
13
                dt_min = min(dxi/lmax(1), deta/lmax(2));
14
                if (dt_min<dt)</pre>
15
                     dt = dt_min;
16
                end
17
18
            end
    end
19
  end
```

5 Get λ_{max}

```
function l_max = lambda_max(u,v, p, rho, xi_x, xi_y, eta_x, eta_y)
2
       gamma = 1.4;
       a = sqrt(qamma*p/rho);
3
       if (¬isreal(a))
4
           а
5
6
           rho
7
           р
8
           return;
      end
9
       1_xi = abs(xi_x*u+xi_y*v) + a*sqrt(xi_x^2 + xi_y^2);
10
       l_{eta} = abs(eta_x*u+eta_y*v) + a*sqrt(eta_x^2 + eta_y^2);
11
12
       l_max = [l_xi, l_eta];
13
  end
```

6 Post Processing

post_process.m

```
1 contourf(x,y,u_n,'LevelStep',0.1,'LineStyle','--')
2 xlim([-1.5,1.5])
3 ylim([-1.5,1.5])
4 title("$u$","Interpreter","latex")
5 xlabel("$x$","interpreter","latex");
6 ylabel("$y$","interpreter","latex");
7 c=colorbar;
8 c.TickLabelInterpreter="latex";
```

```
set(groot, 'defaultAxesTickLabelInterpreter', 'latex');
9
   set (qca, ...
10
       "FontSize", 18, ...
11
       "FontName", "Computer Modern Roman");
   saveas(gcf, "Final_Project/u", "epsc");
13
14
  contourf(x, y, v_n, 'LevelStep', 0.1, 'LineStyle', '--')
15
16 \times lim([-1.5, 1.5])
17 \text{ ylim}([-1.5, 1.5])
18 title("$v$","Interpreter","latex")
19 xlabel("$x$","interpreter","latex");
20 ylabel("$y$","interpreter","latex");
21 c=colorbar;
22 c.TickLabelInterpreter="latex";
  set (groot, 'defaultAxesTickLabelInterpreter', 'latex');
   set (qca, ...
24
       "FontSize", 18, ...
25
       "FontName", "Computer Modern Roman");
26
   saveas(gcf, "Final_Project/v", "epsc");
27
28
   contourf(x,y,sqrt(u_n.^2+v_n.^2),'LevelStep',0.1,'LineStyle','--')
29
30 x \lim ([-1.5, 1.5])
31 \text{ ylim}([-1.5, 1.5])
32 title("\$|U| = \sqrt{u^2+v^2},"Interpreter","latex")
xlabel("$x$","interpreter","latex");
ylabel("$y$","interpreter","latex");
35 c=colorbar:
  c.TickLabelInterpreter="latex";
  set(groot, 'defaultAxesTickLabelInterpreter', 'latex');
37
38
   set (gca, ...
39
       "FontSize", 18, ...
       "FontName", "Computer Modern Roman");
40
   saveas(gcf, "Final_Project/velmag", "epsc");
41
42
  contourf(x,y,p_n,'LevelStep',0.1,'LineStyle','--')
43
44 x \lim ([-1.5, 1.5])
45 vlim([-1.5, 1.5])
46 title("$p$","Interpreter","latex")
47 xlabel("$x$", "interpreter", "latex");
48 ylabel("$y$", "interpreter", "latex");
49 c=colorbar:
50 c.TickLabelInterpreter="latex";
  set(groot, 'defaultAxesTickLabelInterpreter', 'latex');
51
52
   set (gca, ...
       "FontSize", 18, ...
53
       "FontName", "Computer Modern Roman");
54
   saveas(gcf, "Final_Project/p", "epsc");
55
56
57 contourf(x,y,rho_n,'LevelStep',0.1,'LineStyle','--')
58 \times lim([-1.5, 1.5])
  ylim([-1.5, 1.5])
60 title("$\rho$", "Interpreter", "latex")
61 xlabel("$x$","interpreter","latex");
62 ylabel("$y$","interpreter","latex");
63 c=colorbar;
64 c.TickLabelInterpreter="latex";
65 set(groot, 'defaultAxesTickLabelInterpreter', 'latex');
  set(gca, "FontSize", 18, ...
66
       "FontName", "Computer Modern Roman"); saveas(gcf, "Final_Project/rho", "epsc");
67
```

7 Compute Drag Coefficient

compute_drag.m

```
1 M = 0.4; rho_inf = 1; U_inf = M; F_D =0;
2 for i=1:N-1
       M = [xi_x(i,1) xi_y(i,1); eta_x(i,1) eta_y(i,1)];
3
4
       if (i==1)
           D = [0; p_n(i,1) * sqrt((x(i+1,1)-x(N-1,1))^2 + (y(i+1,1)-y(N-1,1))^2)];
5
6
           D = [0; p_n(i,1) * sqrt((x(i+1,1)-x(i-1,1))^2 + (y(i+1,1)-y(i-1,1))^2)];
7
8
      end
9
      A = M \setminus D;
      F_{-}x_{-}i = A(1);
10
       F_D = F_D + F_x_i;
11
12 end
13
14 L=0;
15 for i=1:N-1
       L = L+sqrt((x(i+1,1)-x(i,1))^2+(y(i+1,1)-y(i,1))^2);
16
17 end
18
19 C_D = F_D/0.5/rho_inf/U_inf^2;
```