

# Categorical Data Analysis

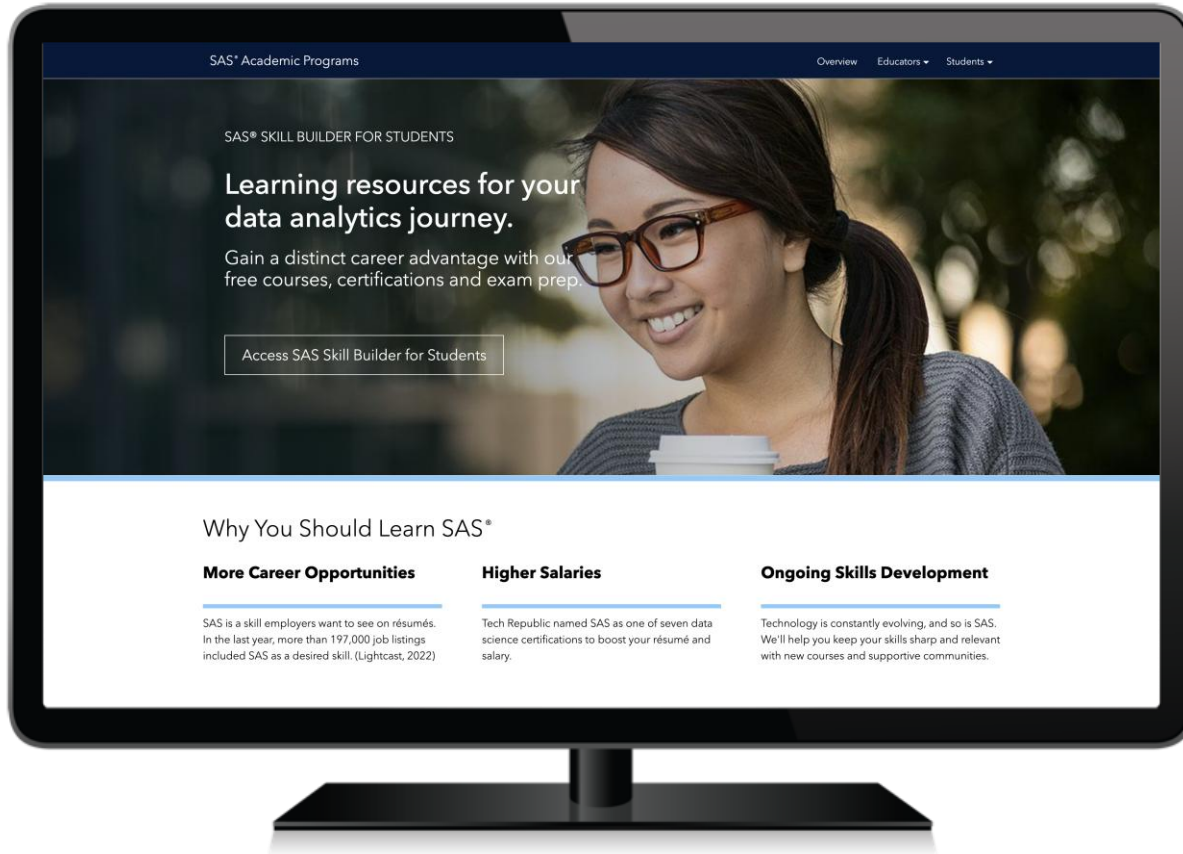
Research Triangle SAS User Group

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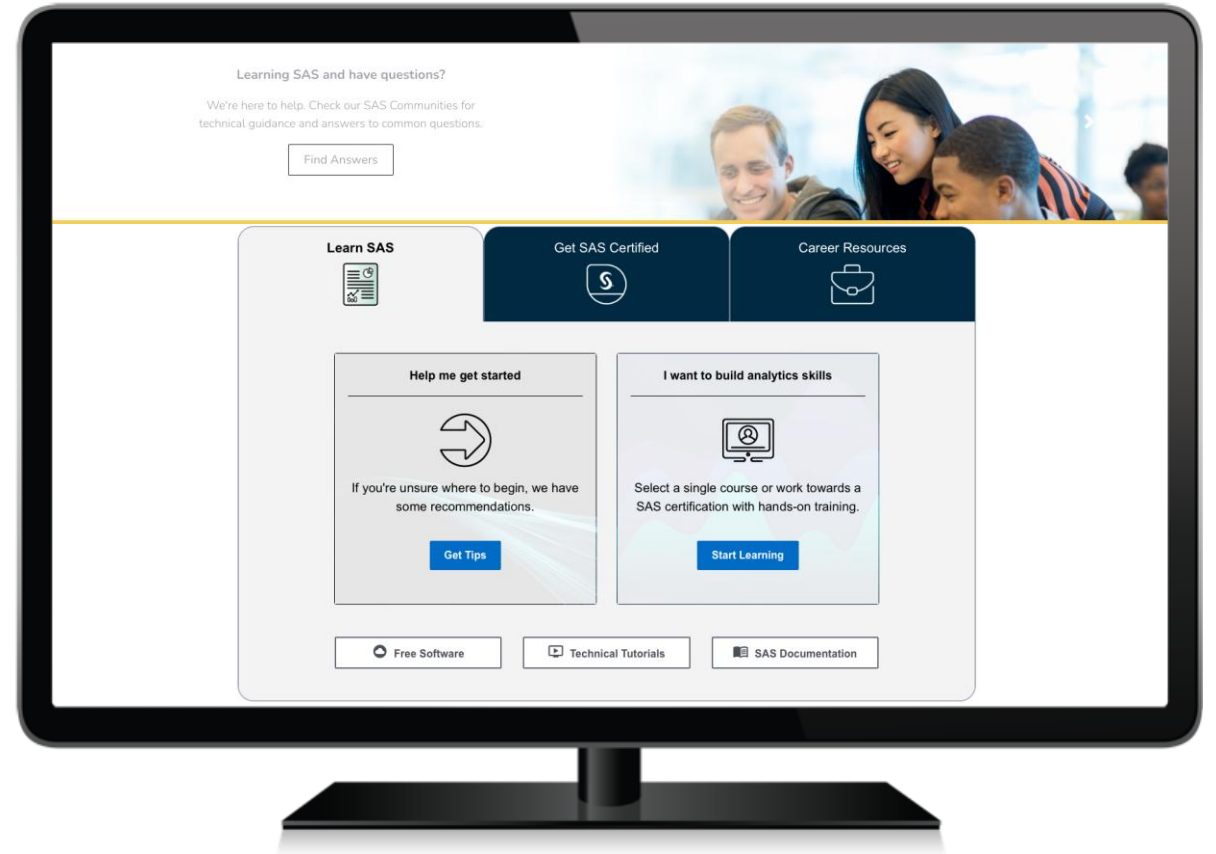


# SAS Skill Builder for Students



Web Experience - Example

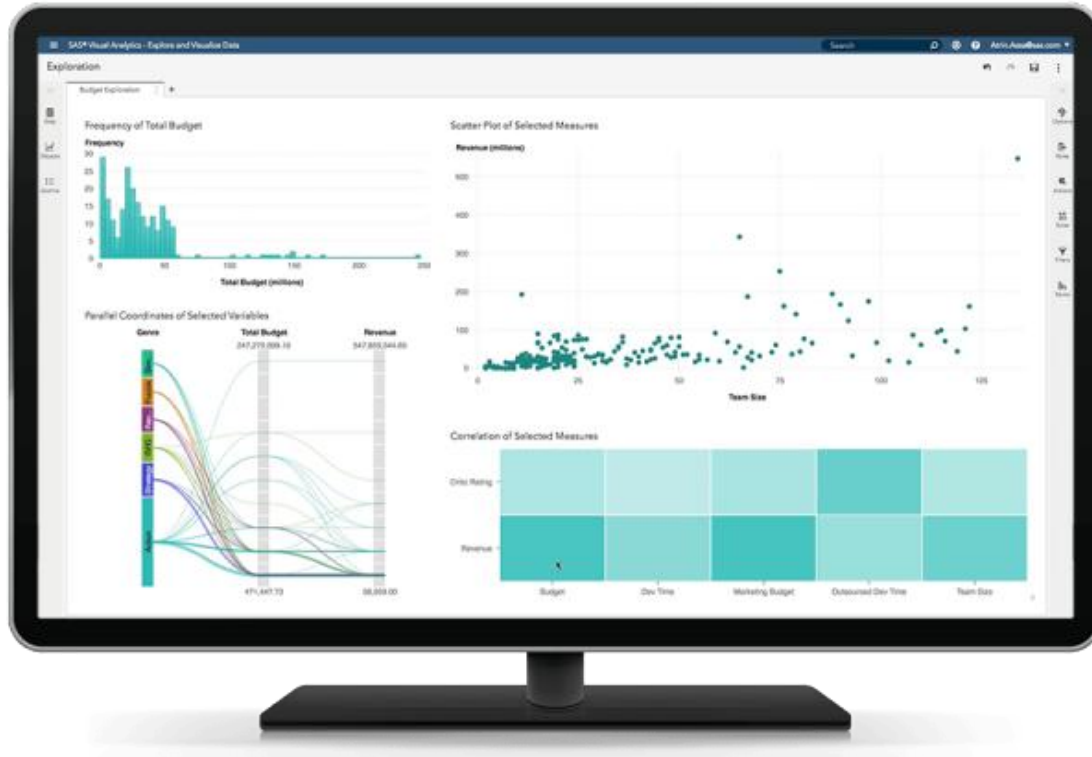
# SAS Educator Portal



Virtual Learning Environment - Example

# Free Academic Software: Viya Advanced

## Getting Started with SAS Viya for Learners



# SAS Certification

## SASCertification

Earning a SAS certification gets you one step closer to the future you've always envisioned

Choose a credential

Schedule an exam



# Categorical Data

- *Categorical data* represent categories, classes and classifications, groups, or qualitative characteristics or attributes.
  - respondent gender (**male** or **female**)
  - product disposition (**conforming** or **nonconforming**)
  - patient mortality (**survived** or **died**)
- *Continuous* data represent measurements.
  - length, time, temperature, concentration
- Categorical data are *qualitative*, continuous data are *quantitative*.
- Categorical data values are *discrete* and the distance between categories is unknown.

# Frequency Table Analysis

- Frequency tables are useful because they can do the following:
  - help detect erroneous data points
  - can be used to assess associations among categorical variables
  - are helpful in determining where possible problems might occur in a logistic regression model

# The FREQ Procedure

- General form of the FREQ procedure:

```
PROC FREQ DATA=SAS-data-set;  
          TABLES table-requests </ options>;  
RUN;
```

# Titanic Insurance Co., Inc.

## Data on Passengers

Variable Name	Details
Age	Age
Cabin	Cabin
Embarked	Port of Embarkation (C = Cherbourg; Q = Queenstown; S = Southampton)
Fare	Passenger Fare (British pound)
Name	Name
Parch	Number of Parents/Children Aboard
<u>PassengerId</u>	Passenger ID
<u>Pclass</u>	Passenger Class (1 = 1st; 2 = 2nd; 3 = 3rd)
Sex	Sex
<u>SibSp</u>	Number of Siblings/Spouses Aboard
Survived	Survival (0 = No; 1 = Yes)
Ticket	Ticket Number



# Demo – Titanic Data – Explore & Visualize

[https://www.sas.com/en\\_us/learn/academic-programs.html](https://www.sas.com/en_us/learn/academic-programs.html)

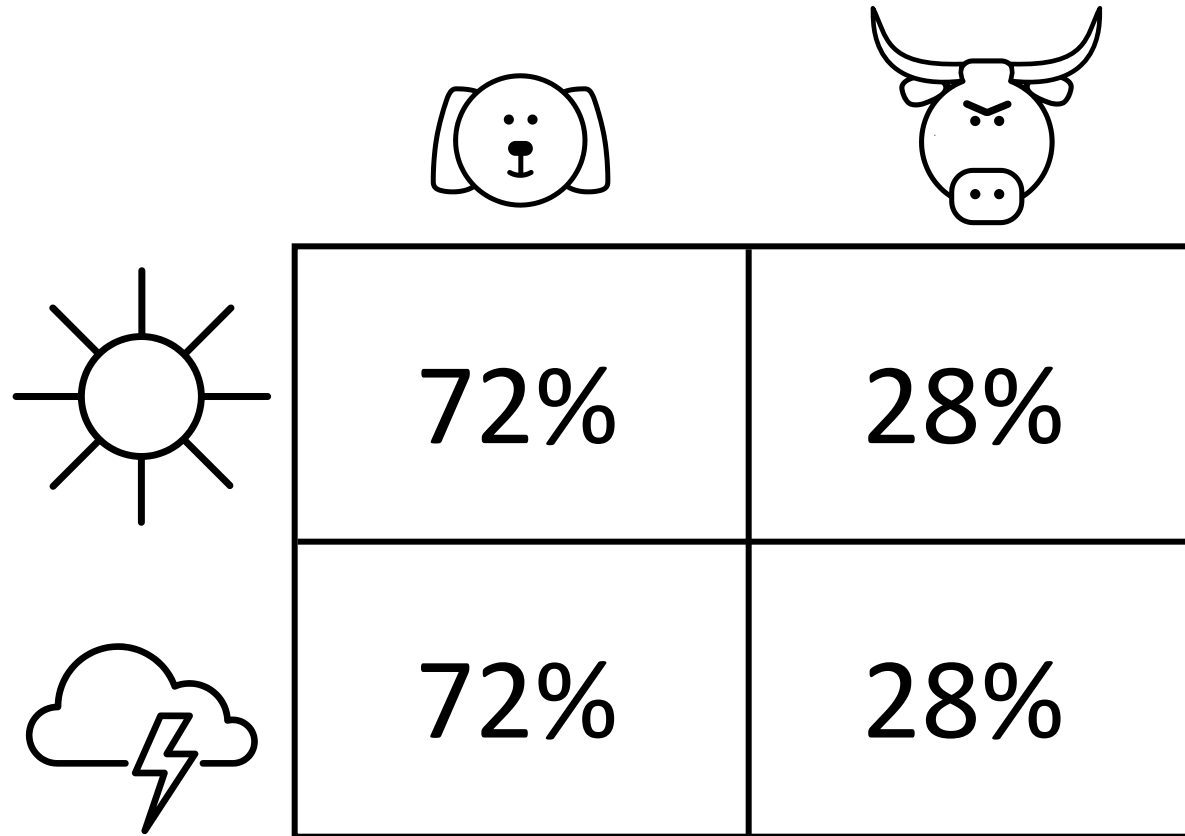


# Categorical Variables Association

- An association exists between two categorical variables if the distribution of one variable changes when the level (or value) of the other variable changes.
- If there is no association, the distribution of the first variable is the same regardless of the level of the other variable.

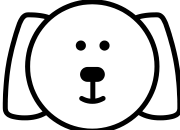
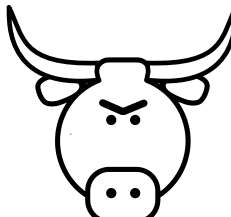
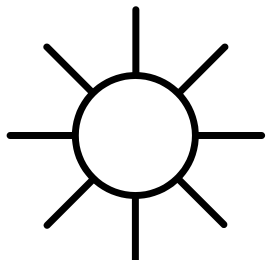

# No Association

- 



Is your manager's mood associated  
with the weather?

# Association

		
	82%	18%
	60%	40%

Is your manager's mood associated  
with the weather?

# Null Hypothesis

- There is ***no*** association between the weather and your boss's mood.
- The probability of your boss being in a good mood is the same on cloudy and sunny days.

## • Alternative Hypothesis

- There ***is*** an association between weather and your boss's mood.
- The probability of your boss being in a good mood is ***not*** the same on cloudy and sunny days.

# Chi-Square Test

## ***NO ASSOCIATION***

observed frequencies = expected frequencies

## ***ASSOCIATION***

observed frequencies  $\neq$  expected frequencies

**Note:** The expected frequencies are calculated by this formula:  
(row total \* column total) / sample size.

# Chi-Square Tests

- Chi-square tests and the corresponding  $p$ -values can do the following:
  - determine whether an association exists
  - do not measure the strength of an association
  - depend on and reflect the sample size

$$\chi^2 = \sum_{i=1}^R \sum_{j=1}^C \frac{(Obs_{ij} - Exp_{ij})^2}{Exp_{ij}}$$

# Demo – Titanic Data – SAS Studio

## Task – Table Analysis

[https://www.sas.com/en\\_us/learn/academic-programs.html](https://www.sas.com/en_us/learn/academic-programs.html)





# Odds Ratios

- An *odds ratio* indicates how much more likely, with respect to odds, a certain event occurs in one group relative to its occurrence in another group.
- Example: How do the odds of males surviving compare to those of females?

$$\text{Odds} = \frac{p_{event}}{1 - p_{event}}$$

# Probability versus Odds of an Outcome

	Outcome		Total
	No	Yes	
Group A	20	60	80
Group B	10	90	100
Total	30	150	180

Probability of Yes  
in Group B = 0.90

÷

Probability of No  
in Group B = 0.10

Odds of Yes in Group B = 0.90 ÷ 0.10 = 9

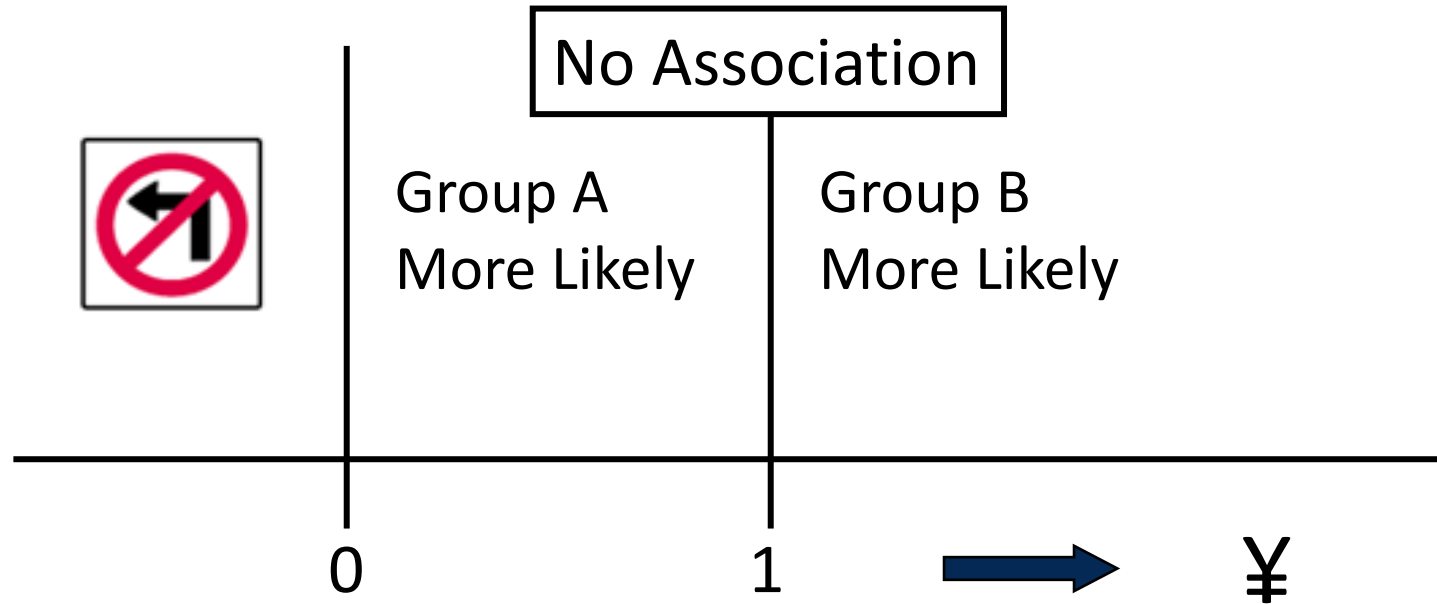
# Odds Ratio

	Outcome		Total
	No	Yes	
Group A	20	60	80
Group B	10	90	100
Total	30	150	180

$$\frac{\text{Odds of Yes in Group B} = 9}{\text{Odds of Yes in Group A} = 3}$$

$$\text{Odds Ratio, B to A} = 9 \div 3 = 3$$

# Properties of the Odds Ratio, B to A



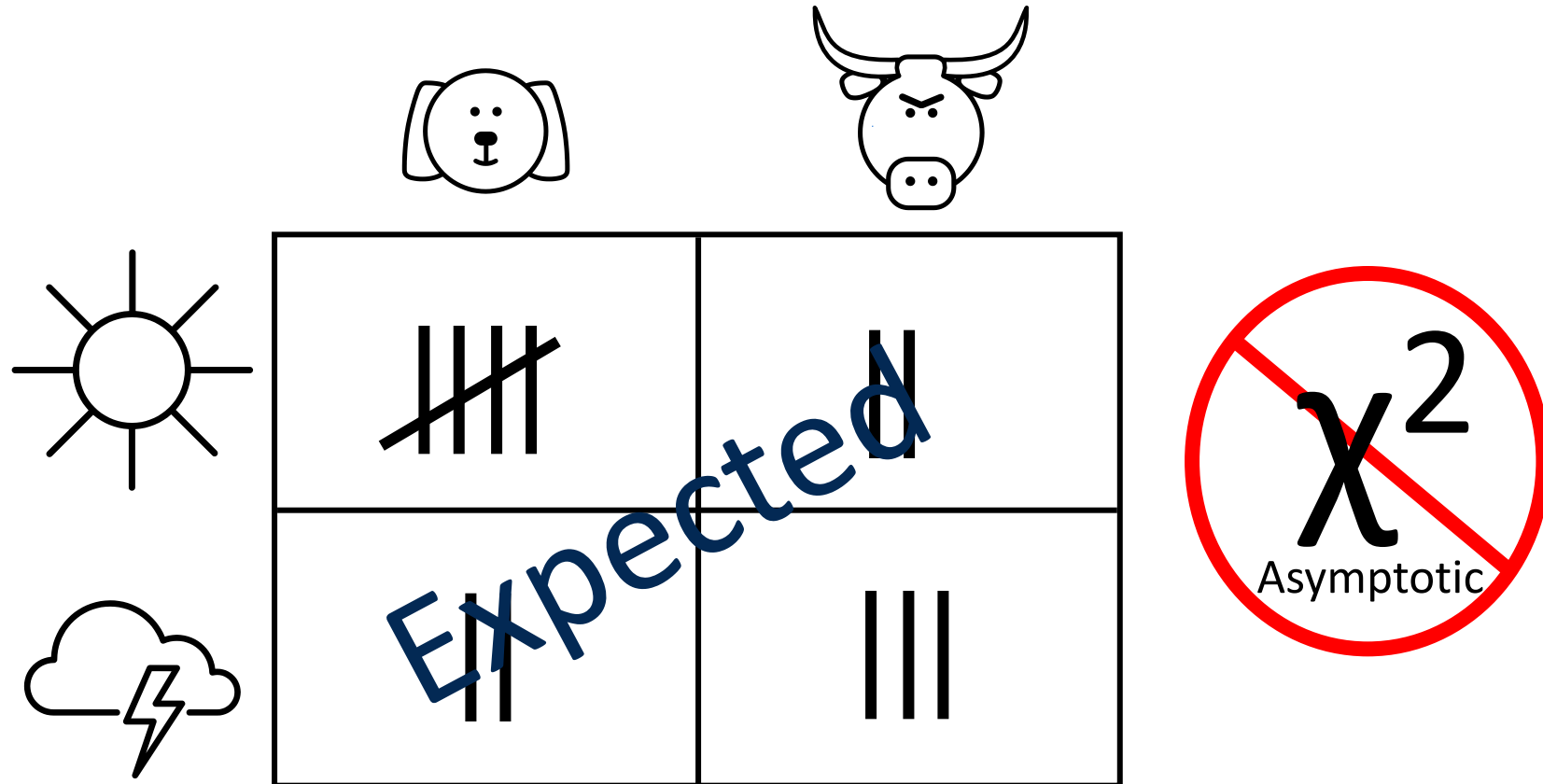
# Tests of Association

<u>Row Variable</u>	<u>Column Variable</u>	<u>R x C table</u>	<u>2 x 2 table</u>
Ordinal	Ordinal	Mantel-Haenzel $\chi^2$	CI for odds ratio
Nominal	Ordinal	Mean score Statistic	CI for odds ratio
Nominal	Nominal	Pearson $\chi^2$	CI for odds ratio

# Measures of Association Strength

<u>Row Variable</u>	<u>Column Variable</u>	<u>R x C table</u>	<u>2 x 2 table</u>
Ordinal	Ordinal	Spearman Correlation	Odds Ratio
Nominal	Ordinal	Uncertainty Coefficient c r	Odds Ratio
Nominal	Nominal	Uncertainty Coefficient c r	Odds Ratio

# When Not to Use the Asymptotic $\chi^2$



**When more than 20% of cells have  
expected counts less than five**

# Demo – adding Odds Ratios

[https://www.sas.com/en\\_us/learn/academic-programs.html](https://www.sas.com/en_us/learn/academic-programs.html)





# Logistic Regression

[https://www.sas.com/en\\_us/learn/academic-programs.html](https://www.sas.com/en_us/learn/academic-programs.html)

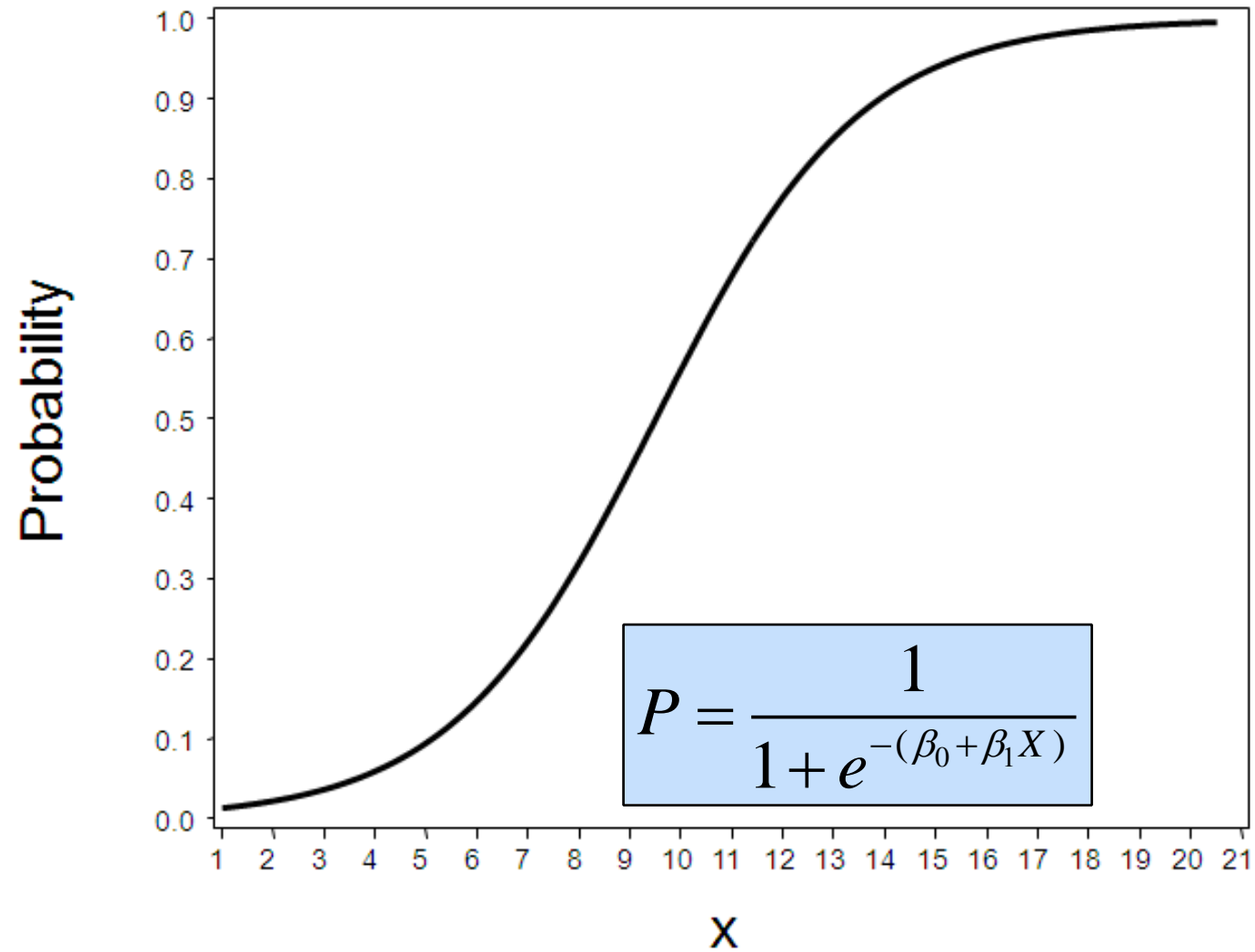


# Why Not Ordinary Least Squares Regression?

$$OLS \text{ Regression: } Y_i = \beta_0 + \beta_1 X_{1i} + \varepsilon_i$$

- - The random error term  $\varepsilon$  has a normal distribution with a mean of zero.
  - The random error term has a constant variance.
  - The errors  $\varepsilon_i$  are independent.
  - The model is correctly specified.
- In logistic regression, the first two assumptions are violated. Therefore, OLS is not the best method for parameter estimation.

# Logistic Regression Model



# Logit Transformation

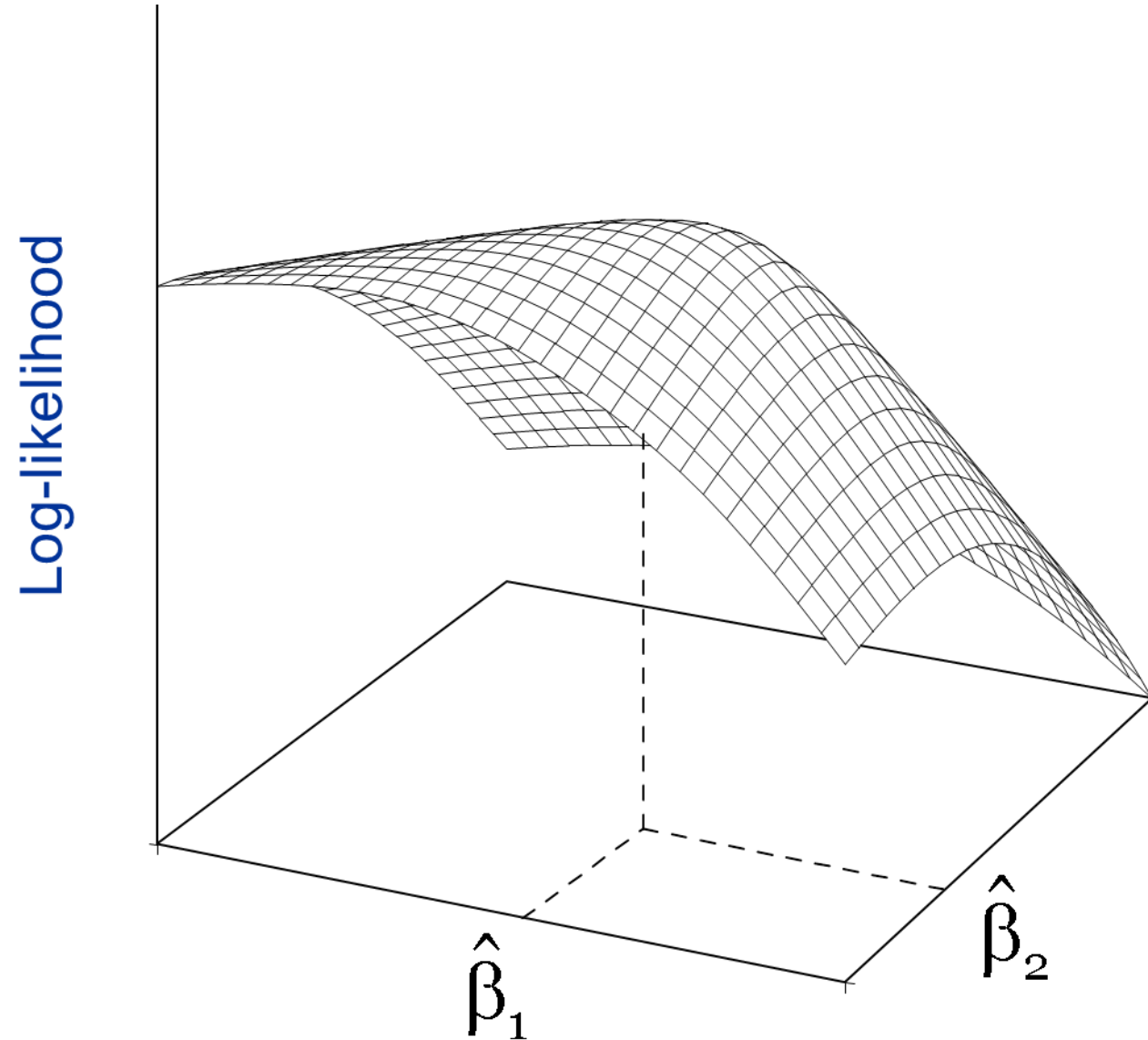
- Logistic regression models transformed probabilities, called *logits*\*,

- 

$$\text{logit}(p_i) = \ln \left( \frac{p_i}{(1 - p_i)} \right) = \beta_0 + \beta_1 X$$

- where
- $i$  indexes all cases (observations)
- $p_i$  is the probability that the event (a sale, for example) occurs in the  $i^{\text{th}}$  case
- $\ln$  is the natural log (to the base e).
- \* The logit is the natural log of the odds.

# Maximum Likelihood Estimation



# Model Fit Statistics

Model Fit Statistics		
Criterion	Intercept Only	Intercept and Covariates
AIC	1416.620	1415.301
SC	1421.573	1425.207
-2 Log L	1414.620	1411.301

- Akaike's information criterion (AIC)

$$AIC = -2\text{Log} (L) + 2k$$

- Schwarz Bayesian information criterion (SC)

$$SC = -2\text{Log} (L) + k \log (n)$$

- Smaller values indicate a better model.

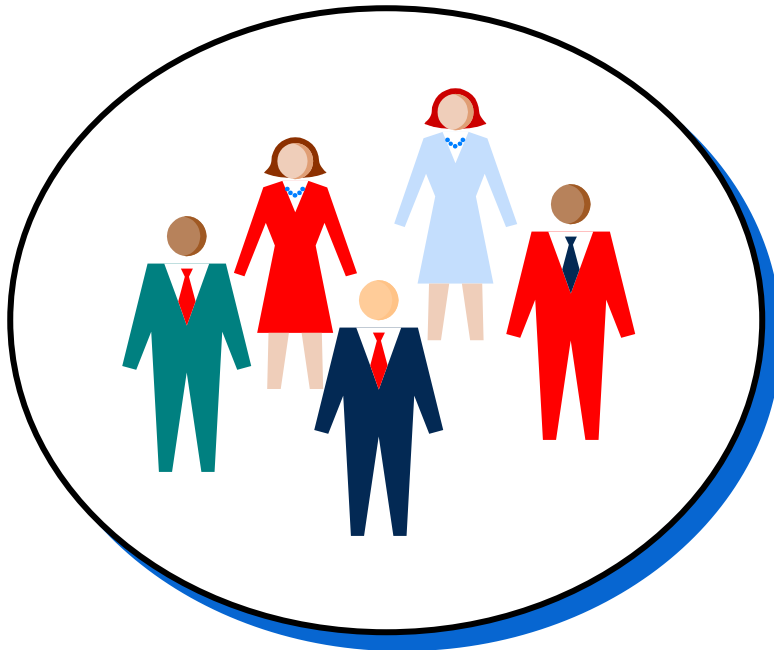
# Predictive Accuracy

- Examining the percentage of concordant, discordant, and tied pairs is a way to assess the predictive accuracy of the model.
- In general, you want a high percentage of concordant pairs and a low percentage of discordant pairs.

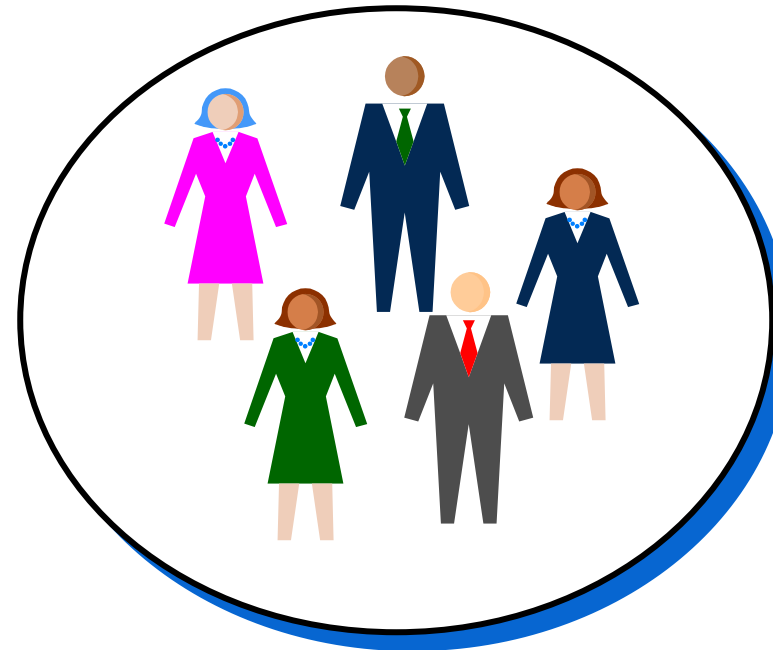
# Comparing Pairs

- To find concordant, discordant, and tied pairs, compare everyone who had the outcome of interest against everyone who did not.

Died



Survived





# Concordant Pair

- Compare a 20-year-old who survived with a 30-year-old who did not.

Died, Age 30



$P(\text{Survived}) = .4077$

Survived, Age 20



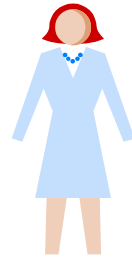
$P(\text{Survived}) = .4272$

The actual sorting agrees with the model.  
This is a **concordant** pair.

# Discordant Pair

- Compare a 45-year-old who survived with a 35-year-old who did not.

Died, Age 35



$P(\text{Survived}) = .3981$

Survived, Age 45



$P(\text{Survived}) = .3791$

The actual sorting disagrees with the model.  
This is a **discordant** pair.

# Tied Pair

- Compare two 50-year-olds. One survived and the other did not.

Died, Age 50



$P(\text{Survived}) = .3697$

Survived, Age 50



$P(\text{Survived}) = .3697$

The model cannot distinguish between the two.  
This is a **tied** pair.

# Model: Concordant, Discordant, and Tied Pairs

Association of Predicted Probabilities and Observed Responses			
Percent Concordant	51.3	Somers' D	0.050
Percent Discordant	46.4	Gamma	0.051
Percent Tied	2.3	Tau-a	0.024
Pairs	264313	c	0.525

# Quasi-Complete Separation

Model Convergence Status
Quasi-complete separation of data points detected.

Analysis of Maximum Likelihood Estimates						
Parameter		DF	Estimate	Standard Error	Wald Chi-Square	Pr > ChiSq
Intercept		1	0.2007	0.4495	0.1993	0.6553
Group	A	1	-1.5870	0.6169	6.6172	0.0101
Group	B	1	-13.7451	225.5	0.0037	0.9514

Odds Ratio Estimates			
Effect	Point Estimate	95% Wald Confidence Limits	
Group A vs C	0.205	0.061	0.685
Group B vs C	<0.001	<0.001	>999.999

# Quasi-Complete Separation

Table of Group by Outcome			
Group	Outcome		
Frequency	0	1	Total
A	28	7	35
B	15	0	15
C	9	11	20
Total	52	18	70

# LOGISTIC Procedure

```
PROC LOGISTIC <options>;  
  CLASS variable</v-options>;  
  MODEL response = <effects></options>;  
  CONTRAST 'label' effect values</options>;  
  EXACT <'label'><Intercept><effects></options>;  
  ODDSRATIO <'label'> variable </ options>;  
  ROC <'label'> <specification> </ options>;  
  ROCCONTRAST <'label'><contrast></ options>;  
  SCORE <options>;  
  STRATA effects</options>;  
  UNITS predictor1=list1 </option>;  
  OUTPUT <OUT=SAS-data-set> keyword=name...  
    keyword=name></option>;  
  
RUN;
```



# Fitting Simple Binary Logistic Regression Models

This demonstration illustrates the concepts discussed previously.



# Demo – Viya – Building Models

[https://www.sas.com/en\\_us/learn/academic-programs.html](https://www.sas.com/en_us/learn/academic-programs.html)



# Questions?

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