

# Diskrētā matemātika. Mājasdarbs.

## Matemātiskā indukcija un rekurentie vienādojumi

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1. uzdevums: 
$$\sum_{k=1}^n (3k^2 + k - 2) = n^3 + 2n^2 - n$$

1. Indukcijas bāze: 
$$\sum_{k=1}^1 (3k^2 + k - 2) \stackrel{?}{=} 1^2 + 2 \cdot 1^2 - 1$$

$$(3 \cdot 1^2 + 1 - 2) \stackrel{?}{=} 1 + 2 - 1$$

$$4 - 2 \stackrel{?}{=} 3 - 1$$

$$2 = 2$$

2. Indukcijas pieņēmums: 
$$\sum_{k=1}^{m-1} (3k^2 + k - 2) = (m-1)^3 + 2 \cdot (m-1)^2 - (m-1)$$

3. Indukcijas pāreja: 
$$\sum_{k=1}^m (3k^2 + k - 2) \stackrel{?}{=} m^3 + 2m^2 - m$$

$$\sum_{k=1}^{m-1} (3k^2 + k - 2) + (3m^2 + m - 2) \stackrel{?}{=} m^3 + 2m^2 - m$$

$$(m-1)^3 + 2 \cdot (m-1)^2 - (m-1) + (3m^2 + m - 2) \stackrel{?}{=} m^3 + 2m^2 - m$$

$$\underline{\underline{m^3 - 3m^2}} + \underline{\underline{3m}} - \underline{\underline{1}} + \underline{\underline{2m^2 - 4m}} + \underline{\underline{2 - m}} + \underline{\underline{1}} + \underline{\underline{3m^2}} + \underline{\underline{m}} - \underline{\underline{2}} \stackrel{?}{=} m^3 + 2m^2 - m$$

$$m^3 + 2m^2 - m = m^3 + 2m^2 - m$$

**Q.E.D**

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2. uzdevums:  $u_{n+2} = 10u_{n+1} - 24u_n; \quad u_0 = 0; \quad u_1 = 2$

$$r^2 = 10r - 24$$

$$r^2 - 10r + 24 = 0$$

$$r_1 = 2; r_2 = 10 \quad (\text{Vjeta teorēma})$$

$$u_n = C_1 \cdot 2^n + C_2 \cdot 10^n$$

$$\begin{cases} u_0 = C_1 \cdot 2^0 + C_2 \cdot 10^0 \\ u_1 = C_1 \cdot 2^1 + C_2 \cdot 10^1 \end{cases}$$

$$\begin{cases} u_0 = C_1 \cdot 2^0 + C_2 \cdot 10^0 = 0 \\ u_1 = C_1 \cdot 2^1 + C_2 \cdot 10^1 = 2 \end{cases}$$

$$u_0 = C_1 \cdot 2^0 + C_2 \cdot 10^0 = 0$$

$$\Rightarrow C_1 + C_2 = 0$$

$$\Rightarrow C_1 = -C_2$$

$$u_1 = C_1 \cdot 2^1 + C_2 \cdot 10^1 = 2$$

$$\Rightarrow 2C_1 + 10C_2 = 2$$

$$\Rightarrow 10C_2 - 2C_2 = 2$$

$$\Rightarrow 8C_2 = 2$$

$$\Rightarrow C_2 = 0,25$$

$$\Rightarrow C_1 = -0,25$$

$$u_n = 0,25 \cdot 2^n - 0,25 \cdot 10^n$$