Diskrētā matemātika. Mājasdarbs.

Matemātiskā indukcija un rekurentie vienādojumi

Iaroslav Viazmitin, 7. grupa, 27. variants

1. uzdevums: $\sum_{k=1}^{n}(3k^2+k-2) = n^3+2n^2-n$

1. Indukcijas bāze: $\sum_{k=1}^{1}(3k^2+k-2) \stackrel{?}{=} 1^2+2\cdot 1^2-1$

$$(3\cdot 1^2+1-2) \stackrel{?}{=} 1+2-1$$

$$4-2 \stackrel{?}{=} 3-1$$

$$2=2$$

2. Indukcijas pieņēmums: $\sum_{k=1}^{m-1}(3k^2+k-2) = (m-1)^3+2\cdot(m-1)^2-(m-1)$

3. Indukcijas pāreja: $\sum_{k=1}^{m}(3k^2+k-2) \stackrel{?}{=} m^3+2m^2-m$

$$\sum_{k=1}^{m-1}(3k^2+k-2)+(3m^2+m-2) \stackrel{?}{=} m^3+2m^2-m$$

$$(m-1)^3+2\cdot(m-1)^2-(m-1)+(3m^2+m-2) \stackrel{?}{=} m^3+2m^2-m$$

$$m^3\underline{\underline{-3m^2}}+\underline{3m}\underbrace{-1}+\underline{\underline{2m^2}}\underline{-4m}+\underbrace{2}\underline{-m}+\underbrace{1}+\underline{\underline{3m^2}}+\underline{m}\underbrace{-2} \stackrel{?}{=} m^3+2m^2-m$$

$$m^3+2m^2-m=m^3+2m^2-m \qquad \textbf{Q.E.D}$$

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2. uzdevums: $u_{n+2}=10u_{n+1}-24u_n;\ \ u_0=0;\ u_1=2$

$r^2=10r-24$

$r^2-10r+24=0$

$r_1=2; r_2=10 \quad$ (Vjeta teorēma)

$u_n=\mathrm{C}_1\cdot 2^n+\mathrm{C}_2\cdot 10^n$

$$\begin{cases} u_0=\mathrm{C}_1\cdot 2^0+\mathrm{C}_2\cdot 10^0 \\ u_1=\mathrm{C}_1\cdot 2^1+\mathrm{C}_2\cdot 10^1 \end{cases}$$

$u_0=\mathrm{C}_1\cdot 2^0+\mathrm{C}_2\cdot 10^0=0$

$\Rightarrow \mathrm{C}_1+\mathrm{C}_2=0$

$\Rightarrow \mathrm{C}_1=-\mathrm{C}_2$

$u_1=\mathrm{C}_1\cdot 2^1+\mathrm{C}_2\cdot 10^1=2$

$\Rightarrow 2\mathrm{C}_1+10\mathrm{C}_2=2$

$\Rightarrow 10\mathrm{C}_2-2\mathrm{C}_2=2$

$\Rightarrow 8\mathrm{C}_2=2$

$\Rightarrow \mathrm{C}_2=0,25$

$\Rightarrow \mathrm{C}_1=-0,25$

$$\boxed{u_n=0,25\cdot 2^n-0,25\cdot 10^n}$$