

Varbūtība

25.02.2026 | Praktiskā nodarbība | Aija Pola

↗ 5.1. uzdevums

1)

uztaisīt blīvuma f-jas grafiku

$$\int_{-\infty}^{+\infty} f(x)dx = 1$$

$$\int_0^1 a(1 - x^3)dx = 1$$

$$a \int_0^1 (1 - x^3)dx = a \left(\int_0^1 1dx - \int_0^1 x^3dx \right) = a \cdot \left(x - \frac{x^4}{4} \right) \Big|_0^1 =$$

$$= a \left(\frac{3}{4} - 0 \right)$$

$$1 = a \frac{3}{4}$$

$$a = \frac{4}{3}$$

2)

$$F(x) = \begin{cases} \int_{-\infty}^x 0dx & = 0 & x \leq 0 \\ \int_{-\infty}^0 0dx + \int_0^x \frac{4}{3}(1 - x^3)dx & = \frac{4}{3} \left(x - \frac{x^4}{4} \right) & 0 < x \leq 1 \\ \int_{-\infty}^0 0dx + \int_0^1 \frac{4}{3}(1 - x^3)dx + \int_1^x 0dx & = 1 & x > 1 \end{cases}$$

3)

$$\boxed{P(x_1 < X < x_2) = F(x_2) - F(x_1)}$$

$$P\left(\frac{1}{3} < X < \frac{2}{3}\right) = F\left(\frac{2}{3}\right) - F\left(\frac{1}{3}\right) = \frac{4}{3} \left(\frac{2}{3} - \frac{1}{4} \left(\frac{2}{3} \right)^4 \right) - \frac{4}{3} \left(\frac{1}{3} - \frac{1}{4} \left(\frac{1}{3} \right)^4 \right) \approx 0,38$$

4)

$$\boxed{P(X < x) = F(x)}$$

$$P(X < 0,5) = F(0,5) = \frac{4}{3} \left(0,5 - \frac{1}{4} (0,5)^4 \right) \approx 0,65$$

↗ 5.2. uzdevums

2)

$$f(x) = F'(x) \begin{cases} 0 & x \leq 0 \\ 0,2 + 2bx & 0 < x \leq 10 \\ 0 & x > 10 \end{cases}$$

3)

$$\begin{aligned} E(X) &= \int_{-\infty}^{+\infty} x \cdot f(x) dx = \\ &= \int_{-\infty}^0 x \cdot 0 dx + \int_0^{10} x \cdot (0,2 + 2bx) dx + \int_{10}^{+\infty} x \cdot 0 dx \approx 3,3(3) \end{aligned}$$

↗ 5.3. uzdevums

1)

$f(x)$:  [Desmos](#)

$$A = 3$$

2)

$$E(X) = 1,5$$

3)

$$D(X) = E(X^2) - E^2(X)$$

$$E(X^2) = \int_1^{\infty} x^2 \frac{3}{x^4} dx = 3 \int_1^{\infty} \frac{1}{x^2} dx = 3 \left(-\frac{1}{x} \Big|_1^{\infty} \right) = 3$$

$$D(X) = 3 - 1,5^2 = 0,75$$

4)

$$\sigma = \sqrt{D(X)} \approx 0,866$$

5)

$$F(X) = \int f(x) dx = \begin{cases} 0 & x < 1 \\ \frac{1}{x^3} & x \geq 1 \end{cases}$$

6)