Assignment 2: Finite Difference Time Domain Approach for 1D Heat Equation

$$\frac{dU}{dt} = \alpha \frac{d^2U}{dx^2}$$

where,

- U is a function U(x,t)
- x is a spatial point having total length N and discretization size dx
- t is a temporal point having total time T and discretization size dt
- α is the thermal diffusivity given as α = k / (c ρ)
- k is the thermal conductivity (Air = 0.026 W m^{-1} K^{-1}, Water = 0.6089 W m^{-1} K^{-1}, Concrete = 0.92 W m^{-1} K^{-1}, Copper = 384.1 W m^{-1} K^{-1}, Diamond = 895 W m^{-1} K^{-1})
- c is the heat capacity (Air = 1.0035 J g^{-1} K^{-1}, Water = 4.1813 J g^{-1} K^{-1}, Concrete = 0.880 J g^{-1} K^{-1}, Coppper = 0.385 J g^{-1} K^{-1}, Diamond = 0.5091) J g^{-1} K^{-1})
- ρ is the density of material (Air = 1.184 Kg m^{-3}, Water = 997.0479 Kg m^{-3}, Concrete = 2400 Kg m^{-3}, Copper = 8940 Kg m^{-3}, Diamond = 3500 Kg m^{-3})

For this ODE, do the following:

1. Implement simulation in C/C++. Your code should be able to plot the results for each time-step. A number of **image files** should be generated (Use T = 20, dt = 0.05, N = 1, dx = 0.0005, Apply Heat = 1 exactly at N/2).

2. Plot the Mean Residual Error over Time

Plot Function for Generating Images (1dheat_images_xx.jpg)

```
/*   Code for simple Plot:
     Steps     = Number of Iterations
     dt        = timestep
     x         = Your 1D-array in which all values are stored
*/
void plot(int steps, double dt, double *x)
{
     FILE *gplot = popen("gnuplot -persistent", "w");
     fprintf(gplot, "set term jpeg\n");
     fprintf(gplot, "set output \"1dheat_images_%d.jpg\"\n", steps);
     fprintf(gplot, "plot '-' u 1:2 title 'x' with lines\n");
     int i;
     for (i = 0; i <= steps; i++) {
          fprintf(gplot,"%lf %lf\n", i*dt, x[i]*100/3000);
     }
     fprintf(gplot,"e");
}
```

Sample Output

A sample video is generated from image files generated from the above code.

Additional Cases: 1D Wave Equation (Not compulsory; An Attempt can Earn you Bonus Marks)

$$\frac{d^2U}{dt^2} = c^2 \frac{d^2U}{dx^2}$$

where,

- U is a function U(x,t)
- x is a spatial point having total length N and discretization size dx
- t is a temporal point having total time T and discretization size dt
- c is a constant

For this ODE, do the following:

1. Implement simulation in C/C++. Your code should be able to plot the results for each time-step. A number of image files should be generated (Use T = 20, dt = 0.05, N = 1, dx = 0.0005).

2. Plot the Mean Residual Error over Time