Lecture 3
Search Space Pruning

Today

Pruning techniques for enumerative search

- Equivalence reduction
- Top-down specification propagation

The problem statement

Behavioral constraints = examples

```
[1,4,7,2,0,6,9,2,5]  →  [1,2,4,7,0]
[0] → [0]
[5,1] → [1,5,0]
```

Search strategy?

Structural constraints = grammar

```
L ::= sort(L)  |  L[N..N]
   |  L + L  |  [N]  |  x
N ::= find(L,N)  |  0
```

Enumerative search

=

Explicit / Exhaustive Search

Idea: Sample programs from the grammar one by one and test them on the examples

```
L ::= sort(L)    |
      L[N..N]    |
      L + L      |
      [N]        |
      x
N ::= find(L,N)  |
      0
```

bottom-up

```
x   0

sort(x)   x[0..0]   x + x   [0]
find(x,0)

sort(sort(x))   sort(x[0..0])
sort(x + x)     sort([0])
x[0..find(x,0)]   ...
```

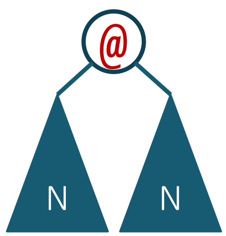
top-down

```
L

x   sort(L)   L[N..N]   L + L   [N]

sort(x)   sort(sort(L))   sort([N])
sort(L[N..N])   sort(L + L)
x[N..N]   (sort L)[N..N]   ...
```

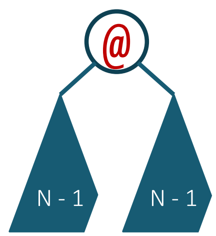
How to make it scale

Prune

Discard useless subprograms



$m * N^2$



$m * (N - 1)^2$

Prioritize

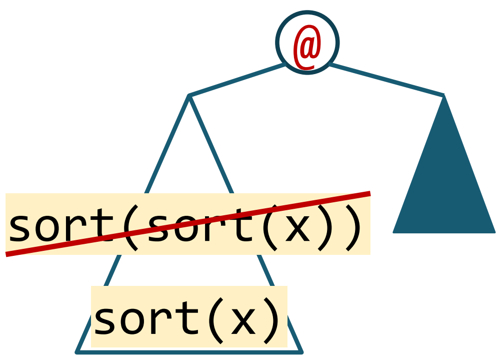
Explore more promising candidates first

```
P = { [0][N..N] ,
      x[N..N]   ,
      ... }
```

dequeue this first

When can we discard a subprogram?

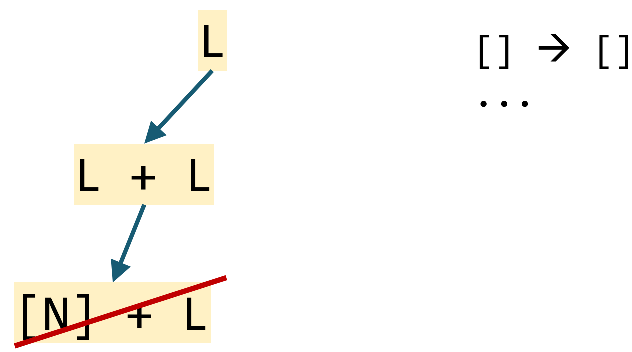
It's equivalent to something we have already explored



Equivalence reduction

(also: symmetry breaking)

No matter what we combine it with, it cannot satisfy the spec



Top-down propagation

Equivalent programs

```
L ::= sort(L)     |
      L[N..N]     |
      L + L       |
      [N]         |
      x
N ::= find(L,N)   |
      0
```

bottom_up →

```
x   0

sort(x)  x[0..0]  x + x  [0]  find(x,0)

sort(sort(x))  sort(x + x)  sort(x[0..0])
sort([0])  x[0..find(x,0)]  x[find(x,0)..0]
x[find(x,0)..find(x,0)]  sort(x)[0..0]
x[0..0][0..0]  (x + x)[0..0]  [0][0..0]
x + (x + x)  x + [0]  sort(x) + x  x[0..0] + x
(x + x) + x  [0] + x  x + x[0..0]  x + sort(x)
...
```

Equivalent programs

```
L ::= sort(L)     |
      L[N..N]     |
      L + L       |
      [N]         |
      x
N ::= find(L,N)   |
      0
```

bottom_up

```
x  0

sort(x)  x[0..0]  x + x  [0]  find(x,0)

sort(sort(x))  sort(x + x)  sort(x[0..0])
sort([0])  x[0..find(x,0)]  x[find(x,0)..0]
x[find(x,0)..find(x,0)]  sort(x)[0..0]
x[0..0][0..0]  (x + x)[0..0]  [0][0..0]
x + (x + x)  x + [0]  sort(x) + x  x[0..0] + x
(x + x) + x  [0] + x  x + x[0..0]  x + sort(x)

...
```

Equivalent programs

```
L ::= sort(L)      |
      L[N..N]      |
      L + L        |
      [N]          |
      x
N ::= find(L,N)    |
      0
```

bottom_up →

```
x   0

sort(x)   x[0..0]   x + x   [0]   find(x,0)

sort(x + x)
x[0..find(x,0)]

x + (x + x)   x + [0]   sort(x) + x
[0] + x   x + sort(x)

...
```

Bottom-up + equivalence reduction

```
bottom-up (<T, N, R, S>, [i → o]) {
  P := [t | t in T && t is nullary]
  while (true)
    forall (p in P)
      if (whole(p) && p([i]) = [o])
        return p;
    P += grow(P);
}

grow (P) {
  P' := []
  forall (A ::= rhs in R)
    P' += [rhs[B -> p] | p in P, B →* p]
  return [p' in P' | forall p in P: !equiv(p, p')];
}
```

How do we implement **equiv**?

- In general undecidable
- For SyGuS problems: expensive
- Doing expensive checks on every candidate defeats the purpose of pruning the space!

Observational equivalence

```
bottom-up (<T, N, R, S>, [i → o])
{ ... }

equiv(p, p’) {
  return p([i]) = p’([i])
}
```

```
[[0] → [0]]

x    0

sort(x)  x[0..0]  x + x  [0]  find(x,0)

sort(x + x)
x[0..find(x,0)]

x + (x + x)  x + [0]  sort(x) + x
[0] + x  x + sort(x)
```

In PBE, all we care about is equivalence on the given inputs!

- easy to check efficiently
- even more programs are equivalent

Observational equivalence

```
bottom-up (<T, N, R, S>, [i → o])
{ ... }

equiv(p, p’) {
  return p([i]) = p’([i])
}
```

```
[[0] → [0]]

x    0

sort(x)   x[0..0]   x + x   [0]   find(x,0)

sort(x + x)
x[0..find(x,0)]

x + (x + x)   x + [0]   sort(x) + x
[0] + x   x + sort(x)
```

Observational equivalence

```
bottom-up (<T, N, R, S>, [i → o])
{ ... }

equiv(p, p’) {
  return p([i]) = p’([i])
}
```

[[0] → [0]]

x 0

x[0..0] x + x

x + (x + x)

Observational equivalence

Proposed simultaneously in two papers:

- Udupa, Raghavan, Deshmukh, Mador-Haim, Martin, Alur: TRANSIT: specifying protocols with concolic snippets. PLDI'13
- Albarghouthi, Gulwani, Kincaid: Recursive Program Synthesis. CAV'13

Variations used in most bottom-up PBE tools:

- **ESolver** (baseline SyGuS enumerative solver)
- **Lens** [Phothilimthana et al. ASLPOS'16]
- **EUSolver** [Alur et al. TACAS'17]

User-specifies equations

[Smith, Albarghouthi: VMCAI'19]

Equations

```
sort(sort(l)) = sort(l)
(l1 + l2) + l3 = l1 + (l2 + l3)
n = n + 0
n + m = m + n
```

derived automatically

Term-rewriting system (TRS)

1. `sort(sort(l))` → `sort(l)`
2. `(l1 + l2) + l3` → `l1 + (l2 + l3)`
3. `n + 0` → `n`
4. `n + m` →$_{\texttt{(n > m)}}$ `m + n`

`x` `0`

`sort(x)` `x[0..0]` `x + x` `[0]` `find(x,0)`

~~`sort(sort(x))`~~ rule 1 applies, not in *normal form*

Built-in equivalences

For a predefined set of operations, equivalence reduction can be hard-coded in the tool or built into the grammar

```
L ::= sort(L)    |
      L[N..N]    |
      L + L      |
      [N]        |
      x
N ::= find(L,N)  |
      0
```

```
L  ::= L1  |  L1 + L
L1 ::= sort(L)   |
       L[N..N]   |
       [N]       |
       x
N  ::= find(L,N) |
      0
```

Built-in equivalences

Used by:

- **λ^2** [Feser et al.'15]
- **Leon** [Kneuss et al.'13]

Leon's implementation using *attribute grammars* described in:

- Koukoutos, Kneuss, Kuncak: An Update on Deductive Synthesis and Repair in the LeonTool [SYNT'16]

Equivalence reduction: comparison

Observational
- Very general, no user input required
- Finds more equivalences
- Can be costly (with many examples, large outputs)
- If new examples are added, has to restart the search

User-specified
- Fast
- Requires equations

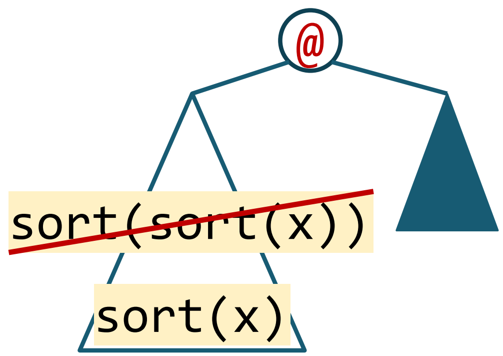
Built-in
- Even faster
- Restricted to built-in operators
- Only certain symmetries can be eliminated by modifying the grammar

Q1: Can any of them apply to top-down?

Q2: Can any of them apply beyond PBE?

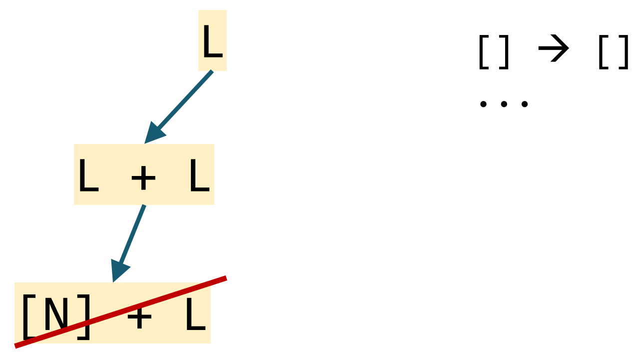
When can we discard a subprogram?

It's equivalent to something we have already explored



Equivalence reduction

No matter what we combine it with, it cannot fit the spec



Top-down propagation

Top-down search: reminder

generates a lot of non-ground terms
only discards ground terms

iter 0: `L`

iter 1: `x` `L[N..N]`

iter 2: `L[N..N]`

iter 3: `x[N..N]` `L[N..N][N..N]`

iter 4: `x[0..N]` `x[find(L,N)..N]` `L[N..N][N..N]`

iter 5: `x[0..0]` `x[0.. find(L,N)]` `x[find(L,N)..N]` …

iter 6: `x[0.. find(L,N)]` `x[find(L,N)..N]` … …

iter 7: `x[0.. find(x,N)]` `x[0.. find(L[N..N],N)]` … … …

iter 8: `x[0.. find(x,0)]` `x[0.. find(x,find(L,N))]` … … … …

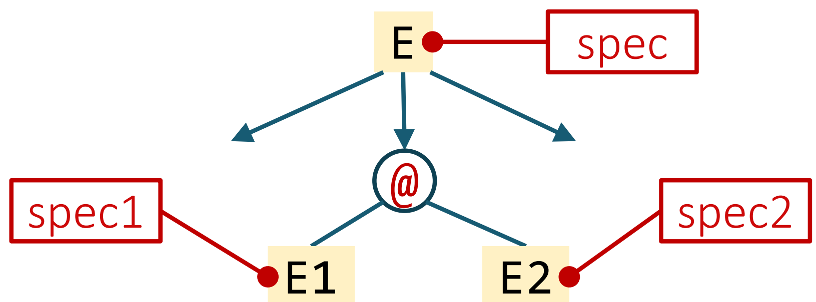
iter 9:

```
L ::= L[N..N]     |
      x
N ::= find(L,N)   |
      0
```

`[[1,4,0,6] → [1,4]]`

Top-down propagation

Idea: once we pick the production, infer specs for subprograms

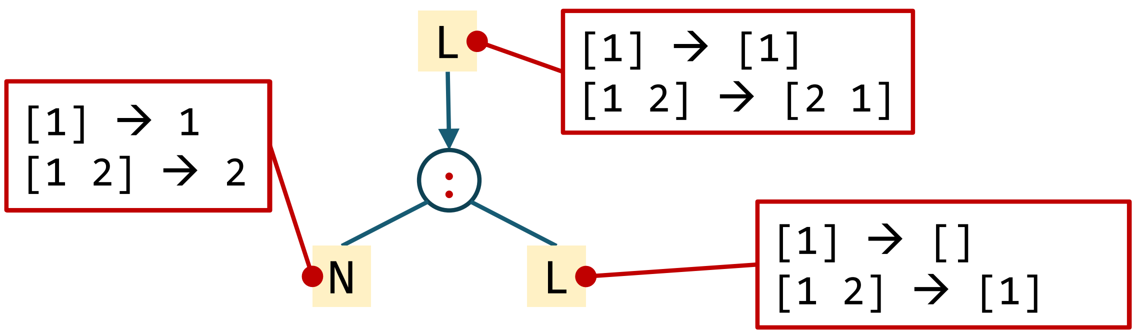


If `spec1 = ⊥`, discard `E1 @ E2` altogether!

For now: spec = examples

When is TDP possible?

Depends on @!

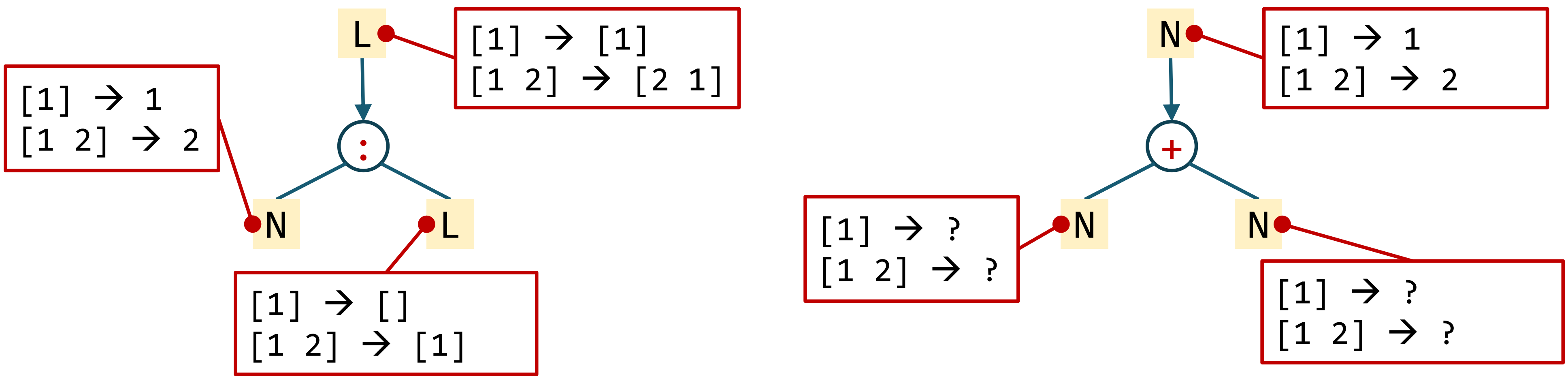


Works when the function is injective!

Q: when would we infer ⊥? A: If at least one of the outputs is [ ]!

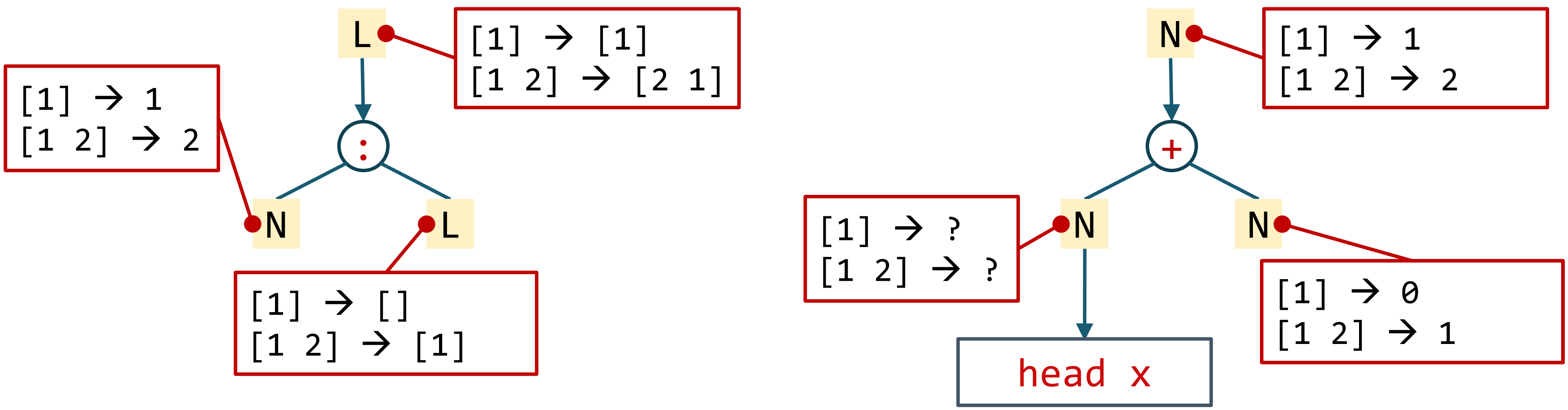
When is TDP possible?

Depends on @!

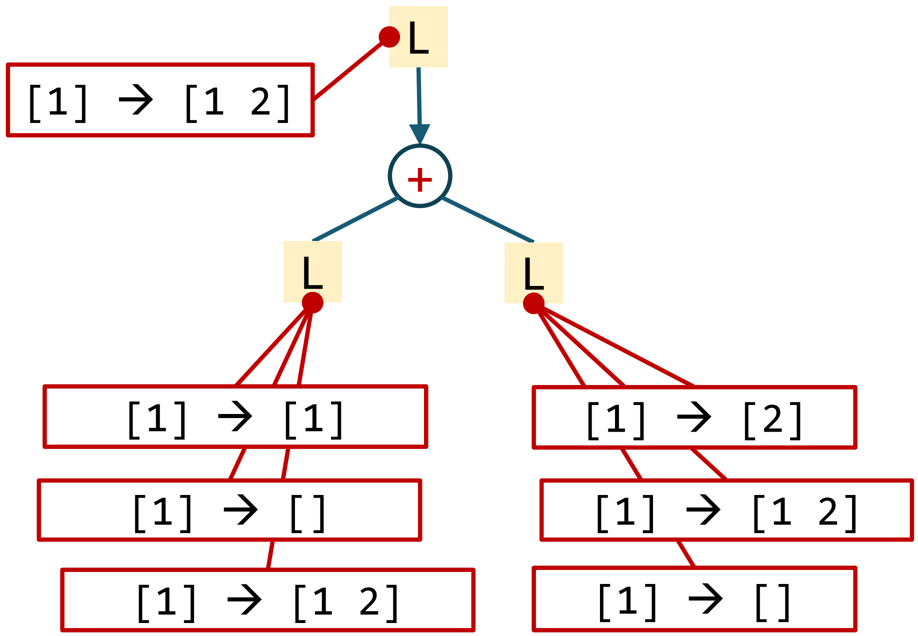


When is TDP possible?

Depends on @!



Something in between?



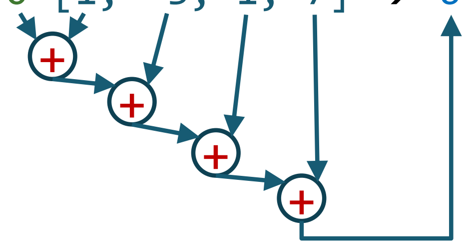
Works when the function is "sufficiently injective"

- output examples have a small pre-image

λ²: TDP for list combinators

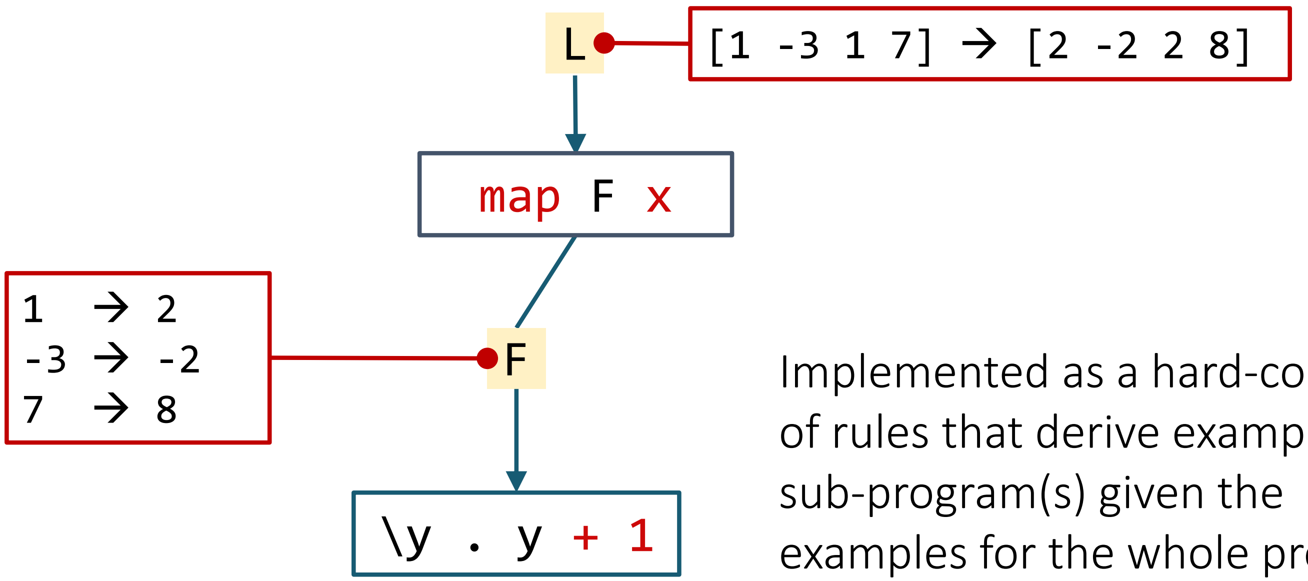
[Feser, Chaudhuri, Dillig '15]

```
map f x           map (\y . y + 1) [1, -3, 1, 7] → [2, -2, 2, 8]

filter f x        filter (\y . y > 0) [1, -3, 1, 7] → [1, 1, 7]

fold f acc x      fold (\y z . y + z) 0 [1, -3, 1, 7] → 6
```



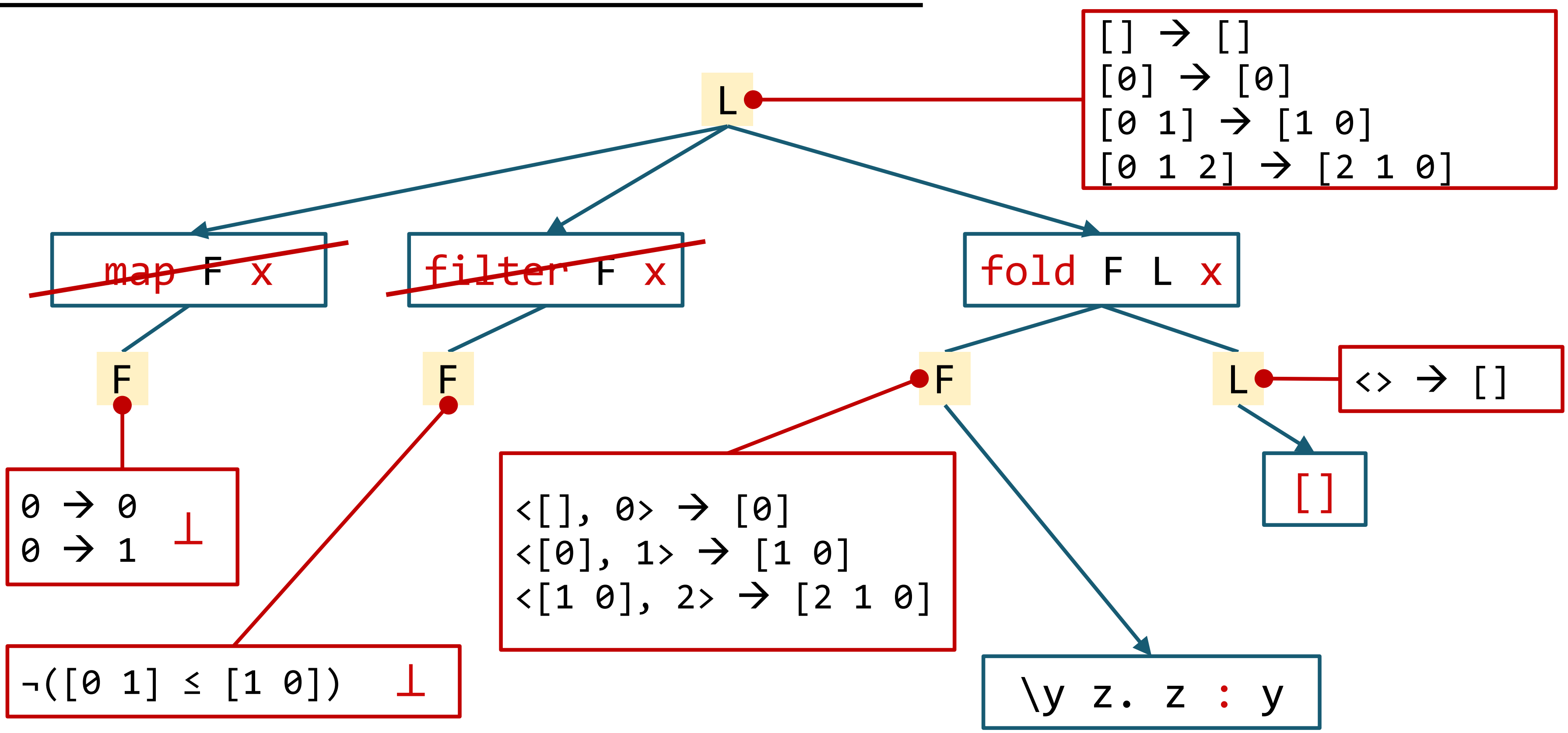

```
                  fold (\y z . y + z) 0 [] → 0
```

λ²: TDP for list combinators



Implemented as a hard-coded set of rules that derive examples for sub-program(s) given the examples for the whole program and the combinator

λ²: TDP for list combinators



Condition abduction

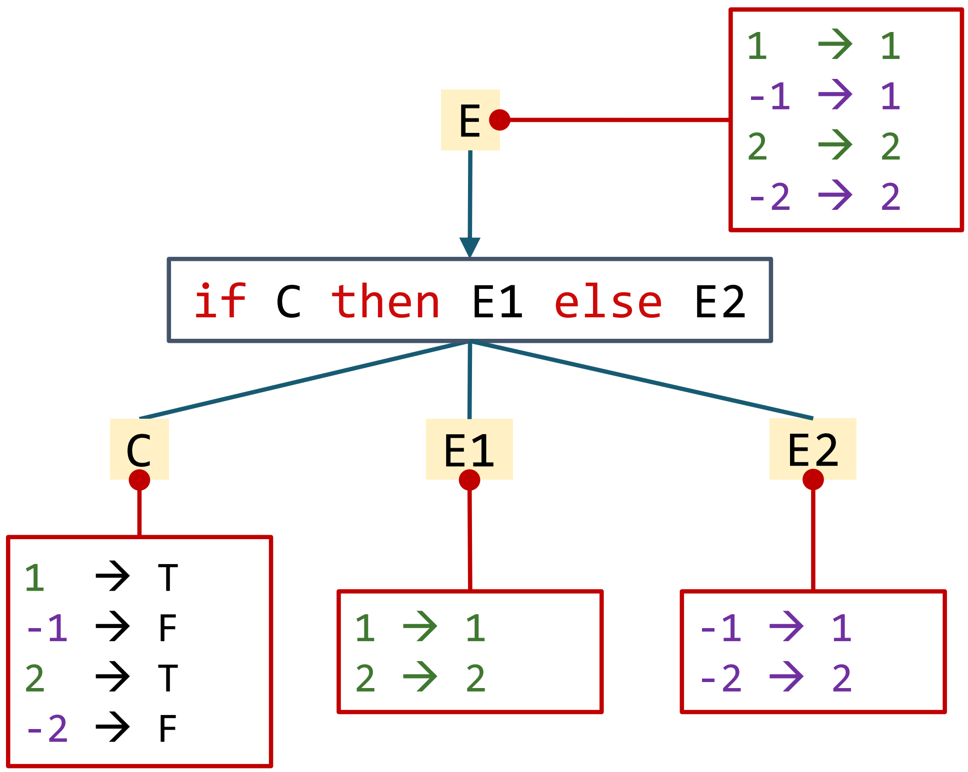
Smart way to synthesize conditionals

Used in many tools (under different names):

- **FlashFill** [Gulwani '11]
- **Escher** [Albarghouthi et al. '13]
- **Leon** [Kneuss et al. '13]
- **Synquid** [Polikarpova et al. '13]
- **EUSolver** [Alur et al. '17]

In fact, an instance of TDP!

Condition abduction



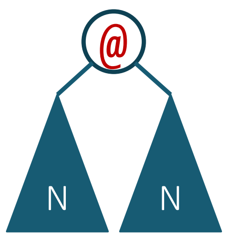
Q: How does EUSolver decide how to split the inputs?

Q: How does EUSolver generate `C`?

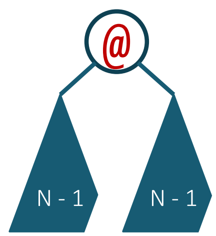
How to make it scale

Prune

Discard useless subprograms



$m * N^2$



$m * (N - 1)^2$

Prioritize

Explore more promising candidates first

```
P = { [0][N..N] ,
      x[N..N]   ,
      ... }
```

dequeue this first