CSCI 420 Computer Graphics
Lecture 11

Lighting and Shading

Light Sources
Phong Illumination Model
Normal Vectors
[Angel Ch. 5]

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Outline

- Global and Local Illumination
- Normal Vectors
- Light Sources
- Phong Illumination Model
- Polygonal Shading
- Example

Global Illumination

- Ray tracing
- Radiosity
  - (diffuse lighting)
- Photon Mapping
  - (rendering equation)
- Follow light rays through a scene
  - Light can bounce hundreds of times.
- Accurate, but expensive (off-line)
  - All light comes from a physical light source.
  - (remember how OpenGL works in contrast!)

Tobias R. Metoc

Raytracing Example

Martin Moeck,
Siemens Lighting

Raytracing Example

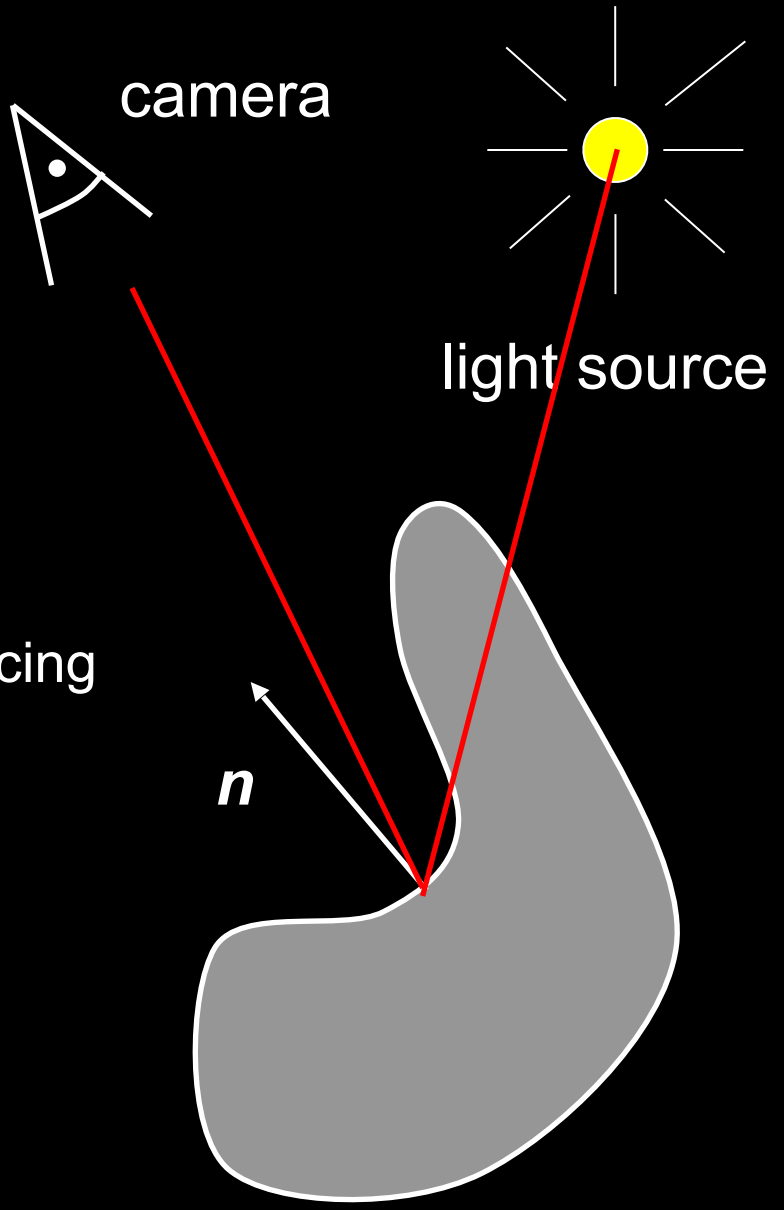
Alita Battle Angel, Weta Digital

Radiosity Example

Restaurant Interior. Guillermo Leal, Evolucion Visual

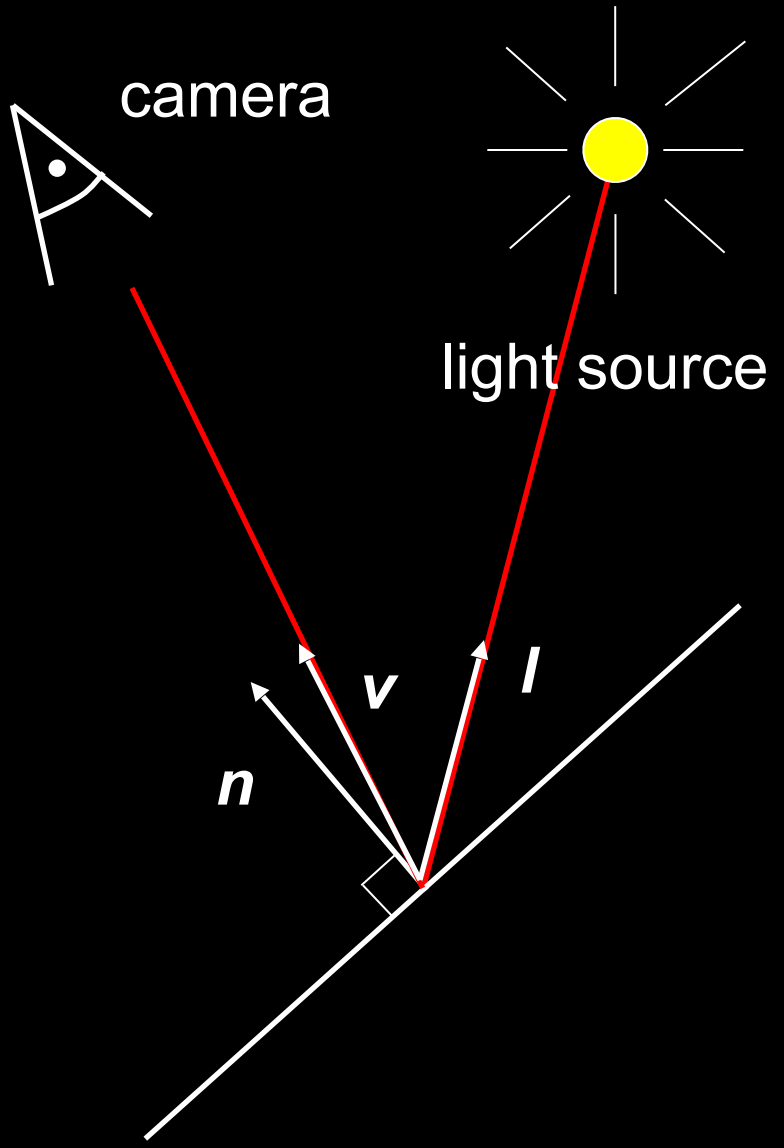
Local Illumination

- Approximate model
- Local interaction between light, surface, viewer
  - No rays, no actual reflection/bouncing
- Phong model (this lecture): fast, supported in OpenGL
- GPU shaders
- Real-time graphics
  - (there are exceptions)



Local Illumination

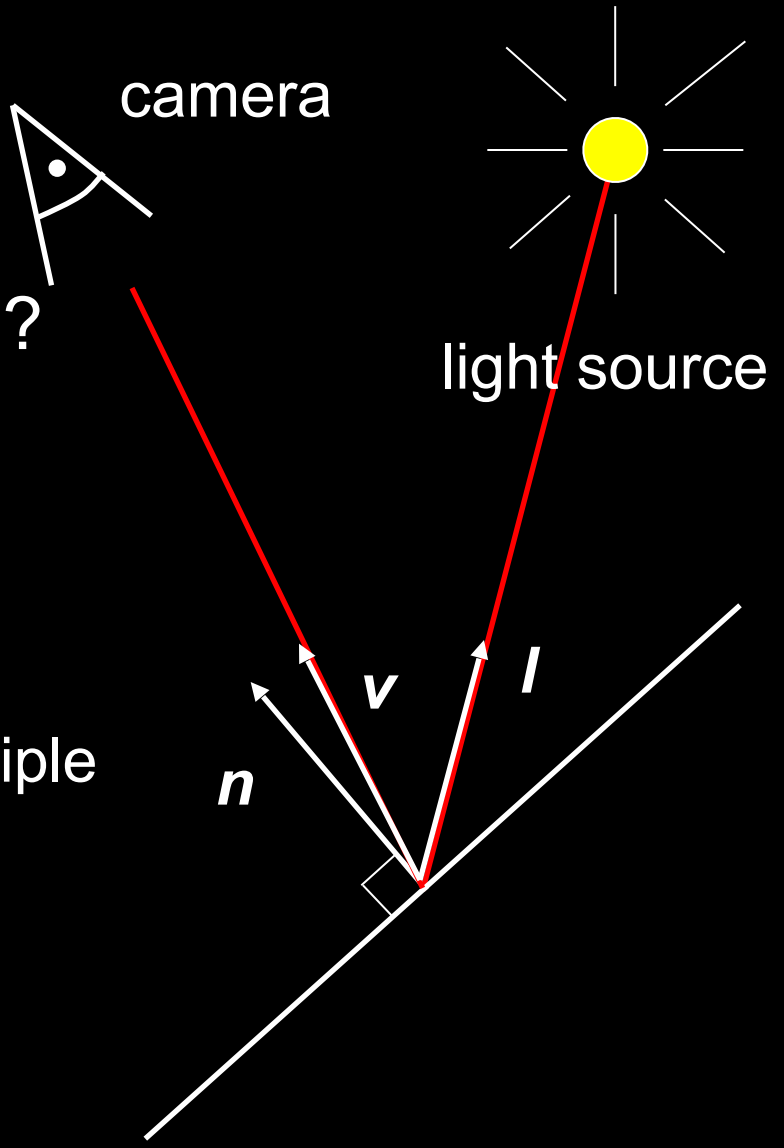
- Approximate model
- Local interaction between light, surface, viewer
- Color determined only based on surface normal, relative camera position and relative light position
- What effects does this ignore?



Local Illumination

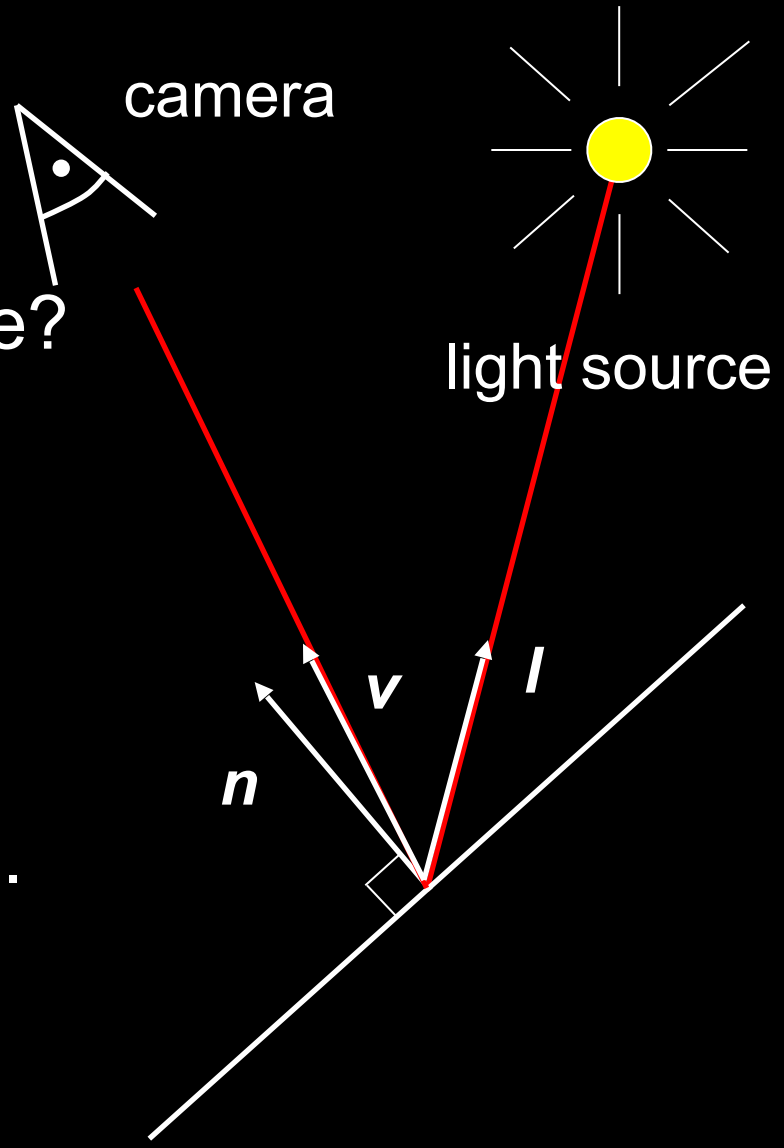
- What effects does this ignore?

- Here are a few, but there are way more:
  - Obstructions in the ray's path
  - Rays hitting the light after multiple bounces
  - Scattering
  - Reflections
  - Sub-surface scattering
  - Transmission
  - Refraction
  - Composite materials



Local Illumination

- What effects does this ignore?
- But it is usually very easy to efficiently compute!
- Modern GPUs add some of these missed effects back in.
  - nvidia RTX ray tracing



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Normal Vectors

- For lighting effects, you need to know the shape's normal vector.

- Must calculate and specify the normal vector
  - Even in OpenGL!
  - Compute the normals on the CPU.
  - Must provide the normals to the GPU so they can be used in the shader.

- Two examples for simple shapes: plane and sphere

Normals of a Plane, Method I

- Method I: given by $ax + by + cz + d = 0$
- Let p_0 be a known point on the plane
- Let p be an arbitrary point on the plane
- Recall: $u \cdot v = 0$ if and only if u orthogonal to v
- $n \cdot (p - p_0) = n \cdot p - n \cdot p_0 = 0$

- Consequently $n_0 = [a \;\; b \;\; c]^T$
- Normalize to $n = n_0/|n_0|$

Normals of a Plane, Method II

- Method II: plane given by p_0, p_1, p_2
- Points must not be collinear
- Recall: u x v orthogonal to u and v

- $n_0 = (p_1 - p_0) \times (p_2 - p_0)$

- Order of cross product determines orientation
- Normalize to $n = n_0/|n_0|$

Normals of Sphere

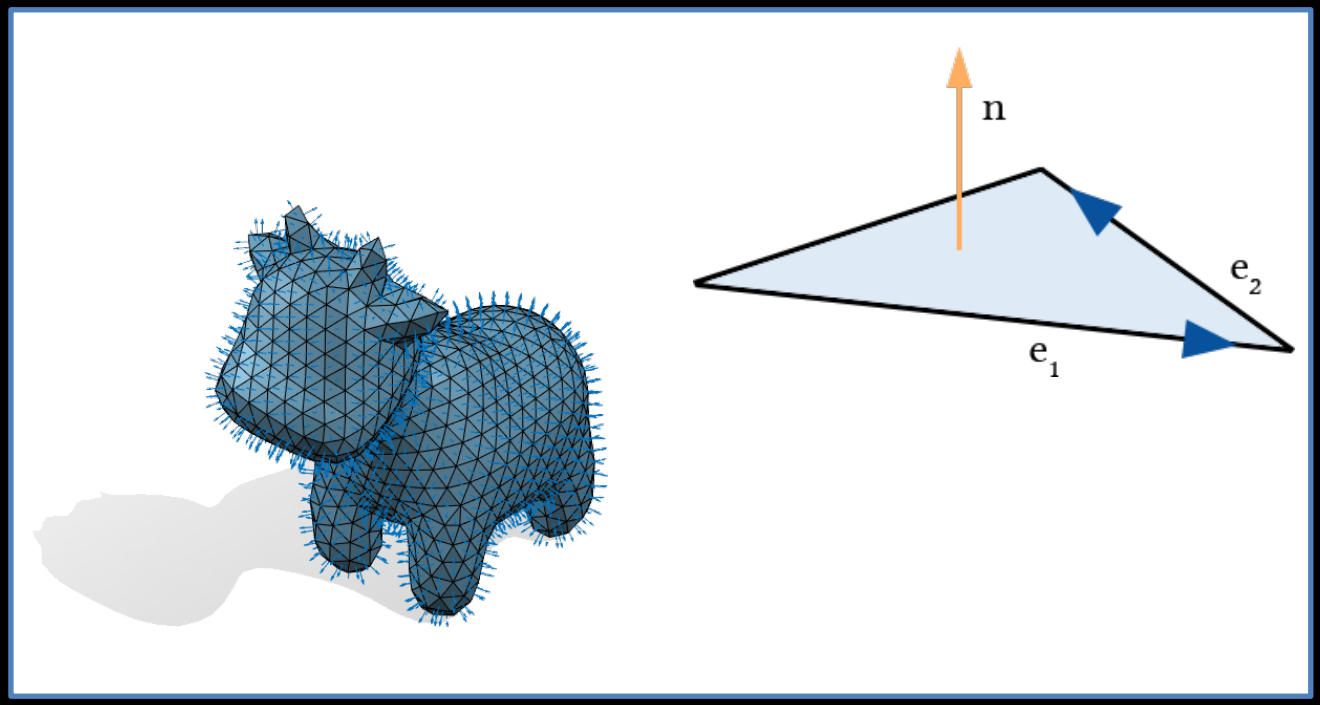
- Implicit Equation $f(x, y, z) = x^2 + y^2 + z^2 - 1 = 0$
- Vector form: $f(p) = p \cdot p - 1 = 0$
- Normal given by gradient vector

$$n_0 = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ \frac{\partial f}{\partial z} \end{bmatrix} = \begin{bmatrix} 2x \\ 2y \\ 2z \end{bmatrix} = 2p$$

- Normalize $n_0/|n_0| = 2p/2 = p$

Normals of a Triangle

- If your geometry is just triangles, this is easy.
- The normal vector $\mathbf{n}$ is the unit-length perpendicular vector to a triangle and positively oriented.
- $\tilde{\mathbf{n}} = \mathbf{e}_1 \times \mathbf{e}_2$, $\mathbf{n} = \tilde{\mathbf{n}}/\|\tilde{\mathbf{n}}\|$



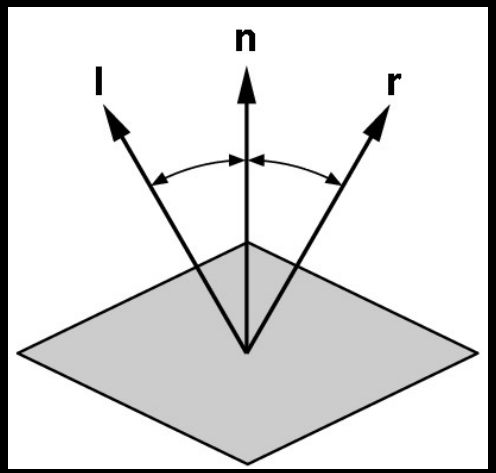
A little bit more linear algebra...

- We already know so much about normal vectors.

- Last two things we need to learn about them today:
  - How do we reflect a vector on another vector?
  - As we transform vertices with our modelview matrix, how do normal vectors transform?

Reflected Vector

- Perfect reflection: angle of incident equals angle of reflection
- Also: ***l***, ***n***, and ***r*** lie in the same plane
- Assume |***l***| = |***n***| = 1, guarantee |***r***| = 1



$$\boldsymbol{l} \cdot \boldsymbol{n} = \cos(\theta) = \boldsymbol{n} \cdot \boldsymbol{r}$$

$$\boldsymbol{r} = \alpha\, \boldsymbol{l} + \beta\, \boldsymbol{n}$$

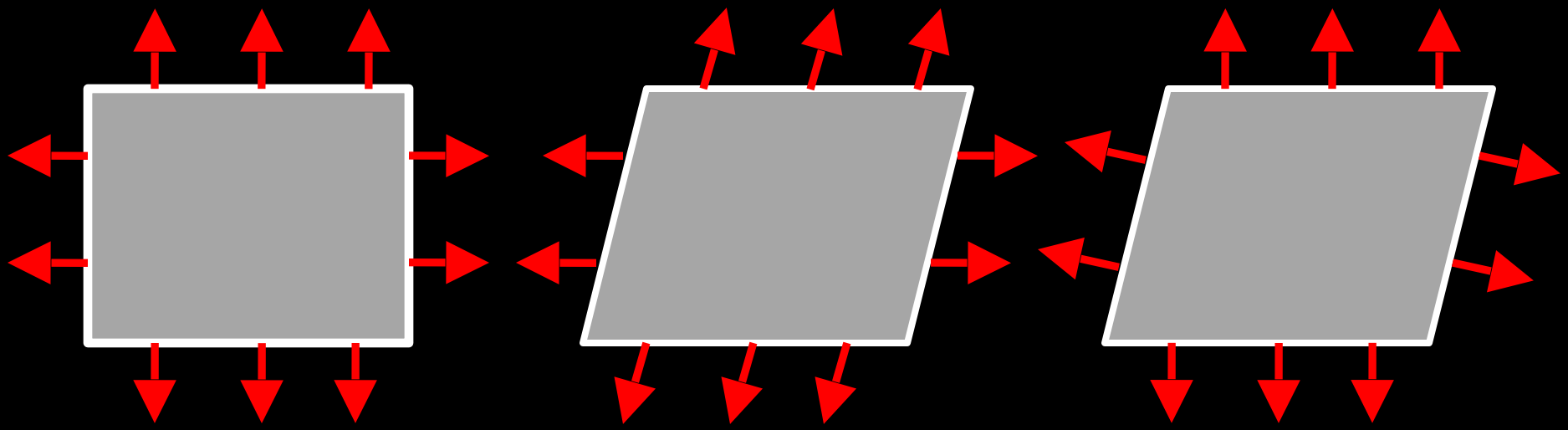
Solution: $\alpha = -1$ and $\beta = 2\,(\boldsymbol{l} \cdot \boldsymbol{n})$

$$\boldsymbol{r} = 2\,(\boldsymbol{l} \cdot \boldsymbol{n})\,\boldsymbol{n} - \boldsymbol{l}$$

Normals Transformed by Modelview Matrix

Modelview matrix *M* (shear in this example)

Only keep linear transform in *M* (discard any translation).



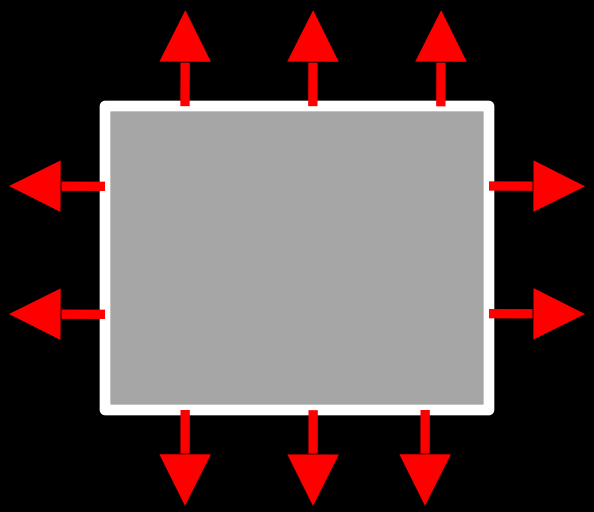
Undeformed

Transformed with *M* (incorrect)

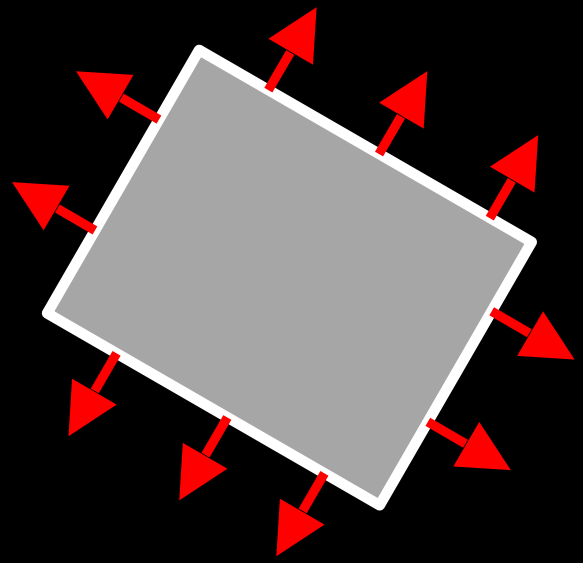
Transformed with $(M^{-1})^T$ (correct)

Normals Transformed by Modelview Matrix

When M is rotation, M = $(M^{-1})^T$



Undeformed



Transformed
with M = $(M^{-1})^T$

(correct)

Normals Transformed by Modelview Matrix
(proof of $(M^{-1})^T$ *transform)*

Point (x,y,z,w) is on a plane in 3D (homogeneous coordinates) if and only if

$a\,x + b\,y + c\,z + d\,w = 0$, or $[a\ b\ c\ d]\,[x\ y\ z\ w]^T = 0$.

Now, let's transform the plane by *M*.

Point (x,y,z,w) is on the transformed plane if and only if

$M^{-1}\,[x\ y\ z\ w]^T$ is on the original plane:

$[a\ b\ c\ d]\,M^{-1}\,[x\ y\ z\ w]^T = 0$.

So, equation of transformed plane is

$[a'\ b'\ c'\ d']\,[x\ y\ z\ w]^T = 0$, for

$[a'\ b'\ c'\ d']^T = (M^{-1})^T\,[a\ b\ c\ d]^T$.

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Light Sources and Material Properties

- Appearance depends on
  - Light sources, their locations and properties
  - Material (surface) properties:

  - Viewer position

Types of Light Sources

- Ambient light: no identifiable source or direction
  - What you have done so far in HW1.
- Point source: given only by point
- Distant light: given only by direction
- Area light: Allows for soft shadows and directionality
- Spotlight: from source in direction
  - Cut-off angle defines a cone of light
  - Attenuation function (brighter in center)

Point Source

- Given by a point p_0
- Light emitted equally in all directions
- Intensity decreases with square of distance

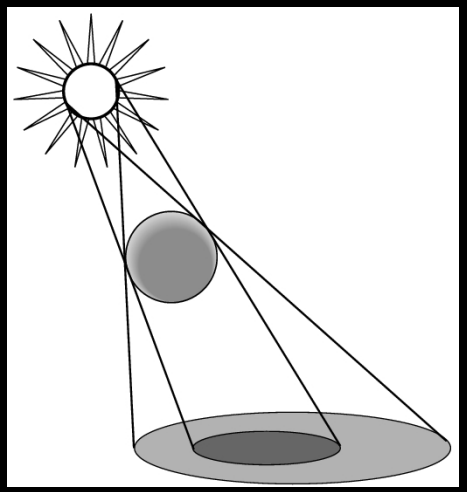
$$I \propto \frac{1}{|p - p_0|^2}$$

Limitations of Point Sources

- Shading and shadows inaccurate
- Example: penumbra (partial “soft” shadow)
- Similar problems with highlights
- Compensate with attenuation

$$\frac{1}{a + bq + cq^2}$$

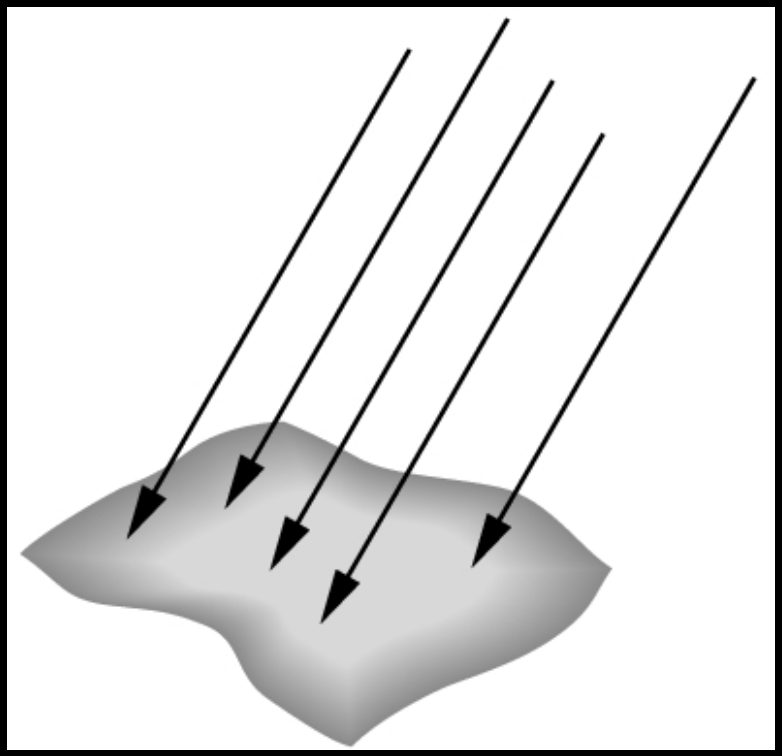
q = distance $|p - p_0|$
a, b, c constants



- Softens lighting
- Better with ray tracing
- Better with radiosity

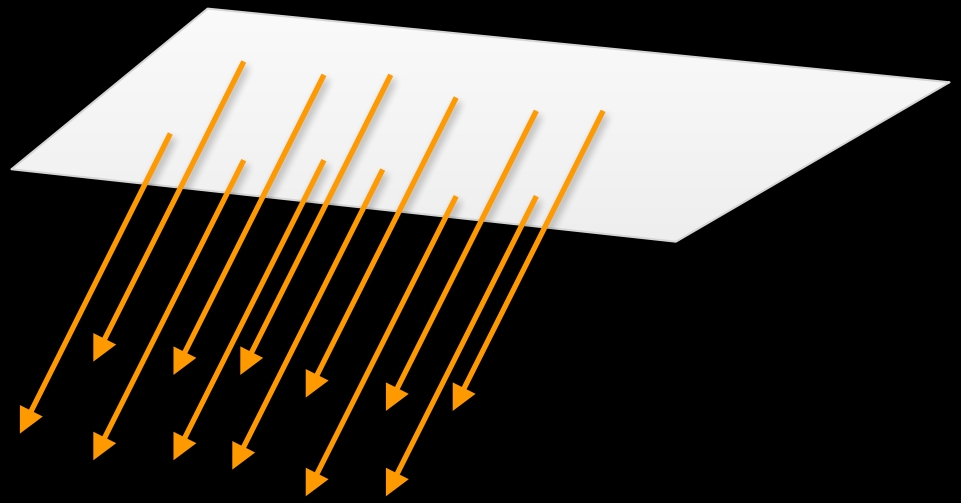
Distant Light Source

- Given by a direction vector [x y z]
- Sun in blender (with artificial soft shadows through sun disk size)



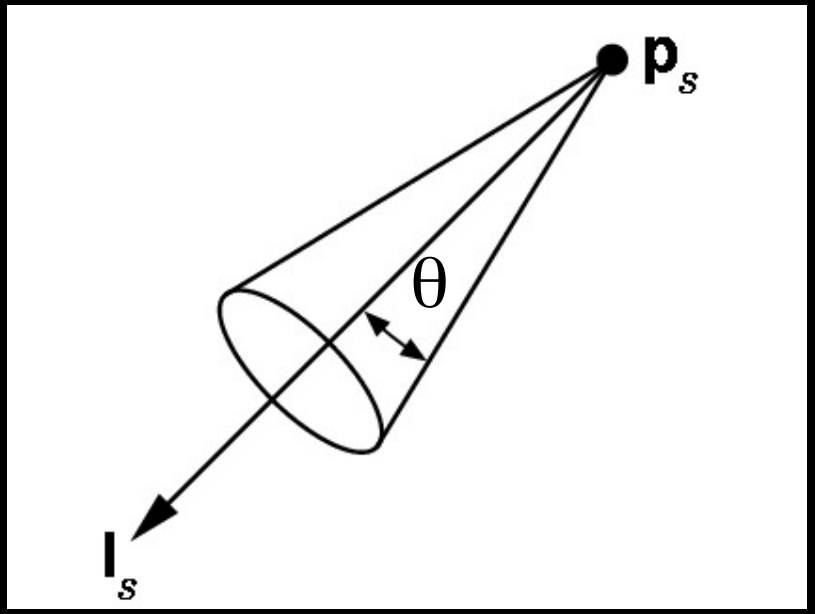
Area Light source

- Light emanated in only one direction from planar source



Spotlight

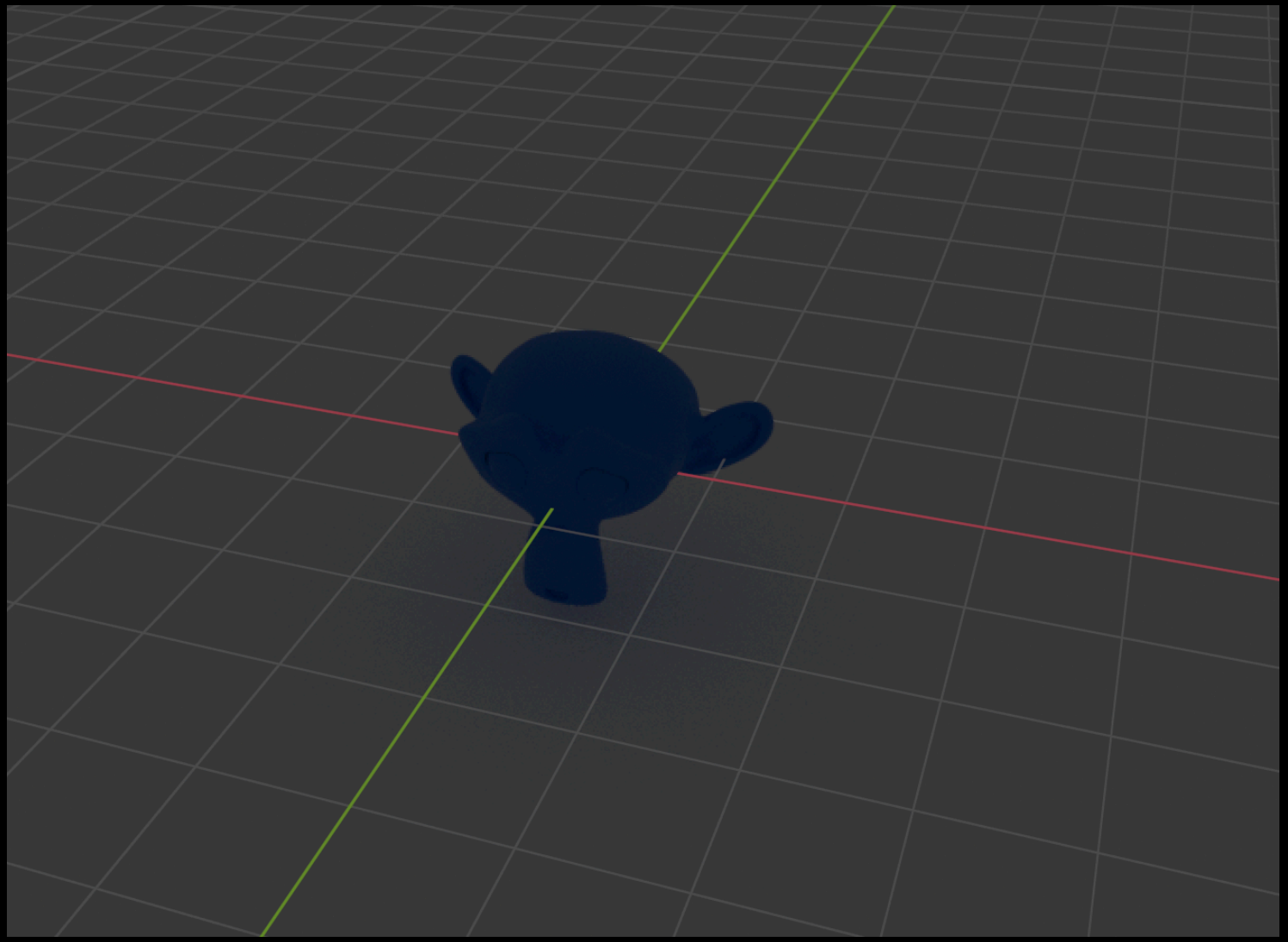
- Light still emanates from point
- Cut-off by cone determined by angle θ



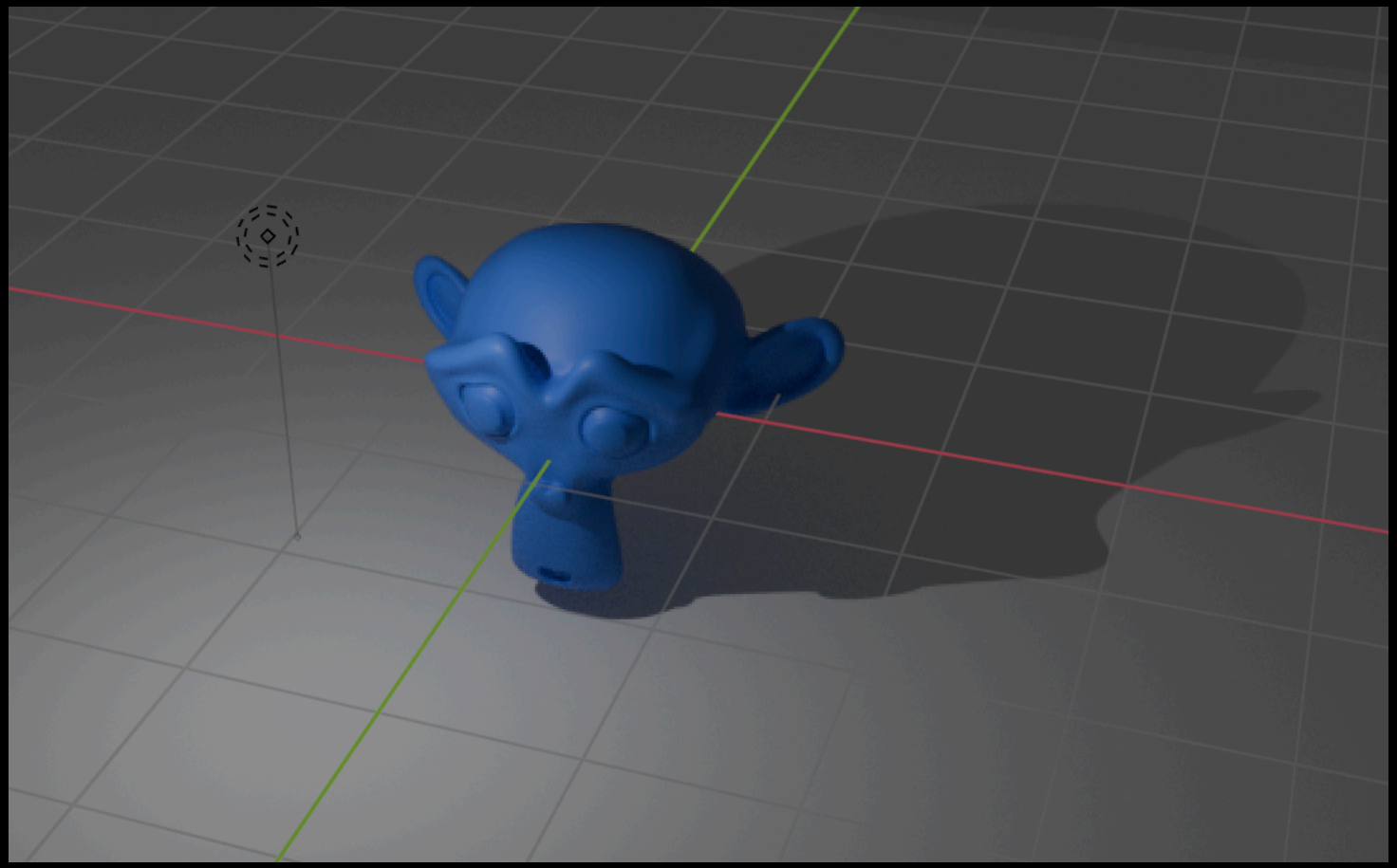
Global Ambient Light

- Independent of light source
- Lights entire scene
- Computationally inexpensive
- Simply add $[G_R\ G_G\ G_B]$ to every pixel on every object
- Not very interesting on its own.
  A cheap hack to make the scene brighter.

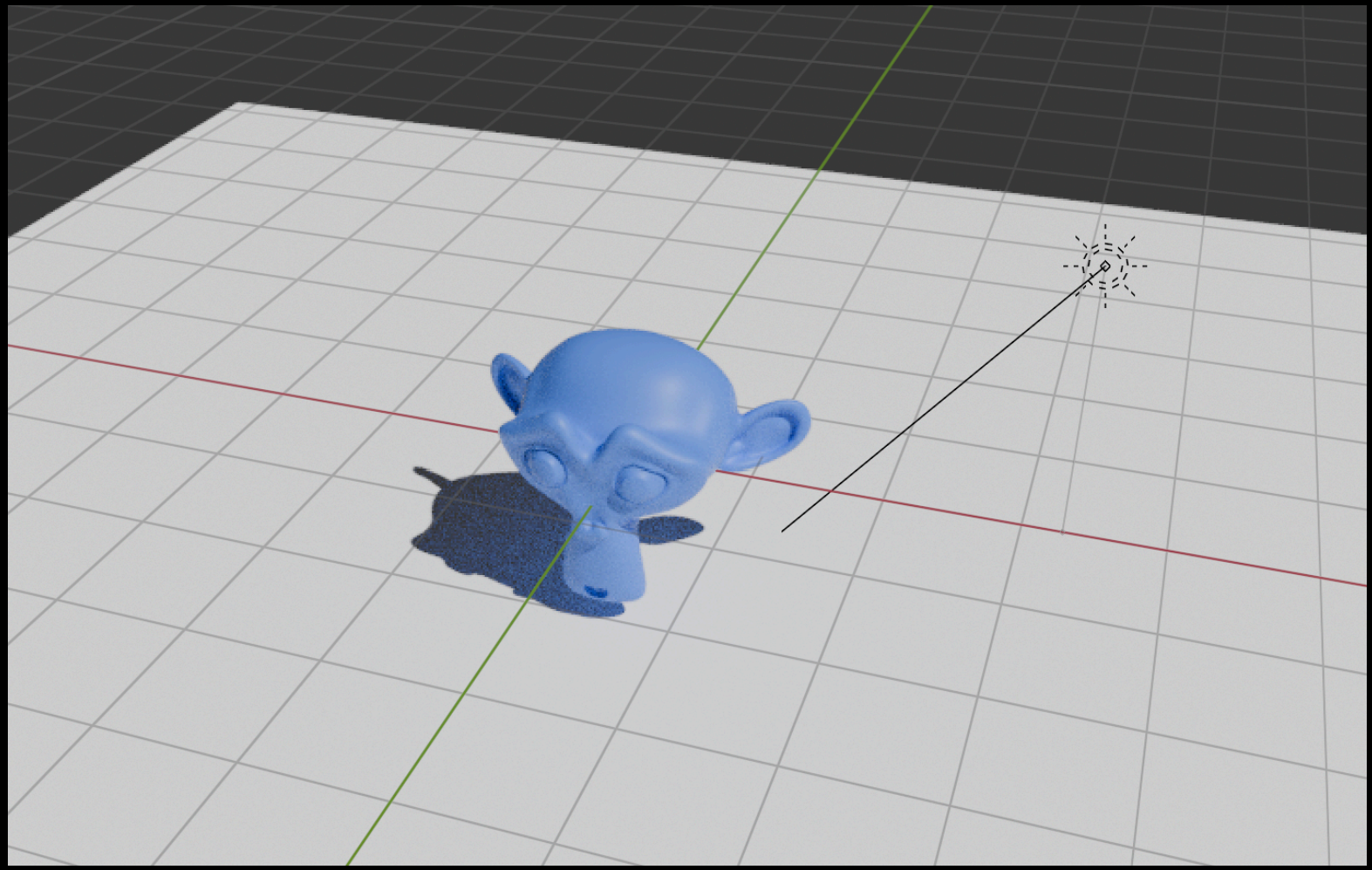
Blender example: Ambient



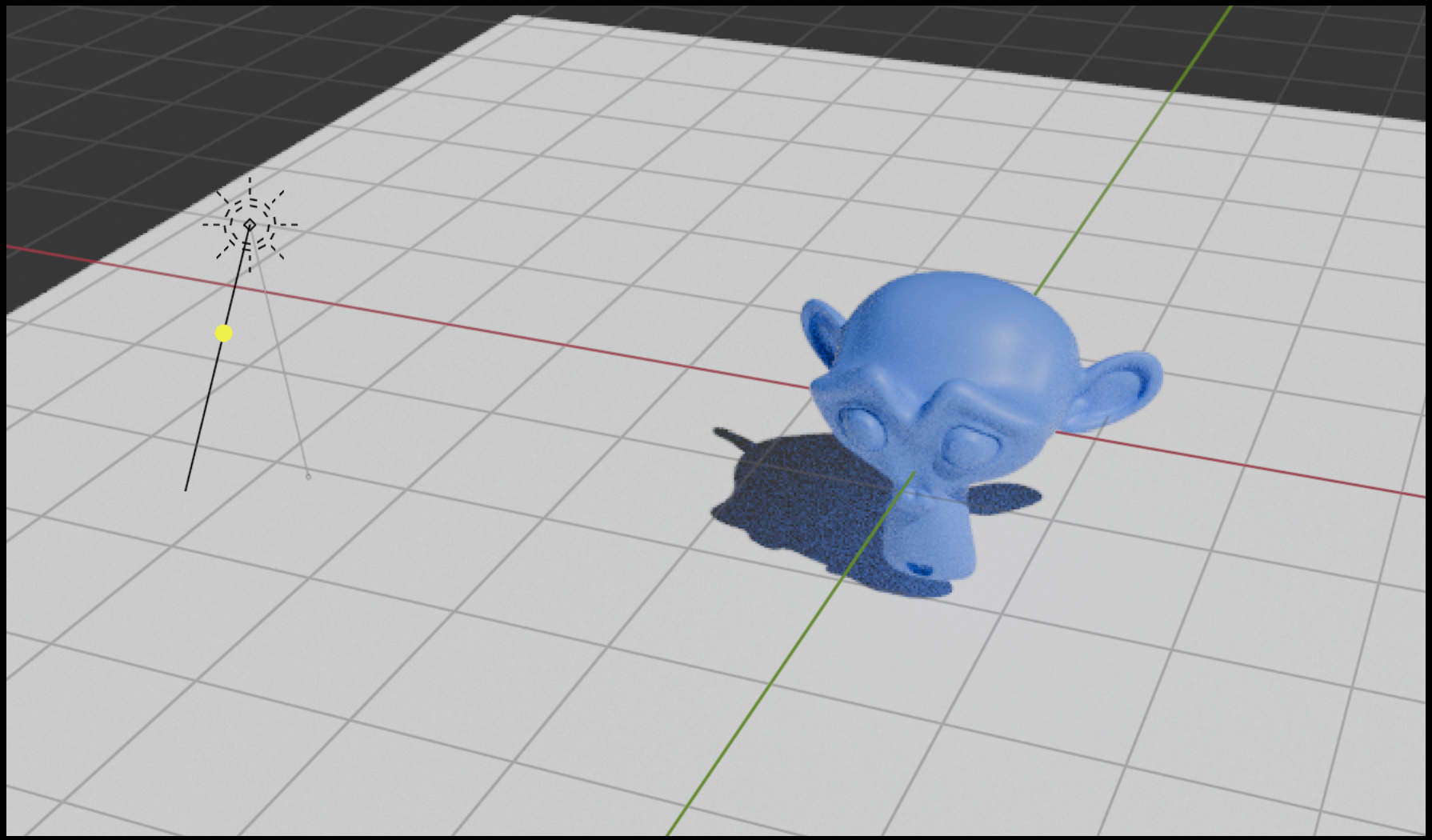
Blender example: Point



Blender example: Sun

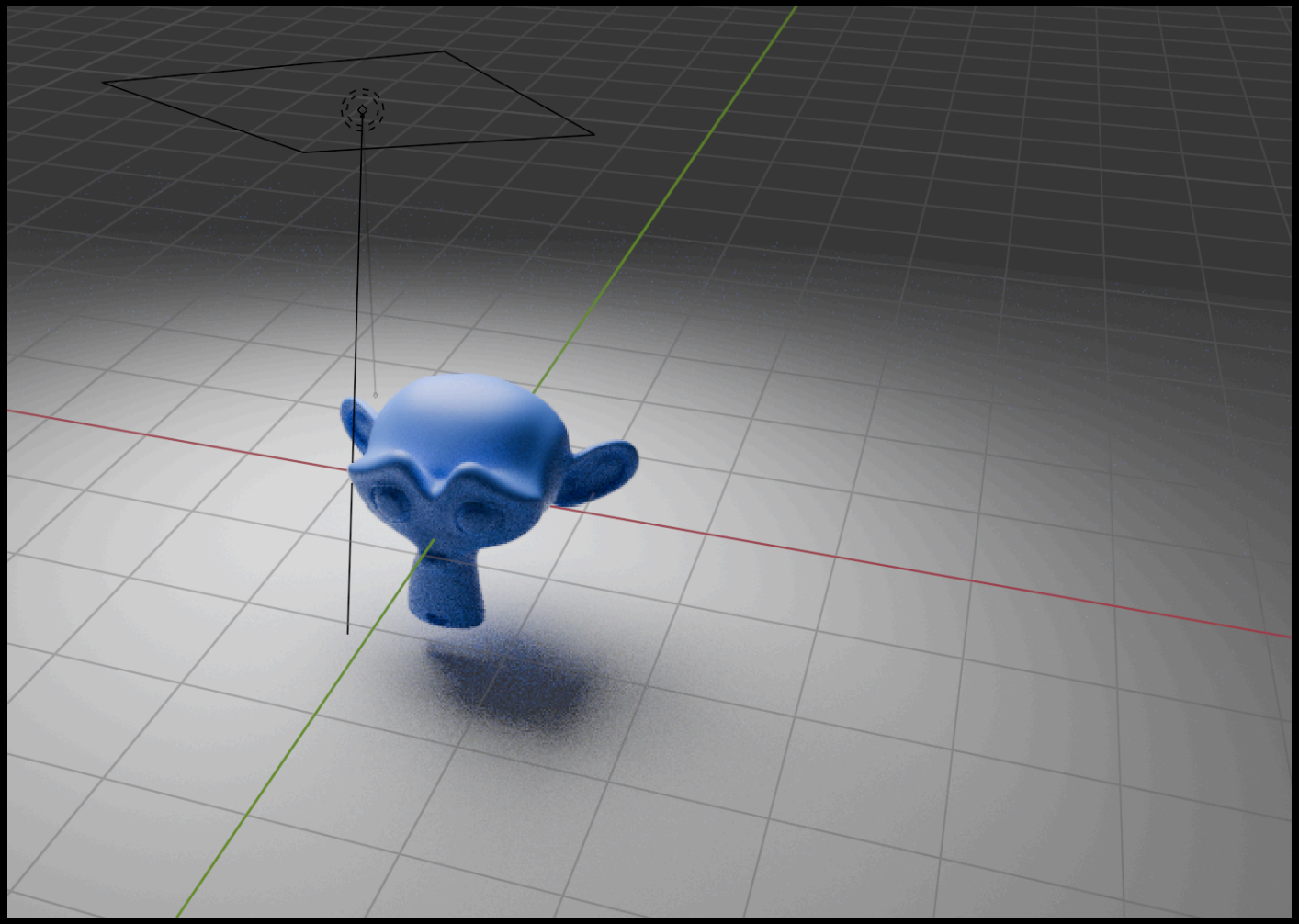


Blender example: Sun



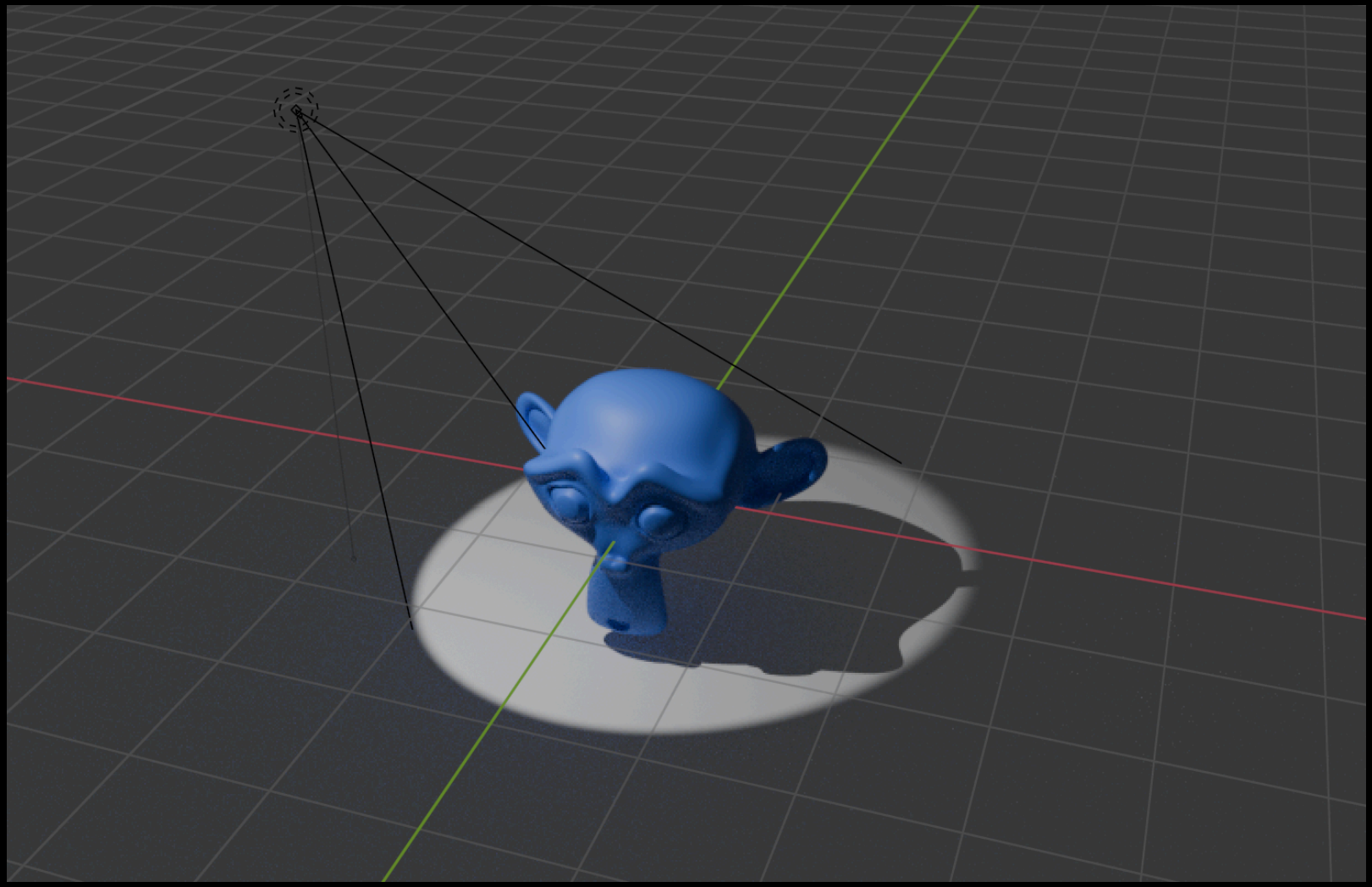
- the position of the sun object does not matter

Blender example: Area



- very expensive, it's still noisy

Blender example: Spot



Secret bonus lighting source: environmental

- a large 360° image that emanates light from every pixel in the image
- wrapped around your scene

polyhaven.com

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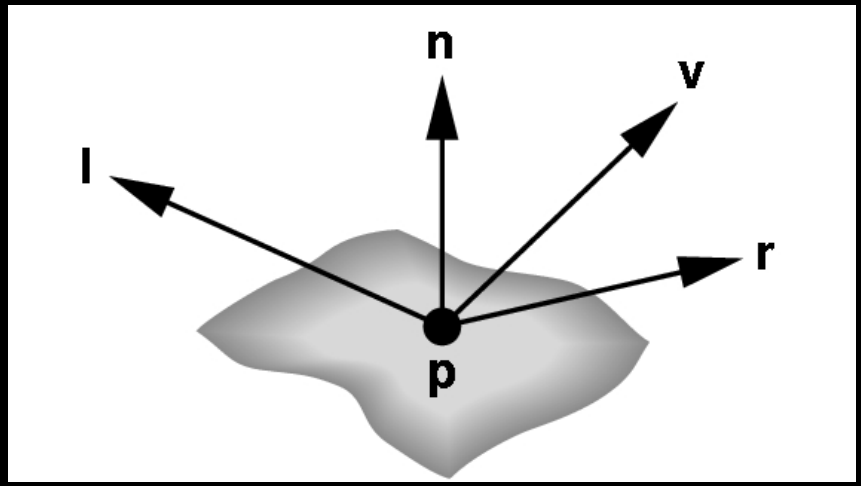
Phong Illumination Model

- Calculate color for arbitrary point on surface
- Compromise between realism and efficiency
- Local computation (no visibility calculations)
- Basic inputs are material properties and **l**, **n**, **v**:

l = unit vector to light source
n = surface normal
v = unit vector to viewer
r = reflection of **l** at **p**
(determined by **l** and **n**)



Phong Illumination Overview

1. Start with global ambient light [G_R G_G G_B]
2. Add contributions from each light source
3. Clamp the final result to [0, 1]

- Calculate each color channel (R,G,B) **separately**
- Light source contributions decomposed into
  - Ambient reflection
  - Diffuse reflection
  - Specular reflection
- Based on ambient, diffuse, and specular lighting and material properties
- You choose how to set them up in your shader!
  - per-vertex? uniform? something exotic?

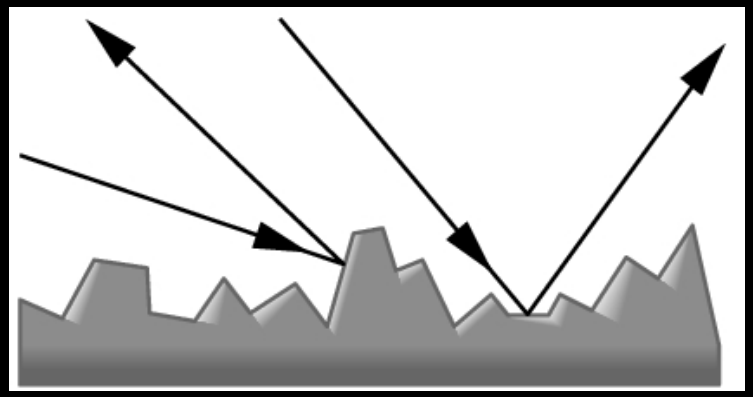
Ambient Reflection

$I_a = k_a L_a$

- Intensity of ambient light is uniform at every point
- Ambient reflection coefficient $k_a \geq 0$
- May be different for every surface and r,g,b
- Determines reflected fraction of ambient light
- L_a = ambient component of light source
  (can be set to different value for each light source)
- Note: L_a is not a physically meaningful quantity
  - (even though it looks like it's the "ambient light color")
  - play with it to achieve mood effects

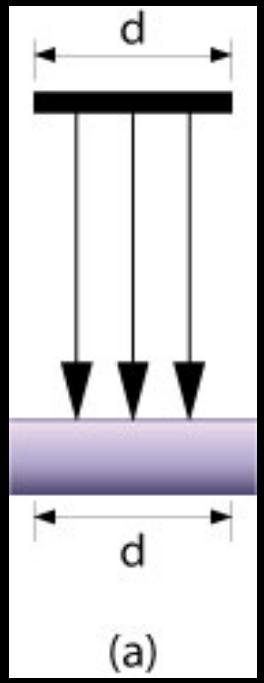
Diffuse Reflection

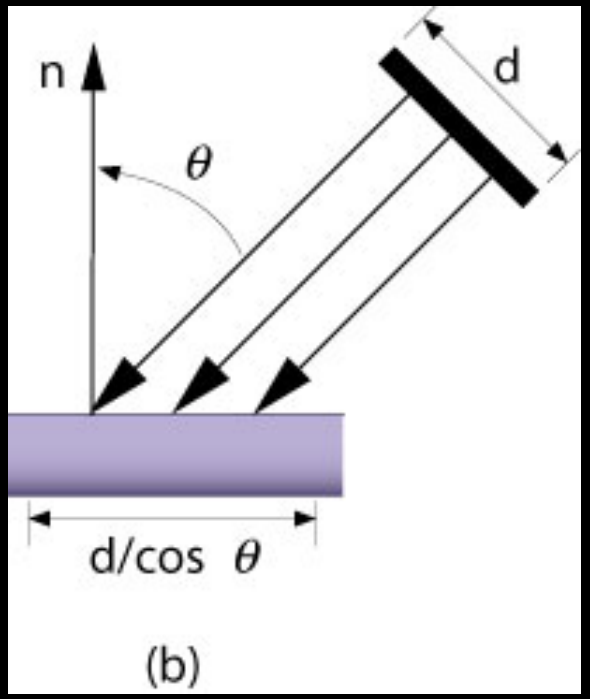
- Diffuse reflector scatters light
- Assume equally all direction
- Called Lambertian surface
- Diffuse reflection coefficient $k_d \geq 0$
- Angle of incoming light is important



Lambert's Law

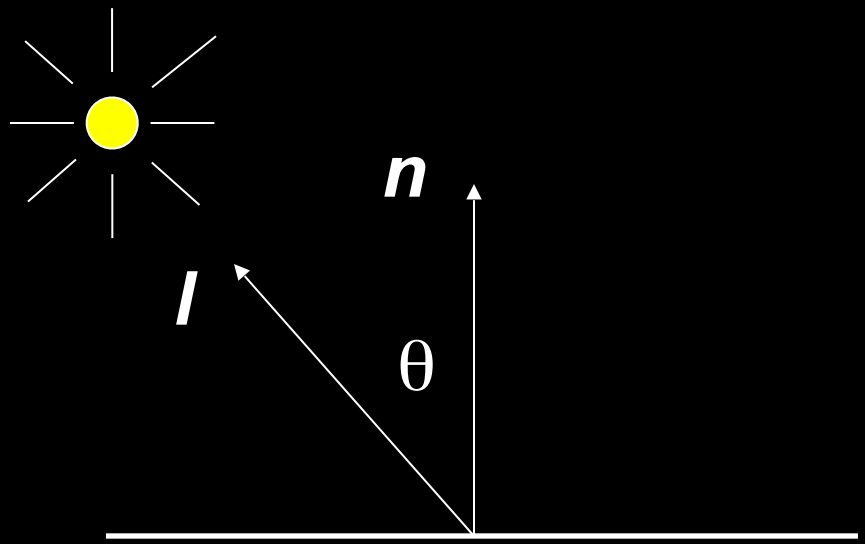
Intensity depends on angle of incoming light.





Diffuse Light Intensity Depends On Angle Of Incoming Light

- Recall
  - $\boldsymbol{l}$ = unit vector to light
  - $\boldsymbol{n}$ = unit surface normal
  - θ = angle to normal
- $\cos\theta = \boldsymbol{l} \cdot \boldsymbol{n}$



- $\boxed{I_d = k_d L_d (\boldsymbol{l} \cdot \boldsymbol{n})}$
- With attenuation:

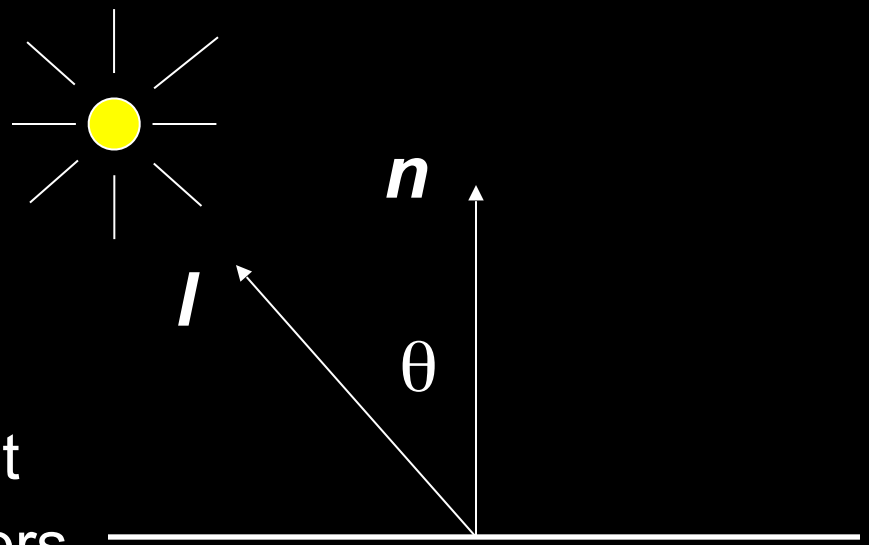
$$I_d = \frac{k_d L_d}{a + bq + cq^2}(l \cdot n)$$

q = distance to light source,
L_d = diffuse component of light

Diffuse Light Intensity Depends On Angle Of Incoming Light

- With attenuation:

$$I_d = \frac{k_d L_d}{a + bq + cq^2}(l \cdot n)$$



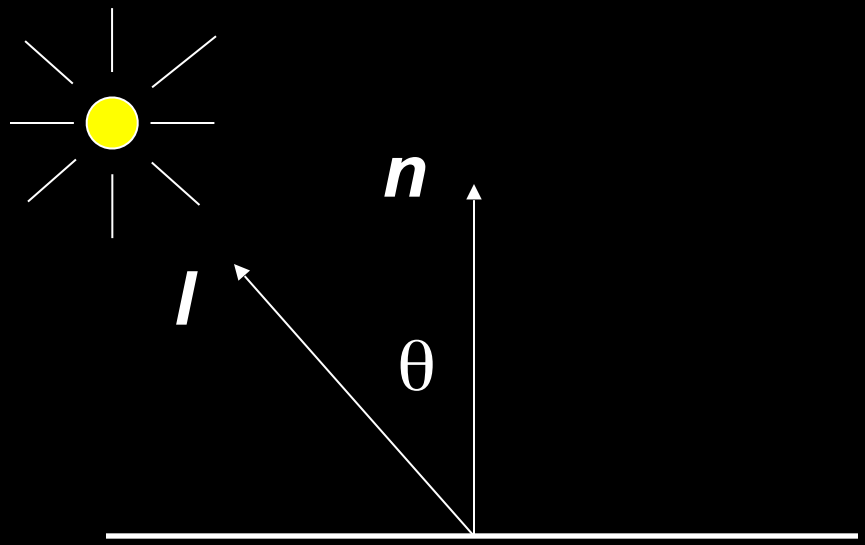
q = distance to light source,
L_d = diffuse component of light
a,b,c = user-defined parameters

- Why attenuate?
  - Physics: farther away from a light source, the light is weaker.
  - What is the physically correct attenuation formula?

Diffuse Light Intensity Depends On Angle Of Incoming Light

- With attenuation:

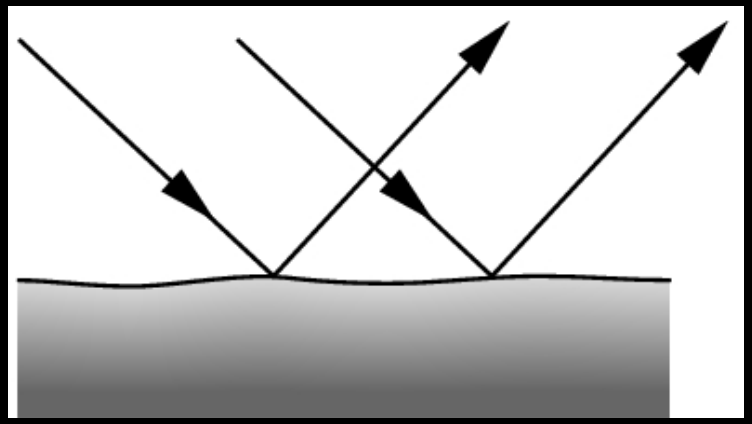
$$I_d = \frac{k_d L_d}{a + bq + cq^2}(l \cdot n)$$



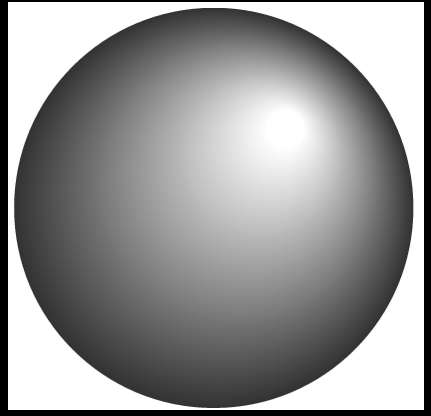
- What is the physically correct attenuation formula?
  - "real" physics would demand only quadratic attenuation
  - does not really look good in real life...
- In fact, in practice, a=b=1, c=0 is a reasonable starting point.

Specular Reflection

- Specular reflection coefficient $k_s \geq 0$
- Shiny surfaces have high specular coefficient
- Used to model specular highlights
- Does not give the mirror effect (no actual reflection) (need other techniques)



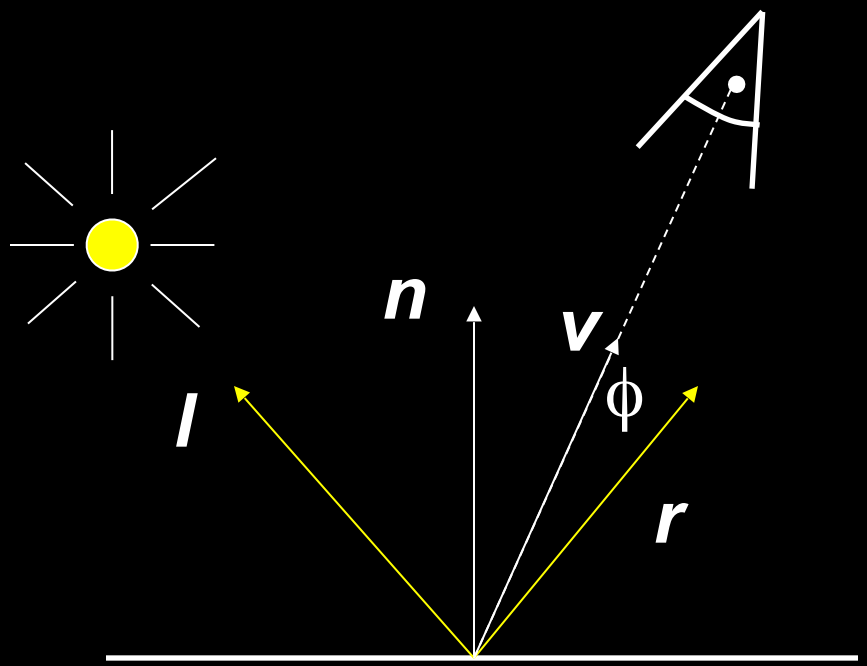
specular reflection



specular highlights

Specular Reflection

- Recall
  - ***v*** = unit vector to camera
  - ***r*** = unit reflected vector
  - ϕ = angle between ***v*** and ***r***
- $\cos \phi = \boldsymbol{v} \cdot \boldsymbol{r}$
- **now *v* also matters!**

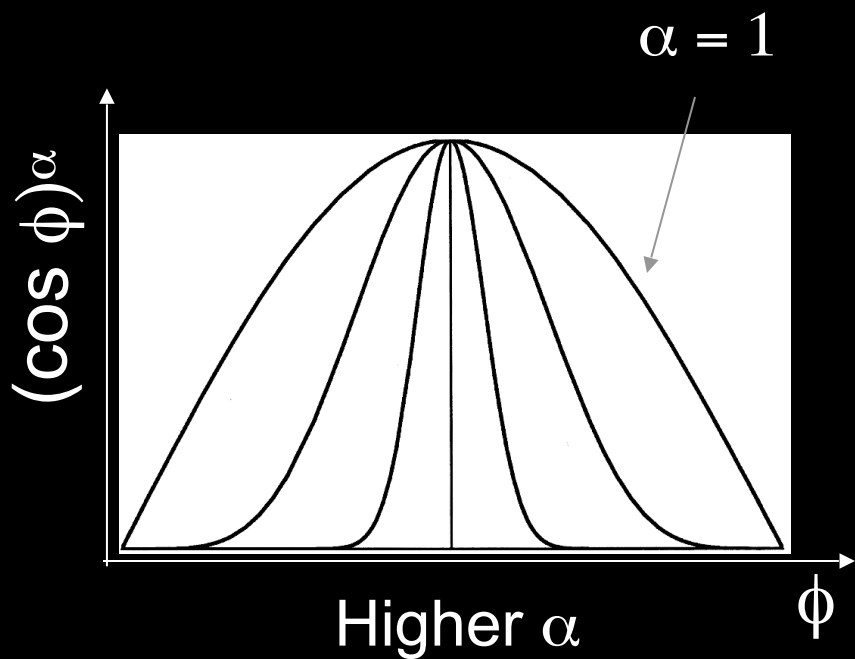


- $I_s = k_s L_s (\cos \phi)^{\alpha}$

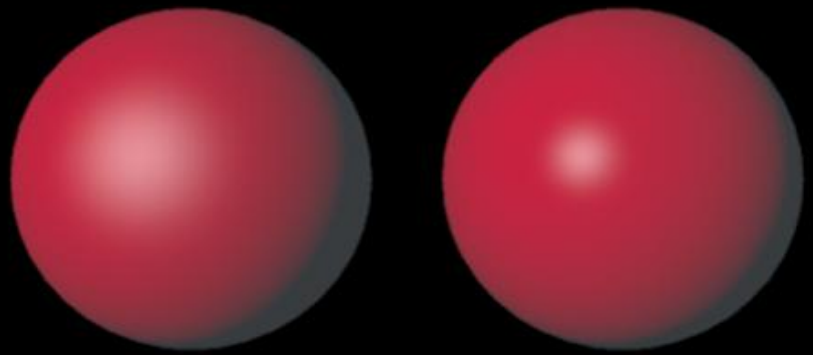
- L_s is specular component of light
- α is shininess coefficient
- Can add distance term as well

Shininess Coefficient

- $I_s = k_s L_s (\cos \phi)^{\alpha}$
- α is the shininess coefficient



Higher α
gives narrower curves



low α high α

Source:
Univ. of Calgary

Summary of Phong Model

- Light components for each color:
  - Ambient (L_a), diffuse (L_d), specular (L_s)
- Material coefficients for each color:
  - Ambient (k_a), diffuse (k_d), specular (k_s)
- Distance q for surface point from light source

$$I = \frac{1}{a + bq + cq^2}\left(k_d L_d (l \cdot n) + k_s L_s (r \cdot v)^{\alpha}\right) + k_a L_a$$

l = unit vector to light
n = surface normal
r = ***l*** reflected about ***n***
v = vector to viewer

Preview for later... shaders

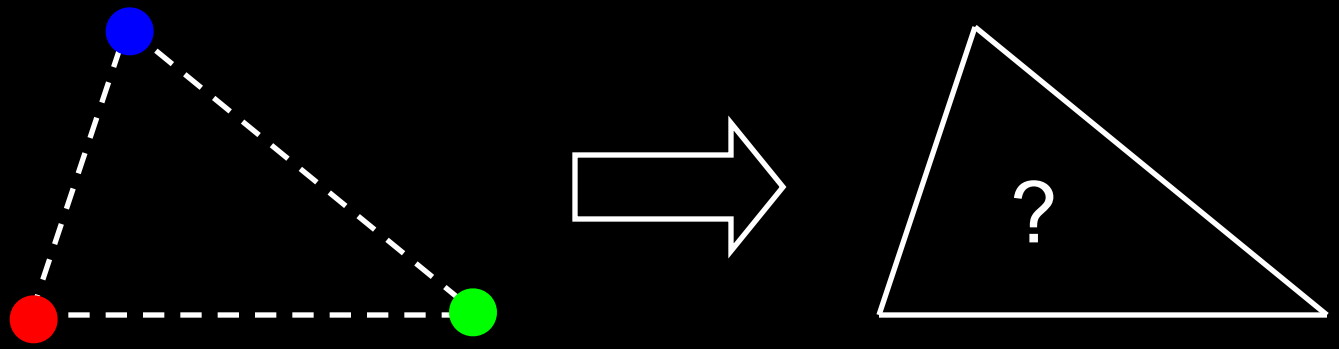
- Where does all of this math happen?
  - CPU
  - Vertex shader
  - Fragment shader
  - Tesselation / geometry shader

Outline

- Global and Local Illumination
- Normal Vectors
- Light Sources
- Phong Illumination Model
- Polygonal Shading
- Example

Polygonal Shading

- Now we know vertex colors
  - either via OpenGL lighting,
  - or by setting directly via glColor3f if lighting disabled
- We have light parameters as uniform shader variables.
- How do we shade the interior of the triangle ?



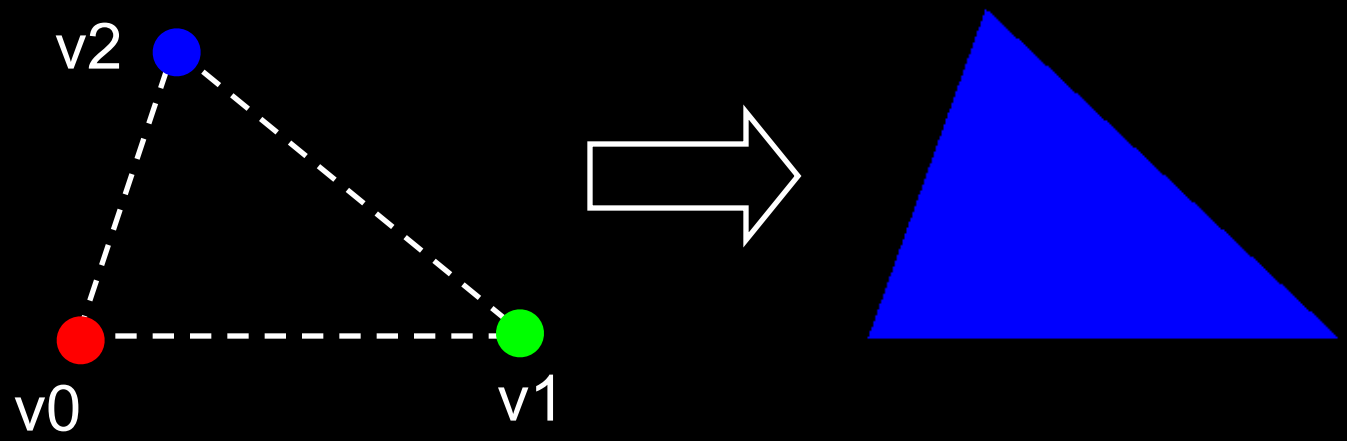
Polygonal Shading

- Curved surfaces are approximated by polygons

- How do we shade?
  - Flat shading
  - Interpolative shading
  - Gouraud shading
  - Phong shading (different from Phong illumination!)

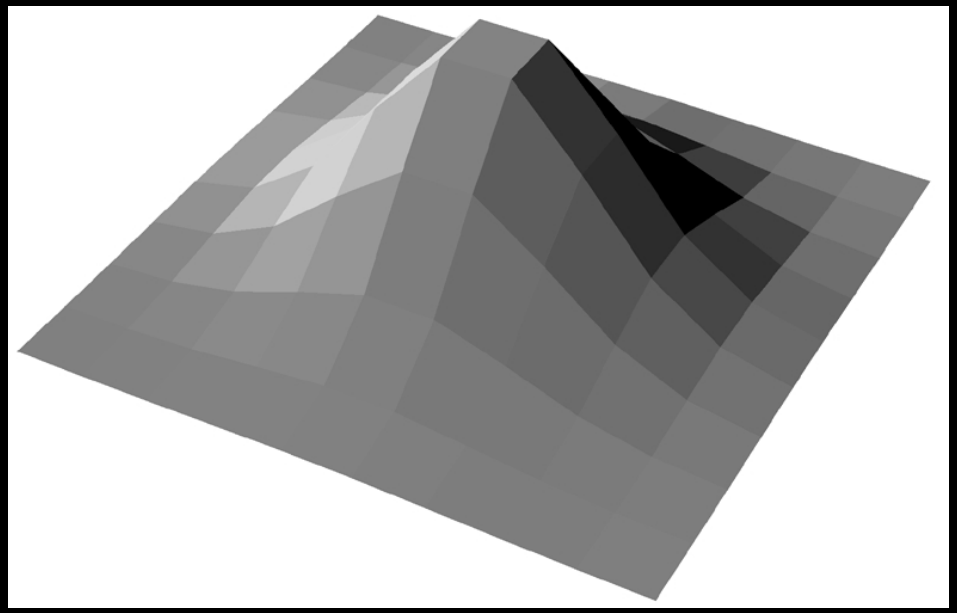
Flat Shading

- Shading constant across polygon
- Core profile: Use interpolation qualifiers in the fragment shader (`flat in` before attribute)
- Compatibility profile: Enable with glShadeModel(GL_FLAT);
- Color of last vertex determines interior color
- Only suitable for *very* small polygons



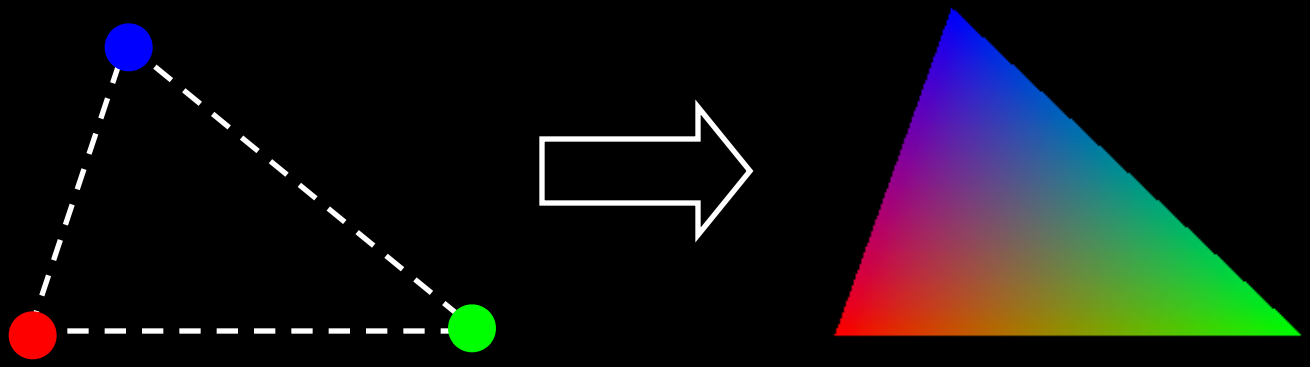
Flat Shading Assessment

- Inexpensive to compute
- Appropriate for objects with flat faces
- Less pleasant for smooth surfaces



Interpolative Shading

- Interpolate color in interior
- Computed during scan conversion (rasterization)
- Core profile: enabled by default (or do `in` before attribute)
- Compatibiltiy profile: enable with glShadeModel(GL_SMOOTH);
- Much better than flat shading
- More expensive to calculate (but not a problem)



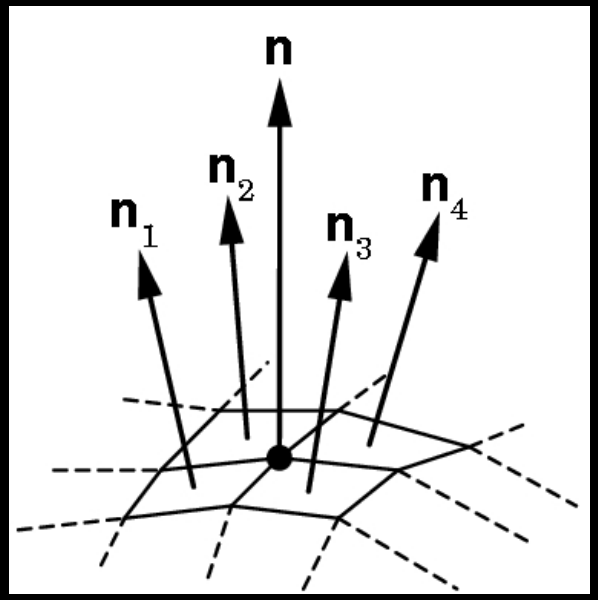
Gouraud Shading

Invented by Henri Gouraud, Univ. of Utah, 1971

- Special case of interpolative shading
- How do we calculate vertex normals for a polygonal surface? Gouraud:
  1. average all adjacent face normals

$$n = \frac{n_1 + n_2 + n_3 + n_4}{|n_1 + n_2 + n_3 + n_4|}$$

  2. use *n* for Phong lighting
  3. interpolate vertex colors into the interior



- Lighting calculations **in vertex shader**

Data Structures for Gouraud Shading

- Sometimes vertex normals can be computed directly (e.g. height field with uniform mesh)
- More generally, need data structure for mesh
- For simple triangle meshes: associate a normal with each vertex in an attribute buffer.
- In general: you need to tell your vertex shader somehow what the normal is at the vertex.

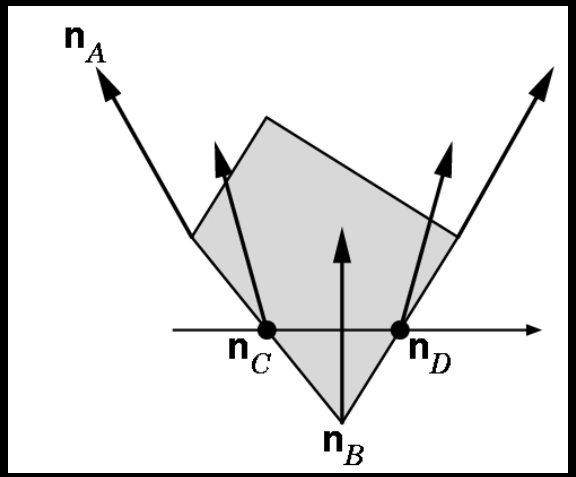
Phong Shading ("per-pixel lighting")

Invented by Bui Tuong Phong, Univ. of Utah, 1973

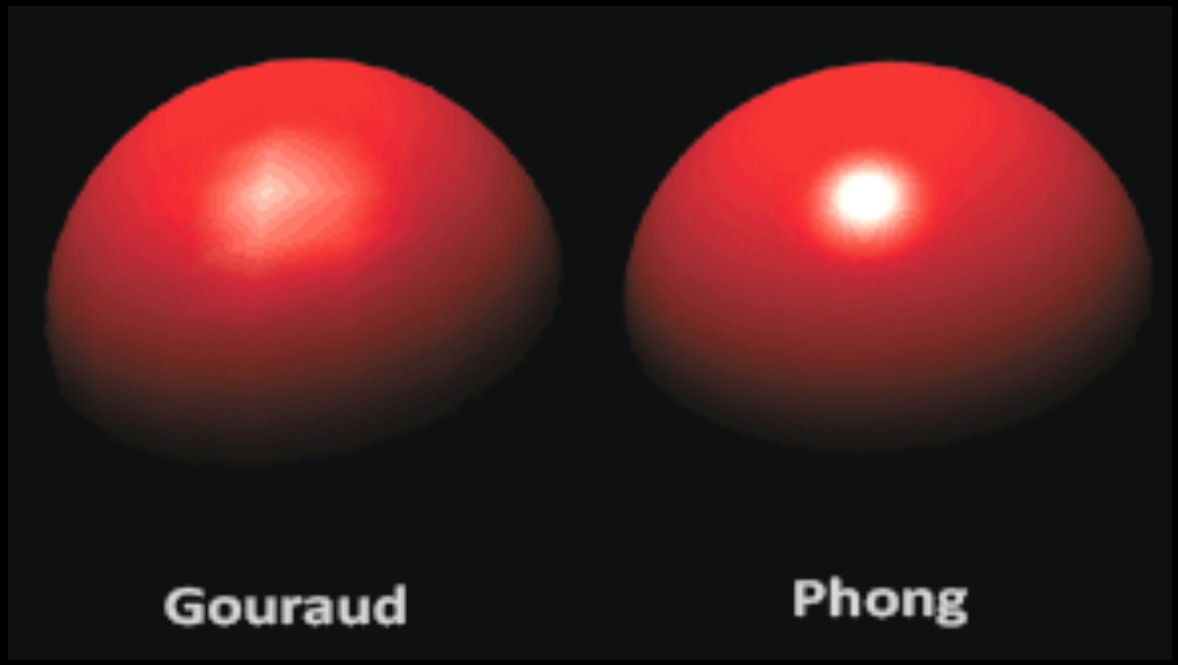
- *At each pixel* (as opposed to at each vertex) :
  1. Interpolate *normals* (rather than colors)
  2. Apply Phong lighting to the interpolated normal

- Significantly more expensive

- Calculation happens **in the fragment shader.**



Gouraud vs. Phong shading



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Gouraud Shading
Colors are computed per-vertex and then interpolated

Phong Shading
Normals are computed per-vertex, then interpolated, and colors are computed per-fragment

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Phong Shader: Vertex Program

```
#version 150

in vec3 position;
in vec3 normal;

out vec3 viewPosition;
out vec3 viewNormal;

uniform mat4 modelViewMatrix;
uniform mat4 normalMatrix;
uniform mat4 projectionMatrix;
```

input vertex position and normal, in world-space

vertex position and normal, in view-space

these will be passed to fragment program (interpolated by hardware)

transformation matrices

Phong Shader: Vertex Program

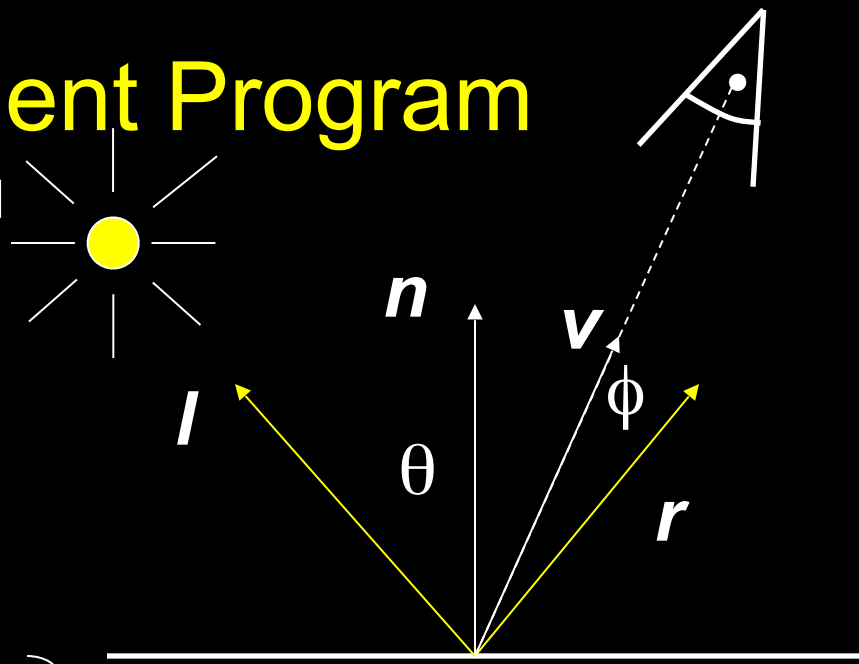
```
void main()
{
  // view-space position of the vertex
  vec4 viewPosition4 = modelViewMatrix * vec4(position, 1.0f);
  viewPosition = viewPosition4.xyz;

  // final position in the normalized device coordinates space
  gl_Position = projectionMatrix * viewPosition4;
  // view-space normal
  viewNormal = normalize((normalMatrix*vec4(normal, 0.0f)).xyz);
}
```

Phong Shader: Fragment Program

```
in vec3 viewPosition;
in vec3 viewNormal;
```

interpolated from vertex program outputs

```
out vec4 c; // output color
```




```
uniform vec4 La; // light ambient
uniform vec4 Ld; // light diffuse
uniform vec4 Ls; // light specular
uniform vec3 viewLightDirection;
```

properties of the directional light

In view space

```
uniform vec4 ka; // mesh ambient
uniform vec4 kd; // mesh diffuse
uniform vec4 ks; // mesh specular
uniform float alpha; // shininess
```

mesh optical properties

Phong Shader: Fragment Program

```
void main()
{
  // camera is at (0,0,0) after the modelview transformation
  vec3 eyedir = normalize(vec3(0, 0, 0) - viewPosition);
  // reflected light direction
  vec3 reflectDir = -reflect(viewLightDirection, viewNormal);
  // Phong lighting
  float d = max(dot(viewLightDirection, viewNormal), 0.0f);
  float s = max(dot(reflectDir, eyedir), 0.0f);
  // compute the final color
  c = ka * La + d * kd * Ld + pow(s, alpha) * ks * Ls;
}
```

VBO and VAO setup

```
During initialization:
// Compute the unit normals (3 components per vertex).
// …

// Put the normals coordinates into a VBO.
// 3 values per vertex, namely x,y,z components of the normal.
VBO vboNormals(numVertices, 3, normals,
   GL_STATIC_DRAW);

// Connect the shader variable “normal” to the VBO.
vao->ConnectPipelineProgramAndVBOAndShaderVariable(
    pipelineProgram, &vboNormals, “normal”);
```

Upload the light direction vector to GPU

```
void display()
{
  glClear (GL_COLOR_BUFFER_BIT|GL_DEPTH_BUFFER_BIT);
  openGLMatrix->SetMatrixMode(OpenGLMatrix::ModelView);
  openGLMatrix->LoadIdentity();
  openGLMatrix->LookAt(ex, ey, ez,  fx, fy, fz,  ux, uy, uz);

  float view[16];
  openGLMatrix->GetMatrix(view); // read the view matrix

  …
```

Upload the light direction vector to GPU

```
  float lightDirection[3] = { 0, 1, 0 }; // the "Sun" at noon
  float viewLightDirection[3]; // light direction in the view space
  // the following line is pseudo-code:
  viewLightDirection = (view * float4(lightDirection, 0.0)).xyz;

  // upload viewLightDirection to the GPU
  pipelineProgram->SetUniformVariable3fv("viewLightDirection",
       viewLightDirection);


  // continue with model transformations
  openGLMatrix->Translate(x, y, z);
  ...

  renderBunny(); // render, via VAO
  glutSwapBuffers();
}
```

Upload the normal matrix to GPU

```
// in the display function:

float n[16];
  matrix->SetMatrixMode(OpenGLMatrix::ModelView);
  matrix->GetNormalMatrix(n); // get normal matrix

  pipelineProgram->SetUniformVariableMatrix4fv(
    ”normalMatrix", GL_FALSE, m);
```

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