CSCI 420 Computer Graphics
Lecture 14

Rasterization

Scan Conversion
Antialiasing
[Angel Ch. 6]

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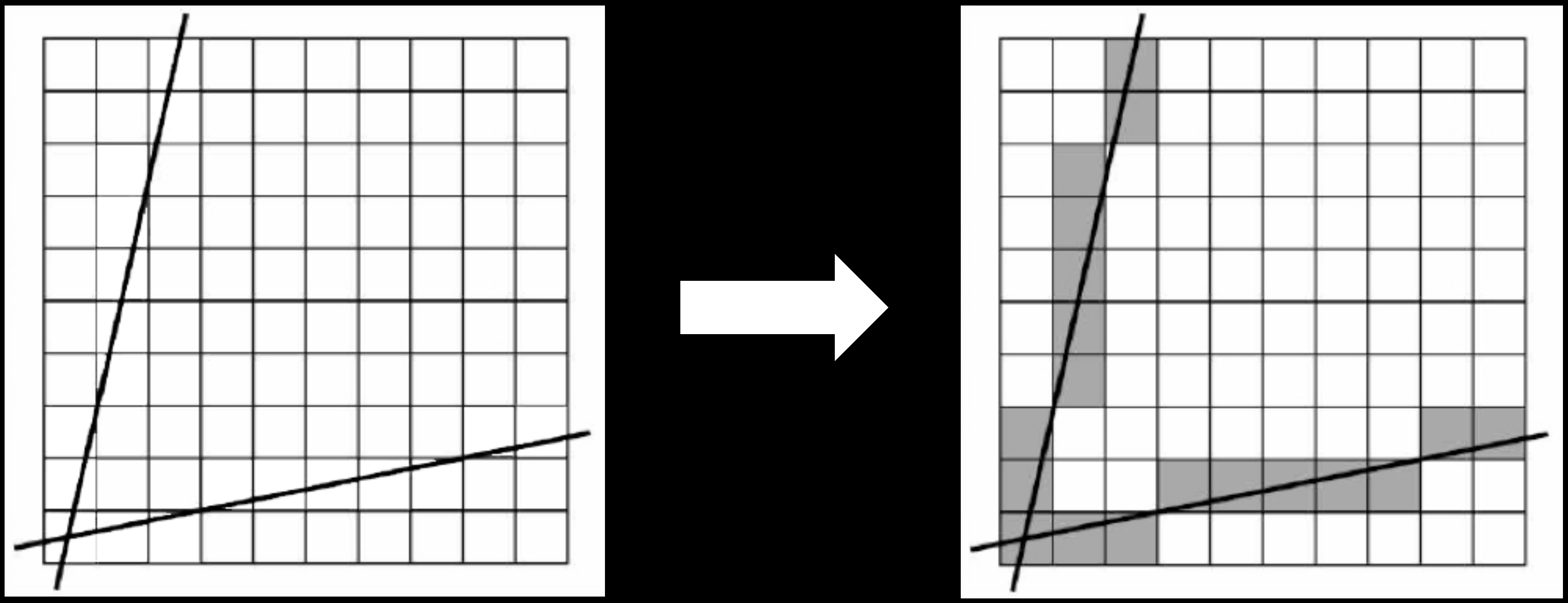
Rasterization (scan conversion)

- Final step in pipeline: rasterization
- From screen coordinates (float) to pixels (int)
- Writing pixels into frame buffer
- Separate buffers:
  - depth (z-buffer),
  - display (frame buffer),
  - shadows (stencil buffer),
  - blending (accumulation buffer)

Rasterizing a line

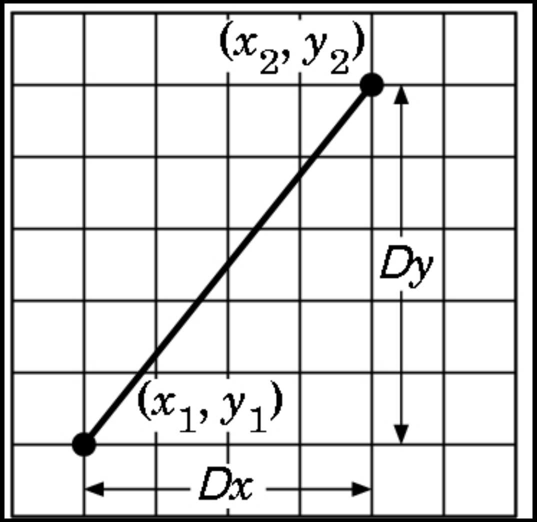


Digital Differential Analyzer (DDA)

- Represent line as

$$y = mx + h \qquad \text{where} \qquad m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$

- Then, if $\Delta x = 1$ pixel, we have $\Delta y = m\,\Delta x = m$

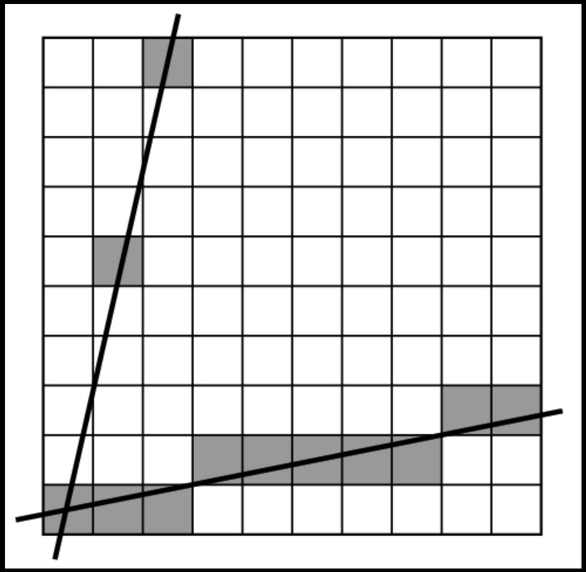


Digital Differential Analyzer

- Assume write_pixel(int *x*, int *y*, int *value*)

```
for (i = x1; i <= x2; i++)
{
   y += m;
   write_pixel(i, round(y), color);
}
```

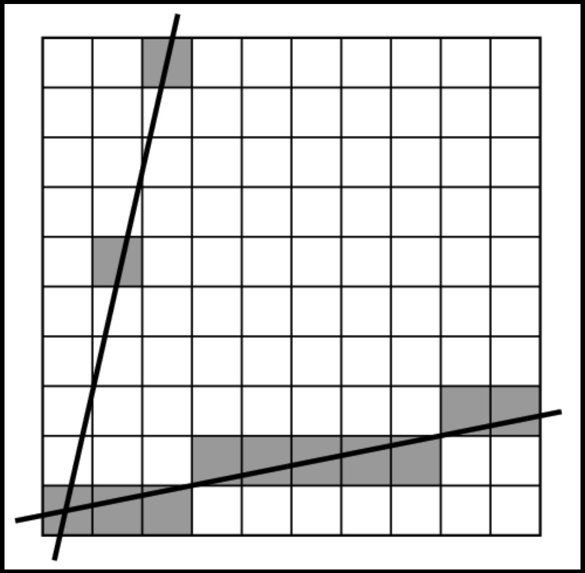
- Problems:
  - Requires floating point addition
  - Missing pixels with steep slopes: slope restriction needed

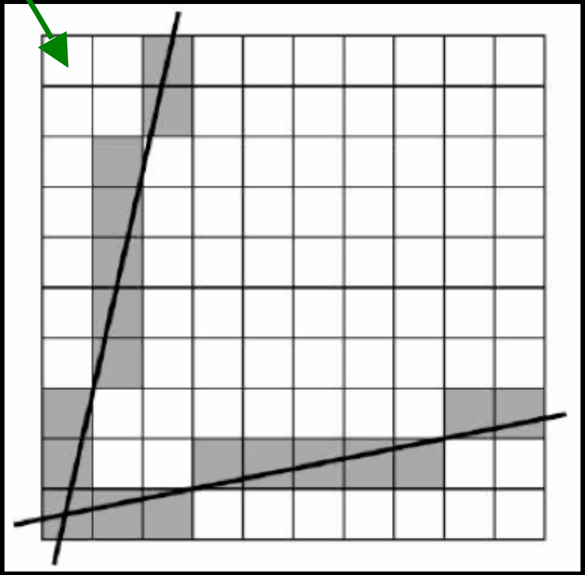


Digital Differential Analyzer (DDA)

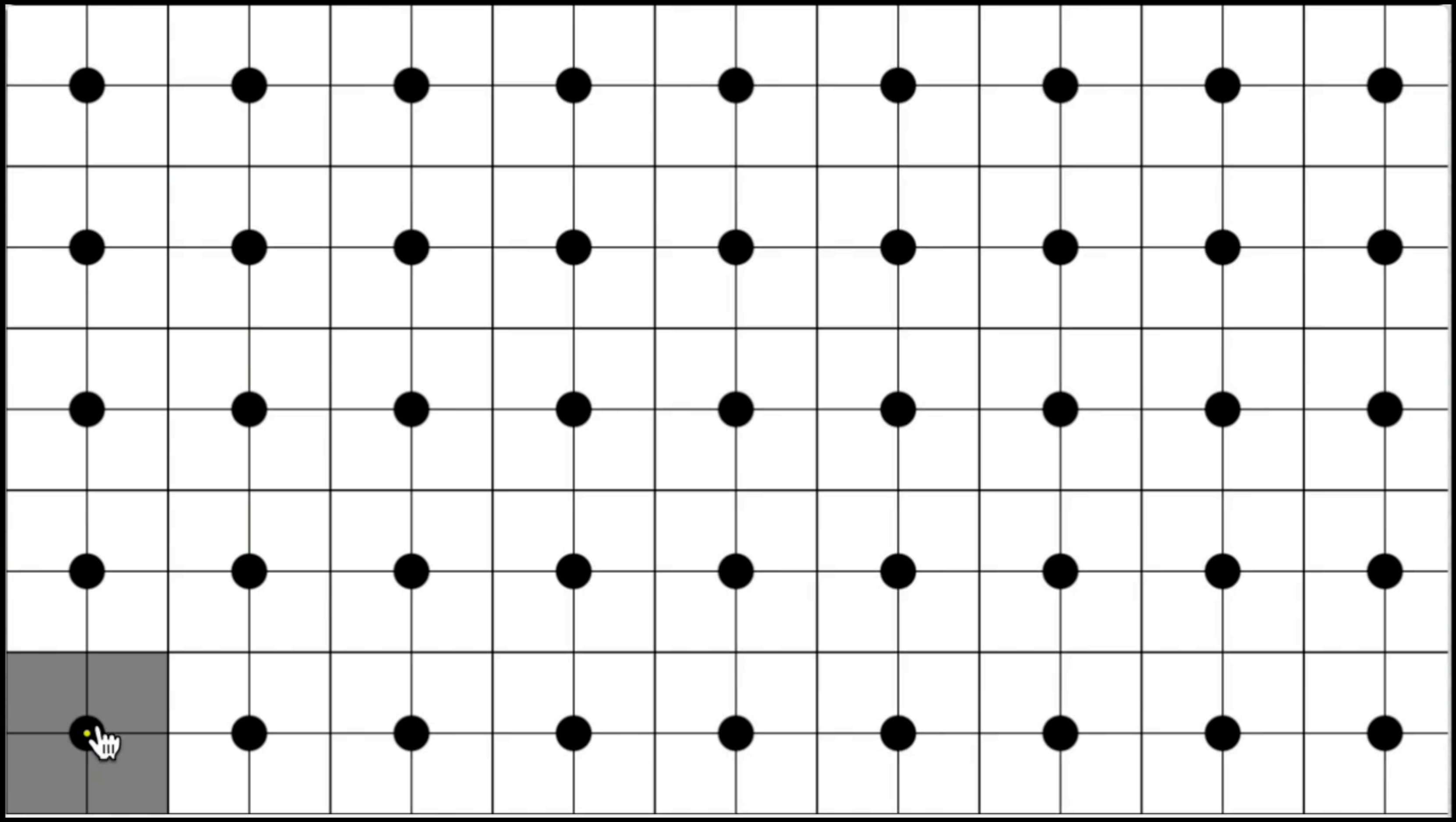
- Assume $0 \leq m \leq 1$
- Exploit symmetry
- Distinguish special cases

But still requires floating point additions!





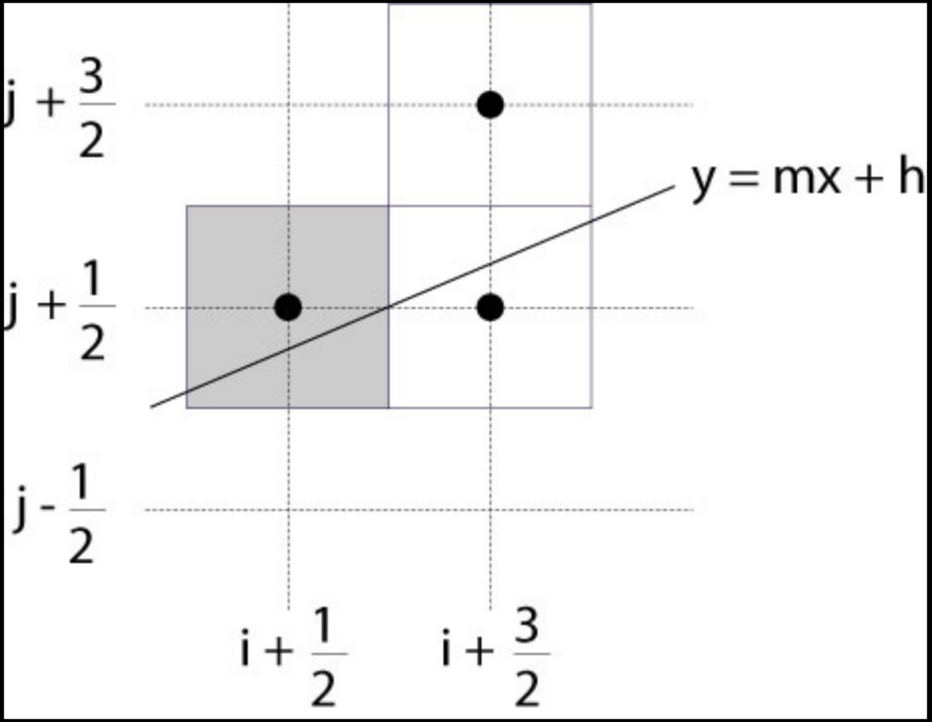
Bresenham's Algorithm



Credits: Pratik Dilip Dhende, CSCI 420 Spring 2024

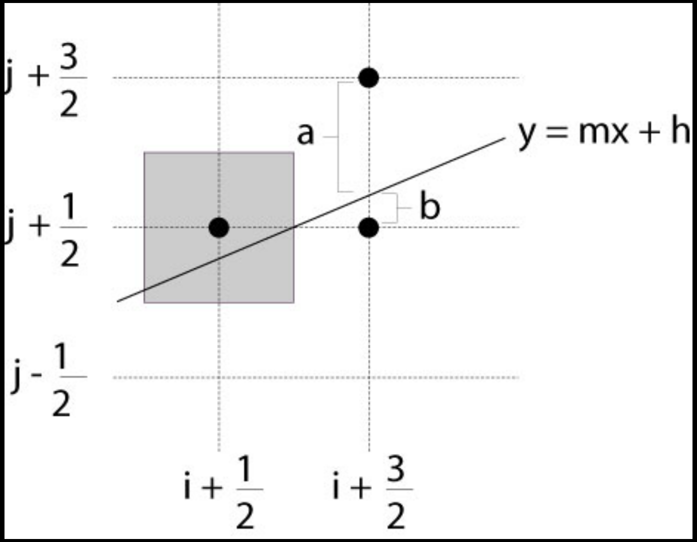
Bresenham's Algorithm I

- Eliminate floating point addition from DDA
- Assume again $0 \leq m \leq 1$
- Assume pixel centers halfway between integers



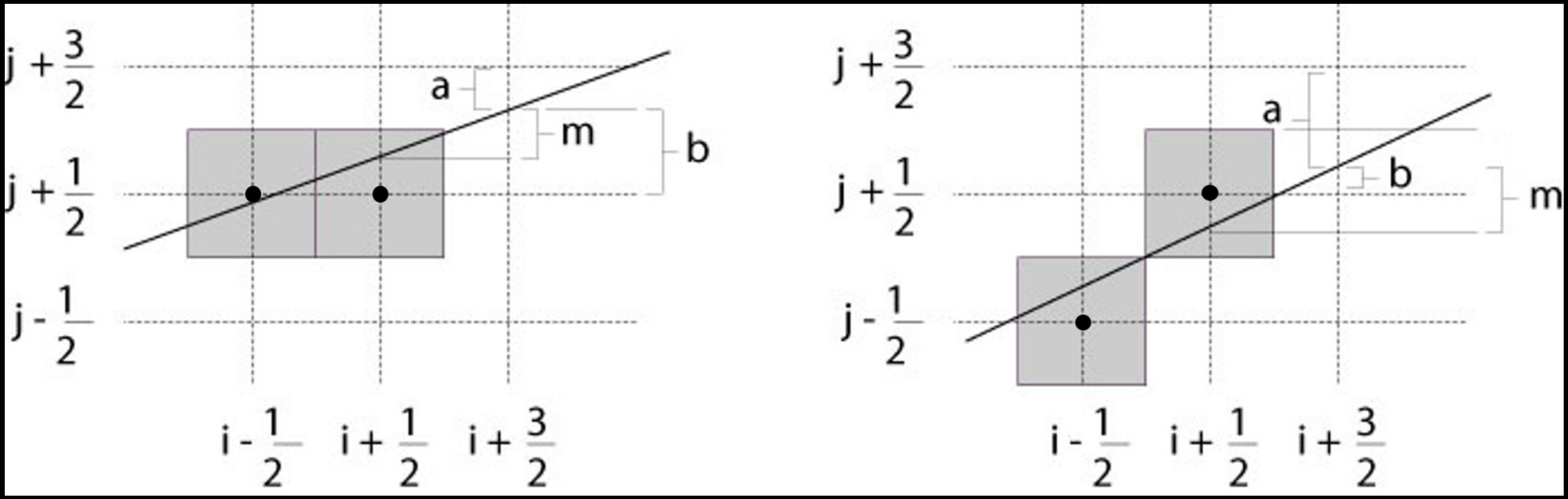
Bresenham's Algorithm II

- Decision variable a – b
  - If a – b > 0 choose lower pixel
  - If a – b ≤ 0 choose higher pixel
- Goal: avoid explicit computation of a – b
- Step 1: re-scale $d = (x_2 – x_1)(a – b) = \Delta x(a – b)$
- d is always integer



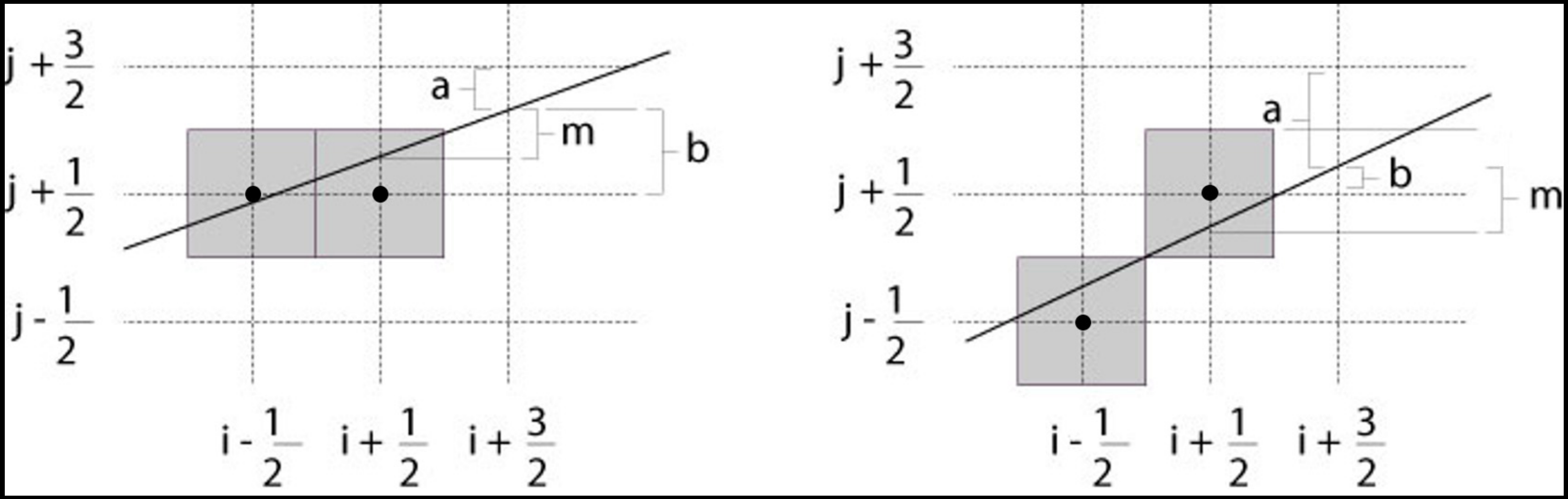
Bresenham's Algorithm III

- Compute d at step k+1 from d at step k!
- Case: j did not change ($d_k > 0$)
  - a decreases by m, b increases by m
  - (a – b) decreases by 2m = 2(Δy/Δx)
  - Δx(a-b) decreases by 2Δy



Bresenham's Algorithm IV

- Case: j did change ($d_k \leq 0$)
  - a decreases by m-1, b increases by m-1
  - (a – b) decreases by 2m – 2 = $2(\Delta y/\Delta x - 1)$
  - Δx(a-b) decreases by $2(\Delta y - \Delta x)$



Bresenham's Algorithm V

- So $d_{k+1} = d_k - 2\Delta y$ if $d_k > 0$
- And $d_{k+1} = d_k - 2(\Delta y - \Delta x)$ if $d_k \leq 0$
- Final (efficient) implementation:

```
void draw_line(int x1, int y1, int x2, int y2) {
    int x, y = y1;
    int twice_dx = 2 * (x2 - x1), twice_dy = 2 * (y2 - y1);
    int twice_dy_minus_twice_dx = twice_dy - twice_dx;
    int d = twice_dx / 2 - twice_dy;

    for (x = x1 ; x <= x2 ; x++) {
        write_pixel(x, y, color);
        if (d > 0) d -= twice_dy;
        else {y++; d -= twice_dy_minus_twice_dx ;}
    }
}
```

Bresenham’s Algorithm VI

- Need different cases to handle m > 1
- Highly efficient
- Easy to implement in hardware and software
- Widely used

Outline

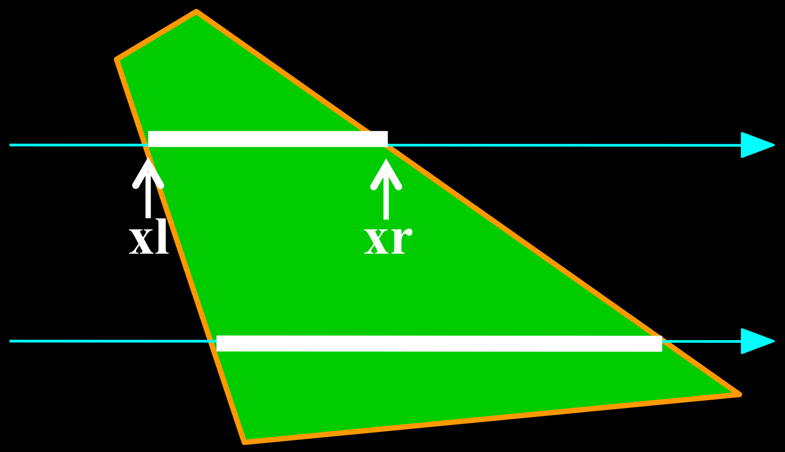
- Scan Conversion for Lines
- Scan Conversion for Polygons
- Antialiasing

Scan Conversion of Polygons

- Multiple tasks:
  - Filling polygon (inside/outside)
  - Pixel shading (color interpolation)
  - Blending (accumulation, not just writing)
  - Depth values (z-buffer hidden-surface removal)
  - Texture coordinate interpolation (texture mapping)
- Hardware efficiency is critical
- Many algorithms for filling (inside/outside)
- Much fewer that handle all tasks well

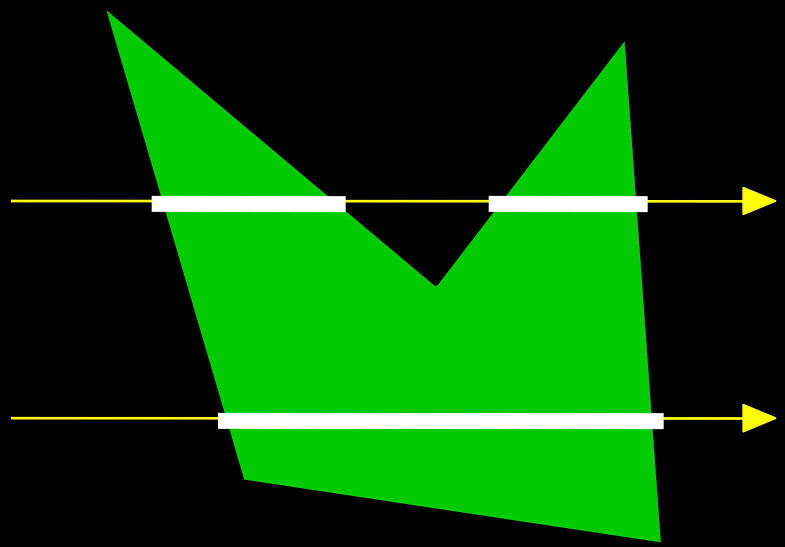
Filling *Convex* Polygons

- Find top and bottom vertices
- List edges along left and right sides
- For each scan line from bottom to top
  - Find left and right endpoints of span, xl and xr
  - Fill pixels between xl and xr
  - Can use Bresenham's algorithm to update xl and xr



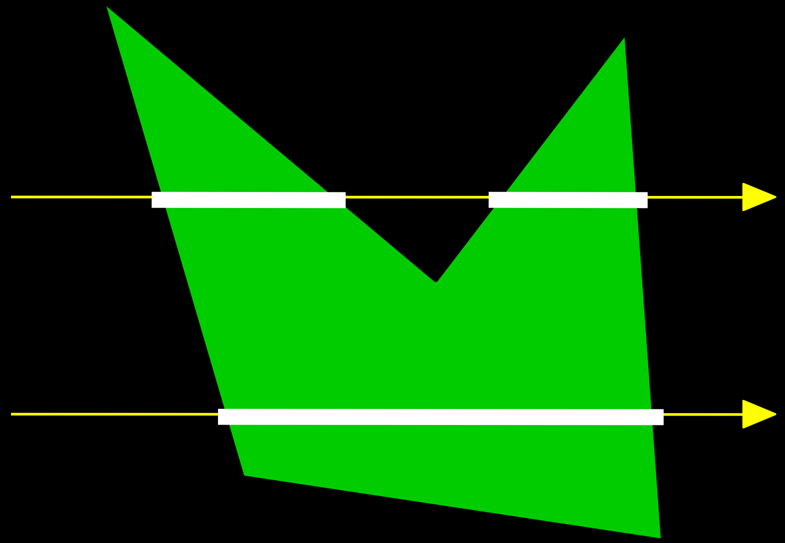
Concave Polygons: Odd-Even Test

- Approach 1: odd-even test
- For each scan line
  - Find all scan line/polygon intersections
  - Sort them left to right
  - Fill the interior spans between intersections
- Parity rule: inside after an odd number of crossings



Edge vs Scan Line Intersections

- Brute force: calculate intersections explicitly
- Incremental method (Bresenham's algorithm)
- Caching intersection information
  - Edge table with edges sorted by y_{min}
  - Active edges, sorted by x-intersection, left to right
- Process image from smallest y_{min} up



Concave Polygons: Tessellation

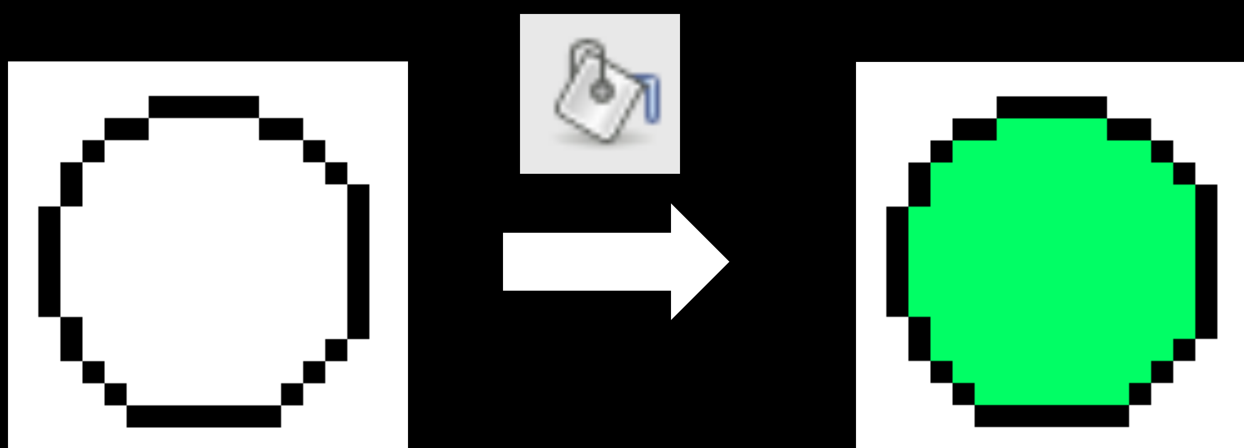
- Approach 2: divide non-convex, non-flat, or non-simple polygons into triangles
- OpenGL specification
  - Need accept only simple, flat, convex polygons
  - Tessellate explicitly with tessellator objects
  - Implicitly if you are lucky
- Most modern GPUs scan-convert only triangles

Or how it's actually done...

- Triangulate your concave polygon before rasterization!

Flood Fill

- Draw outline of polygon
- Pick color seed
- Color surrounding pixels and recurse
- Must be able to test boundary and duplication
- More appropriate for drawing than rendering

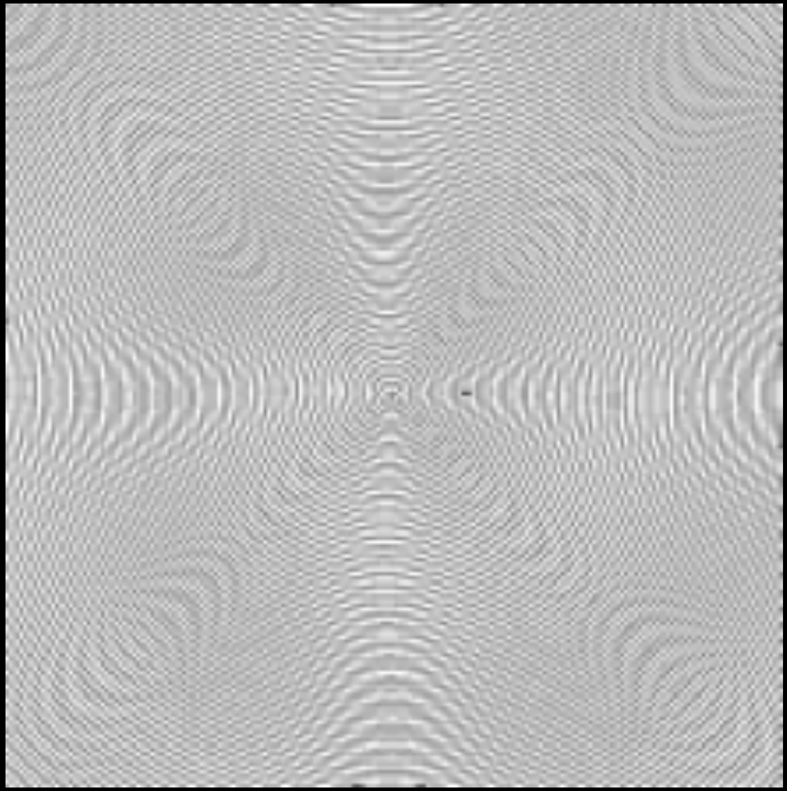


Outline

- Scan Conversion for Lines
- Scan Conversion for Polygons
- Antialiasing

Aliasing

- Artifacts created during scan conversion
- Inevitable (going from continuous to discrete)
- Aliasing (name from digital signal processing): we sample a continues image at grid points
- Effect
  - Jagged edges
  - Moire patterns

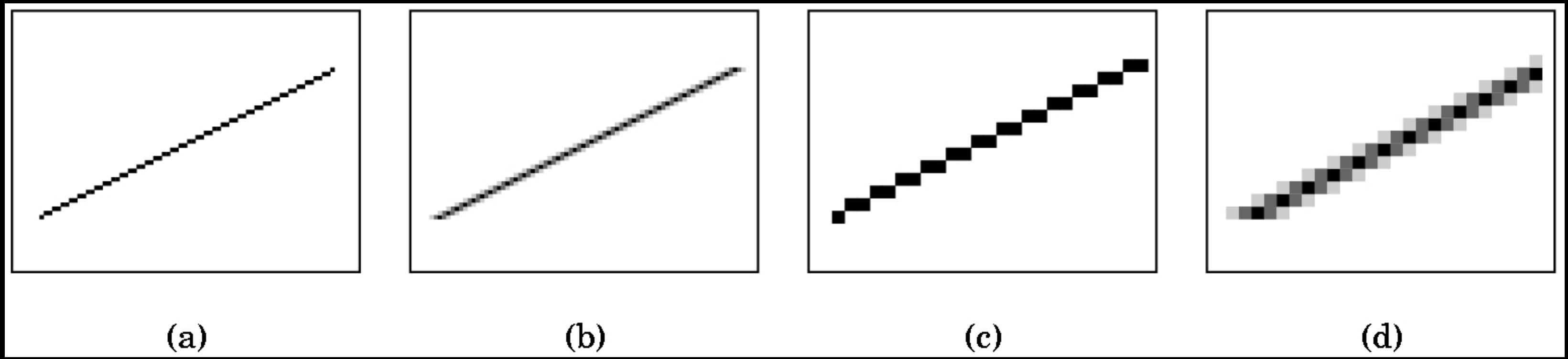


Moire pattern from sandlotscience.com

More Aliasing

Antialiasing for Line Segments

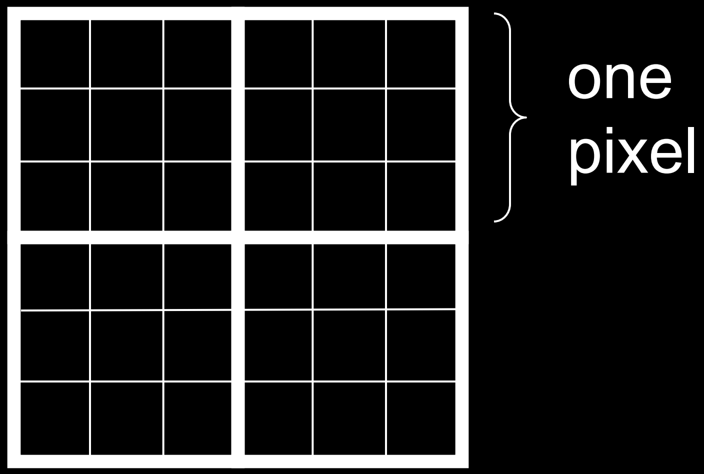
- Use area averaging at boundary



- (a) is aliased; (b) is antialiased
- (c) is aliased + magnified
- (d) is antialiased + magnified

Antialiasing by Supersampling

- Mostly for off-line rendering (e.g., ray tracing)
- Render, say, 3x3 grid of mini-pixels
- Average results using a filter
- Can be done adaptively
  - Stop if colors are similar
  - Subdivide at discontinuities

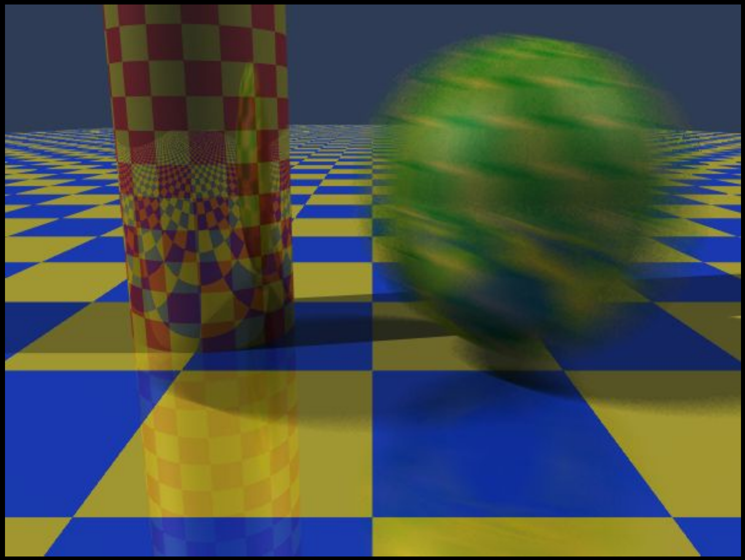


Supersampling Example

- Other improvements
  - Stochastic sampling: avoid sample position repetitions
  - Stratified sampling (jittering) :
    perturb a regular grid of samples

Temporal Aliasing

- Sampling rate is frame rate (30 Hz for video)
- Example: spokes of wagon wheel in movies
- Solution: supersample in time and average
  - Fast-moving objects are blurred
  - Happens automatically with real hardware (photo and video cameras)
    - Exposure time is important (shutter speed)
  - Effect is called motion blur



Motion blur

Wagon Wheel Effect

Source: YouTube

Motion Blur Example

Achieve by stochastic sampling in time

T. Porter, Pixar, 1984
16 samples / pixel / timestep

Depth of Field

Wide depth of field

Narrow depth of field

digital-photography-school.com

Summary

- Scan Conversion for Polygons
  - Basic scan line algorithm
  - Convex vs concave
  - Odd-even rules, tessellation
- Antialiasing (spatial and temporal)
  - Area averaging
  - Supersampling
  - Stochastic sampling