CS 520: Assignment 3 - Probabilistic Search (and Destroy)

Haoyang Zhang, Han Wu, Shengjie Li, Zhichao Xu

November 20, 2018

1 Introduction, group members and division of workload

In this group project, we implemented a minesweeper solver that far exceeded our expectation. Not only can our program solve the normal minesweeper, but also it can solve minesweeper with inaccurate information. Our program also has a gorgeous GUI and can visualize the progress of solving minesweeper by animation.

Name RUID	Workload
Haoyang Zhang 188008687	Implemented the minesweeper solver. Finished the writing of report for most of the questions. Wrote *Solution Algorithm Explanation.html* and *Uncertainty Explanation.html* which are two documents about our algorithm
Han Wu 189008460	Ran tests and generated figures for question 2.4. Finished the writing of report for question 2.4
Shengjie Li 188008047	Designed and implemented the GUI of our program. Implemented a function that can generate animation of the progress of solving minesweeper. Finished the format design of whole report using LaTeX.
Zhichao Xu 188008912	Proofread the report. Ran tests and generated figures for question 2.5 and question 4.1. Finished the writing of report for question 2.5 and question 4.1.

2 A Stationary Target

1. Given observations up to time t (Observations$_t$), and a failure searching Cell$_j$ (Observations$_t+1 =$ Observations$_t \wedge$ Failure in Cell$_j$), how can Bayes' theorem be used to efficiently update the belief state, i.e., compute:

$$\mathbb{P}(\text{Target in Cell}_i|\text{Observations}_t \wedge \text{Failure in Cell}_j). \tag{1}$$

When $i \neq j$,

$$\begin{aligned}&\mathbb{P}(\text{Target in Cell}_i|\text{Observations}_t \wedge \text{Failure in Cell}_j)\\ &= \alpha\mathbb{P}(\text{Target in Cell}_i|\text{Observations}_t).\end{aligned} \tag{2}$$

Denote $\mathbb{P}$ (Target in Cell$_i$|Observations$_t$) by a_{it},

$(2) = \alpha \cdot a_{it}$.

When $i = j$,

$$\begin{aligned}&\mathbb{P}(\text{Target in Cell}_j|\text{Observations}_t \wedge \text{Failure in Cell}_j)\\ &= \alpha\mathbb{P}(\text{Target in Cell}_j|\text{Observations}_t) \cdot \mathbb{P}(\text{Target not found in Cell}_j|\text{Target in Cell}_j).\end{aligned} \tag{3}$$

Denote $\mathbb{P}$ (Target not found in Cell_j|Target in Cell_j) by q,

$$(3) = \alpha \cdot a_{jt} \cdot q.$$

$$\alpha = \frac{1}{\sum_{i \neq j}(\alpha \cdot a_{it}) + \alpha \cdot a_{jt} \cdot q} = \frac{1}{1 - a_{jt}(1-q)}.$$

$$a_{i0} = \frac{1}{2500} \text{ for } i = 0, \ldots, 2499$$

2. Given the observations up to time t, the belief state captures the **current probability the target is in a given cell**. What is the probability that the target will be **found** in Cell_i if it is searched:

$$\mathbb{P}(\text{Target found in Cell}_i|\text{Observations}_t)? \tag{4}$$

$$\begin{aligned}
&\mathbb{P}(\text{Target found in Cell}_i|\text{Observations}_t)\\
&= \mathbb{P}(\text{Target in Cell}_i|\text{Observations}_t)(1 - \mathbb{P}(\text{Target not found in Cell}_i|\text{Target in Cell}_i))\\
&= a_{it} \cdot (1-q)
\end{aligned}$$

$$a_{i0} = \frac{1}{2500} \text{ for } i = 0, \ldots, 2499$$

3. Consider comparing the following two decision rules:

   - Rule 1: At any time, search the cell with the highest probability of containing the target.
   - Rule 2: At any time, search the cell with the highest probability of finding the target.

   For either rule, in the case of ties between cells, consider breaking ties arbitrarily. How can these rules be interpreted / implemented in terms of the known probabilities and belief states?

   For a fixed map, consider repeatedly using each rule to locate the target (replacing the target at a new, uniformly chosen location each time it is discovered). On average, which performs better (i.e., requires less searches), Rule 1 or Rule 2? Why do you think that is? Does that hold across multiple maps?

4. Consider modifying the problem in the following way: at any time, you may only search the cell at your current location, or move to a neighboring cell (up/down, left/right). Search or motion each constitute a single 'action'. In this case, the 'best' cell to search by the previous rules may be out of reach, and require travel. One possibility is to simply move to the cell indicated by the previous rules and search it, but this may incur a large cost in terms of required travel. How can you use the belief state and your current location to determine whether to search or move (and where to move), and minimize the total number of actions required? Derive a decision rule based on the current belief state and current location, and compare its performance to the rule of simply always traveling to the next cell indicated by **Rule 1** or **Rule 2**. Discuss.

   In this question, we need to move to the cell that we want to search and motion also counted as one step. We call this "search and motion case". We use 3 rules to guide our search. Rule 1 means just traveling to the "best" cell which is indicated by the original Rule 1. Rule 2 is the same case. Rule 4 is invented by ourselves. It uses new criterion to supervise our search. We generate 5000 maps and use different rules to search the target in the maps. The result is shown in Fig 1.

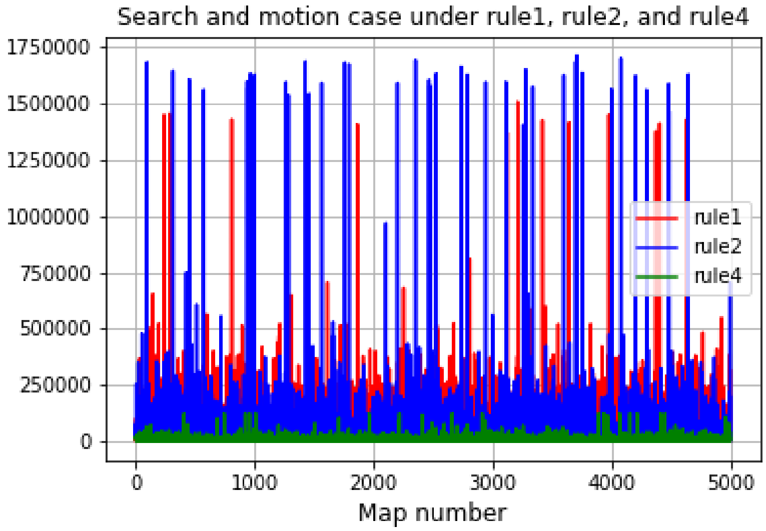

Figure 1: Number of actions in search and motion case

From the figure, we can see that the performance of Rule 4 is better than Rule 1 and Rule 2. The means of Rule1, Rule 2, and Rule 4 are 58475.71, 45925.69, and 9745.66. It is easy to find that the average number of actions of Rule 4 is much smaller than Rule 1 and Rule 2. The variances of Rule1, Rule 2, and Rule 4 are 7920337425.95, 5069004957.26, and 119653468.99. From this view, we can also conclude that Rule 4 is better. When the number of search actions is beyond 100000, we call it unsolvable because it is extremely hard to find the target under this circumstance. The fail rate of Rule1, Rule 2, and Rule 4 are 0.0026, 0.0076, and 0.003. Rule 4 also performs well.

5. An old joke goes something like the following:

   A policeman sees a drunk man searching for something under a streetlight and asks what the drunk has lost. He says he lost his keys and they both look under the streetlight together. After a few minutes the policeman asks if he is sure he lost them here, and the drunk replies, no, and that he lost them in the park. The policeman asks why he is searching here, and the drunk replies, "the light is better here".

   In light of the results of this project, discuss.

3 A Moving Target

In this section, the target is no longer stationary, and can move between neighboring cells. Each time you perform a search, if you fail to find the target the target will move to a neighboring cell (with uniform probability for each). However, all is not lost - whenever the target moves, surveillance reports to you that the target was seen at a **Type1** $\times$ **Type2** border where Type1 and Type2 are the cell types the target is moving between (though it is not reported which one was the exit point and which one the entry point.

Implement this functionality in your code. How can you update your search to make use of this extra information? How does your belief state change with these additional observations? Update your search accordingly, and again compare **Rule 1** and **Rule 2**.

Re-do question 4) above in this new environment with the moving target and extra information.

1. Update search results
   For a given block, there are 2 different search results: 'True'(find the target) and 'False'(Target

not found). If the result is 'True', this board is done. If the result is 'False', we have to update the 'prob' to represent this result.

Using the notion of particle filter, assuming that each block have p_i samples. If we have not found a target in $block_k$, some samples should be cast off, i.e. $p'_k = p_k(1 - q_k)$. Then, resample all blocks by multiplying α, where $\alpha = \frac{1}{1 - p_k q_k}$

In a nutshell, after searching $block_k$: $p'_i = \alpha p_i$, where $i \neq k$; $p'_k = \alpha p_k(1 - q_k)$ $\alpha = \frac{1}{1 - p_k q_k}$

The pseudo code is as follows:

Algorithm 1 updateP(pos)

```
prob[pos] = prob[pos] * failP[cell[pos]]
normalize(prob)
return
```

Algorithm 2 normalize(prob)

```
normalize(prob):
sumP = sum(prob)
prob = prob / sumP
return
```

In this problem, the target is moving and we can search the cell that we are interested in directly. We don't need to move to the cell that we decide to search. We call this case "Moving case". We use Rule 1, Rule 2, and Rule 5 to supervise our search. Rule 1 and Rule 2 are the same as we mentioned before. Rule 5 is designed by ourselves. 5000 maps are generated and searched under 3 rules. The result is shown in Fig 2.

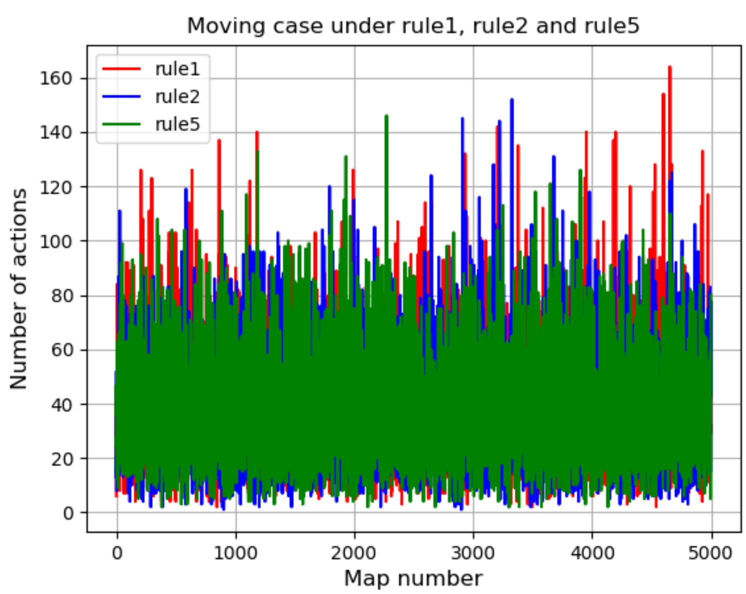

Figure 2: Number of actions in moving case

From the figure above, we find that the performance of 3 rules are similar to each other. The means of Rule1, Rule 2, and Rule 5 are 39.25, 37.82, and 38.04, which is close to each other. The variances of Rule1, Rule 2, and Rule 5 are 379.82, 347.52, and 351.06, which is also similar.

2. Update with additional observations
   Using the notion of particle filter, we can update reports easily. For a given cell, if it is not a possible previous block, all its samples should be ruled out. If it is a possible previous block, and if there are some possible neighbors according to the reports, its samples will migrate to them. But if there are no possible neighbors, all its samples will also be trapped in this block and die out. Then, resample all blocks.

   Yet, notice that reports are related to each other. For example, 2 consecutive reports H-F, C-H tell far more than "the target first move from Hilly to Flat or from Flat to Hilly, and then move form Cave to Hilly or from Hilly to Cave". We know that the target must move from Flat to Hilly and then to Cave.

   Therefore, we should consider each consecutive pair of reports and try to figure out the real moving direction. Once we know the real direction, we can pin down the direction of future reports, but also all previous reports.

   Notice that if we know the real direction of previous reports, we should re-filter all previous observations because observations have become more accurate.

   The pseudo code is as follows:

Algorithm 3 updateR(report)

```
reUpFlag, directions = solveReprot(report)
if reUpFlag then
  reUpdateReport(searchHistory, targetHistory)
else
  updateReport(directions)
end if
return
```

Algorithm 4 solveReport(report)

```
if reportSolved then
  direction = trackTarget(report, targetHistory[-1])
  reUpFlag = False
else
  common = findCommon(report, reportHistory[-1])
  if len(common) == 1 then
    backtrack(common, reportHistory)
    direction = tarckTarget(report, targetHistory[-1])
    reUpFlag = True
  else
    direction = (report, report.reverse())
    reUpFlag = False
  end if
end if
reportHistory.append(report)
return reUpFlag, direction
```

Algorithm 5 reUpdateReprot(searchHistory, targetHistory)

```
for i in range(searchHistory) do
    updateP(searchHistory[i])
    updateReport(((targetHistory[i], targetHistory[i+1]), ))
end for
return
```

Algorithm 6 updateReprot(directions)

```
tempProb = zeros_like(prob)
for prev, post in directions do
    for each block pos do
        if cell[pos] == prev then
            nPosList = where(border[pos, post])
            factor = len(nPosList)
            if factor then
                for each nPos in nPosList do
                    tempPorb[nPos] = tempPorb[nPos] + prob[pos] / factor
                end for
            end if
        end if
    end for
end for
prob = normalize(tempProb)
return
```

Algorithm 7 trackTarget(report, prev)

```
post = report - prev
targetHistory.append(post)
return post
```

Algorithm 8 backtrack(post, reportHistory)

```
targetHisory.insert(0, post)
for report in reportHistory.reverse() do
    prev = reprot - post
    targetHistory.insert(0, prev)
end for
return
```

In this problem, the target is moving and motion also counted as one step. We call this case "moving and motion". We use three rules to guide our search — Rule 1, Rule 2, and Rule 5. Rule 1 and Rule 2 are just the original rules where motion is one step now. Rule 5 is designed for this specific problem. We generate 5000 maps and use these 3 rules to search the target in the maps. The result is shown in Fig 3.

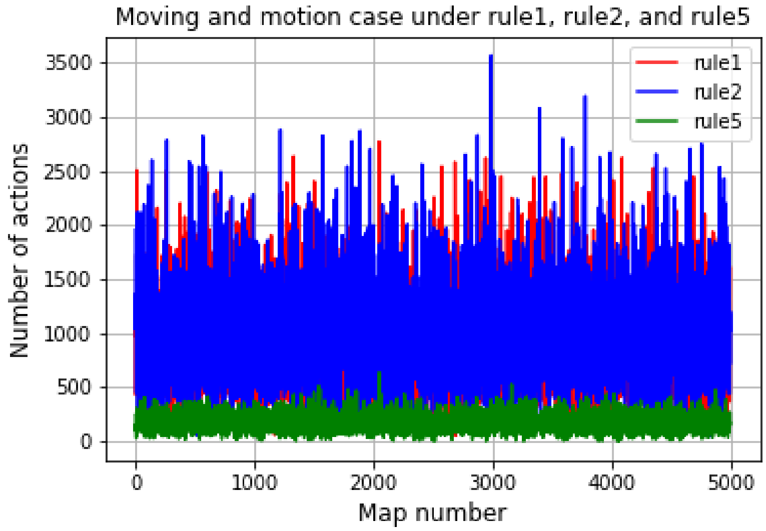

Figure 3: Number of actions in moving and motion case

From the figure above, it is easy to find that the performance of Rule 5 is better than Rule 1 and Rule 2. The means of Rule1, Rule 2, and Rule 5 are 938.88, 991.54, and 177.54. We can see that the average number of actions of Rule 5 is smaller than Rule 1 and Rule 2. The variances of Rule1, Rule 2, and Rule 5 are 154395.57, 176673.58, and 5534.82. From this view, we can also conclude that Rule 5 is better.