Homework #1

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Part 1

a. The estimated cost of cheapest solution through the east neighbor of the agent is

$$f(east) = g(east) + h(east) = 1 + h(east) = 1 + 2 = 3$$

, but the estimated cost of cheapest solution through the north neighbor of the agent is

$$f(north) = g(north) + h(north) = 1 + h(north) = 1 + 4 = 5$$

According to the rule of A* search, because $f(east) < f(north)$, we choose to move the agent to the east at the first step given that the agent doesn't know initially which cells are blocked.

b. We keep a closed list and the agent never comes back to the cells in the closed list. Because the grid world is finite, we can put at most finite number of cells into the closed list, which means we can only move finite steps to find the target or report that it is impossible to reach the target if we cannot find unblocked neighbors at one cell. Suppose there are n unblocked cells in the grid world. According to the repeated A* algorithm this project use, the length of the path we compute every time is at most n. And we can compute path at most for n times, which means we need to compute path after each move. Therefore, that the number of moves of the agent until it reaches the target or discovers that this is impossible is $O(n \times n) = O(n^2)$.

Part 2

We set 2 different modes of priority queue for the open list in A* search. If we set mode=0, we use $f(s) + g(s)$ as priorities to break ties in favor of cells with smaller g-values. If we set mode=1, we use $f(s) - g(s)$ as priorities to break ties in favor of cells with larger g-values. For the same maze we generate, we run A* search using two different modes of priority queue. Here is 10 results of 10 different experiment:

mode= 0	8147	855	9062	2364	14177	2562	4587	5762	10474	943
mode=1	767	216	949	677	1321	889	1402	1885	1090	461

From the table we can know the number of expanded cells of mode=0 is always bigger than the number of expanded cells of mode=1, which means the Repeated Forward A* with larger g-value tie break strategy is much faster than Repeated Forward A* with smaller g-value tie break strategy.
The reason is that if we choose larger g-value to break ties, the agent will choose

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Answer. Let $x_{ij} = 1$ if number i is hashed to bucket j. Let n_j denotes the number of elements in bucket j. For bucket j, we know the bubble sort algorithm of sorting this bucket j is $O(n_j^2)$. Thus, the total expected running time is

$$E(\sum_{j=1}^{n} O(n_j^2)) = \sum_{j=1}^{n} O(E(n_j^2))$$

We know

$$E(n_j^2) = \sum_{i=1}^{n}\sum_{k=1}^{n} E(x_{ij}x_{kj} = \sum_{i=1}^{n} E(x_{ij}^2)) + 2\sum_{i=1}^{n}\sum_{k=i+1}^{n} E(x_{ij}x_{kj}) = n \times \frac{1}{n} + 2 \times \binom{n}{2}\frac{1}{n^2} = O(1)$$

Therefore, the expected running time is

$$E(\sum_{j=1}^{n} O(n_j^2)) = \sum_{j=1}^{n} O(E(n_j^2)) = n \times O(1) = O(n)$$

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a. Let $H = \{constant(a)|0 \leq a \leq m-1\}$ This set of hash functions is uniform because for all x and i$(0 \leq i \leq m-1)$, $Prob(h(x) = i) = \frac{1}{m}$. This set of hash functions is not universal because for all $x \neq y$, $Prob(h(x) = h(y)) = 1 \neq \frac{1}{m}$. Therefore, this set of hash functions is uniform but not universal.

b. Let $H = \{h_{a,b}(x,y) = ax + by \pmod m | 0 \leq a, b \leq m-1\}$. This set of hash function is universal from class, that is, for input $(x,y) \neq (x',y')$, we assume $x \neq x'$, if $ax + by = ax' + by' \pmod m$, we get $a = \frac{b(y'-y)}{x-x'} \pmod m$, if b is fixed, a is fixed according to this equation. Thus, m of hash functions can cause collision of (x,y) and (x',y'), and there are m^2 hash functions in total. Thus the probability of $Prob(h(x,y) = h(x',y') = \frac{m}{m^2} = \frac{1}{m})$.
This set of hash functions is not strongly universal because if $h(x,y) = ax + by \equiv i \pmod m$ and $h(x',y') = ax' + by' \equiv j \pmod m$, we can get $a = \frac{(i-j)-b(y-y')}{x-x'} \pmod m$. When b is fixed, a is fixed according to this equation.Thus, there are m hash functions satisfying $h(x,y) = i \ \& \ h(x',y') = j$. Thus, $Prob(h(x,y) = i \ \& \ h(x',y') = j) = \frac{m}{m^2} = \frac{1}{m} \neq \frac{1}{m^2}$. Therefore, this set $H = \{h_{a,b}(x,y) = ax + by \pmod m | 0 \leq a, b \leq m-1\}$ is universal but not strongly universal.