Algorithm 1: Transformation from Parameters to the Normal Form

Input: $m \in \mathbb{R}^2$, $C \in \mathbb{R}^{2\times 2}$, *where* $C = C^\top$, $\sigma \in \mathbb{R}$
Output: $\alpha \in [0, \pi/2]$, $\kappa \in \mathbb{R}^{>1}$, $\sigma \in \mathbb{R}^+$

Rotate the coordinate system
$A \leftarrow [v_1\, v_2]$ *where* $\quad Cv_i = \lambda_i v_i, \quad v_i = \frac{v_i}{\|v_i\|}$ ▷ Eigendecomposition
$C \leftarrow A^T C A$
$m \leftarrow A^T m$

Rescale the coordinate system
$f_\mathrm{s} \leftarrow \frac{1}{\sqrt{\det|C|}}$
$C \leftarrow f_\mathrm{s} \cdot C$
$m \leftarrow f_\mathrm{s} \cdot m$

Rescale to set $|m|$ to 1
$f_\mathrm{d} \leftarrow \frac{1}{|m|}$
$m \leftarrow f_\mathrm{d} \cdot m$
$\sigma \leftarrow f_\mathrm{d} \cdot \sigma$

Flip and swap axis if necessary
if $m_{(1)} < 0$ **then**
  $m \leftarrow \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \cdot m$
if $m_{(2)} < 0$ **then**
  $m \leftarrow \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot m$
if $C_{(1,1)} < 0$ **then**
  $C \leftarrow \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \cdot C \cdot \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
  $m \leftarrow \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \cdot m \cdot \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Aggregate return values
$\alpha \leftarrow \arccos(m_{(1)})$
$\kappa \leftarrow C_{(1,1)}$
return α, κ, σ