Randomized Graph Cluster Randomization With R Package: RGCR

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- Ugander, J. and Yin, H., 2023. Randomized graph cluster randomization. *Journal of Causal Inference*, 11(1), p.20220014.
- You can download 'RGCR' from https://github.com/RUC-ChangHao/RGCR.

Classic causal inference and randomization

- Binary treatment $Z_i \in \{0, 1\}$ ($Z \sim P(z)$, CR, Bernoulli design, etc)
- Potential outcomes $Y_i(1)$ and $Y_i(0)$ (SUTVA assumption)
- Causal effects of interest: average causal effect (ACE)

$$\mathrm{ACE} \stackrel{\text{def}}{=} \frac{1}{n}\sum_{i=1}^{n}\{Y_i(1) - Y_i(0)\}$$

- Estimators: ht (ipw), hajek, aipw, etc

$$\hat{\tau}_{\text{ht}} = \frac{1}{n}\sum_{i=1}^{n}\frac{Y_i(1)Z_i}{P(Z_i=1)} - \frac{1}{n}\sum_{i=1}^{n}\frac{Y_i(0)(1-Z_i)}{P(Z_i=0)}$$

- Technically, a missing data problem

	1	2	3	$\cdots$	n
$Y_i(1)$	36.5	?	?	$\cdots$	38.0
$Y_i(0)$	?	38.0	37.0	$\cdots$	?

- Question: To get better estimation of ATE, how do we assign treatment Z? (i.e., seek the 'optimal' experiment design $Z \sim P(z)$)

Causal inference under interference

- Violation of SUTVA
- Common in advertising, epidemiology and educational studies
- Potential outcomes $Y_i(\mathbf{z})$, where $\mathbf{z} \in \{0,1\}^n$
- Causal effects of interest: total treatment effect (TTE)

$$\mathrm{TTE} \overset{\text{def}}{=} \frac{1}{n}\sum_{i=1}^{n}\{Y_i(\mathbf{1}) - Y_i(\mathbf{0})\}$$

- Classic randomization does not work here (why?)

Table: Bias (SD) of HT Estimators

sample size	16^2	24^2	32^2	48^2	64^2
SUTVA	0.00 (0.22)	-0.04 (0.16)	0.02 (0.13)	0.00 (0.08)	0.01 (0.05)
interference	0.09 (7.91)	-1.00 (3.74)	-0.47 (8.47)	-0.39 (2.33)	0.22 (4.61)

General framework for interference

- A social network $G = (V, E)$
  - through which individuals interfere each other
  - observable and correctly measured
- An exposure mapping
  - determines the extent and intensity of the interference
  - technically reduces the number of potential outcomes
  - canonical examples (minor notation abuse)
    - (no interference) $Y_i(z) = Y_i(z_i)$
    - (neighborhood interference) $Y_i(z) = Y_i(z_{\mathcal{N}_i})$
    - (arbitrary interference) $Y_i(z) = Y_i(z)$
    - ("individualized" interference) $Y_i(z) = Y_i(?)$
- Estimators: ht, hajek, difference-in-means, etc
- Experimental designs $Z \sim P(z)$: complete randomization, bernoulli randomization, cluster randomization, etc

A toy example

clustering $\boldsymbol{c}_1$ clustering $\boldsymbol{c}_2$

- Under full neighborhood interference

$$\mathrm{TTE} \stackrel{\text{def}}{=} \frac{1}{n}\sum_{i=1}^{n}\{Y_i(\mathbf{1}) - Y_i(\mathbf{0})\} = \frac{1}{n}\sum_{i=1}^{n}\{Y_i(\mathbf{1}_{\mathcal{N}_i}) - Y_i(\mathbf{0}_{\mathcal{N}_i})\}$$

  - what really matters is valid sample size
  - what's a good design? intuitively (under proper social networks)
    - gives valid sample size a lower bound (increasing with sample size)
    - gives every node a positive exposure probability (due to ht estimators)

Randomized graph cluster randomization

- Randomized graph clustering
  - a weight indicating which nodes are more important
    - related to the variance of $\hat{\tau}_{\mathrm{ht}}$, roughly speaking, related to the inverse exposure probabilities $1/\pi_i^1$ and $1/\pi_i^0$
    - types: uniform, degree, spectral weighting
  - a randomized clustering design deduced by the weight
    - types: 3-net and 1-hop-max (heuristically)
  - a clustering results generated from the clustering design
- Randomization at the level of clusters
  - types: complete design and bernoulli design
- Remark: no literature really solves this problem, all algorithms are heuristic in some sense

$$d^*, \hat{\tau}^* = \arg \min_{d \in \mathcal{D}, \hat{\tau} \in \mathcal{M}} \max_{\{Y_i(z)\} \subseteq \mathcal{Y}} E_{\{Y_i(z)\}, d, \hat{\tau}} \left[\mathrm{dist} \left(\hat{\tau}_d(Y, Z), \tau(\{Y_i(z)\}) \right) \right]$$

Weighted 3-net clustering

Algorithm 4: Weighted 3-net clustering.

Input: Graph $G = (V, E)$, node weights $\boldsymbol{w} \in \mathbb{R}_+^n$.
Output: Graph clustering $\boldsymbol{c} \in \mathbb{R}^n$

for $i \in V$ **do**
  $X_i \leftarrow \beta(w_i, 1)$
$\pi \leftarrow \arg\mathrm{sort}([X_i]_{i \in V}, \mathit{descend})$
$S \leftarrow \emptyset$, unmark all nodes
for $i \in \pi$ **do**
  if *i is unmarked* **then**
    $S \leftarrow S \cup \{i\}$
    for $j \in B_2(i)$ **do**
      mark node j if it is unmarked yet
for $i \in V$ **do**
  $c_i \leftarrow \arg\min\{j \in S, j \rightarrow \mathbf{dist}(i, j)\}$, i.e., the id of the node in S with shortest graph distance to i (arbitrary tie breaking)
return $\boldsymbol{c}$

Weighted 1-hop-max clustering

Algorithm 3: Weighted 1-hop-max clustering.

Input: Graph $G = (V, E)$, node weights $\boldsymbol{w} \in \mathbb{R}^n_+$.
Output: Graph clustering $\boldsymbol{c} \in \mathbb{R}^n$

```
for i ∈ V do
    X_i ← β(w_i, 1)
for i ∈ V do
    c_i ← max([X_j for j ∈ B_1(i)])
return c
```

R package 'RGCR'

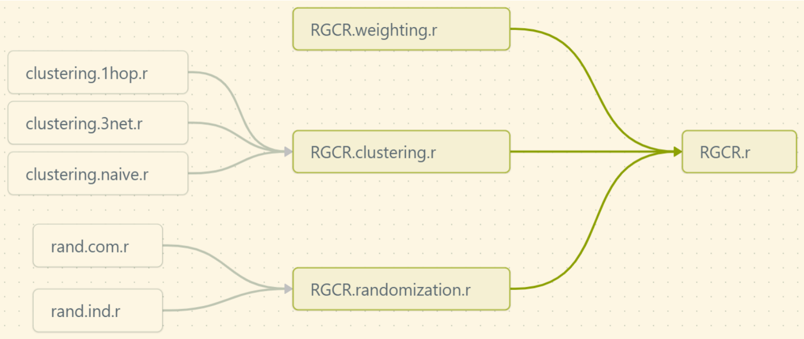
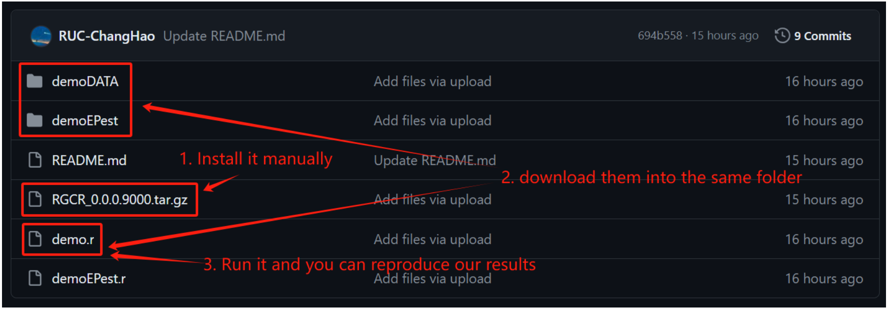


Figure: The Structure of functions

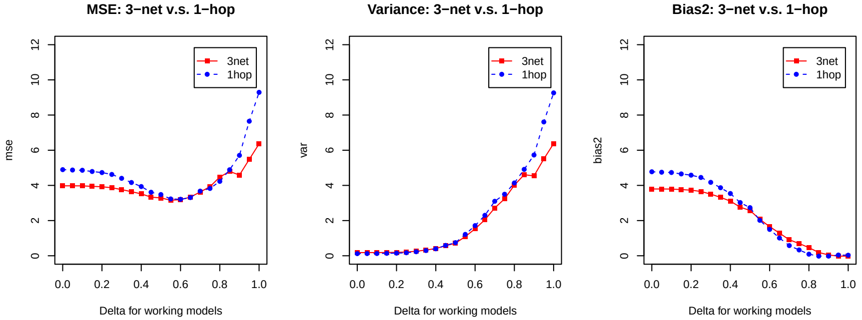
- Source: https://github.com/RUC-ChangHao/RGCR

A short guidance to use 'RGCR'



Application of ‘RGCR’

- The real data analysis of experimental design is actually a simulation



- Conclusion: the design ‘spectral weighting + 3-net clustering + complete randomization’ is recommended (after numerous simulations...)

Thank you!