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Coupled quantum wells as a novel source of optical anisotropies in nanostructured systems

Doctoral Thesis in Applied Sciences
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Statement of authorship

I, Oscar Ruiz Cigarrillo, student of the Graduate Program in Applied Sciences of the School of Sciences of the Universidad Autonoma de San Luis Potosi, as author of the thesis "Coupled quantum wells as a novel source of optical anisotropies in nanostructured systems", declare that the thesis is an original, unpublished, authentic, personal work, that the corresponding sources have been cited and that in its execution the legal provisions in force that protect the copyright and intellectual and industrial property rights were respected. The ideas, doctrines, results and conclusions I have reached are my absolute responsibility.

Abstract

In the present work, it is proposed a new source of IOAs occurring in asymmetric coupled quantum wells ACQWs, namely a reduction of the symmetry from D_{2d} to C_{2v} as imposed by asymmetry along the growth direction. We report on reflectance anisotropy spectroscopy (RAS) of double GaAs quantum wells structures coupled by a thin (< 2 nm) tunneling barrier (CQWs). Two groups of DQWs systems were studied: one where both QWs have the same thickness (SCQWs) and another one where they have different thicknesses (ACQWs). RAS measures the in-plane optical anisotropies (IOAs) arising from the intermixing of the heavy- and light- holes in the valence band when the symmetry of the DQW system is lowered from D_{2d} to C_{2v} . If the CQWS are symmetric, residual IOAs stem from the asymmetry of the QW interfaces; for instance, associated to Ga segregation into the AlGaAs layer during the epitaxial growth process. In the case of an ACWQs with QWs with different thicknesses, the AlGaAs layers (that are sources of anisotropies) are not distributed symmetrically at both sides of the tunneling barrier. Thus, the system losses its inversion symmetry yielding an increase of the RAS strength. The RAS line shapes were compared with reflectance spectra in order to assess the heavy- and light- hole mixing induced by the symmetry breakdown. The energies of the optical transitions were calculated by numerically solving the one-dimensional Schrödinger equation using a finite-differences method. Our results are useful for interpretation of the transitions occurring in both, symmetric and asymmetric CQWs.

ACKNOWLEDGEMENT

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ABBREVIATIONS

BS	Band structure
BZ	<i>Brillouin zone</i>
QS	Quantum Structures
QW	Quantum Well
SQW	Single Quantum Well
CQWs	Coupled Quantum Wells
VB	Valence Band
CB	Conduction Band
SCQWs	Symmetric coupled quantum wells
ACQWs	Asymmetric coupled quantum wells
RAS	Reflectance Anisotropy Spectroscopy
PL	Photoluminescence spectroscopy
PR	Photoreflectance spectroscopy
R	Reflectance spectroscopy
PRD	Photo-Reflectance Differential Spectroscopy
FDM	Finite difference method
CCD	Charge coupled device
0D	Zero-dimensional
1D	One-dimensional
2D	Two-dimensional
3D	Three-dimensional
fcc	Face-centered cubic
2DEG	Two-dimensional electron gas
BL	Beer-Lambert-Law
TB	Tight-Binding method
PD	Photo-Detector
PEM	Photo-Elastic Modulator
QM	Quantum Mechanics
$k \cdot p$	Semiempirical theoretical tool to calculate band-structure
TB	Semiempirical Tight-Binding Method
DFT	Density Functional Theory
SOC	Spin-Orbit Coupling, also called Spin-Orbit interaction
LFLM	Dr. Luis Felipe Lastras Martinez group.
EFA	Envelope function Approximation
EMA	Effective Mass Approximation
FKOs	Franz Keldysh oscillations
IOA	In-plane Optical Anisotropy

LIST OF CODES AND PACKAGES

This list denote the *Open-Source* packages, codes, tools, and repositories for the development of this work. All inside of this work as images or numerical calculations are subject to the *Open-Source* ideology. Our codes are housed in our own GitHub repository, both personal and laboratory repository. It's important to say that without the development of the *Open-Source* codes like contents in this list, our codes, they couldn't be enhanced.

cqws-codes Repository of our codes implemented in this work. [1]

kp-lflm-group $k \cdot p$ julia [2] package developed by Our group research [3]

ASE The Atomic Simulation Environment (ASE) is a set of tools and Python modules for setting up, manipulating, running, visualizing and analyzing atomistic simulations. [4]

Spglib Software library for crystal symmetry search [5]

SOLCORE A multi-scale, Python-based library for modelling solar cells and semiconductor materials [6]

Aestimo One-dimensional (1D) self-consistent Schrödinger-Poisson solver for semiconductor heterostructures [7]

VESTA 3D visualization program for structural models, volumetric data such as electron/nuclear densities, and crystal morphologies. [8]

PGF/TikZ PGF is a macro package for creating graphics. It is platform- and format-independent and works together with the most important TeXbackend drivers, including pdfTeXand dvips. It comes with a user-friendly syntax layer called TikZ. [9]

pst-optexp PStricks package to drawing optical experimental setups. [10]

SYMBOLS

X⁻ Negative Trion

X⁺ Positive Trion

X Direct Exciton

IX Indirect Exciton

$\text{Al}_x\text{Ga}_{1-x}\text{As}$ AlGaAs semiconductor as a function of Al concentration x

\hbar Planck's constant (eV)

m_0 electron effective mass

(hkl) Family of lattice planes with Miller indices h , k and l

E_g Energy bandgap

e electron

hh heavy-hole

lh light-hole

$e_n - hh_n$ **or** $e_n - lh_n$ Electronic transitions

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1

PHYSICAL BACKGROUND

This chapter bears the basis of fundamental physics in semiconductors and quantum structures that were implemented to understand the results in this work.

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QUANTUM MECHANICS concerns electronic behavior that exhibits many phenomena non explained by the classical regime. Quantum structures (QS) are artificially systems conformed by semiconductors where electrons exhibit quantum nature, and is a great platform to propose novel devices. Nowadays, the progress in creation of QS consist in precisely deposition of thin films, in which electrons show interesting electrical and optical properties [12]. Most of these properties consist in quantum behavior as the energy confinement, which is the principal interest in the study of electrons and their consequent interactions. Therefore, their study is still an ongoing emerging topic.

In this chapter, it is presented the fundamental concepts to describe the physical phenomena resultant in this work, without intention to replicate concepts and models already explained in publications with major impact. Therefore, the aim of this chapter is to highlight the subtle concepts behind the results of this thesis.

1.1 Semiconductor Band Structure

We start describing the band structure (BS) of zincblende semiconductors. The BS dictates the electron behavior in a solid; therefore, we will need to invoke the Schrödinger equation. Inside a solid, around around 10^{23} valence electrons contribute to the bonding in each cubic centimeter, which results in a many-body complex problem [13], with a general hamiltonian [14, 15]:

$$H = \frac{1}{2M} \sum_{i=1}^{N_n} \mathbf{P}_j^2 + \frac{1}{2m_0} \sum_{j=1}^{N_e} \mathbf{p}_j^2 + \frac{Z^2}{2} \sum_{i,j=1, i \neq j}^{N_n} V_c(\mathbf{R}_i - \mathbf{R}_j) - Z \sum_{i=1}^{N_n} \sum_{j=1}^{N_e} V_c(\mathbf{r}_j - \mathbf{R}_i) + \frac{1}{2} \sum_{i,j=1, i \neq j}^{N_e} V_c(\mathbf{r}_i - \mathbf{r}_j), \quad (1.1)$$

where N_n is the number of atomic nuclei, N_e is the number of electrons with mass m_0 , asumming that each nuclei has mass M , and charge Z_e . As a result, this Hamiltonian is too complicated as the sum of five terms which consists in: kinetic energies to electrons and nuclei, the nucleus-nucleus, nucleus-electron and electron-electron Coulomb interactions; also \mathbf{R}_i are the positions of the nuclei and \mathbf{r}_j are the position of the electrons, the operators \mathbf{P} and \mathbf{p} are momentum operators to nuclei and electrons respectively. Finally, the Coulomb potential V_c [14] is also considered.

Fortunately, because the QS are formed by crystalline materials, the Bloch theorem provide with most important tool to develop the required equations. The Bloch theorem establishes a periodic potential $U(\mathbf{r})$ for electrons, accounting for the material's periodicity (definition

of crystal structure) and the Schrödinger equation can thus be described in terms of single electron picture as:

$$\left[-\frac{\hbar^2}{2m_0} \nabla^2 + U(\mathbf{r}) \right] \psi(r) = \mathbf{E}\psi(\mathbf{r}). \quad (1.2)$$

The key reason that the periodic potential in a crystal structure is highly important is their translational invariance concept and the consequent symmetry operations that are possible in a crystalline solid. The symmetry concept, as a tool to understand solids, is discussed with major detail in the next chapter. In accordance to Bloch's theorem, we can associate a wave vector \mathbf{k} with each energy state, $E_n(\mathbf{k})$. Thus, it is useful to display the energies $E_n(\mathbf{k})$ as a function of the wave vector \mathbf{k} . This result also known as dispersion relation, but in general terms is the electron band structure of the given solid [13].

Even if the calculation of electron band structure in solids are very complex by the distance between atoms that composes it, and the Bloch's theorem provides the most important tool to reduce the problem to crystalline structures, there exist several methods to calculate the realistic bandstructure for semiconductors that are categorized into two groups: Atomistic methods* (Tight-binding, orthogonalized plane wave methods) and Perturbative methods ($\mathbf{k} \cdot \mathbf{p}$)[†]. These two main categories with theirs respective methodologies, have special characteristics which becomes in the reasons to choose them. The reasons have to do how to described BS which means, in case of Atomistic methods, the entire bands (both valence and conduction) can be described, whereas in perturbative methods are focuses to near bandedge BS. Hence, each of these methods can be chosen and enhanced as the system to study requires. We will not enter in details regarding which of these methods are the best for our case and the reasons are simple: each method is very powerful and we must bear in mind that the complexity of solutions requires that these are solved by numerical techniques, given very good approximations. Thus, we basically conclude that the electron behavior inside a semiconductor consist in solutions of the Schrödinger equation [16].

1.1.1 Valence and Conduction Bands

In general, the bands of semiconductors composed by valence and conduction bands separated by a region known as bandgap. The bandgap, which is proportional to separation energy of valence and conduction bands also is called as forbidden region, this is because electron states are nominally forbidden, therefore this gap energy determine the electron conduction in a semiconductor and the difference they have with the insulators and metals as shown in Figure 1.1.

*In this category can include the ab initio methods, these are the most complex methods due to propose solutions of the many-body problem

[†]In fact, both TB and $\mathbf{k} \cdot \mathbf{p}$ also consider in same kind, because both are *semi-empirical* methods due to they consider experimental parameters.

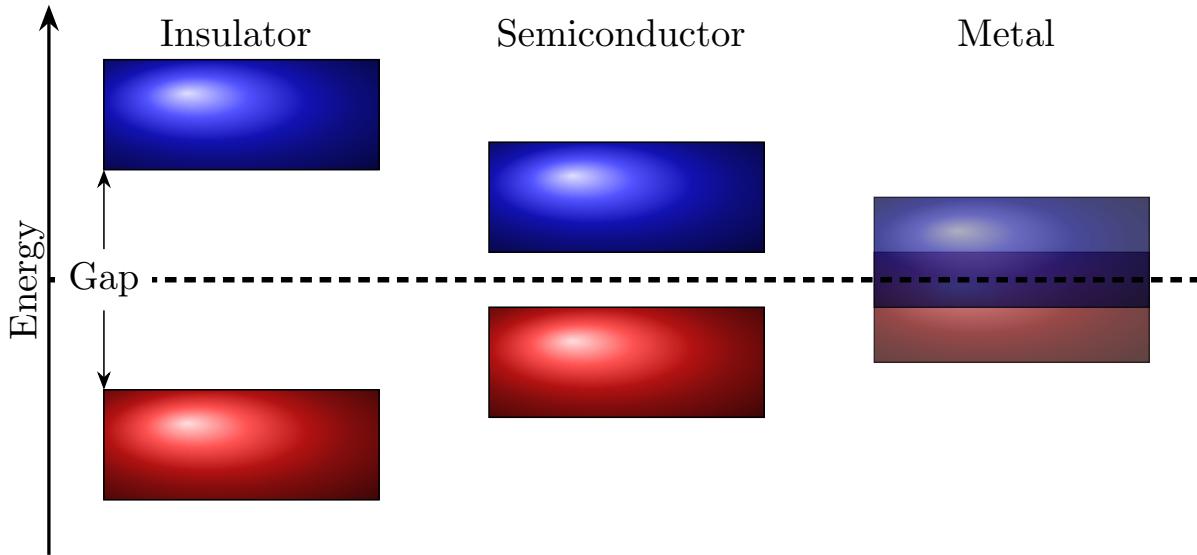


Figure 1.1: Band energy diagram for insulators (left), semiconductors (center) and metals (right). The principal difference is the gap energy, for insulators this is longer than semiconductors, although in semiconductors gap energy depends on materials, finally in metals doesn't exist gap energy instead exist an overlap bands characterize these. Dashed line determines Fermi level.

The bandgap determines many characteristics and functionalities in semiconductors. In addition the bandgap energy classifies in direct and indirect semiconductors, but they do not only depend on this energy but the BS is the liable signature for this difference. As pointed out before, it is so difficult to describe electron behavior in solids due to the many body interactions that exists rendering the Schrödinger equation very hard to solve. Starting by describing bulk semiconductors, for example GaAs, that belongs to the family III-V cubic semiconductor with a lattice composed by two sublattices with a single atom each (Figure 1.2). For this case when the atoms in two sublattice are different the crystal structure is then called *zinc-blende* [17].

To calculate BS of bulk semiconductors it is important to define specific symmetry directions. For each three-dimensional wave vector \mathbf{k} , then the plot energy as a function of \mathbf{k} is along of different high-symmetry directions [13]. GaAs is a direct semiconductor for which the [001] direction is of high-symmetry denoted by Γ point ($\mathbf{k} = 0$)

The most “exact” computation is carried out by DFT theory. These calculations are commonly called atomistic even some semiempirical models can be regarded as atomistic, but the semiempirical models are good approximations in comparison with the DFT theory. So, which is the reason to call “exact” solutions to the DFT results? The answer leads us to great discussion and it's not intended to get into controversy, but in general the DFT calculations have the capacity to calculate in terms of electrons interaction and the empirical methods are based in the potential choice.

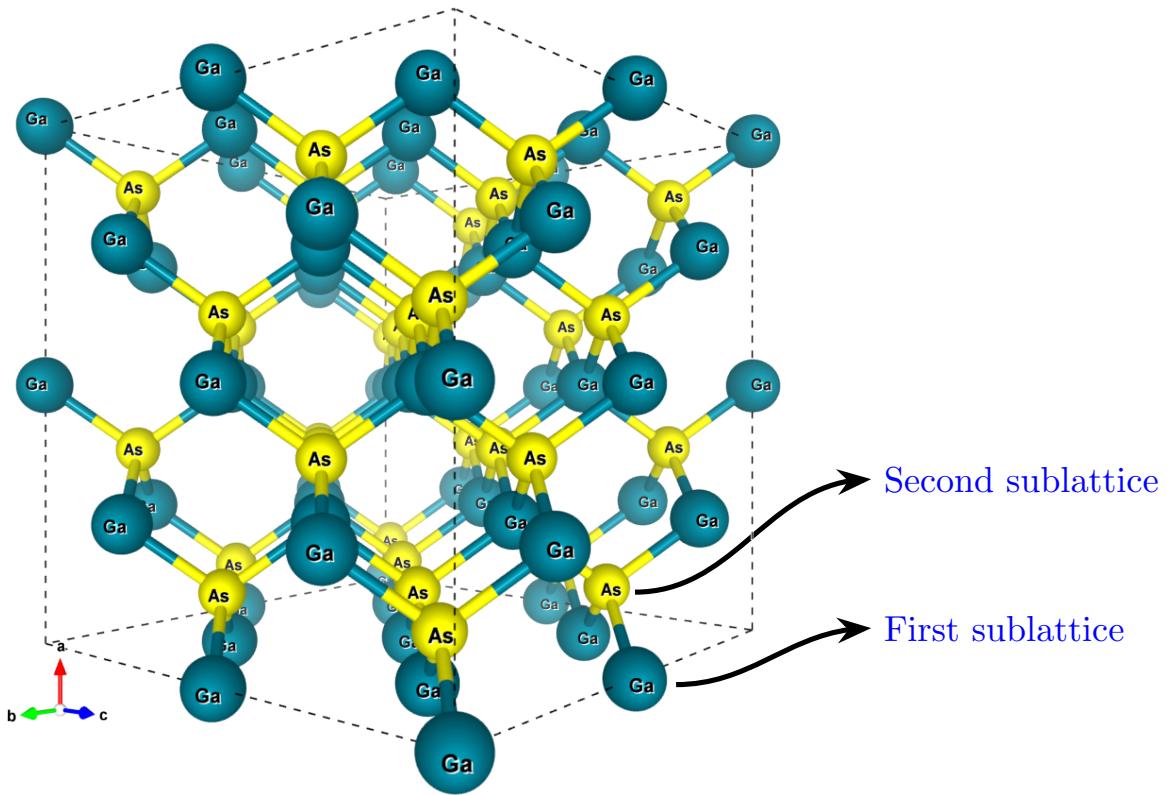


Figure 1.2: GaAs crystal structure, where each sublattice corresponds to Ga and As atoms respectively.

We will not get into details regarding BS theories and models, but to give a brief reference and the importance to this work. The models much preferred in semiconductor heterostructures such as GaAs/AlGaAs are semiempirical, this is because DFT theory and their derived models are very limited to carry out into large structures, and their electron interaction nature need high computational performance. Therefore, in comparison with empirical models where the main role is the potential of semiconductor structures, reduces computational effort. Hence, with empirical models where the main role is the potential of semiconductor structures, this reduces computational effort. Then, the most models used in semiconductor BS are empirical models, these models are distinguished by low computational requires for this reason are considered like approximations. The importance to discuss these concepts will take relevance when we discuss the physics model proposed in this work.

Section 1.1.1 shows the results of calculations of TB model as discussed in [18] and the code was implemented by R. Muller [19]. The model proposed by Vogl et al. takes into account a small number of localized pseudo-orbitals and is based on the empirical parameters to be substituted on a TB Hamiltonian. The importance to obtain BS is to get detailed optical properties of solid structures.

Figure 1.3(a) shows that the of GaAs is a direct band semiconductor. In Figure 1.3(b)

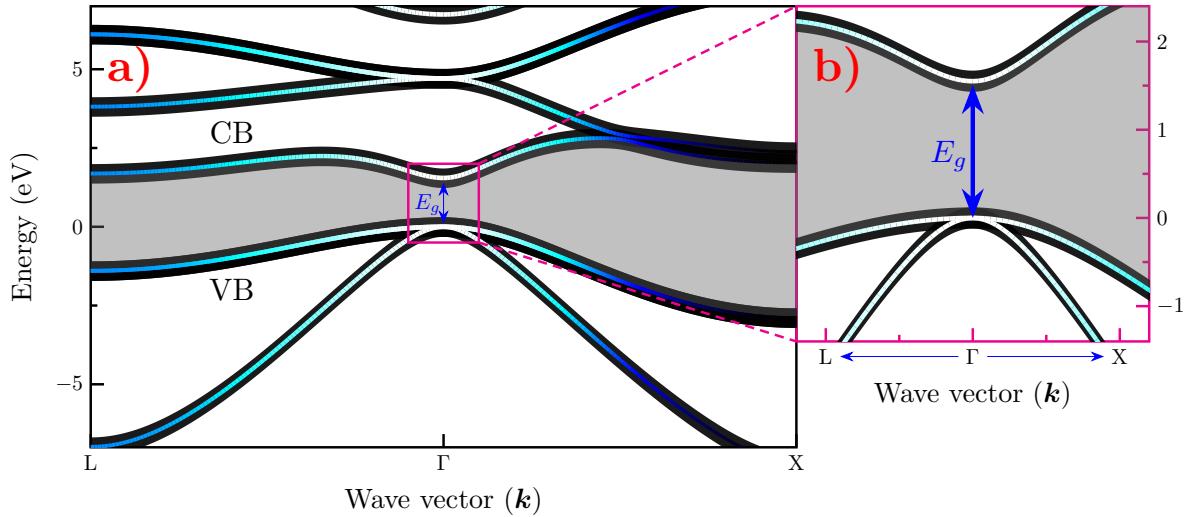


Figure 1.3: Band structure of GaAs, (a) shows the zoom around of Γ to denote the direct band gap and the electrons energy needed to jump from valence to conduction band. (b) denotes the two directions to dispersion of the bands corresponds to Brillouin zone: $\Gamma \rightarrow X$ and $\Gamma \rightarrow L$. [11]

it is plotted, the band dispersion around Γ point, which exhibits the highest symmetry. It is well-known that the band dispersion increasing \mathbf{k} along two different directions of the Brillouin zone, from $\mathbf{k} = (0, 0, 0)$ to X point $\mathbf{k} = (2\pi a_L)(1, 0, 0)$ and L point $\mathbf{k} = (2\pi a_L)(1, 1, 1)$. These figures, are the typical representation of a direct gap III-V semiconductors around $\mathbf{k} = 0$ and the shape of the dispersion is parabolic. The BS of GaAs shown in Section 1.1.1 does not take into account the contribution of the spin*. It is important to mention that the characteristic curvature $E-\mathbf{k}$ of the dispersion bands correspond to an electron (e) in case of positive curvature, while the negative curvature corresponds to hole states; heavy (hh) and light hole (lh) bands, so, this denotes that the transitions, are of dipole nature [11, 15].

All the above discussion refers to a basic quantum mechanisms in solids. The electron absorption, specifically interband absorption, gives a way to a fundamental physical process involving the principle of many basic studies of semiconductors, applications, and the importance to understand the electron behavior in a semiconductor structures disputed in this work. Then, the Figure 1.4 schematizes the two typical transitions in GaAs bulk, this transitions are near to $k=0$, so that is called interband absorption. Is important to remark that the interband transitions are observed in all solids, but the mechanisms are different dependently of their bandstructure, for what, being repetitive when mentioning that the bandstructure is the key to study solids, trough the very well known Fermi's golden rule. Later this is disputed according to highlight the model and results obtained. It has been mentioned that the direct transitions are of dipole nature, as before mentioned the CB is type s -like while the VB is p -like, then it's electric-dipole allowed transitions $p \rightarrow s$. The

*This spin contribution is called as split-off (so) hole band.

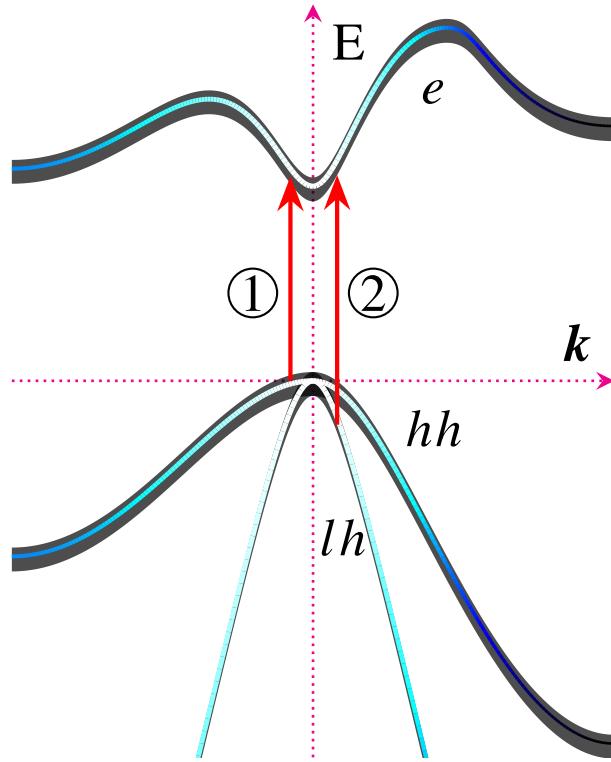


Figure 1.4: Two typical transitions for GaAs near $k = 0$. The first one correspond to the heavy-hole and second one to the light-hole.

excitation of electron in the CB creates a hole VB, then the final quasiparticle is called **electron-hole pair**, which is the main subject of the following sections.

1.1.2 Excitons

The importance to study bandstructure of semiconductors is very clear so far, so that could be said that absorption process is the source of optical properties of solids. It's due to this that the importance to study of semiconductors in the optoelectronics applications. But, this process give rise to formation of one of the most important excitations in the crystal structures. The photon absorption process carries an electron from CB to VB, this generates an empty location in VB which has positive charge. This positive empty location called as hole, therefore the electron and hole have opposite charge then it's to be expected that they are attracted, so, this creates a bound state called an exciton [20].

From Figure 1.4 and Figure 1.5 is clearly that excitons are commonly presented in direct band gap semiconductors as GaAs, this was denoted by absorption experiments, after mentioned in ?? this is the cause in photoluminescence mechanism.

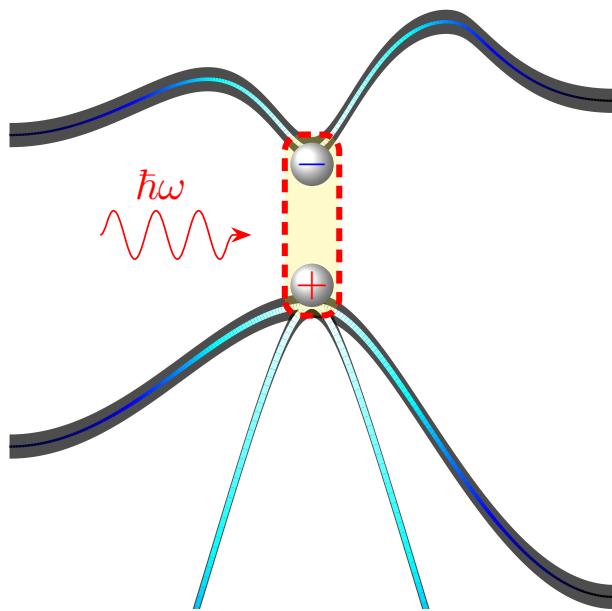


Figure 1.5: Qualitative scheme of exciton creation in GaAs as direct gap.

1.2

Semiconductor Low-Dimensional Structures

The previous section engaged to explain the principles of semiconductors, this is the bandstructure, the importance of these is practically the fingerprint of all semiconductor, without bandstructure the understanding of these would be improbable. The first approximations were based in GaAs bulk, their cubic symmetry practically defines their nature and consequently the physical effects as excitons existence. But, what happens if joined several semiconductors with same symmetry and structural parameters? The bulk properties and physical properties are the same?. The answers they are well-known, when two materials with relatively same structural parameters, as lattice constant can create a heterojunction, the union of several heterojunction make up a heterostructure.

The Figure 1.6 is a general scheme of a heterostructure, in this case it presents three species of atoms Al, As, Ga. These atoms can locate in columns III-V of the periodic table, hence its name of III-V semiconductors. The principal characteristic of these atoms is that it can create matched structures as GaAs, AlAs and ternary alloys as $\text{Al}_x\text{Ga}_{1-x}\text{As}$ with specific Al concentration. The matched semiconductors produce a material with new properties based principally in the difference of bandgap which involves the alloys. As can see in Figure 1.6 it's consisting a GaAs/ $\text{Al}_x\text{Ga}_{1-x}\text{As}$ heterostructure, these interface is well-matched due to the lattice parameters is relatively equals, therefore and thanks to powerful growth technics as MBE it's possible to get high-quality quantum structures.

Also, the heterostructure composed by two semiconductors with different band gaps

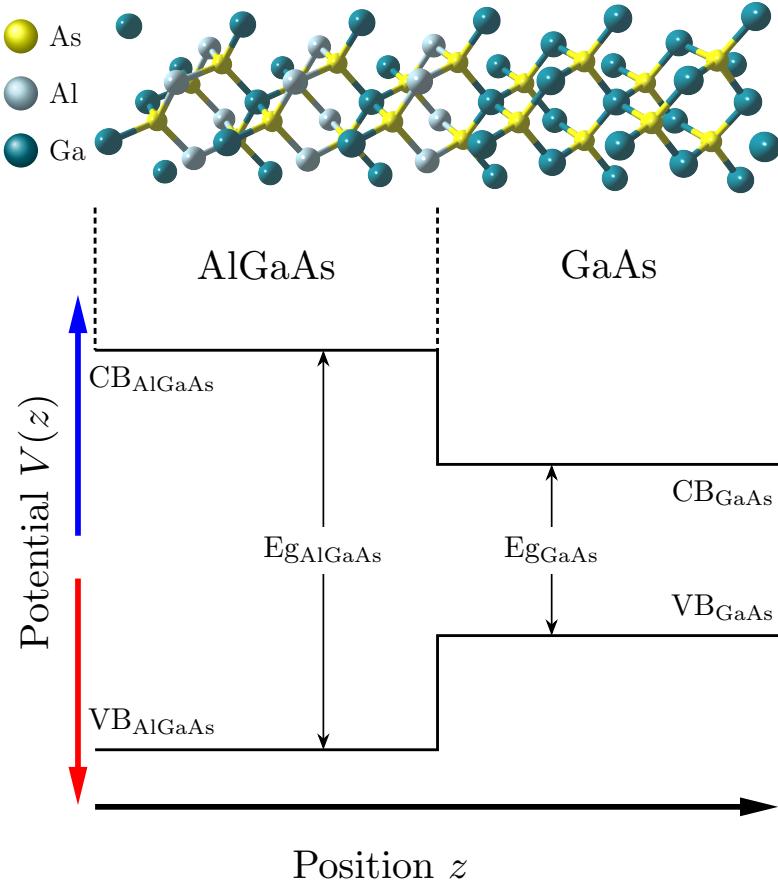


Figure 1.6: General scheme of GaAs/ $\text{Al}_x\text{Ga}_{1-x}\text{As}$ heterostructure. Scheme of atomic arranged of this heterjunction, the dashed lines are the matched between two dissimilar materials (top). Band-edge profile (bottom).

generate a discontinuity in either the conduction or the valence band can be represented by a constant potential term [21]. The theory to treatment the electron behavior in these structures, is relatively simple if we consider the above. Although, in this chapter doesn't have intention to explore the theory of electron behavior in that, worth noting that it get one-dimensional potential $V(z)$ to both bands, so the Schrödinger equation can solve simple.

1.2.1 Quantum wells

The major relevance in nanostructures systems lies in the fact of quantum confinement, the junction of semiconductors results in an interest quantum structures with specific dimensions. From 3D bulk, the dimensions reduce to 2D, 1D and 0D dimensional structures. Therefore, each of that has interest properties and their correspond applications. The Figure 1.7 shows the low dimensional heterostructures from 3D bulk, the first low dimensional from 3D to 2D is the Quantum Wells, then from 2D to 1D it have the Quantum

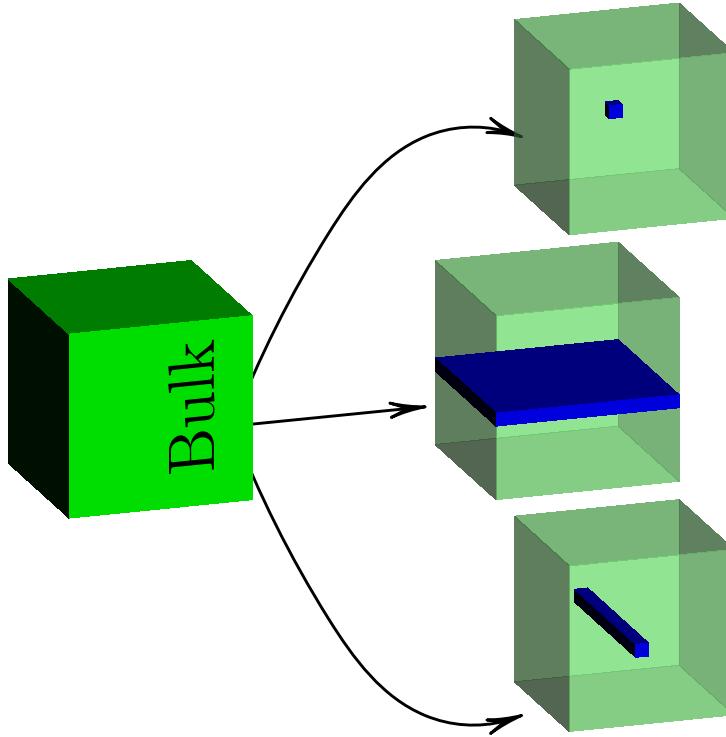


Figure 1.7: Heterostructures from bulk (3D), to Quantum Wells (2D), Quantum Wires (1D) and Quantum Dots (0D).

Wires finally with 0D have the Quantum Dots. The quantum confinement is the principal reason to study that structures, the electron behavior which exhibits in it should can to help understand a great variety of quantum mechanical phenomena as electron interaction on a crystal. Suppose a heterostructure composed with a two semiconductor alloys as sandwich, this 2D quantum structure is called a Single Quantum Well (SQW).

This dissimilar semiconductors in terms of their potential energy ($V(z)$) can be schematized as Figure 1.8. The Gap difference of $\text{Al}_x\text{Ga}_{1-x}\text{As}$ and GaAs is due to x Al concentration in $\text{Al}_x\text{Ga}_{1-x}\text{As}$ therefore it obtains a one dimensional potential profile, with that can confinement electrons in a 2D plane along z direction. All of these carries to quantum mechanics formalism, the electron behavior should be obeyed these rules. If we have an electron closed in two potential barriers an L distance, the wave which describe it will be spatially confined. So if we confined many electrons in these potential, we have two important physical aspects: the first one is known as Pauli's exclusion principle, which as of its Fermion nature prevents carriers with the same spin occupying the same region in of space [21, 22], the second one and one of the most relevant in the birth of the quantum mechanics; the Heisenberg's uncertainty principle. That last, it can say that is the consequence of quantum confinement due to the space reduction of the electrons is expected that momentum increases by an amount of the order \hbar/L . Therefore, the energy of that confined particles increases, and it's referred to as confinement energy [15].

Then the quantum confinement is our started point to understand the optical properties

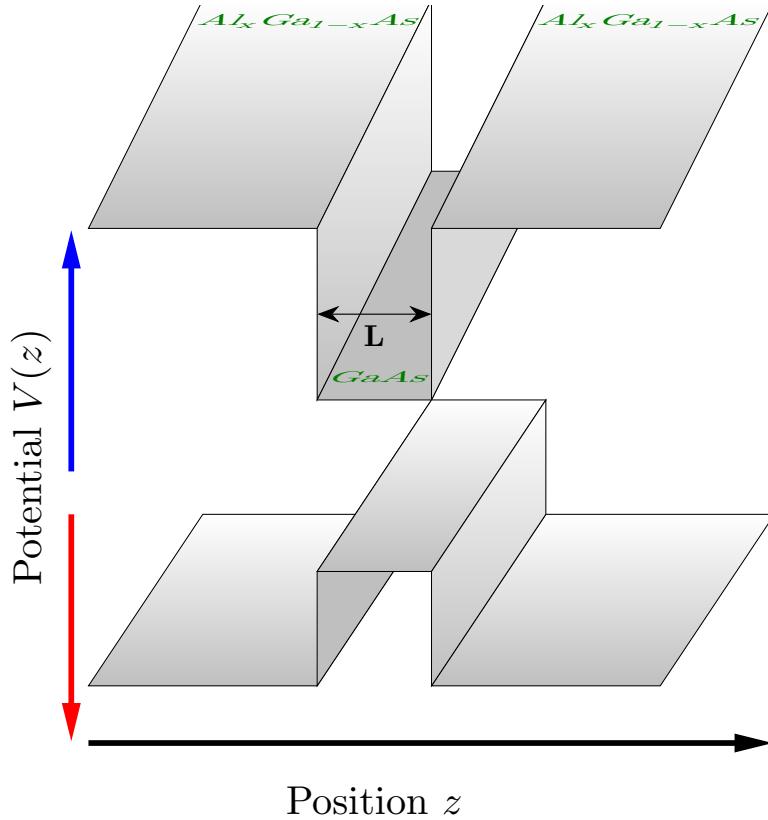


Figure 1.8: $GaAs/Al_xGa_{1-x}As$ Single Quantum Well

in QWs. As is referred in the figure, the uni-dimensional potential profile can well describe by top conduction- and bottom valence-bands, the band offset in these two of correspond gap energy between that, while Al concentration increases their bandgap and the band offset (Q_c to CB and Q_v to VB) also to.

It's so clearly that the QWs have the potential to presents amazing quantum properties, even if all of these are very important we focus on the optical properties, basically our interest is the light-matter interaction through its result mechanisms.

1.2.2 Preliminary approach of Quantum Confinement effect in QWs

As the title describes, here it will try to explain as the first approach the quantum confinement effect in QWs. If it starts with the scoop, which it can be reduced the electrons space, then this mean that in reciprocal space it has two components k_x and k_y . Then say in crystal symmetry properties to the case of $GaAs/Al_xGa_{1-x}As$ QWs it's Γ the central point, as long as $x < 0.4^*$ the bandstructure depends on confinement energ, so say which the bandstructure depends on confinement energy. In this case, the

*It will be explained in the next section, although it's due to the Gap go from direct to indirect, shortly the symmetry $\Gamma \rightarrow X$.

electronic properties in comparison with a bulk semiconductor properties can solve through particle-in-a-box as textbook problem as first approach. Nevertheless, even if usually can solve without much mathematical formalism is very essential that it dedicates a chapter with their solution, this is because will employ a physical formalism exclusively to QWs structures. In general way, as it before mentioned the Schrödinger equation solution is the fundamental pillar to understand, where it's taken into account which in a crystal the periodic potential is the key. Here are important remarks before to continue, when it has a heterostructure starting with the bulk model it's clearly that the system doesn't same, the Quantum Mechanics which is behind take into account the symmetry properties, then it can be developed a Hamiltonian to understand that system. In the next chapter will be explained details and the formalism both physical and mathematical to solve and discuss it's. The model which give the tools to get the solutions is called as Effective Mass Approximation, thus their correspond Schrödinger equation is [11, 21, 23–26]:

$$-\frac{\hbar^2}{2m^*} \frac{\partial^2}{\partial z^2} \psi(z) + V(z)\psi(z) = E\psi(z), \quad (1.3)$$

where the m^* is the effective mass in each material, and $V(z)$ is the potential profile got by heterostructure materials properties. Therefore, that differential equation can solve as in textbooks explained [11, 21, 23, 25, 27–30]. The idea is thinking as a one particle in a finite potential well, where is well important established the boundary conditions and solve the Schrödinger equation in each part of single QW, this means that need to create a potential function. Then it's can obtain the Eigenfunctions and their correspond Eigenenergies.

The principal idea doesn't is reproducing something which is very well known, the objective of this part is established the scoop of the next chapter. Therefore, before to continue, we will finish with to explain the dispersion in-plane of single QW. As in the QW the one-dimensional potential set up the 1D confinement, is important doesn't confuse which the QW is a 2D structure, but their confinement is along of z direction this mean 1D. Then, the particle can motion in the $x - y$ plane. By this reason, even if consider 3D Schrödinger equation and the above is considered it obtain Equation (1.3) therefore, the solutions in the one-dimensional potential produce discrete states of energy $E_z = E_n$ [21], where n is the energy level it which produce subbands as shows in Figure 1.9. In contrast, before it called as “energy bands” in the bulk case, now due to the quantum confinement gets subbands to both conduction- and valence bands.

These subbands are the result of the sum of E_z and $E_{x,y}$, which are the 1D confinement energy and the in-plane momentum $k_{x,y}$ then [21]:

$$E = E_n + \frac{\hbar^2 |\mathbf{k}_{x,y}|^2}{2m^*}. \quad (1.4)$$

From equation the effective mass m^* depends on particle, i.e the effective mass to elec-

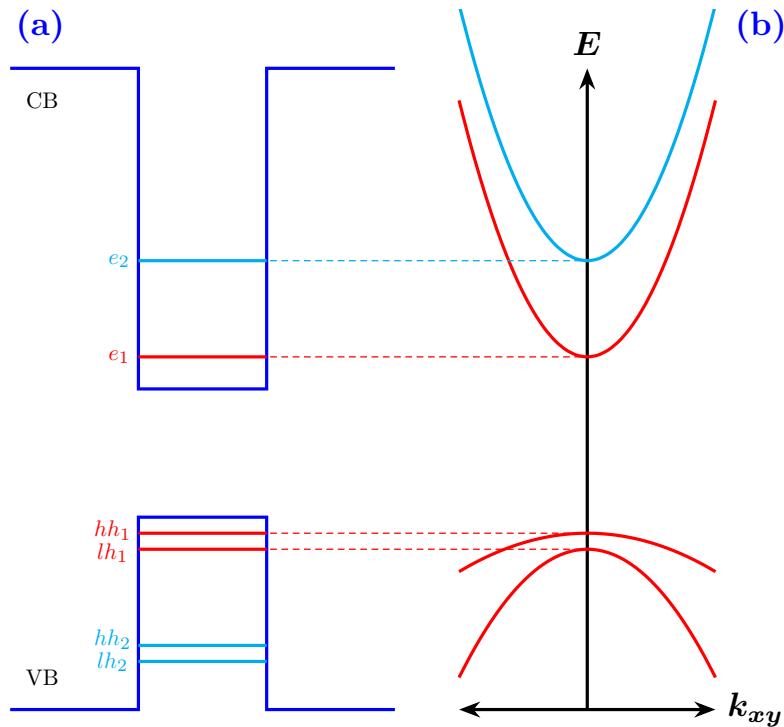


Figure 1.9: General scheme of typical Schrödinger's equation solutions to one-dimensional potential as (a) where the Eigenenergies of both electron and holes are denoted with same color depending on n value.(b) It's plot, of the subbands in the same case of (a) to both particles.

trons in CB and the holes in VB. So, the most relevant in the solutions is the energy E_n (Figure 1.9(a)) is discrete, this is the quantum confinement in the low-dimensional heterostructures.

1.3 Summary

In this chapter, was exposed the generalities of semiconductor band structure and low-dimensional heterostructures, highlighting or taking in major GaAs/Al_xGa_{1-x}As that's the semiconductors of major importance in this work. The band structure interpretation usually be so hard, and their calculations even more, but the impact and relevance in optical properties of semiconductors starts from that interpretation, from here arises the mathematical arsenal to right physical interpretation. Another significant concept which was treatment as first approach is the effective mass concept, even if, when solved, the bulk Hamiltonian it considers the mass as constant parameter or depending on semiconductor material, contrary in low-dimensional structures have an important role.

In generally, the band structure of semiconductors is the key to understand quantum

properties of solids, in this work the relevant is the light-matter. Remember that light-matter interaction in solids can be studied by process resulting in it, as absorption, reflection, transmission, diffraction, scattering, and others [31]. Although, the light-matter interactions are fundamentally quantum electrodynamical, also, can be studied in quantum way through before mentioned process. Firstly, is the photon absorption process can help to understand or calculate the fundamental parameter in semiconductors as is bandgap energy. This parameter is the started point in the study of semiconductors, this is the start point in the map called bandstructure, if it ignores the gap value in the semiconductor to study it couldn't possibly get the principal optical properties of that.

Then, the bandstructure of semiconductors is the map to understand them, without these routes or fundamental parameters couldn't have quantum devices.

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