The following formulas have consented to their use on this exam. At least one will be relevant.

$$\kappa(t) = \frac{||\boldsymbol{r}'(t) \times \boldsymbol{r}''(t)||}{||\boldsymbol{r}'(t)||^3} \qquad \qquad \kappa(x) = \frac{|f''(x)|}{[1 + (f'(x))^2]^{3/2}}$$

$$\kappa(s) = ||\boldsymbol{T}'(s)|| \qquad \qquad \frac{\partial z}{\partial x} = -\frac{F_x}{F_z} \qquad \qquad \frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$$

1. [12 pts] Let $\mathbf{r}_1(t) = \langle 2, 1, a \rangle + \langle t, t, 4t \rangle$ and $\mathbf{r}_2(s) = \langle 0, -1, 3 \rangle + \langle 2s, bs, cs \rangle$ be parametrizations of lines.

a) Find specific values of a, b, and c such that the lines are coincident (ie. they are the same (1,1,4) $||\langle 2,b,c\rangle|$ and contain (2,1,a) and (0,-1,3) $||\langle 2,b,c\rangle|$ and contain (2,1,a) and (0,-1,3) $||\langle 2,b,c\rangle|$ and contain (2,1,a) and (0,-1,3) $||\langle 2,b,c\rangle|$ and $||\langle 2,1,a\rangle|$ and $|\langle 2,1$

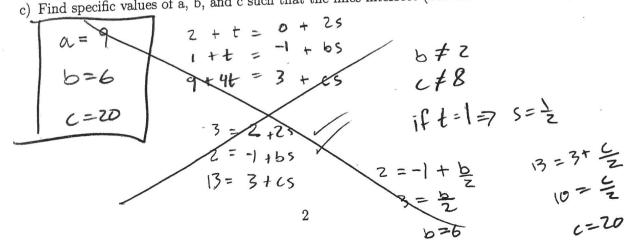
$$2+t=25$$
 $1+t=-1+25$
 $1+t=85$
 $1+t=85$
 $1+t=85$
 $1+t=1+25$
 $1+t=1$

$$a = 9$$

$$b = 2$$

$$c = 8$$

Find specific values of a, b, and c such that the lines intersect (but are not coincident).



2. [10 pts] Find an equation for the tangent plane to the graph of $f(x,y) = \ln(x^2y + x + y)$ at the point (x,y) = (-1,2).

$$Z = \int_X (X - X_0) + \int_Y (y - y_0) + Z_0$$

 $Z_0 = \int_{-1,2} = \ln(2 + (-1) + 2) = \ln(3)$

$$\int_{x}(x,y) = \frac{2xy+1}{(x^2y+x+y)} = \frac{2(-2)+1}{2-1+2} = \frac{-3}{3} = -1$$

$$f_{\gamma}(x,y) = \frac{\chi^2 + 1}{(\chi^2 y + \chi + y)}$$
 = $\frac{1+1}{2-1+2} = \frac{2}{3}$

3. [14 pts] Consider the lines
$$r_1(s) = \langle 1+s, 1+4s, -4-6s \rangle$$
 and $r_2(t) = \langle t, -3-5t, 2+t \rangle$.

a) Find the point of intersection of r_1 and r_2 .

b) Find an equation for the plane that contains both lines.

$$-26(0)$$
 $-7(-3)$ $-9(2) = d$

$$\lim_{(x,y)\to(-1,1)}\frac{x^2+2x+1}{(x+1)^2+(y-1)^2}$$

$$\lim_{(x,y)\to(-1,1)} \frac{x^2 + 2x + 1}{(x+1)^2 + (y-1)^2}$$
along $y = -x : \lim_{(x,y)\to(-1,1)} \frac{x^2 + 2x + 1}{(x+1)^2 + (x-1)^2} \Rightarrow \lim_{(x,y)\to(-1,1)} \frac{x^2 + 2x + 1}{x^2 + 2x + 1 + x^2 - 2x + 1} = \lim_{(x,y)\to(-1,1)} \frac{x^2 + 2x + 1}{(x+1)^2 + (x-1)^2} = \lim_{(x,y)\to(-1,1)} \frac{x^$

$$\frac{x^2 + 2x + 1}{(x+1)^2 + (x-1)^2}$$

$$\frac{-\frac{(x_1+x_2)(x+1)}{2x^2+2}}{1}$$

$$\frac{1}{(x,y)-(4,1)} \frac{x+1}{z} = 0$$

along
$$y=1$$
 $\lim_{(x+1)^2} \frac{x^2+2x+1}{(x+1)^2} = \lim_{(x+1)^2} \frac{(x+1)^2}{(x+1)^2} = 1$

Timit DNE

Along $y=1$

Along $x=-1$
 $\lim_{(x+1)^2 to} \frac{(x+1)^2}{(x+1)^2} = 1$
 $\lim_{(x+1)^2 to} \frac{(x+1)^2}{(x+1)^2}$

$$\frac{(x+y)^2}{(x+y)^2} = 1$$

$$\lim_{\lambda \to 0} \frac{|\lambda|^2}{|\lambda|^2}$$

$$n \frac{(\chi + 1)^2}{(\chi + 1)^2 + 0} = 1$$

[6 pts] The expression $e^{xy}\cos x = \arcsin(xyz)$ defines z implicitly as a function of x and y. Determine $\frac{\partial z}{\partial x}$.

termine
$$\frac{\partial^2}{\partial x}$$
.

 $e^{xy}\cos x - \arcsin(xyz) = 0$
 $\frac{d}{dx} \arcsin(xyz) = 0$

$$\frac{\partial z}{\partial x} = \frac{-F_x}{F_x}$$

$$\frac{\partial z}{\partial x} = \frac{-F_x}{F_z}$$

$$\int_{z}^{z} \frac{dz}{dx} = \frac{-F_x}{f_z}$$

$$\frac{ye^{xy}\cos(x) - \frac{yz}{1+x^2}}{\frac{xy}{1+z^2}}$$

6. [20 pts] Consider the helix parametrized by
$$r(t) = \left\langle 4 \sin\left(\frac{t}{3}\right), t, 4 \cos\left(\frac{t}{3}\right) \right\rangle$$
.

a) Which axis does the helix wrap around?

b) Find a parametrization for the tangent line to the curve at $t=2\pi$.

$$\frac{10}{211} = (\frac{4}{3}\cos(\frac{1}{3}), 1, -\frac{4}{3}\sin(\frac{1}{3}))$$

$$= (\frac{4}{3}\cos(\frac{1}{3}), 1, -\frac{4}{3}\cos(\frac{1}{3}))$$

$$= (\frac{4}{3}\cos(\frac{1}{3}), 1, -\frac{4}{3}\cos(\frac{1}{3})$$

$$= (\frac{4}{3}\cos(\frac$$

$$\vec{u}(t) = \vec{r}(2\pi) + \vec{r}(2\pi) \left(t-2\pi\right)$$

c) Find an arc-length parametrization for the given curve.

$$s = \sqrt{\left[\frac{4}{3}\cos\left(\frac{4}{3}\right)\right]^2 + 1 + \left[-\frac{4}{3}\sin\left(\frac{4}{3}\right)\right]^2} dt$$

$$S = \int_{6}^{\frac{1}{4}} \frac{16}{3} \cos(\frac{t}{3})^{2} + 1 + \left[-\frac{4}{3} \sin(\frac{t}{3}) \right]^{2} dt$$

$$S = \int_{6}^{t} \frac{16}{9} (\cos^{2}(\frac{\pi}{3}) + \sin^{2}(\frac{t}{3})) + 1 dt dt$$

$$S = \int_{6}^{t} \frac{16}{9} + 1 dt dt dt$$

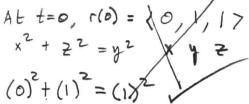
$$5 - \int_{0}^{t} \frac{16}{9} + 1 day$$

$$s = \int_{0}^{t} \frac{16}{9} + 1 day \left(\frac{dt}{3} \right)$$

$$=\frac{5}{3} + -\frac{5}{3} = (0)$$

$$r(s) = \left(4 \sin\left(\frac{s}{5}\right), \frac{3}{5} s, 4 \cos\left(\frac{s}{5}\right)\right)$$

- 7. [6 pts] Consider the curve \mathcal{C} parametrized by $\mathbf{r}(t) = \langle e^{-t} \sin t, e^{-t}, e^{-t} \cos t \rangle$ for $t \geq 0$. Let \mathcal{S} denote the surface $\{(x, y, z) : x^2 + z^2 = y^2\}$.
 - a) Show that the curve $\mathcal C$ lies on the surface $\mathcal S.$



this does not prove it.



- b) What type of surface is S? doubt cont.

 paraboloid going out towards positive direction of y
- 8. [6 pts] Determine whether the statement is True (T) or False (F). No justification is needed.
 - a) The unit tangent vector T(t) is always perpendicular to the vector T'(t).
 - b) The vector projection of a vector v onto a non-zero vector w is always non-zero. The



- c) The linearization of f(x,y) = 2 3x + 4y at any point (a,b) is L(x,y) = 2 3x + 4y. The linearization of f(x,y) = 2 3x + 4y.
- 9. [8 pts] Find a parametrization for the intersection of the surfaces $x^2 z^2 = y 1$ and $x^2 + z^2 = 16$.

$$x^{2} = 16 - Z^{2}$$

$$16 - Z^{2} - Z^{2} = y - 1$$

$$17 - 2z^{2} = y$$

$$2z^{2} = 17 - y$$

$$z^{2} = 17 - y$$

$$x = \sqrt{\frac{4+15}{2}}$$
let $y=t$

$$\Gamma(t) = \left(\frac{1}{2} + \frac{1}{2} \right)^{\frac{1}{2} - t}$$

- 10. [14 pts] Let $f(x, y, z) = x^2yz$, and let $r(t) = \langle t, 2t, 3t^2 \rangle$.
 - a) Use the Chain Rule for Paths to evaluate $\frac{d}{dt}f(\mathbf{r}(t))$ at t=1.

$$\frac{x}{\partial t} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial t} + \frac{\partial f}{\partial z} \cdot \frac{\partial z}{\partial t}$$

$$= \frac{1}{30}$$

$$= \frac{1}{30}$$

$$= \frac{1}{30}$$

$$= \frac{1}{30}$$

b) Find the directional derivative of f(x, y, z) at r(1) in the direction of r'(1).

$$r(1) = \langle 1, 2, 3 \rangle$$
 (point)
 $r'(1) = \langle 1, 2, 66 \rangle = \langle 1, 2, 6 \rangle$ (direction $\Rightarrow \frac{1}{14} \langle 1, 2, 6 \rangle$
 $||r'(1)|| = \sqrt{1 + 4 + 26} = \sqrt{41}$

c) Did you get the same answers for parts (a) and (b)? Why or why not?