

Prior Prefixes

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Question

Write a program that takes an input String s and prints all palindromic substrings of s .

Explanation and Algorithm

Solution: Forward and Backwards Hashing

To understand this solution, it is helpful to understand simple multiplicative hashing and polynomial evaluation.

Multiplicative Hashing: Hashing a String s

```
LargePrime = 6829
Base = 1
for (char c : s) {
    Base = Base * LargePrime + c
}
```

The above rule is analogous to Horner's Rule for Polynomial Evaluation: Given a polynomial of degree n ,

$$\begin{aligned} b_n &= a_n \\ b_{n-1} &= a_{n-1} + x * b_n \\ &\dots \\ b_0 &= a_0 + x * b_1 \end{aligned}$$

Horner's rule uses the above formula to evaluate polynomials more efficiently than classical polynomial evaluation. We can view the above multiplicative hash value as evaluating a polynomial with $x = \text{LargePrime}$ and with the character values as the polynomial coefficients.

Given this Horner's rule view, we can find all palindromic prefixes of a String in linear time. To do this, we use Horner's rule - style hashing, and normal polynomial evaluation - style, to hash the String and its reverse at the same time.

Hints

1. How do you tell if a string is a palindrome?
2. Consider using hashing.
3. Can you hash a string and its reverse in one pass?
4. How many passes will it take to hash all of the substrings?
5. Can you check multiple substrings in a single pass?

Code

```
/*Answer */

public class FindSubstrings{
    public static void main(String [] args){
        String s = args[2];
        for (int i = 0; i <= s.length(); i++){
            int largePrime = 6829;
            int primePower = 1;
            int forwardBase = 0;
            int reverseBase = 0;
            for (int j = i; j < s.length(); j++) {
                forwardBase = forwardBase * largePrime + (int) s[j];
                reverseBase = reverseBase + primePower * (int) s[j];
                primePower *= largePrime;
                if (forwardBase == reverseBase) {
                    System.out.println(s.substring(i,j+1))
                }
            }
        }
    }
}
```

Run time analysis

The run time of this solution is $O(n^2)$, where n is the length of the string. The runtime is polynomial because the hashing strategy gives us all prefixes of a

string in linear time, and there are n different starting indices from which to generate different prefixes.