Question 6.7 from CTCI: The Apocalypse

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Question

In the new post-apocalyptic world, the world queen is desperately concerned about the birth rate. Therefore, she decrees that all families should ensure that they have one girl or else they face massive fines. If all families abide by this policy-that is, they have continue to have children until they have one girl, at which point they immediately stop-what will the gender ratio of the new generation be? (Assume that the odds of someone having a boy or a girl on any given pregnancy is equal.) Solve this out logically and then write a computer simulation of it.

Explanation and Algorithm

This can be solved in a straight forward fashion using expected value. Remember, expected value works by multiplying the quantity of events you are measuring happening by the probability of that event happening. So:

0 * (1/2) + 1 * (1/4) + 2 * (1/8) + 3 * (1/16) + 4 * (1/32) +... yields the answer for this question which is 1/2.

To elucidate this further, each whole number represents the number of boys that are born in a given case and each fraction represents the probability of an outcome. So, the first term is 0 * (1/2) because there is a 1/2 that no males are born. The second term expresses the probability of one boy being born then a girl being born which requires two specific 1/2 probabilities (hence 1/4). The weighted value is 1 since the event we are measuring are boys being born. This continues for all terms to infinity since technically an infinite amount of boys can be born with the probability of that event occuring halving after each son is born.

Note: Every fraction represent the probabilities of an overall outcome not the probability of a boy or a girl being born given the previous amount of boys and

girls born. Each birth is an independent event which means, no matter what the event, there is a 50/50 chance for a boy or a girl being born which never changes.

Hints

1. Observe that each family will have exactly one girl. pre-order and post-order traversal?
2. Think about writing each family as a sequence of Bs and Gs.
3. You can attempt this mathematically, although the math is pretty difficult. You might find it easier to estimate it up to families of, say, 6 children . This won 't give you a good mathematical proof, but it might point you in the right direction of what the answer might be.
4. Logic might be easier than math . Imagine we wrote every birth into a giant string of Bs and Gs. Note that the groupings of families are irrelevant for this problem. What is the probability of the next character added to the string being a B versus a G?
5. Observe that biology hasn't changed; only the conditions under which a family stops having kids has changed. Each pregnancy has a 50% odds of being a boy and a 50odds of being a girl.

Code

```
/* Java CTCI Answer 1 */

double runNfamilies(int n){

   int boys = 0;
   int girls = 0;

   for int (int i = 0; i < n; i++){
      int[]genders = runOneFamily();
      girls += genders[0];
      boys += genders[1];
   }
   return girls / (double) (boys + girls);

}

int[] runOneFamily(){
   Random random = new Random();
```

```
   int boys = 0;
   int girls = 0;

   while(girls == 0){
      if(random.nextBoolean()){
         girls += 1;
      }
      else{
         boys += 1;
      }
   }
   int[] genders = {girls, boys};
   return genders;
}
```

```
/* Simple no array solution */

int girls = 0;
int boys = 0;

double fam(int n){
   for(int i = 0; i < n; i++){
      while (random.nextBoolean())
         boys++;
      girls++;
   }
   return (girls / (double) (girls + boys));
}
```

Run Time analysis

Technically, the run time will be a sum of n values with each value being the amount of times it took for a particular family to have a girl. At least, this will mean n runs of the for loop up, but theoretically the algorithm could run forever since it depends on random number generation.

Sources

Question, answer and other material taken from Cracking the Coding Interview 6th edition by Gayle Laakmann McDowell.