Question 6.6 from CTCI: Blue-Eyed Island

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Question

Blue-Eyed Island: A bunch of people are living on an island, when a visitor comes with a strange order: all blue-eyed people must leave the island as soon as possible. There will be a flight out at 8:00 pm every evening. Each person can see everyone else's eye color, but they do not know their own (nor is anyone allowed to tell them). Additionally, they do not know how many people have blue eyes, although they do know that at least one person does. How many days will it take the blue-eyed people to leave?

Explanation and Algorithm

Imagine all the people in the island looking at each other at the same time. Each person will have knowledge of every other persons' eye color except their own. Since they all know at least one person either one of two things will happen: They will see no one with blue eyes , know they are the only blue eyed person on the island and leave that night or they will see one or more blue-eyed people and stay. Now remember, in the latter case everyone has infomation on everyone else. That means that every person observing another blue-eyed person will assume that either that person is the only blue-eyed person and will leave the first night or that they will still be there the next day. If they are then that means there must be at least one other person with blue-eyes. If a given person, on the second night, sees only one other person with blue-eyes they will automatically figure out they are the second person with blue eyes and thus that person and the person they observed with blue eyes will leave. This logic extends to all other numbers n. If n people have blue eyes on the island then on the nth day each blue eyed person will realize one more day has passed than the amout of blue-eyed people they've seen and will all individually realize they must leave and will all do so that night together. At the same time, every other person who is not blue-eyed will assume either every blue eyed person will leave that night or they are also a blue eyed person that will be revealed to them the

next day when no one has left. Thus, it takes n days for all blue eyed people to leave the island where n is the amount of blue eyed people on the island.

Hints

1. This is a logic problem, not a clever word problem. Use logic/ math/ algorithms to solve it.
2. Suppose there were exactly one blue-eyed person . What would that person see? When would they leave?
3. Now suppose there were two blue-eyed people. What would they see? What would they know? When would they leave? Remember your answer from the prior hint. Assume they know the answer to the earlier hint.
4. Build up from this. What if there were three blue-eyed people? What if there were four blue-eyed people?

Sources

Question, answer and other material taken from Cracking the Coding Interview 6th edition by Gayle Laakmann McDowell.