

Question 6.2 from CTCL: Dominos

September 20, 2017

Question

There is an 8x8 chess board in which two diagonally opposite corners have been cut off. You are given 31 dominos, and a single domino can cover exactly two squares. Can you use the 31 dominos to cover the entire board? Prove your answer (by providing an example, or showing why it's impossible)

Explanation and Algorithm

No, you cannot use the 31 dominos to cover the entire chessboard:

We're told that we have an 8 x 8 chessboard with opposite corners cut off. So we have, in total, 62 squares that we need to cover. We also know that we are given 31 dominos, each of which covers two squares. In a chess board, no two adjacent squares are the same color, so each domino must cover one white square and one black square. In a chessboard without opposite corners cut off, we would have 32 black squares and 32 white squares. However, since we are given a chessboard with opposite corners cut off, we know that the the squares on the corners that were cut off must have been the same color. So, we have 2 less squares of one color. Let's say that those are white squares. We would now have 30 white squares and 32 black squares.

When we lay down the dominos, they take up one white square and one black square. Remember, no two adjacent squares in a chessboard are the same color. So, 31 dominos would take up 31 white squares and 31 black squares exactly. However, we know that we have 30 white squares and 32 black square. There is then no possible way to cover the entire board. There can never be a situation where a domino covers 2 squares of the same color.

Hints

1. Consider how many squares are in the given chessboard. What properties do the squares have in a chess board? Think about the colors of adjacent squares.

2. Consider how a domino lies on a chess board. How might this relate to the colors of adjacent squares?
3. Consider the number of white and black squares covered by the 31 dominos. How many squares of each color should be covered in total?

Sources

Question, answer and other material taken from Cracking the Coding Interview 6th edition by Gayle Laakmann McDowell.