Question 6.1 from CTCI:
The Heavy Pill

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Question

The Heavy Pill: You have 20 bottles of pills. 19 bottles have 1.0 gram pills, but one has pills of weight 1.1 grams. Given a scale that provides an exact measurement, how would you find the heavy bottle? You can only use the scale once.

Explanation and Algorithm

Each pill has a certain number of pills (let's say 50) to start with. In order, to distinguish which bottle has the heavier pills you must use the number of pills in the bottles as an unique identifier. The only way to do that is by making each bottle have a unique number of pills. The number of pills acts both as a unique identifier for each bottle. For simplicity sake, let's imagine the first bottle has 1 pill, the second 2, the third 3, all the way to the 20th with 20. Now, weight them all. You will get a number X.

If all the pills were weighing 1.0 grams X would be 1+2+3+...+20 (let's call this Y). However one of them is tipping the scale more. So what you need to is subtract X-Y. The difference will be equal to some multiple between 1-20 * 1.1. Which multiple it is tells you which bottle has the heavier pills.

Hints

1. You can only use the scale once. This means that all, or almost all, of the bottles must be used . They also must be handled in different ways or else you couldn't distinguish between them.

2. What happens if you put one pill from each bottle on the scale? What if you put two pills from each bottle on the scale?

3. Imagine there were just three bottles and one had heavier pills. Suppose you put different numbers of pills from each bottle on the scale (for example, bottle 1 has 5 pills, bottle 2 has 2 pills, and bottle 3 has 9 pills). What would the scale show?

4. You should be able to have an equation that tells you the heavy bottle based on the weight.

Sources