1. (10pts) The probability that a vehicle entering Michigan has Canadian license plates is 0.12; the p
that a vehicle entering Michigan is an SUV is 0.28; and the probability that a vehicle entering is an S
with Canadian license plates is 0.09.

A. An SUV is entering Michigan, what is the probability that it has Canadian plates?

B. A vehicle with Canadian plates is entering Michigan, what is the probability it is an SUV?

A)

$P_{suv} = .28 \qquad P_{can} = .12 \qquad P_{s+c} = .09$

A) Probability given that it is an suv

where A = is an SUV

B = has canadian plates

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{.09}{.28} = .3214$$

B) Probability $\overset{suv}{}$ given that it is canadian

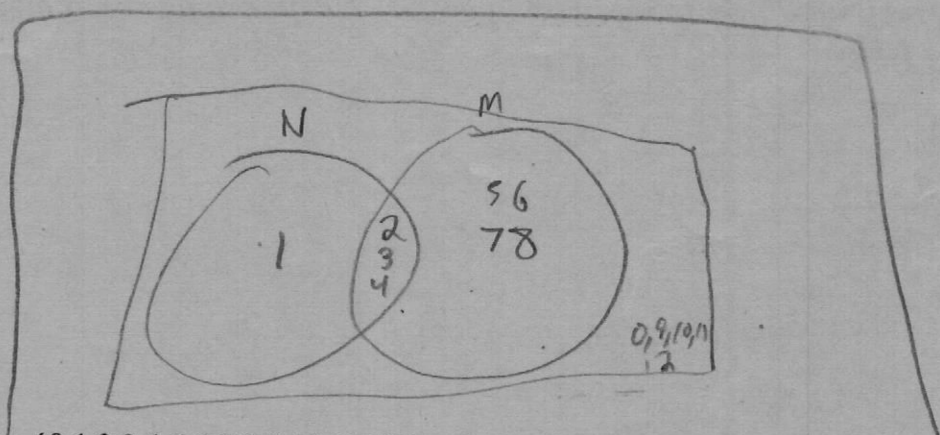
where C = is an SUV

D = canadian plates

$$P(C|D) = \frac{P(D \cap C)}{P(D)} = \frac{.09}{.12} = .75$$

A: .3214

B: .75



2. (15pts) Consider the sample space $S = \{0,1,2,3,4,5,6,7,8,9,10,11,12\}$ and the events $M = \{2,3,4,5,6,7,8\}$ and $N = \{1,2,3,4\}$. Find:

or

A. $M \cup N$ $\{1,2,3,4,5,6,7,8\}$

and

B. $M \cap N$ $\{2,3,4\}$

and

C. $M' \cap N'$ $\{0,9,10,11,12\}$

out side both

ISyE 3770: Exam 1

Total: 75 **out of 100**

Last Name: | C | u | r | r | i | e | r |

First Name: | C | o | l | l | i | n |

GTID: 902553414

This exam is given under the Georgia Tech Honor Code System. You must observe and sign the Honor Pledge: *"I have neither given nor received aid on this exam."*
Your signature below signifies your compliance with this honor code.

Signature: ______________________

Directions: Show all work. DO NOT detach any pages of this exam. Write legibly; if the grader cannot read your response it will be marked incorrect. BOX YOUR FINAL ANSWER or write in the space provided. All numeric responses should be given in decimal or fractional form to receive full credit. No questions are allowed. Do your best. GOOD LUCK!

2 catagories

Transactions to a computer database can be classified as either new items (additions) or changes to [existing] items (modifications). Adding a new item can be completed in less than 100 milliseconds 90% of the time. Changing an existing item can be completed in less than 100 milliseconds 20% of the time. If 30% of transactions are changes, what is the probability that a transaction can be completed in less than 100 milliseconds?

.3 ch → .20

.7 add → .90 → chance to complete w/in 100 msec

Sum of probability of both cases

$$\underbrace{(.7)(.9)}_{\text{addition}} + \underbrace{(.3)(.2)}_{\text{mod.}} = .69$$

$\boxed{\text{.69 probability a transaction can be completed in less than 100 milliseconds}}$

4. (10pts) The probability that a patient recovers from a delicate heart operation is 0.9. What is the probability that exactly 5 of the next 7 patients having this operation recover?

-10

Bernoulli trials → negative binomial R.V. in form of $f(x) = \binom{x-1}{r-1}(1-p)^{x-r}p^r$

$r = 5$

$x = 7$

$p = .9$

$$\binom{7-1}{5-1}(1-.9)^2(.9)^5 = \cancel{.0885735}$$

$\boxed{\text{P = .0885735 that 5 out of next 7 patients recover}}$

I assume all integration constants are zero

5. (20pts) Weekly consumption of Sprite from a local chain of convenience stores is a continuous ra variable X having *pdf* $f(x) = 2(x-1),\ 1 < x < 2$. Find the mean and variance of X.

−10

For a Continuous R.V. pdf of $f(x)$, $\mu = \int_{-\infty}^{\infty} x f(x)\,dx$ $\sigma^2 = \int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu^2$

Solve for μ

$$2\int (x^2 - x)\,dx = \frac{2x^3}{3} - \cancel{\frac{2x^2}{2}}\Big|_1^2 = \left(\frac{16}{3} - \frac{12}{3}\right) - \left(\frac{2}{3} - 1\right) = \frac{5}{3}$$

Solve for σ^2

$$2\int (x^3 - x^2)\,dx = \frac{2x^4}{4} - \frac{2x^3}{3}\Big|_1^2 = \left(8 - \frac{16}{3}\right) - \left(\frac{2}{4} - \frac{2}{3}\right) = \frac{17}{6}$$

Mean: 5/3

Variance: 17/6

6. (10pts) A student council cabinet consists of a President, Vice President, Treasurer and Secretary. If there are 65 students in the class, how many different cabinets can be formed?

1 2 3 4

−5

$$\binom{65}{4} = \boxed{677{,}040 \text{ different cabinets}}$$

assume each person can hold 1 office

7. (5pts) Suppose A and B are mutually exclusive events. Then $P(A \cap B) = P(A) + P(B)$. True or False?

False if they are mutually exclusive $P(A \cap B) = \emptyset$ $\Rightarrow$ assume $P(A) \neq \emptyset$ and $P(B) \neq \emptyset$

8. (20pts) A shipment of 7 TV sets contains 2 defective sets. A hotel purchases 3 TVs randomly selected from the shipment. If x is the number of defective sets purchased by the hotel, find the probability distribution (*pmf*) of X.

7 TV 2 broken 3 purchased

X = # of defective sets
0, 1, 2

$$P(0) = \left(\frac{5}{7}\right)\left(\frac{4}{6}\right)\left(\frac{3}{5}\right) \times 1 = \frac{2}{7}$$

$$P(1) = \left(\frac{5}{7}\right)\left(\frac{4}{6}\right)\left(\frac{2}{5}\right) \times 3 = \frac{4}{7}$$

$$P(2) = \left(\frac{2}{7}\right)\left(\frac{1}{6}\right)\left(\frac{5}{5}\right) \times 3 = \frac{1}{7}$$

Probability #ways this could occur

$$\boxed{\text{pmf} = \begin{cases} 2/7 & x=0 \\ 4/7 & x=1 \\ 1/7 & x=2 \\ 0, & \text{otherwise} \end{cases}}$$