

**請尊重智慧財產權**  
**勿私自轉載或移作他用**

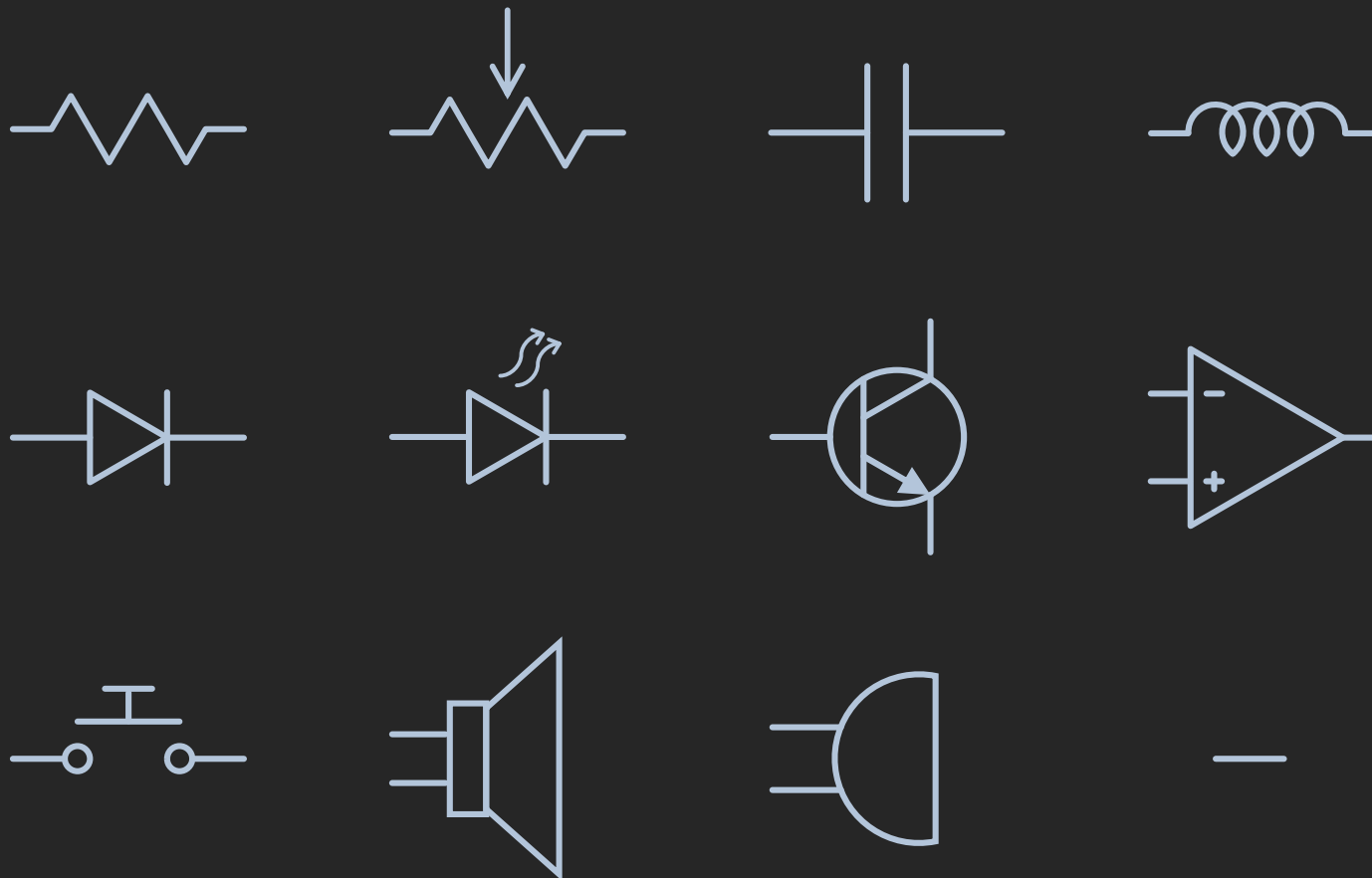
by RUI030 2023.09.16

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Pre-class

# Circuit Analysis

# 元件



# Content

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## Symbols

- ① Ground
- ② Resistor
- ③ Capacitor
- ④ Inductor

## Laws & Methods

- ⑤ KCL
- ⑥ Impedance
- ⑦ Transfer Function
- ⑧ Example

Components

# Branch

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- ▶ A single element.

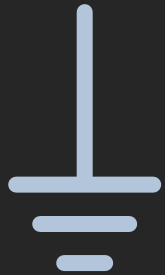
# Node

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- ▶ Point of connection between multiple branches.

# Ground (GND)

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- ▶ A selected voltage reference node ( $V_{\text{GND}} = 0$ ).
- ▶ A common return path for electric current.
- ▶ A direct physical connection to the Earth.



# Resistor

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## ► Ohm's Law

$$R \triangleq \frac{V}{I}$$



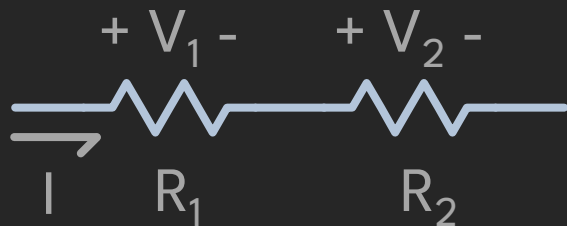
$R$  : Resistance ( $\Omega$ )

$V$  : Voltage ( $V$ )

$I$  : Current ( $A$ )



## Connected in Series

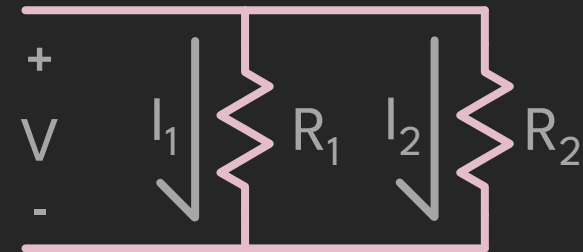


$$I = \frac{V_1}{R_1} = \frac{V_2}{R_2}$$

$$V_1 + V_2 = IR_{eq}$$

$$R_{eq} = \frac{V_1}{I} + \frac{V_2}{I} = R_1 + R_2$$

## Connected in Parallel



$$V = I_1 R_1 = I_2 R_2$$

$$I_1 + I_2 = \frac{V}{R_{eq}}$$

$$\frac{1}{R_{eq}} = \frac{I_1}{V} + \frac{I_2}{V}$$

$$= \frac{1}{R_1} + \frac{1}{R_2}$$

## Connected in Series

---

- ▶ n resistors connected in series.
- ▶ Same current.

$$I = \frac{V_n}{R_n}$$

$$\sum V_n = IR_{eq}$$

$$R_{eq} = \sum R_n$$

## Connected in Parallel

---

- ▶ n resistors connected in parallel.
- ▶ Same voltage.

$$V = I_n R_n$$

$$\sum I_n = \frac{V}{R_{eq}}$$

$$\frac{1}{R_{eq}} = \sum \frac{1}{R_n}$$

## Connected in Series

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$$R_{eq} = R_1 + R_2 + \dots + R_n$$

## Connected in Parallel

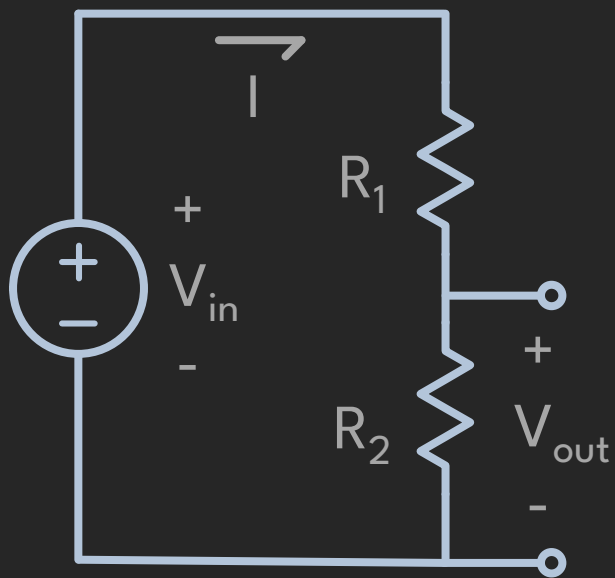
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$$R_{eq} = R_1 || R_2 || \dots || R_n$$

# Voltage Divider

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- Two resistors connected in series form a voltage divider.

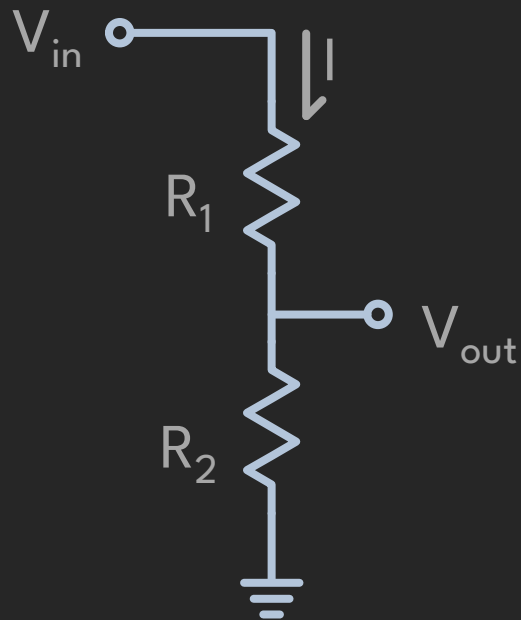
$$V_{in} = I(R_1 + R_2)$$

$$V_{out} = IR_2$$

$$\Rightarrow \frac{V_{out}}{V_{in}} = \frac{R_2}{R_1 + R_2}$$

# Voltage Divider

---



- Two resistors connected in series form a voltage divider.

$$V_{in} = I(R_1 + R_2)$$

$$V_{out} = IR_2$$

$$\Rightarrow \frac{V_{out}}{V_{in}} = \frac{R_2}{R_1 + R_2}$$

# Potentiometer

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- ▶ Can be used as a **variable resistor**.
- ▶ A three-terminal resistor with a sliding or rotating contact forms an adjustable voltage divider.



$$R = \frac{\rho L}{A}$$

$R$  : Resistance ( $\Omega$ )

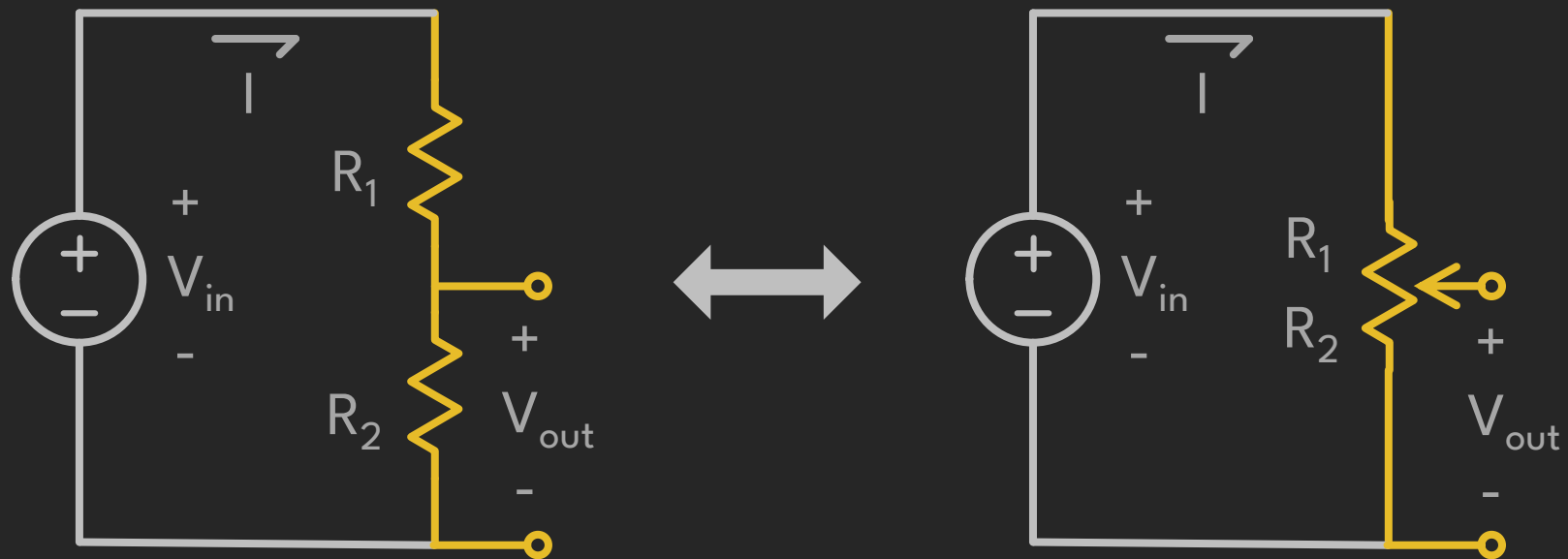
$\rho$  : Resisivity ( $\Omega m$ )

$L$  : Length ( $m$ )

$A$  : Cross section Area ( $A$ )

# Potentiometer

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# Capacitor

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## ► Definition of **capacitance**

$$C \triangleq \frac{Q}{V}$$



$C$  : *Capacitance* (F)

$Q$  : *Charge* (C)

$V$  : *Voltage* (V)



# Capacitor

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- ▶ A capacitor can remove the DC part of a signal.

$$Q = CV$$

$$\Rightarrow i(t) = \frac{dQ}{dt} = C \frac{dv(t)}{dt}$$

# Connected in Series

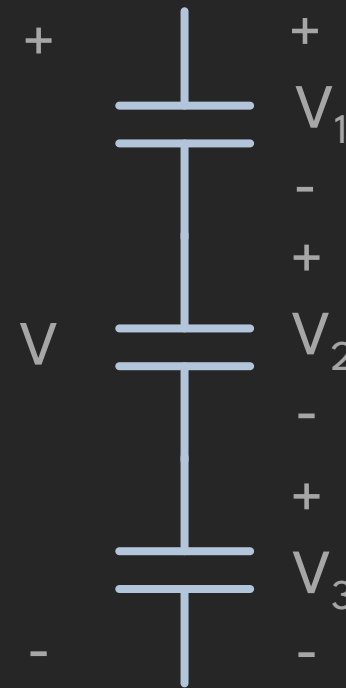
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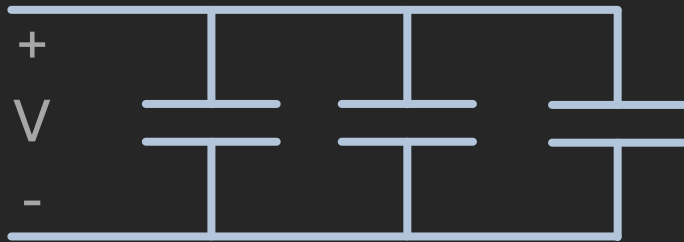
- n capacitors connected in series.

$$V = \sum V_n$$

$$\frac{Q}{C_{eq}} = \sum \frac{Q}{C_n}$$

$$\frac{1}{C_{eq}} = \sum \frac{1}{C_n}$$





## Connected in Parallel

- n capacitors connected in parallel.

$$Q_n = C_n V$$

$$\sum Q_n = C_{eq} V$$

$$C_{eq} = \sum C_n$$

## Connected in Series

---

- n capacitors connected in series.

$$\frac{1}{C_{eq}} = \sum \frac{1}{C_n}$$

## Connected in Parallel

---

- n capacitors connected in parallel.

$$C_{eq} = \sum C_n$$

# Inductor

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► i-v equation



$$v(t) = L \frac{di(t)}{dt}$$

$L$  : Inductance (H)

# Connected in Series

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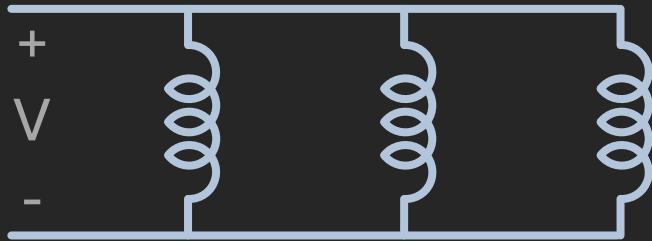
- n inductors connected in series.

$$v(t) = \sum v_n(t)$$

$$L_{eq} \frac{di(t)}{dt} = \sum L_n \frac{di(t)}{dt}$$

$$L_{eq} = \sum L_n$$





## Connected in Parallel

- $n$  inductors connected in parallel.

$$v(t) = L_{eq} \frac{di(t)}{dt} = L_n \frac{di_n(t)}{dt}$$

$$\frac{di(t)}{dt} = \sum \frac{di_n(t)}{dt}$$

$$\frac{v(t)}{L_{eq}} = \sum \frac{v(t)}{L_n}$$

$$\frac{1}{L_{eq}} = \sum \frac{1}{L_n}$$

## Connected in Series

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- n inductors connected in series.

$$L_{eq} = \sum L_n$$

## Connected in Parallel

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- n inductors connected in parallel.

$$\frac{1}{L_{eq}} = \sum \frac{1}{L_n}$$



# Laws & Methods

# Kirchhoff's Current Law (KCL)

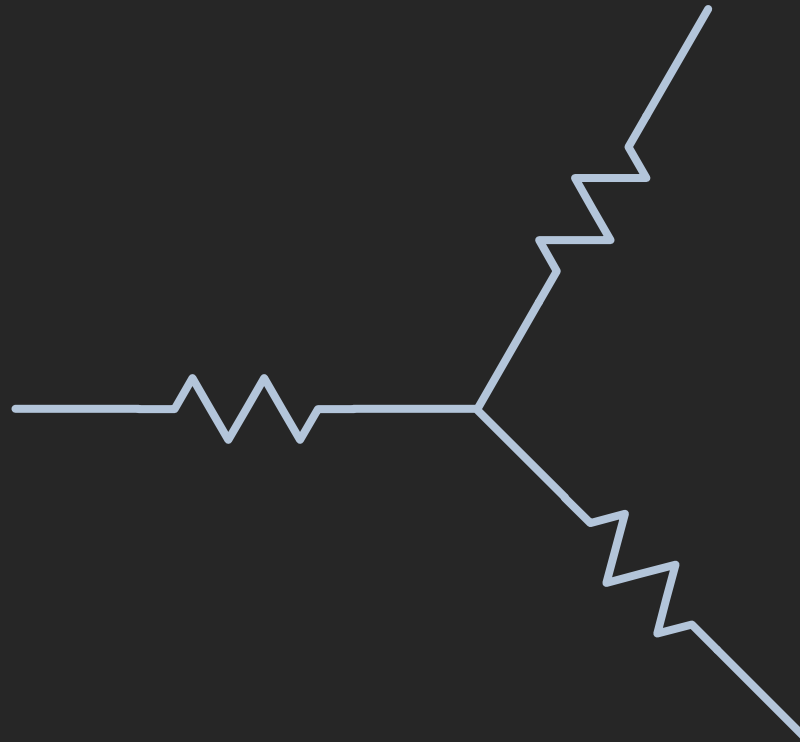
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- ▶ The sum of current that flow in a node is equal to the sum of current that flow out of itself.
- ▶ See also KVL...

$$\sum I_{in} = \sum I_{out}$$

# Kirchhoff's Current Law (KCL)

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## Resistance (R)

- ▶ Real numbers.
- ▶ Electrical energy is dissipated in the form of heat. ( $P=I^2R$ )

## (X) Reactance

- ▶ Imaginary numbers.
- ▶ Opposition presented to AC by inductance or capacitance.

# Impedance (Z)

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► Complex numbers

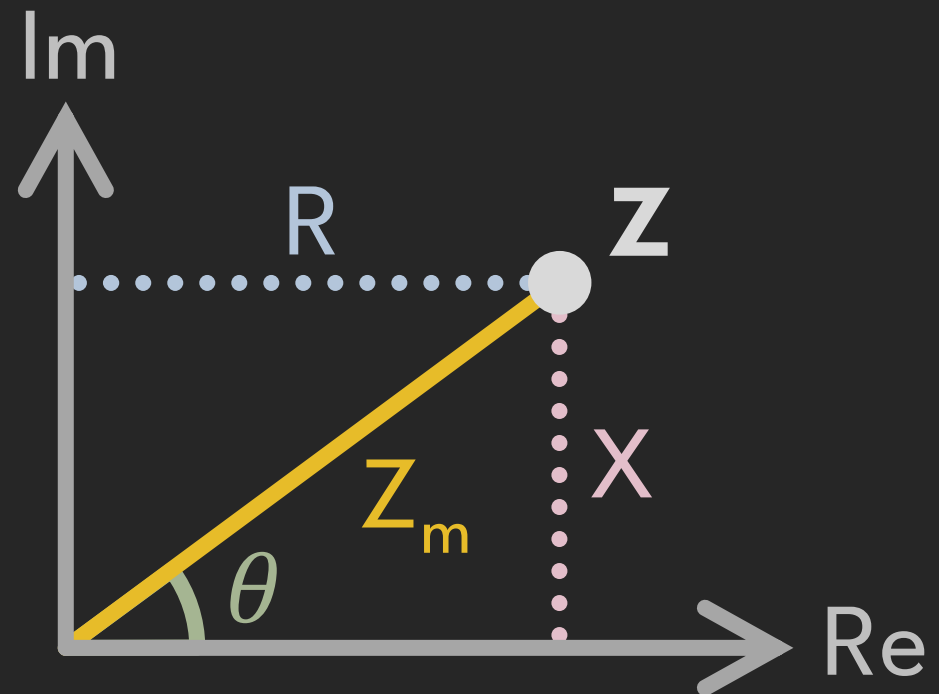
►  $j = \sqrt{-1}$

► Cartesian form:

$$\mathbf{Z} = R + jX$$

► Polar form:

$$\mathbf{Z} = Z_m \angle \theta$$



# Impedance (Z)

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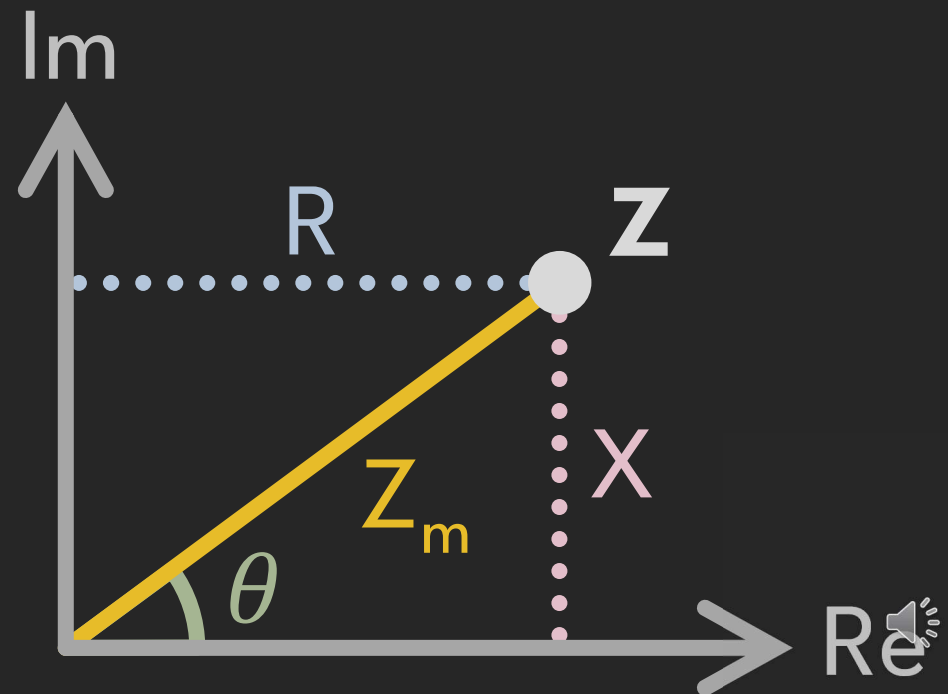
$$\mathbf{Z} = R + jX$$

$$R = \operatorname{Re}\{\mathbf{Z}\} = Z_m \cos \theta$$

$$X = \operatorname{Im}\{\mathbf{Z}\} = Z_m \sin \theta$$

$$Z_m = |\mathbf{Z}| = \sqrt{R^2 + X^2}$$

$$\theta = \tan^{-1} \frac{X}{R}$$



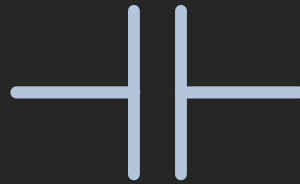
Impedance ( $Z$ )

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$\omega$ : frequency



$$Z_R = R$$



$$Z_C = \frac{1}{j\omega C}$$

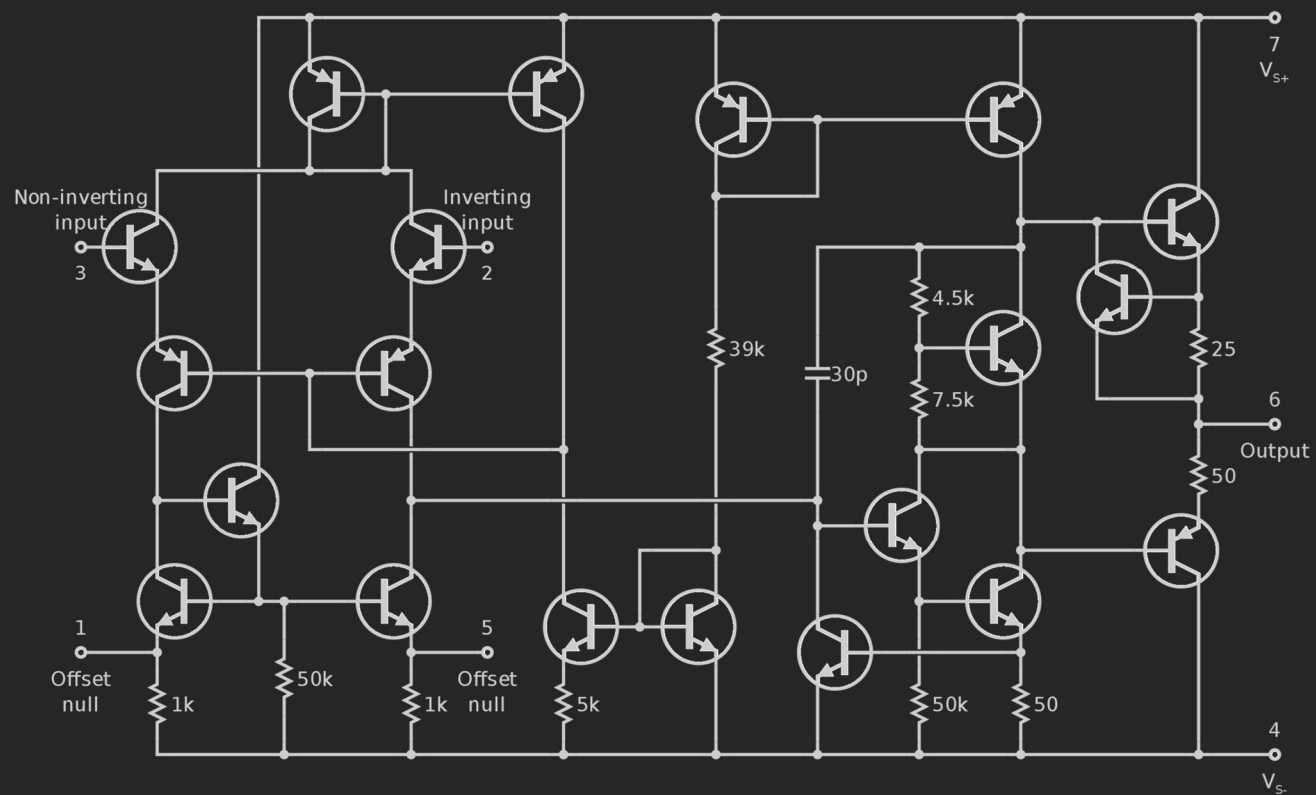


$$Z_L = j\omega L$$

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$$V = IZ$$

# Transfer Function

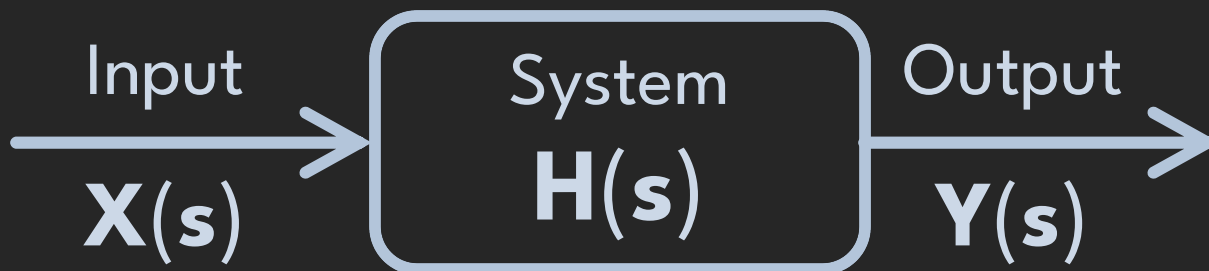




# Transfer Function

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- ▶ A math function that describes the relationship between the input and the output of a system.
- ▶  $H(s) \triangleq \frac{Y(s)}{X(s)}$
- ▶ Mostly, we discuss about  $V_{\text{out}}/V_{\text{in}}$ .



# EXAMPLE

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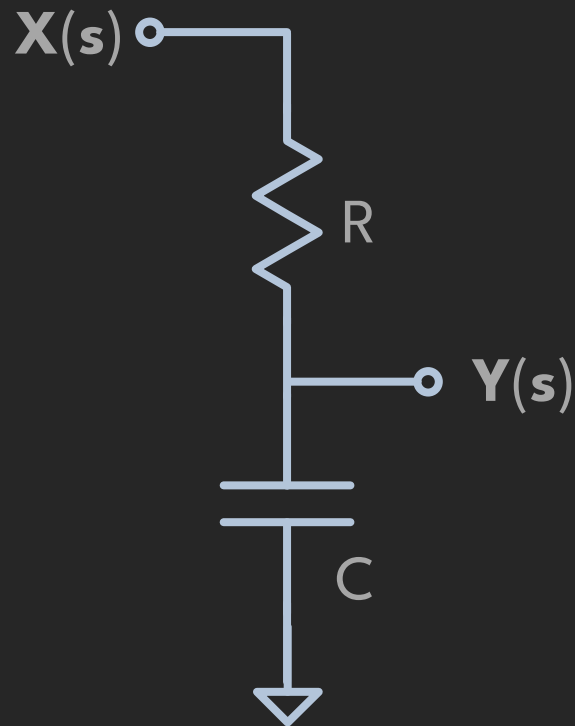
- Calculate the equivalent impedance.



# EXAMPLE

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► Derive the transfer function  $\mathbf{Y(s)}/\mathbf{X(s)}$  as a function of  $\omega$ .



Thank You