Safety Science 82 (2016) 68–76

Contents lists available at ScienceDirect

Safety Science

journal homepage: www.elsevier.com/locate/ssci

Interaction between vehicles and pedestrians at uncontrolled mid-block crosswalks

Peng Chen [a,*], Chaozhong Wu [b], Shunying Zhu [a]

[a] School of Transportation, Wuhan University of Technology, Wuhan 430063, China

[b] Engineering Research Center for Transportation Safety (Ministry of Education), Wuhan University of Technology, Wuhan 430063, China

ARTICLE INFO

Article history:
Received 12 April 2015
Received in revised form 8 August 2015
Accepted 10 September 2015
Available online 23 September 2015

Keywords:
Crosswalk
Pedestrian
Vehicle
Cumulative prospect theory
Evolutionary game

ABSTRACT

Pedestrian crossing safety has attracted increased attention in recent years. However, little research has been conducted for examining the interaction between vehicles and pedestrians at uncontrolled mid-block crosswalks. In this paper, both a decision model and a motion model are developed for simulating this interaction process. Cumulative prospect theory is embedded in the evolutionary game framework for modeling the decision behaviors of drivers and pedestrians during the interactions, which can capture the phenomenon of disagreement among a pedestrian crossing group. Cellular automata-based moving rules are used to depict the motion of vehicles with consideration of the three-second rule, and a modified bidirectional pedestrian model is developed in order to consider the right-moving preference and resolve the deadlock among mixed flows. Results of calibration and validation of the proposed model are also presented. An application is designed for the purpose of illustrating the model's capabilities. The results demonstrate that the proposed model can well replicate actual observed traffic.

© 2015 Elsevier Ltd. All rights reserved.

1. Introduction

The interaction between vehicles and pedestrians at crosswalks may induce not only traffic congestions of vehicle flow but also accidents. Thus, analysis of the interaction between pedestrians and vehicles has been the subject of numerous studies focused on road design, traffic signals, and road users' behaviors (de Lavalette et al., 2009; Guo et al., 2012; Koh et al., 2014; Li, 2013; Liu and Tung, 2014).

In the last decade, limited studies have been conducted for attempting the modeling of mixed pedestrian–vehicle flows at uncontrolled crosswalks. For example, Zhang et al. (2004) used cellular automata to simulate pedestrians' crossing and introduced the concept of a "stop point" for resolving conflicts among pedestrians or between pedestrians and vehicles at a crosswalk. Further, Helbing et al. (2005) proposed a macro model for analytically investigating the oscillations and delays of pedestrian and vehicle flows. Sun et al. (2012) used cellular automata to model the behavior of mutual interferences between pedestrians and vehicles at a crosswalk by introducing conflict interference rules between pedestrians and vehicles. Jin et al. (2013) developed a modified car-following model and pedestrian crossing rules so as to analyze the interaction between vehicle traffic and pedestrian flow. Xin et al. (2014) proposed a pedestrian–vehicle cellular automata model to study the characteristics of the mixed traffic, in which the heterogeneity of pedestrians is taken into account.

The aforementioned studies focused primarily on modeling of the behavior characteristics of vehicle drivers and pedestrians. None of these studies, however, considered the decision process of vehicle drivers and pedestrians during the interaction. This shortcoming resulted in a failure to understand the interaction between vehicles and pedestrians from the perspective of the entire system. The relationship between conflicting traffic streams at a mid-block crosswalk is essentially a game of competing for limited time and space resources. Thus, game theory is applicable to the analysis of the interaction between vehicles and pedestrians. Classical game theory is based on rational behavior exhibited in interpersonal conflicting situations. In order to deal with the limited rationality in the selection and decision processes, evolutionary game theory is proposed as an extension of the classical paradigm toward bounded rationality. However, the payoff matrix in evolutionary game theory is composed of the payoff based on expected utility theory, which is contrary to the assumption of bounded rationality. In order to overcome the shortcoming of expected utility theory, prospect theory is proposed for handling the decision behavior under risk and uncertainty. Most studies on prospect the-

* Corresponding author at: School of Transportation, Wuhan University of Technology, 1174, Heping Avenue, Wuhan 430063, Hubei, China. Tel.: +86 18995626798.

E-mail address: seupc@163.com (P. Chen).

http://dx.doi.org/10.1016/j.ssci.2015.09.016
0925-7535/© 2015 Elsevier Ltd. All rights reserved.

ory in the field of transportation focus on the route choice, network equilibrium, and departure time choice (Gao et al., 2010; Hjorth and Fosgerau, 2012; Jou et al., 2008; Kemel and Paraschiv, 2013). A recent study (Wang et al., 2013) embedded the prospect theory model into the replicator dynamics framework in order to model the evolution of the traveler route choice under risk. The framework for that study was developed at the macro level.

Different from the work of Wang et al. (2013), in the present work, we model the interaction between vehicles and pedestrians from a micro-dynamics viewpoint, in order to provide a more detailed analysis of individual behavior. The micro-level dynamics of the system is usually defined by strategy update rules. A huge variety of microscopic update rules has been defined and applied in game theory (Szabo and Fath, 2007). These update rules can be divided into two categories of learning paradigms: one is the social learning paradigm, in which players update their strategies based on imitation of strategies of those players who have performed better in the past, and the other is the individual learning paradigm, which is based on an individual's learning and updating of strategies based only on his/her own experience (Arifovic and Karaivanov, 2010). In this study, we consider mainly social learning paradigms in order to describe the player's learning process, which may help us better understand the herd mentality (Jin et al., 2013) and capture the phenomenon of disagreement among a pedestrian crossing group, which previous works failed to reflect.

Moreover, the aforementioned studies on the modeling of the motion of pedestrians and vehicles have the following three shortcomings. First, in China, pedestrians prefer to walk or move along the right-hand side of the road (Yang et al., 2008). However, this asymmetrical behavior has been ignored in the abovementioned studies. Second, a waiting pedestrian may not cross the conflict area with a vehicle and exchange his/her position with an opposing crossing pedestrian in accordance with the existing model, which results in the pedestrian and the vehicle stopping in the conflict area. Therefore, the deadlock among mixed flows should be resolved. In this study, we develop improved rule sets for simulating the motion of pedestrians. Finally, our observation suggests that drivers driving on the road usually follow the "three-second rule" (New Jersey Motor Vehicle Commission, 2015) to avoid tailgating, which should be considered in the vehicle-following model.

The remainder of this paper is organized as follows. Section 2 describes the framework of the proposed model. Section 3 describes the decision model of the interaction between vehicles and pedestrians at uncontrolled mid-block crosswalks. Section 4 introduces a motion model for vehicles and pedestrians. Sections 5 and 6 respectively present the calibration and validation of the proposed model. Section 7 describes an application of the proposed model. Finally, Section 8 concludes the paper with a summary and outlook for further research.

2. Model framework

The model consists of two modules: a decision model and a motion model (see Fig. 1). The objective of the decision model is to describe the perception and judgment of pedestrians and drivers during street crossing. Then, the motion model determines the microscopic movements of vehicles and pedestrians. The moving rules for vehicle and pedestrian flows are extended from the existing cellular automata model. In the following, each model is described in detail.

3. Decision model

In this section, we describe the decision model of the interaction between vehicles and pedestrians at uncontrolled mid-block crosswalks. First, we define the interaction within the framework of game theory. Then, we present three key components in the decision model based on the evolutionary game: payoff based on cumulative prospect theory (Tversky and Kahllemna, 1992), dynamic topology, and microscopic update rules.

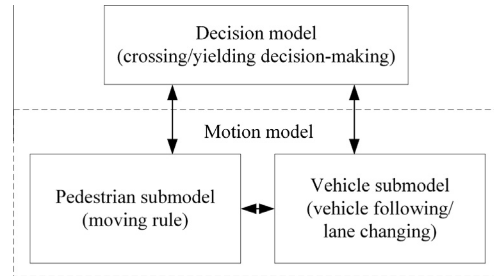

Fig. 1. Model framework.

3.1. Definition and notation

We formulate the game to take into account the population, players, strategy, and payoff.

(1) *Population:* The two populations are vehicles and pedestrians. Let C denote the population of vehicles and let S denote the population of pedestrians.
(2) *Players:* For a certain uncontrolled mid-block crosswalk, each user in the population is a player of the game. Let C_i and S_i ($i = 1, \ldots, l$) denote the players in the populations of C and S, respectively.
(3) *Strategy:* The vehicle chooses whether or not to yield to the pedestrian. The pedestrian chooses whether or not to accept the gap. For simplicity, the set of strategies for all players is taken as {crossing, yielding}. Let $M = \{(p_1, p_2)|p_j \geq 0, \sum_{j=1}^{2} p_j = 1.\}$ be the set of probability distributions over the two pure strategies, where p_j represents the proportion of users choosing strategy j.
(4) *Payoff:* The payoffs for the vehicle and pedestrian are described in Section 3.2.

3.2. Payoffs based on cumulative prospect theory

3.2.1. Assumption

(1) *Delay costs:* When the vehicle and pedestrian are in conflict, the driver may yield to the pedestrian or the pedestrian may wait for a chance to cross; thus, at least one of the two sides would spend more time, which can be described by the delay. Let t_C^D and t_S^D denote the delay costs for the vehicle and pedestrian, respectively.
(2) *Risk costs:* When the vehicle and pedestrian are in conflict, both the driver and the pedestrian may choose to cross, and thus, both of them would face potential losses, which can be described by the risk. For drivers, accidents will lead to economic losses. For pedestrians, accidents will lead to physical injury or loss of life. Let t_C^R and t_S^R denote the risk costs for the vehicle and pedestrian, respectively. Note that $t_C^R < t_S^R$.

We also assume that each player is bounded rational and chooses the strategy according to cumulative prospect theory.

3.2.2. Payoff matrix

According to the above assumptions, the payoff matrix for the decision model in our study is as presented in Table 1. In such a bimatrix form, a_{ij} and b_{ij} ($i, j = 1, 2$) denote the payoffs for the vehicle and pedestrian, respectively, for different strategy profiles.

For ease of calculation and analysis, nondimensional delay costs and risk costs that do not indicate the real losses are set according to the relationship between delays and risks in different situations. For pedestrians, risk costs are $t_S^R = 20$, which means that life is priceless. For drivers, the higher the vehicle speed, the greater are the risk costs (Liu, 2013). Risk costs for drivers under three grades of vehicle speed are listed in Table 2.

Both pedestrians and drivers will become impatient with an increase in waiting time. Based on observations and the work of Liu (2013), the waiting time can be divided into three waiting stages. In the first stage, road users are emotionally stable. In the second stage, road users suffer from mood swings. Finally, in the third stage, road users may become extremely impatient. Delay costs for pedestrians and drivers in these three waiting stages are presented in Table 3.

Thus, the payoffs in the payoff matrix can be determined as follows.

(1) If the driver and pedestrian choose {crossing}, then $a_{11} = -t_C^R$ and $b_{11} = -t_S^R$.
(2) If the driver chooses {crossing} and the pedestrian chooses {yielding}, then $a_{12} = 1$ and $b_{12} = -t_S^D$.
(3) If the driver chooses {yielding} and the pedestrian chooses {crossing}, then $a_{21} = -t_C^D$ and $b_{21} = 1$.
(4) If the driver and pedestrian choose {yielding}, then $a_{22} = -1$ and $b_{22} = -1$.

3.2.3. Prospect value of payoff

In accordance with cumulative prospect theory (Tversky and Kahllemna, 1992), the prospect of a certain alternative is defined as

$$V(f) = \sum_{j=0}^{n} \pi_j^+(f^+)\,v(x_j) + \sum_{j=-m}^{0} \pi_j^-(f^-)\,v(x_j) \tag{1}$$

where f^+ is the positive part of f; f^- is the negative part of f; $-m \leqslant j \leqslant n$; positive parts indicate positive outcomes; negative parts indicate negative outcomes; the zero indicates a neutral outcome; m and n denote the number of prospects with outcomes as "losses" or "gains," respectively; and the decision weights $\pi_j^+(f^+) = (\pi_0^+, \ldots, \pi_n^+)$ and $\pi_j^-(f^-) = (\pi_{-m}^-, \ldots, \pi_0^-)$ are defined by

$$\pi_n^+ = w^+(p_n),\ \ \pi_{-m}^- = w^-(p_{-m}), \tag{2}$$

$$\pi_j^+ = w^+(p_j + \cdots + p_n) - w^+(p_{j+1} + \cdots + p_n), \quad 0 \le j \le n-1, \tag{3}$$

$$\pi_j^- = w^-(p_{-m} + \cdots + p_j) - w^-(p_{-m} + \cdots + p_{j-1}), \quad 1-m \le j \le 0 \tag{4}$$

In Eqs. (2)–(4), $w^+(p_j)$ and $w^-(p_j)$ are the weighting functions. The functional forms for gains and losses are respectively expressed as follows:

Table 1
Payoff matrix.

Vehicle	Pedestrian	
	Crossing	Yielding
Crossing	a_{11}, b_{11}	a_{12}, b_{12}
Yielding	a_{21}, b_{21}	a_{22}, b_{22}

Table 2
Risk costs for drivers.

Vehicle speed grade	Low speed ($v \leqslant 35$ km/h)[a]	Medium speed ($35 < v \leqslant 55$ km/h)[a]	High speed ($v > 55$ km/h)[a]
Risk costs t_C^R	2	6	8

[a] Sourced from Liu (2013).

Table 3
Delay costs for pedestrians and drivers.

Waiting stage	Pedestrians		Drivers	
	Waiting time	Delay costs, t_S^D	Waiting time	Delay costs, t_C^D
First stage	$t \leqslant 15$ s	1	$t \leqslant 5$ s	1
Second stage	$15 < t \leqslant 30$ s	2	$5 < t \leqslant 10$ s	2
Third stage	$t > 30$ s	4	$t > 10$ s	4

$$\text{For gains}: w^+(p_j) = \frac{p_j^\gamma}{\left(p_j^\gamma + (1-p_j)^\gamma\right)^{1/\gamma}} \tag{5}$$

$$\text{For losses}: w^-(p_j) = \frac{p_j^\delta}{\left(p_j^\delta + (1-p_j)^\delta\right)^{1/\delta}} \tag{6}$$

where p_j is the probability of receiving x_j as the outcome. The median values of γ and δ are 0.61 and 0.69, respectively.In Eq. (1), the value function $v(x_j)$ reflects the value of the outcome x_j. A general form for the value function is

$$v(x) = \begin{cases} x^\alpha & x \ge 0 \\ -\lambda(-x)^\beta & x < 0 \end{cases} \tag{7}$$

where x is the outcome relative to the reference; $\alpha, \beta > 0$ measure the curvatures of the value functions for gains and losses, respectively; and λ is the coefficient of loss aversion. The median values of α, β, and λ are 0.88, 0.88, and 2.25, respectively.

According to the general form of the prospect mentioned above, the prospect of the payoff for the player choosing a certain strategy can be computed as follows:

The prospect for the driver choosing {crossing}, V_C^C, is computed as:

$$V_C^C = \pi_{-1}^- v(a_{11}) + \pi_1^+ v(a_{12}) = w^-(p_1)v(a_{11}) + w^+(p_2)v(a_{12}) \tag{8}$$

where p_1 and p_2 denote the probabilities of the pedestrian choosing crossing and yielding, respectively.

The prospect for the driver choosing {yielding}, V_C^Y, is computed as:

$$V_C^Y = \pi_{-2}^- v(a_{21}) + \pi_{-1}^- v(a_{22}) = w^-(p_1)v(a_{21}) + (w^-(p_1+p_2) - w^-(p_1))v(a_{22}) \tag{9}$$

The prospect for the pedestrian choosing {crossing}, V_S^C, is computed as:

$$V_S^C = \pi_{-1}^- v(b_{11}) + \pi_1^+ v(b_{21}) = w^-(p_3)v(b_{11}) + w^+(p_4)v(b_{21}) \tag{10}$$

where p_3 and p_4 denote the probabilities of the driver choosing crossing and yielding, respectively.

The prospect for the pedestrian choosing {yielding}, V_S^Y, is computed as:

$$V_S^Y = \pi_{-2}^- v(b_{12}) + \pi_{-1}^- v(b_{22}) = w^-(p_3)v(b_{12}) + (w^-(p_3+p_4) - w^-(p_3))v(b_{22}) \tag{11}$$

Then, the driver would compare V_C^C and V_C^Y, and the pedestrian would compare V_S^C and V_S^Y. Finally, the players choose the strategy with the highest prospect.

3.2.4. Probability of selection

In order to determine the probabilities of the driver (or pedestrian) choosing crossing and yielding mentioned in Section 3.2.3, the binary logistic regression model (Popuri et al., 2011) is employed in this study, because it provides ease of interpretation and ease of implementation in a simulation environment. The crossing and yielding behaviors of the driver (or pedestrian) could be measured as a binary variable with two values: 1 = crossing and 0 = yielding. The probability of the crossing response can be obtained as follows:

$$P(y_i = 1|U_i) = p_i = \frac{e^{U_i}}{1 + e^{U_i}} \quad (12)$$

$$U_i = \alpha + \sum_{i=1}^{m} \beta_i x_i \quad (13)$$

where intercept α and parameter β_i describe the effects of m explanatory variables x_i on the crossing response. The model parameters can be obtained through maximum likelihood estimation in SPSS statistical software.

The following explanatory variables are considered as predictors in the model: the distance between the vehicle and the pedestrian along the vehicle movement direction (DISTANCE), the vehicle speed (VEHSPEED), the pedestrian speed (PEDSPEED), the number of pedestrians arriving at the crossing point (PEDNUMBER), and the vehicle type (VEHTYPE). The variable PEDNUMBER is applied only to predict the driver crossing behavior, and the variable VEHTYPE is applied only to predict the pedestrian crossing behavior.

3.3. Dynamic topology

In this study, the connectivity between players can be represented by a bipartite graph (Tian et al., 2012), where the nodes represent players, and the edges, which connect the nodes, refer to the connection between the corresponding players. A given bipartite graph $G = (T, \perp, E)$ consists of two disjoint sets of nodes—T (top nodes) and $\perp$ (bottom nodes)—and a set of edges, E. Here, the interaction between the vehicle and the pedestrian can be viewed this way with T being the set of vehicles, $\perp$ being the set of pedestrians, and each vehicle being linked to pedestrians (see Fig. 2). In this realistic social network, the connectivity structure is dynamic. After the vehicle or the pedestrian has crossed the crosswalk, the connectivity between the vehicle and the pedestrian changes, which is followed by a new structural evolution.

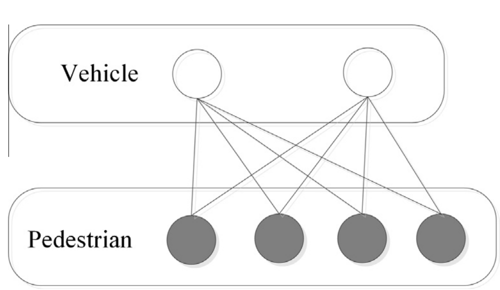

Fig. 2. Schematic connectivity structure for interaction between vehicles and pedestrians.

3.4. Microscopic update rules

In this study, we adopted the imitation rule to reflect the interaction among the pedestrian crossing group. The update rule is defined as follows (Zhang et al., 2011):

When player h (C_i or S_i) (with the prospect of the payoff as V_h) is selected for strategy update, a neighbor g (with the prospect of the payoff as V_g) is drawn at random from among all neighbors. Player h will adopt the strategy of player g with the probability

$$w(V_h - V_g) = \frac{1}{1 + \exp\left[(V_h - V_g)/k\right]} \quad (14)$$

where k is the measure of stochastic uncertainties (noise) allowing for irrational choices. It must be pointed out that it is difficult to obtain the noise, because we do not know the nature of the pedestrian. Therefore, in this study, we adopt reverse engineering: we first assign values to the noise arbitrarily and then obtain the noise by adjusting these values until the simulation results correspond to the real observation.

4. Motion model

In order to describe the movement behavior specifically, we present the following moving rules for vehicles and pedestrians.

4.1. Moving rules for vehicles

The cellular automata model is used to depict the motion of vehicles. In this model, time, space, and velocity are all discrete. The road is divided into a lattice of cells. The cell size is set to 25 cm $\times$ 25 cm. Each cell is either empty or occupied by one vehicle or pedestrian. The vehicular traffic model consists of a vehicle-following submodel and a lane changing submodel.

For a vehicle-following submodel, the system is updated at each time step with the following modified Nagel–Schreckenberg rules (Nagel and Schreckenberg, 1992), which redefine the value of the limiting velocity for the ith vehicle:

(1) *Acceleration:* If $v_i < v_{Li}$, $v_i \to v_i + a_a$.
(2) *Deceleration:* If $v_i > v_{Li}$, $v_i \to v_{Li}$.
(3) *Randomization:* If $v_i > 0$, with probability p_R, $v_i \to \max(v_i - a_d, 0)$.
(4) *Vehicle movement:* At t, the vehicle positions are updated.

Here, v_i denotes the velocity of the ith vehicle, a_a is the acceleration rate, a_d is the deceleration rate, and p_R is a randomization parameter. v_{Li} is the limiting velocity value for the ith vehicle, which can be determined on the basis of two factors: the maximal velocity v_{max} and the safety velocity v_{is}. Thus, v_{Li} is calculated as

$$v_{Li} = \min(v_{max}, v_{is})$$

where $v_{is} = l_{is}/3$, and l_{is} is the distance between the ith vehicle and the vehicle (or the pedestrian) ahead of it; this equation means that vehicle drivers use the "three-second rule" to avoid tailgating.

For a lane changing submodel, it is assumed that a vehicle changes to another lane if the following set of conditions is satisfied (Rahman et al., 2013; Zheng, 2014):

Condition 1: $\text{gap}_i < \min(v_i + a_a, v_{max})$
Condition 2: $\text{gap}_{i,o} > \min(v_i + a_a, v_{max})$
Condition 3: $\text{gap}_{i,ob} > v_{max}$

where gap_i is the number of empty cells ahead in the same lane, $\text{gap}_{i,o}$ is the number of empty cells ahead in the other lane, and $\text{gap}_{i,ob}$ is the number of empty cells behind in the other lane.

The first and second inequalities or conditions mentioned above check the present lane and target lane, respectively, for achieving favorable speed conditions. Then, the availability of sufficient space for performing lane change is checked by the third condition. The possibility of lane change is expressed by a certain probability. Thus, the lane change rule can be described as follows (Hua et al., 2011):

If the three conditions for lane change are satisfied and $p_s < p_{\mathrm{lane}l}$, then lane(l) → lane($1 - l$), where p_s is the probability of a driver opting for lane change; $p_{\mathrm{lane}l}$ is the critical probability of a vehicle changing from lane l to the adjacent lane; and $l = 0, 1$ (lane 0 is the inner lane and lane 1 is the outer lane).

4.2. Moving rules for pedestrians

In this study, pedestrians are considered to occupy an area of 0.5 m^2. The maximum crossing velocity of a pedestrian is set to 2 m/s (v_{maxp} = 8 cells/s). Based on the bidirectional pedestrian model developed by Blue and Adler (2001), we propose a modified rule set for describing bidirectional pedestrian flows at uncontrolled mid-block crosswalks. There are two improvements in the model. First, the right-moving preference is taken into account in the modified rule set. Second, in order to resolve the deadlock among mixed flows, a crossing pedestrian would avoid a waiting pedestrian by choosing an available lane.

The modified rule set consists of two parallel updates. The first is lane assignment, and the second is that forward motions change the positions of all pedestrians in two parallel update stages.

Parallel update 1: Lane change

(1) *Eliminate conflicts:* If two pedestrians that are laterally adjacent sidestep into each another, then an empty cell between them is available to one of them with 50/50 random assignment.

(2) *Identify gaps:* The same lane or the adjacent (left or right) lane is chosen that best advances forward movement up to v_{maxp} according to the gap computation subprocedure that follows the step forward update.

(a) For dynamic multiple lanes:

(1) Step out of the lane of a pedestrian from the opposite direction by assigning gap = 0 if the opposing pedestrian is within 16 cells (16 = 2 ∗ largest v_{maxp}).

(2) Step behind a same-direction pedestrian when avoiding an opposite-direction pedestrian by choosing any available lane with $\mathrm{gap}_{\mathrm{same,\ dir}} = 0$ when gap = 0.

(3) IF gap = 0 or 1 AND gap = $\mathrm{gap}_{\mathrm{opp}}$ (cell occupied by an opposing pedestrian) AND the velocity of one of the entities is zero (i.e., the pedestrian is waiting for crossing the conflict area with the vehicle) THEN the pedestrian crossing the conflict area with the vehicle would avoid the waiting pedestrian by choosing any available lane.

(b) Ties of equal maximum gaps ahead are resolved according to the following:

(4) Two-way tie between adjacent lanes: 38/62 random assignment for staying in the left/right lane, considering the right-moving preferences (Ren et al., 2012)

(5) Two-way tie between present lane and single adjacent lane: 85/15 random assignment for staying in the present/adjacent lane

(6) Three-way tie: 80/8/12 assignment for staying in the present/left/right lane

(3) *Move:* Each pedestrian p_n is moved by 0, +2, or −2 lateral sidesteps after (1) and (2) are completed.

Parallel update 2: Step forward

(1) *Update velocity:* Let $v(p_n)$ = gap, where "gap" is obtained from the gap computation subprocedure described below.

(2) Exchanges: IF gap = 0 or 1 AND gap = $\mathrm{gap}_{\mathrm{opp}}$ (cell occupied by an opposing pedestrian) AND the velocities of both entities are positive numbers THEN $v(p_n)$ = gap + 2.

(3) Move: Each pedestrian p_n is moved $v(p_n)$ cells forward on the lattice after (1) and (2) are completed.

Subprocedure: Gap computation

(1) Same direction: Look ahead a max of 8 cells (largest v_{maxp}) IF occupied cell is found with same direction THEN set $\mathrm{gap}_{\mathrm{same}}$ to number of cells between entities ELSE $\mathrm{gap}_{\mathrm{same}} = 8$.

(2) Opposite direction: Look ahead a max of 16 cells (16 = 2 ∗ largest v_{maxp}) IF occupied cell found with opposite direction THEN set $\mathrm{gap}_{\mathrm{opp}}$ to INT (0.5∗number of cells between entities) ELSE $\mathrm{gap}_{\mathrm{opp}} = 8$.

(3) Assign gap = min ($\mathrm{gap}_{\mathrm{same}}$, $\mathrm{gap}_{\mathrm{opp}}$, v_{maxp})

5. Model calibration

In order to replicate the interaction between vehicles and pedestrians, four types of parameters should be calibrated: the vehicle characteristic, pedestrian characteristic, the coefficients in the binary logistic regression model, and the noise parameter in the imitation rule.

Data were collected by the counting and camera recording methods. Jianshe First Road in Wuhan, China, was chosen as the observation site. Fig. 3 shows a sketch of the site. Data collection was performed in evening peak hours (17:15–18:15) on workdays of October 21–23, 2013. In this two-way four-lane site, the width of each lane is 3 m. The width of the crosswalk is 5 m, and the heavy vehicle percentage is 12%. In this study, we focus on a typical situation with two lanes in one direction, and we assume the vehicle arrival distribution and pedestrian arrival distribution to follow the Poisson distribution. It is commonly assumed that the vehicle velocity and pedestrian velocity obey a normal distribution. Results of the calibration set of data are presented in Table 4.

A total of 1217 interaction events were collected for calibration of the logistics regression model. The forward stepwise algorithm with a threshold of $p = 0.05$ was applied and the SPSS program was used to construct the logistics regression model. The performance of this model was determined by a 5-fold cross-validation procedure (Berger, 2004). This procedure partitions the data into 5 disjoint subsets of nearly equal size. One of the subsets is reserved for testing, whereas the remaining data constitute the training sample. This procedure is repeated 5 times. The best logistic regression model was extracted on the basis of results of the goodness-of-fit test and the prediction accuracy.

The specifications of the best logistic regression model are presented in Table 5. Variables with estimated coefficients having a

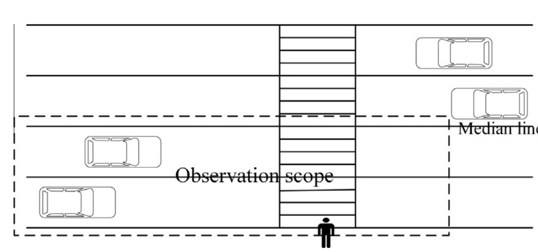

Fig. 3. Sketch of observation site.

significance value (Sig.) of less than 0.05 were accepted as influential predictor variables (see Table 5). To identify possible collinearity among the variables in the fitted model, the variance inflation factor (VIF) was estimated. A VIF value greater than 10 indicates a serious multicollinearity problem (Sze et al., 2014). In this study, values of VIF of the variables included in the model were all smaller than 2 (see Table 5). No evidence could be established for the existence of collinearity among the variables. The Hosmer–Lemeshow test showed that the goodness of fit of the model can be accepted, because the significance of chi-square is larger than 0.05 (see Table 5). The value of Nagelkerke R^2 showed that the independent variables can explain the dependent variables: the values for the two models (model 1 and model 2) are 0.701 and 0.933, respectively. In addition, the overall model performs classification successfully and accurately. The accuracies of predicting the driver crossing behavior and pedestrian crossing behavior are 92.6% and 96.3%, respectively.As mentioned in Section 3.4, we determine the noise parameter in the imitation rule according to two measures: the mean percent error (MPE) of delays for vehicles and pedestrians and the frequency of occurrence of disagreement among the pedestrian crossing group. In this study, the MPE of two delays is expressed as

$$\mathrm{MPE} = \frac{1}{2}\left(\frac{|y_1^s - y_1^o|}{y_1^o} + \frac{|y_2^s - y_2^o|}{y_2^o}\right) \tag{15}$$

where y_1^s is the simulated value of the average delay for vehicles, y_1^o is the corresponding observed value of the average delay for vehicles, y_2^s is the simulated value of the average delay for pedestrians, and y_2^o is the corresponding observed value of the average delay for pedestrians.

Fig. 4 shows the MPE of delays for vehicles and pedestrians under different noise parameters. From the figure, it can be seen that the MPE reached the minimum value when the noise parameter was 0.5, 2, 7, or 30. Hence, we compared the field data with the frequency of occurrence of disagreement among the pedestrian crossing group for these four noise parameters. From Table 6, we can see that the simulation result with the noise parameter of 7 agreed well with the observation result.

Table 4
Parameters used for model and calibration of data.

Parameter	Calibrated or typical value
Light vehicle (e.g., car) length, L_{lv} (m)	4.5 (=18 cells)
Light vehicle (e.g., car) width, W_{lv} (m)	1.8 (≈7 cells)
Heavy vehicle (e.g., bus) length, L_{hv} (m)	10 (=40 cells)
Heavy vehicle (e.g., bus) width, W_{hv} (m)	2.5 (=10 cells)
Maximum vehicle velocity, v_{max} (m/s)	9.7
Average velocity of vehicle, v_v (m/s)	7.5
Standard deviation of velocity of vehicle, σ_v (m/s)	2
Maximum acceleration rate, a_a (m/s^2)	2
Maximum deceleration rate, a_d (m/s^2)	2
Randomization parameter[a], p_R	0.3
Critical probability of changing from lane 0 to lane 1[b], p_{lane0}	0.6
Critical probability of changing from lane 1 to lane 0[c], p_{lane1}	1
Width of pedestrian, W_p (m)	0.5 (=2 cells)
Average velocity of pedestrian, v_p (m/s)	1.38
Standard deviation of velocity of pedestrian, σ_p (m/s)	0.27
Vehicle arrival rate, λ_v (veh/s)	0.30
Pedestrian arrival rate at roadside, λ_{pr} (ped/s)	0.16
Pedestrian arrival rate at median, λ_{pm} (ped/s)	0.086

[a] Sourced from Xie et al. (2012).
[b] Sourced from Jing et al. (2012).
[c] Sourced from Hua et al. (2011).

Table 5
Specifications of logistic regression.

Variable	Model 1		Model 2		VIF
	b_i (S.E.)	Wald (Sig.)	b_i (S.E.)	Wald (Sig.)	
DISTANCE	−0.144 (0.050)	8.311 (0.004)	0.495 (0.241)	4.209 (0.040)	1.340
VEHSPEED	0.558 (0.259)	4.645 (0.031)	−3.135 (1.573)	3.970 (0.046)	1.397
PEDSPEED	−3.696 (0.914)	16.335 (0.000)	17.915 (8.102)	4.889 (0.027)	1.084
Constant	5.326 (1.634)	10.619 (0.001)	−13.292 (6.000)	4.907 (0.027)	–
Hosmer–Lemeshow test					
Chi-square (Sig.)	14.828 (0.063)		0.511 (1.000)		

Note: b_i, coefficients; S.E., standard error; Wald, Wald chi-square value; Sig., Significance value; VIF, variance inflation factor.
Model 1 – model of predicting driver crossing behavior.
Model 2 – model of predicting pedestrian crossing behavior.

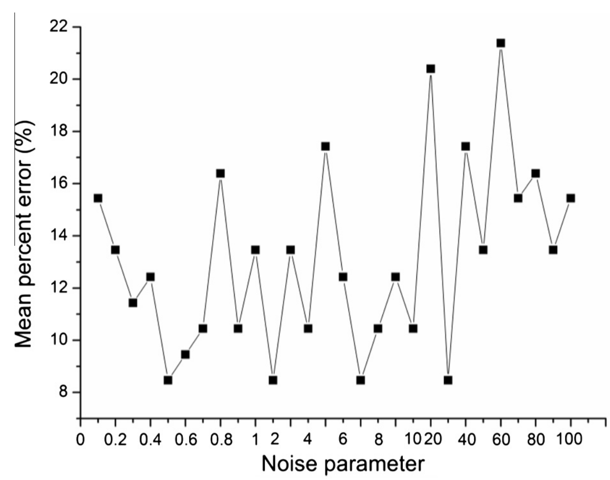

Fig. 4. MPE of delays for different noise parameters.

Table 6
Comparison of frequency of occurrence of disagreement between observations and simulations for four noise parameters.

Output	Field data	Simulation data			
		0.5	2	7	30
Occurrence of disagreement (times/h)	43	28	28	40	48

6. Model validation

In keeping with the approach of Fernandez (2010), the model validation in this study involved the application of the correlated inspection approach (Law, 2007). Two levels of validation were used. The first level consisted of comparing the average outputs obtained from the model with field observations as a way of testing the overall predictive power of the model. The considered outputs were the average steady-state statistics for the following variables: vehicle delay, pedestrian delay, and frequency of occurrence of disagreement among the pedestrian crossing group. Fig. 5 shows an example of disagreement among the pedestrian crossing group obtained from the simulation. From Fig. 5, it can be found that there are three pedestrians at the roadside when the time step is

515, and then, two of these pedestrians move and the remaining one continues to wait at the roadside. As can be seen in Table 7, there is an overall similarity between the average system output and the simulation model output. All differences are within 10%.

The second level of validation was a statistical comparison of the outputs. The method used to achieve this was a confidence interval and hypothesis test for the mean based on the t distribution. The following statistic was defined:

$$t_n = \frac{u(n) - u_0}{\sqrt{S^2(n)/n}} \tag{16}$$

where u_0 is the mean value of the actual system output, $u(n)$ is the mean value of the model output, $S(n)$ is the standard deviation of the values of the model output, and n is the number of observations. Therefore, the null hypothesis to be tested is H_0: $u = u_0$. If $|t_n| \le t_{n-1,1-\alpha/2}$ and the value of the t distribution has $n-1$ degrees of freedom and confidence level α, H_0 cannot be rejected.

Table 8 presents the results of the t test at a confidence level of 95%, where the term "accept H_0" actually means that the test fails to reject the null hypothesis. The values given in parentheses are the critical values of the t distribution (Law, 2007). In summary, in all cases, the statistical tests demonstrated the similarity of the model outputs with the actual outputs with a good level of confidence.

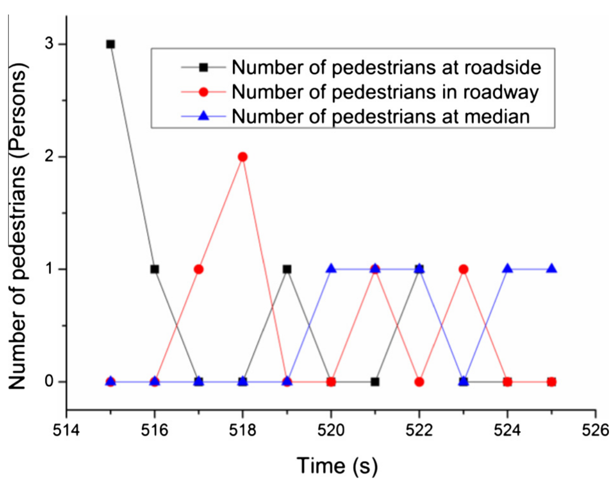

Fig. 5. Example of disagreement among pedestrian crossing group obtained from simulation.

Table 7
Average validation results for observation site.

Output	Field data	Simulation data	Difference (%)
Vehicle delay (s/veh)	2.0	1.8	−10.0
Pedestrian delay (s/ped)	10.1	9.4	−6.9
Occurrence of disagreement (times/h)	43	40	−6.9

Table 8
Statistical validation of proposed model.

Output	n	u_0	$u(n)$	$S(n)$	$\lvert t_n \rvert$	Test result
Vehicle delay	61	2.0 s	1.8 s	0.9 s	1.736 (2.000)	Accept H_0
Pedestrian delay	61	10.1 s	9.4 s	3.6 s	1.519 (2.000)	Accept H_0

7. Applications

As discussed above, the proposed model could simulate the interaction between vehicles and pedestrians accurately. Therefore, we can apply it to the analysis of the dynamic characteristics of vehicle and pedestrian traffic flows.

We assume that the pedestrian arrival rate at the median is 54% of that at the roadside and that it remains constant throughout the simulation. Fig. 6 shows delays for vehicles and pedestrians at different pedestrian arrival rates at the roadside (λ_{pr}) and vehicle arrival rates (λ_v). It can be seen in Fig. 6(a) that the delays for vehicles increase moderately with an increase in the pedestrian arrival rate. This is due to the fact that increasing numbers of pedestrians interrupt the vehicle traffic flow, since crossing takes longer. In contrast, the delays for pedestrians decrease with an increasing flow of pedestrians (see Fig. 6(b)), for the following two reasons: the herd

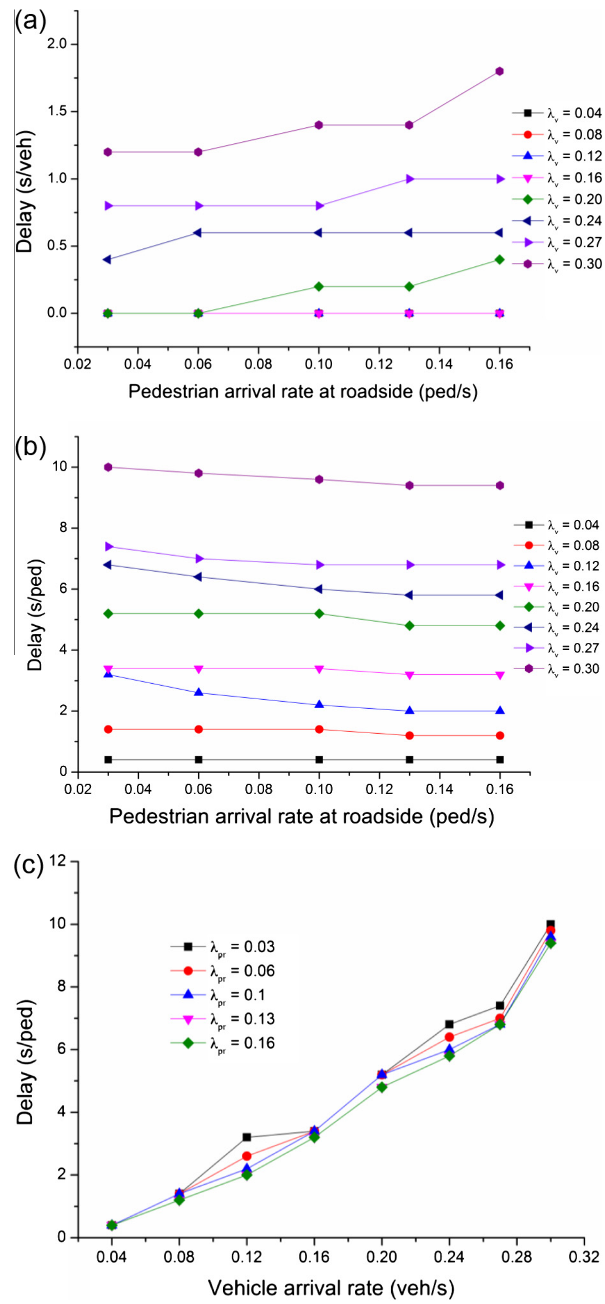

Fig. 6. Delays at various pedestrian arrival rates at roadside and vehicle arrival rates.

mentality effect and lateral discomfort. Herd mentality implies that most waiting pedestrians will follow an aggressive pedestrian, who has a smaller critical gap to cross. Therefore, with an increase in the number of pedestrians, the arrival probability of pedestrians with a smaller critical gap is larger, which shortens the waiting time and delays for pedestrians. Lateral discomfort implies that when there are more waiting pedestrians, there is a deceleration effect on the moving vehicles, and the available gap increases, which leads to easier crossing and a decrease in pedestrian delays. From Fig. 6(c), it is clear that delays for pedestrians increase considerably when the vehicle arrival rate increases from 0.04 to 0.30. The reason for this is that with an increase in the number of vehicles, the time headway becomes smaller, and so, most of the pedestrians are unable to cross the road and they spend more time waiting for a sufficient time gap.

On the basis of the results shown in Fig. 6, we can say that our results are in agreement with those of Jin et al. (2013) in terms of the effect of the interaction between vehicles and pedestrians on the delays for them. However, unlike the proposal of Jin et al. (2013), the model proposed in this paper can capture the phenomenon of disagreement among a pedestrian crossing group, which could help us in better understanding the herd mentality.

8. Conclusions

In this paper, we have presented a new approach for modeling the interactions between vehicles and pedestrians at uncontrolled mid-block crosswalks. In order to take into account the decision process of vehicle drivers and pedestrians during the interaction, evolutionary game theory and cumulative prospect theory are employed for addressing the crossing decision behavior under bounded rationality and risk. A cellular automata-based model is used to depict the motion of vehicles with consideration of the three-second rule, and a modified rule set for pedestrians is proposed in order to take into account the right-moving preference and resolve the deadlock among mixed flows. The developed model is calibrated and validated using real data collected at Jianshe First Road in Wuhan, China. The results show that the proposed model can well simulate the interaction between vehicles and pedestrians. Finally, we apply the proposed model to the analysis of the effect of the interaction between vehicles and pedestrians on the delays for them. It is found that the simulation results show satisfactory agreement with the actual observations.

Although the findings of this study are promising, further studies need to be conducted for improving the performance of the developed model. In this study, we considered only the social learning paradigm in game theory for the crossing decision. As a follow-up study, the individual learning paradigm could be incorporated in game theory, which may help us better understand pedestrians' crossing behavior. Moreover, we must exert caution in generalizing from our sample and results to other samples. Additional and convergent evidence from other samples should be sought in future studies. Finally, the proposed model needs more parameters for simulations than do the existing models in the literature. Thus, we intend to try out other methods such as rough set theory to obtain the probabilities of the driver (or the pedestrian) choosing crossing and yielding, which may reduce the number of parameters and preserve the behavior of the model.

Acknowledgements

Research for this study was supported by the National Natural Science Foundation of China (Grant Nos. 51208400, 51108362, 51078299). The authors thank the reviewers for their useful suggestions.

Appendix A. Supplementary material

Supplementary data associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.ssci.2015.09.016.

References

New Jersey Motor Vehicle Commission, 2015. Chapter 5: Defensive Driving. http://www.state.nj.us/mvc/pdf/Licenses/Driver%20Manual/Chapter_5.pdf (viewed 30 March).
Arifovic, J., Karaivanov, A., 2010. Learning by doing vs. learning from others in a principal–agent model. J. Econ. Dyn. Control 34 (10), 1967–1992.
Berger, P.A., 2004. Rough set rule induction for suitability assessment. Environ. Manage. 34 (4), 546–558.
Blue, V.J., Adler, J.L., 2001. Cellular automata microsimulation for modeling bi-directional pedestrian walkways. Transport. Res. Part B – Methodol. 35 (3), 293–312.
de Lavalette, B.C., Tijus, C., Poitrenaud, S., Leproux, C., Bergeron, J., Thouez, J., 2009. Pedestrian crossing decision-making: a situational and behavioral approach. Saf. Sci. 47 (9SI), 1248–1253.
Fernandez, R., 2010. Modelling public transport stops by microscopic simulation. Transport. Res. Part C – Emerg. Technol. 18 (6), 856–868.
Gao, S., Frejinger, E., Ben-Akiva, M., 2010. Adaptive route choices in risky traffic networks: a prospect theory approach. Transport. Res. Part C – Emerg. Technol. 18 (5SI), 727–740.
Guo, H., Wang, W., Guo, W., Jiang, X., Bubb, H., 2012. Reliability analysis of pedestrian safety crossing in urban traffic environment. Saf. Sci. 50 (4SI), 968–973.
Helbing, D., Jiang, R., Treiber, M., 2005. Analytical investigation of oscillations in intersecting flows of pedestrian and vehicle traffic. Phys. Rev. E 72 (4), 46130.
Hjorth, K., Fosgerau, M., 2012. Using prospect theory to investigate the low marginal value of travel time for small time changes. Transport. Res. Part B – Methodol. 46 (8), 917–932.
Hua, X., Wang, W., Wang, H., 2011. A two-lane cellular automaton traffic flow model with the influence of driving psychology. Acta Phys. Sinica 60 (08), 45028.
Jin, S., Qu, X., Xu, C., Wang, D., 2013. Dynamic characteristics of traffic flow with consideration of pedestrians' road-crossing behavior. Phys. A – Stat. Mech. Appl. 392 (18), 3881–3890.
Jing, M., Deng, W., Wang, H., Ji, Y., 2012. Two-lane cellular automaton traffic model based on car following behavior. Acta Phys. Sinica 61 (24), 450224.
Jou, R., Kitamura, R., Weng, M., Chen, C., 2008. Dynamic commuter departure time choice under uncertainty. Transport. Res. Part A – Policy Pract. 42 (5), 774–783.
Kemel, E., Paraschiv, C., 2013. Prospect theory for joint time and money consequences in risk and ambiguity. Transport. Res. Part B – Methodol. 56, 81–95.
Koh, P.P., Wong, Y.D., Chandrasekar, P., 2014. Safety evaluation of pedestrian behaviour and violations at signalised pedestrian crossings. Saf. Sci. 70, 143–152.
Law, A.M., 2007. Simulation Modelling and Analysis, fourth ed. McGraw-Hill.
Li, B., 2013. A model of pedestrians' intended waiting times for street crossings at signalized intersections. Transport. Res. Part B – Methodol. 51, 17–28.
Liu, B.T., 2013. Non-cooperative dynamic game model between drivers and crossing pedestrian. Master Thesis, Harbin Institute of Technology.
Liu, Y., Tung, Y., 2014. Risk analysis of pedestrians' road-crossing decisions: effects of age, time gap, time of day, and vehicle speed. Saf. Sci. 63, 77–82.
Nagel, K., Schreckenberg, M., 1992. A cellular automaton model for freeway traffic. J. Phys. (France) 2 (12), 2221–2228.
Popuri, Y., Proussaloglou, K., Ayvalik, C., Koppelman, F., Lee, A., 2011. Importance of traveler attitudes in the choice of public transportation to work: findings from the Regional Transportation Authority Attitudinal Survey. Transportation 38 (4), 643–661.
Rahman, M., Chowdhury, M., Xie, Y., He, Y., 2013. Review of microscopic lane-changing models and future research opportunities. IEEE Trans. Intell. Transport. Syst. 14 (4), 1942–1956.
Ren, G., Lu, L., Wang, W., 2012. Modeling bi-direction pedestrian flow by cellular automata and complex network theories. Acta Phys. Sinica 61 (14), 450114.
Sun, Z., Jia, B., Li, X.G., 2012. The study of the interference between pedestrians and vehicles based on cellular automaton model. Acta Phys. Sinica 61 (10), 100508.
Szabo, G., Fath, G., 2007. Evolutionary games on graphs. Phys. Rep.-Rev. Sect. Phys. Lett. 446 (4–6), 97–216.
Sze, N.N., Wong, S.C., Lee, C.Y., 2014. The likelihood of achieving quantified road safety targets: a binary logistic regression model for possible factors. Accid. Anal. Prevent. 73, 242–251.
Tian, L., He, Y., Liu, H., Du, R., 2012. A general evolving model for growing bipartite networks. Phys. Lett. A 376 (23), 1827–1832.
Tversky, A., Kahllemna, D., 1992. Advances in prospect theory: cumulative representation of uncertainty. J. Risk Uncertainty 5 (4), 297–323.
Wang, G., Ma, S., Jia, N., 2013. A combined framework for modeling the evolution of traveler route choice under risk. Transport. Res. Part C – Emerg. Technol. 35 (SI), 156–179.
Xie, D., Gao, Z., Zhao, X., Wang, D.Z.W., 2012. Cellular automaton modeling of the interaction between vehicles and pedestrians at signalized crosswalk. J. Transport. Eng.-ASCE 138 (12), 1442–1452.

Xin, X., Jia, N., Zheng, L., Ma, S., 2014. Power-law in pedestrian crossing flow under the interference of vehicles at an un-signalized midblock crosswalk. Phys. A – Stat. Mech. Appl. 406, 287–297.

Yang, L.Z., Li, J., Liu, S.B., 2008. Simulation of pedestrian counter-flow with right-moving preference. Phys. A – Stat. Mech. Appl. 387 (13), 3281–3289.

Zhang, C.Y., Zhang, J.L., Xie, G.M., Wang, L., 2011. Coevolving agent strategies and network topology for the public goods games. Eur. Phys. J. B 80 (2), 217–222.

Zhang, J., Wang, H., Li, P., 2004. Cellular automata modeling of pedestrian's crossing dynamics. J. Zhejiang Univ. (Sci.) 5 (7), 835–840.

Zheng, Z., 2014. Recent developments and research needs in modeling lane changing. Transport. Res. Part B – Methodol. 60, 16–32.