

Documentation for Minimum Path Sum Solution

Problem Statement

Given an (m times n) grid filled with non-negative numbers, find a path from the top-left to the bottom-right corner which minimizes the sum of all numbers along its path. You can only move either down or right at any point in time.

Examples

Example 1

Input:

```
grid = [  
    [1, 3, 1],  
    [1, 5, 1],  
    [4, 2, 1]  
]
```

Output: 7

Explanation: The path that minimizes the sum is $1 \rightarrow 3 \rightarrow 1 \rightarrow 1 \rightarrow 1$, with a sum of 7.

Example 2

Input:

```
grid = [  
    [1, 2, 3],  
    [4, 5, 6]  
]
```

Output: 12

Explanation: The path that minimizes the sum is $1 \rightarrow 2 \rightarrow 3 \rightarrow 6$, with a sum of 12.

Constraints

- ($m == \text{grid.length}$)
- ($n == \text{grid[i].length}$)
- ($1 \leq m, n \leq 200$)
- ($0 \leq \text{grid[i][j]} \leq 200$)

Solution

The solution uses dynamic programming to compute the minimum path sum. It maintains a 2D list `dp` where `dp[i][j]` represents the minimum path sum to reach the cell `(i, j)`.

Algorithm

1. Initialization:

- If the grid is empty, return 0.
- Initialize `m` and `n` to be the number of rows and columns of the grid, respectively.
- Create a 2D list `dp` of size `m x n` initialized to 0.

2. Base Case:

- Set `dp[0][0]` to `grid[0][0]` because this is the starting point.

3. Fill the First Row:

- For each column `j` from 1 to `n-1`, set `dp[0][j]` to `dp[0][j-1] + grid[0][j]` since you can only move right in the first row.

4. Fill the First Column:

- For each row `i` from 1 to `m-1`, set `dp[i][0]` to `dp[i-1][0] + grid[i][0]` since you can only move down in the first column.

5. Fill the Rest of the dp Table:

- For each cell `(i, j)` where `i > 0` and `j > 0`, set `dp[i][j]` to the minimum of the values from the cell above (`dp[i-1][j]`) and the cell to the left (`dp[i][j-1]`), plus the value of the current cell (`grid[i][j]`).

6. Result:

- The value at `dp[m-1][n-1]` will be the minimum path sum to reach the bottom-right corner of the grid.