

Solution Documentation for Square Root Calculation

Problem Description

Given a non-negative integer x , the task is to return the square root of x rounded down to the nearest integer. The returned integer should also be non-negative. The solution must not use any built-in exponent function or operator (e.g., `pow(x, 0.5)` in C++ or `x ** 0.5` in Python).

Examples

Example 1:

- *Input:* $x = 4$
- *Output:* 2
- *Explanation:* The square root of 4 is 2, so we return 2.

Example 2:

- *Input:* $x = 8$
- *Output:* 2
- *Explanation:* The square root of 8 is approximately 2.82842. Rounded down to the nearest integer, we return 2.

Constraints

- $(0 \leq x \leq 2^{31} - 1)$

Solution

The solution employs a binary search algorithm to find the square root of x . Here's a step-by-step explanation of the approach:

1. Edge Case Handling:

- If x is less than 2, return x directly since the square root of 0 is 0 and the square root of 1 is 1.

2. Binary Search Initialization:

- Initialize two pointers, left and right, to 0 and x respectively.

3. Binary Search Process:

- While left is less than or equal to right:
- Calculate the midpoint mid as the integer division of $(\text{left} + \text{right}) // 2$.
- Compute mid_squared as $\text{mid} * \text{mid}$.
- Compare mid_squared with x :
- If mid_squared is equal to x , return mid as the square root.
- If mid_squared is less than x , move the left pointer to $\text{mid} + 1$.
- If mid_squared is greater than x , move the right pointer to $\text{mid} - 1$.

4. Return Result:

- After exiting the loop, return right as it will be the largest integer for which $\text{right} * \text{right}$ is less than or equal to x .

Explanation of the Code

1. Edge Case Handling:

- The function first checks if x is less than 2. If true, it directly returns x because the square root of x is x itself.

2. Binary Search:

- The left pointer starts at 0 and the right pointer starts at x .
- The loop runs as long as left is less than or equal to right.
- Inside the loop, mid is calculated as the integer division of $(\text{left} + \text{right}) // 2$.
- mid_squared is the square of mid.

3. Conditions Inside the Loop:

- If mid_squared is equal to x , the exact square root is found, and mid is returned.
- If mid_squared is less than x , it means the true square root is larger, so left is moved to $\text{mid} + 1$.
- If mid_squared is greater than x , it means the true square root is smaller, so right is moved to $\text{mid} - 1$.

4. Returning the Result:

- When the loop exits, right is the largest integer such that $\text{right} * \text{right}$ is less than or equal to x , making it the integer square root of x .