

Documentation: Finding a Peak Element in an Array

Problem Overview:

- **Objective:** Given a 0-indexed integer array `nums`, find a peak element and return its index. A peak element is defined as an element that is strictly greater than its immediate neighbors.
- **Definition:** *For an array `nums`, a peak element is an element `nums[i]` such that:*
 - `nums[i] > nums[i-1]` (if `i > 0`)
 - `nums[i] > nums[i+1]` (if `i < n-1`), where `n` is the length of the array.
- **Boundary Conditions:**
 - It is assumed that `nums[-1]` and `nums[n]` are both $-\infty$ (negative infinity), which means elements at the boundaries of the array are considered greater than non-existent neighbors outside the array.
 - Multiple Peaks: The array may contain multiple peak elements. The algorithm is required to return the index of any one of these peaks.

Constraints:

- The length of the array `nums` is between 1 and 1000 inclusive.
- Each element in `nums` is a 32-bit signed integer, ranging from -2^{31} to $2^{31} - 1$.
- No two adjacent elements in `nums` are equal, i.e., `nums[i] != nums[i + 1]` for all valid `i`.

Approach:

- **Algorithm Type:** The problem is optimally solved using a binary search technique, which takes advantage of the logarithmic nature of the search space.

- **Time Complexity Requirement:** The problem explicitly requires an algorithm that operates in $(O(\log n))$ time complexity, indicating that a divide-and-conquer approach like binary search is suitable.

Solution Strategy:

1. Initialization:

- Define two pointers: left initialized to the start of the array and right initialized to the end of the array.

2. Binary Search Process:

- Calculate the middle index mid using the formula: $mid = (left + right) // 2$.
- *Compare the element at mid with its right neighbor ($nums[mid + 1]$):*
 - If $nums[mid]$ is greater than $nums[mid + 1]$, it implies a peak might be on the left side (including mid itself), so update the right pointer to mid .
 - If $nums[mid]$ is less than $nums[mid + 1]$, the peak must lie on the right side, so update the left pointer to $mid + 1$.
- Repeat this process until the left pointer equals the right pointer.

3. Termination Condition:

- The binary search loop continues until the left and right pointers converge to the same index. At this point, the converged index is the peak element's index.

4. Result:

- Return the index at which the left pointer and right pointer converge. This index corresponds to a peak element in the array.

Edge Cases:

- **Single Element Array:** If the array contains only one element, that element is trivially a peak, and the index 0 should be returned.
- **Peak at the Start or End:** If the peak is at the start (`nums[0]`) or the end (`nums[n-1]`) of the array, the algorithm will correctly identify it because of the virtual $-\infty$ assumption for elements outside the array bounds.

Summary:

The solution leverages the binary search algorithm to efficiently locate a peak element by systematically narrowing down the potential peak region. The algorithm's ability to discard half of the array in each iteration ensures that the solution meets the required ($O(\log n)$) time complexity. The flexibility of returning any peak element makes the problem simpler, as multiple correct answers are possible.