

Documentation for Leetcode 474: Ones and Zeroes

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1. Problem Statement

You're given:

- An array of binary strings `strs`
- Two integers `m` and `n`

Your task is to find the size of the largest subset of `strs` such that the total number of '0's is at most `m` and the total number of '1's is at most `n`.

Constraints:

- $1 \leq \text{strs.length} \leq 600$
- $1 \leq \text{strs}[i].\text{length} \leq 100$
- `strs[i]` contains only '0' and '1'
- $1 \leq m, n \leq 100$

2. Intuition

This is a variant of the 0/1 Knapsack problem, where:

- Each string has a "cost" in terms of the number of 0s and 1s.
- We are bounded by two constraints: total 0s $\leq m$, and total 1s $\leq n$.

We aim to maximize the number of strings selected within these constraints.

3. Key Observations

- The count of 0s and 1s in each string acts like a weight.
- We need to track the maximum number of items (strings) that can be picked without exceeding these weights.
- The problem is not about summing values, but about maximizing the count of included elements.

4. Approach

- Use Dynamic Programming (DP).
- Define a 2D DP array $dp[m+1][n+1]$ where $dp[i][j]$ = max number of strings that can be picked with i 0s and j 1s.
- For each string:
 - Count its zero and one.
 - Update the dp table in reverse order to avoid counting the same string multiple times.
- Finally, return $dp[m][n]$.

5. Edge Cases

- Empty strs list: Return 0.
- Strings with only 0s or only 1s.
- $m = 0$ or $n = 0$: Only strings with no 0s or 1s can be included.
- Strings with counts greater than m or n must be excluded.

6. Complexity Analysis

Time Complexity:

- $O(L * m * n)$
 - L = length of strs
 - Each string updates up to $m * n$ entries in the DP table

Space Complexity:

- $O(m * n)$
 - We use a 2D DP table of size $(m+1) \times (n+1)$

7. Alternative Approaches

- Brute-force recursion with memoization:
 - Explore all subsets with recursion and memoize results.
 - More complex to manage and less efficient for large inputs.
- 3D DP: Use $dp[i][j][k]$ for tracking up to i -th string. But this is unnecessary; 2D DP is optimal.

8. Test Cases

✓ Test Case 1:

Input: $strs = ["10", "0001", "111001", "1", "0"]$, $m = 5$, $n = 3$

Output: 4

Explanation: Subset = $["10", "0001", "1", "0"]$

✓ Test Case 2:

Input: $strs = ["10", "0", "1"]$, $m = 1$, $n = 1$

Output: 2

Explanation: Subset = $["0", "1"]$

✓ Test Case 3:

Input: strs = ["0","0","1","1"], m = 2, n = 2

Output: 4

✓ Test Case 4:

Input: strs = ["10"], m = 0, n = 1

Output: 0

9. Final Thoughts

- This problem teaches how to extend classical knapsack DP to multiple constraints.
- Always update the DP table in reverse when doing 0/1 knapsack to avoid reusing items.
- Efficient even for large inputs due to optimal $O(m * n * L)$ complexity.