

□ Complex Number Multiplication – Documentation

1. Problem Statement

Given two complex numbers represented as strings in the format "a+bi", where a and b are integers in the range $[-100, 100]$, return a string representing their product in the same format.

□ The multiplication of two complex numbers $(a + bi)$ and $(c + di)$ is given by:

$$(a + bi)(c + di) = (ac - bd) + (ad + bc)i$$

2. Intuition

If we can extract the real (a, c) and imaginary (b, d) parts from the string inputs, we can directly plug them into the formula for complex multiplication.

3. Key Observations

- Complex number strings are always in the format "real+imaginaryi".
- The multiplication formula simplifies the need for using any complex number library.
- Parsing and integer multiplication suffice to solve the problem.

4. Approach

- Parse the string inputs num1 and num2 to extract integers a, b, c, and d.
- Apply the formula:
 - Real part: $a * c - b * d$
 - Imaginary part: $a * d + b * c$
- Format the result as a string: "real+imaginaryi"

5. Edge Cases

- Negative numbers in real or imaginary parts (e.g., "1+-1i").
- Zero real or imaginary parts (e.g., "0+2i", "3+0i").
- Multiplying any number by "0+0i" should return "0+0i".

6. Complexity Analysis

Time Complexity

- $O(1)$ — Constant time operations (splitting, parsing, multiplying).

Space Complexity

- $O(1)$ — Constant space for storing parsed values and result string.

7. Alternative Approaches

- Using Python's complex number support:

```
def complexNumberMultiply(num1: str, num2: str) -> str:
    a = complex(num1.replace('i', 'j'))
    b = complex(num2.replace('i', 'j'))
    result = a * b
    return f"{int(result.real)} + {int(result.imag)}i"
```

Note: This uses built-in features, but avoids manual control and may be disallowed in interviews.

8. Test Cases

Input	Output	Explanation
"1+1i", "1+1i"	"0+2i"	$(1 + i)(1 + i) = 1 + 2i + i^2 = 0 + 2i$
"1+-1i", "1+-1i"	"0+-2i"	$(1 - i)(1 - i) = 1 - 2i + i^2 = 0 - 2i$
"0+0i", "1+1i"	"0+0i"	Multiplying by zero results in zero
"2+3i", "4+5i"	"-7+22i"	$2*4 - 3*5 = -7; 2*5 + 3*4 = 22$

9. Final Thoughts

This is a classic string parsing + math problem. It teaches:

- How to handle string input formats cleanly.
- Application of algebraic identities in code.
- Edge case awareness when dealing with signs and zero.