Documentation for 507. Perfect Number

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1. 📌 Problem Statement

A perfect number is a positive integer equal to the sum of its positive divisors, excluding itself. Given an integer num, return true if it is a perfect number, otherwise return false.

2. 💡 Intuition

Instead of checking all numbers from 1 to num - 1, we can optimize by checking only up to √num, because divisors come in pairs: if i divides num, then num / i is also a divisor.

3. 🔍 Key Observations

- Every number is divisible by 1.
- Divisors always occur in pairs. If i divides num, then num / i also divides num.
- We must exclude num itself when calculating the sum.
- We should avoid adding the square root twice when i == num // i.

4. ⚙ Approach

- **Early return** for num <= 1, as these can't be perfect numbers.
- Initialize total = 1 since 1 is a universal divisor.
- Loop i from 2 to sqrt(num):
  - If num % i == 0, then both i and num // i are divisors.
  - Add both to the sum, making sure not to double-count when i == num // i.
- Compare the final sum to the original number and return the result.

5. 🚧 Edge Cases

- num = 1 → Should return False (no positive divisors other than itself)
- num = 0 or negative → Should return False
- num is a known perfect number like 6, 28, 496, etc. → Should return True
- num is a large non-perfect number → Should not time out due to the efficient algorithm

6. Complexity Analysis

Time Complexity:

- O(√n): We only iterate up to the square root of num.

Space Complexity:

- O(1): Constant space used for variables.

7. 🔁 Alternative Approaches

- Brute Force:
  - Check all integers from 1 to num - 1.
  - Time Complexity: O(n) — too slow for large num.
- Lookup Table:

- Store known perfect numbers (e.g., 6, 28, 496, 8128, 33550336) in a set.
- Time Complexity: O(1)
- Limitation: Not scalable or general.

8. ✅ Test Cases

Input	Output	Explanation
28	True	Divisors: 1 + 2 + 4 + 7 + 14 = 28
6	True	Divisors: 1 + 2 + 3 = 6
496	True	Known perfect number
1	False	No proper divisors
10	False	Divisors: 1 + 2 + 5 = 8 ≠ 10
8128	True	Known perfect number

9. 🎯 Final Thoughts

- The optimized solution is efficient enough for large inputs (num <= 10^8).
- The perfect number problem connects with number theory and prime-related patterns.
- For deeper study, look into Euclid–Euler Theorem, which links perfect numbers with Mersenne primes.