

Documentation: Unique Paths

Overview

This algorithm calculates the number of unique paths a robot can take to move from the top-left corner to the bottom-right corner of an $m \times n$ grid. The robot is only allowed to move either down or right at any given point.

Problem Statement

Given two integers m and n , representing the dimensions of the grid, the task is to determine the number of unique paths the robot can take to reach the bottom-right corner.

Example

Input:

- $m = 3$
- $n = 7$

Output:

- 28

Explanation: There are 28 unique paths for the robot to move from the top-left corner to the bottom-right corner in a 3×7 grid.

Solution Approach

The problem can be solved using dynamic programming. The solution initializes a 2D array `dp` of size `m x n` to store the number of unique paths. It then iterates through the grid, filling the `dp` array based on the constraints of the problem.

1. **Initialization:** Set the top-left cell of `dp` to 1, as there is only one way to reach it. Fill the first row and first column of `dp` with 1s since the robot can only move right from the top row and down from the leftmost column.
2. **Dynamic Programming:** Iterate through the remaining cells of `dp`. For each cell `(i, j)`, the number of unique paths to reach it is the sum of the number of unique paths to reach the cell above it `(i-1, j)` and the cell to its left `(i, j-1)`.
3. **Return Result:** Finally, return the value stored in the bottom-right corner of `dp`, which represents the total number of unique paths.

Complexity Analysis

- **Time Complexity:** The time complexity of this solution is $O(m * n)$, where `m` and `n` are the dimensions of the grid.
- **Space Complexity:** The space complexity is also $O(m * n)$ since we use a 2D array to store the number of unique paths.