

1. 📖 Problem Statement

Given an integer array `nums` of size `n`, return the minimum number of moves required to make all elements in the array equal.

In one move, you can increment or decrement a single element by 1.

Constraints:

- $1 \leq \text{nums.length} \leq 10^5$
- $-10^9 \leq \text{nums}[i] \leq 10^9$

2. 💡 Intuition

To make all elements equal with the fewest number of increment/decrement operations, we want to minimize the total sum of differences. This naturally leads us to the median, which is the statistical middle value of a sorted list.

3. 🔑 Key Observations

- The sum of absolute differences $|x_i - m|$ is minimized when m is the median.
- Changing all values to the median will require fewer operations than changing to the mean or any arbitrary value.
- This approach works for both even and odd length arrays.

4. 📝 Approach

- Sort the array to easily find the median.
- Choose the median element.
- Compute the sum of absolute differences between each element and the median.
- Return this sum as the minimum number of moves.

5. Edge Cases

- If the array has only one element, no move is needed.
- If all elements are already equal, result is 0.
- Works for negative, zero, or large positive integers as long as within range.

6. Complexity Analysis

□ Time Complexity:

- Sorting takes $O(n \log n)$.
- Calculating the sum of differences takes $O(n)$.
- Total: $O(n \log n)$

□ Space Complexity:

- $O(1)$ extra space (if sorting in place), otherwise $O(n)$ if a new list is used.

7. Alternative Approaches

a. Brute Force:

- Try making all elements equal to every possible number in the array.
- Time complexity: $O(n^2) \rightarrow$ inefficient for large input.

b. QuickSelect for Median (Optimized):

- Use the QuickSelect algorithm to find the median in $O(n)$ average time.
- Total complexity becomes $O(n)$ on average.
- Useful if performance is critical for large inputs.

8. ☐ Test Cases

✓ Test Case 1:

Input: $[1, 2, 3]$

Output: 2

Explanation: $[1, 2, 3] \rightarrow [2, 2, 3] \rightarrow [2, 2, 2]$

✓ Test Case 2:

Input: $[1, 10, 2, 9]$

Output: 16

Explanation: All elements moved to 2 or 9 $\rightarrow \sum(|\text{nums}[i] - \text{median}|)$

✓ Test Case 3:

Input: $[5]$

Output: 0

Explanation: Only one element, no moves needed.

✓ Test Case 4:

Input: $[-1, 0, 1]$

Output: 2

Explanation: Median is 0. Total moves: $|-1-0| + |0-0| + |1-0| = 1 + 0 + 1 = 2$

9. ☐ Final Thoughts

- This problem beautifully demonstrates how median minimizes the sum of absolute differences.
- Sorting provides a straightforward and efficient solution.
- While there's an optimized linear-time version using QuickSelect, the sorted version is often sufficient for coding interviews and real-world use cases.