

Documentation for Longest Word in Dictionary through Deleting

1. Problem Statement

Given:

- A string *s*.
- A word dictionary.

Task:

Return the longest word in the dictionary that can be formed by deleting some characters of *s*.

- If there are multiple answers, return the lexicographically smallest one.
- If no valid word exists, return an empty string.

2. Intuition

To find a word from the dictionary that exists as a subsequence of the main string *s*, we can:

- Check each word using a two-pointer method to see if it's a valid subsequence.
- Prioritize longer and lexicographically smaller words by sorting the dictionary.

3. Key Observations

- The task is essentially about subsequence matching.
- We must process every word in the dictionary.
- Sorting helps us quickly find the correct match without having to compare every match manually.

4. Approach

- Define a helper function `is_subsequence(word)`:
 - Use two pointers to check if the word can be formed by deleting characters in *s*.

- Sort the dictionary:
 - Primary key: $-\text{len}(\text{word}) \rightarrow$ longest first.
 - Secondary key: word \rightarrow lexicographically smallest first.
- Iterate over the sorted dictionary:
 - Return the first word that is a subsequence of s.

5. Edge Cases

- Dictionary is empty \rightarrow return "".
- No word in the dictionary is a subsequence \rightarrow return "".
- All words are of the same length. \rightarrow Pick the one with the smallest lexicographical order.

6. Complexity Analysis

Time Complexity

- Sorting: $O(n * \log n)$ where $n = \text{len}(\text{dictionary})$
- Subsequence check: For each word (length m), check against s (length k) $\rightarrow O(m + k)$ in worst case.
- Total:
 $O(n * \log n + n * k)$ where $k = \text{len}(s)$

Space Complexity

- Sorting and storage: $O(n)$
- No extra space used other than minor variables $\rightarrow O(1)$ auxiliary space.

7. Alternative Approaches

- Trie-based optimization: Build a trie from the dictionary and perform DFS over s to match words.
- Indexed mapping of characters in s: Store indices of characters in s and binary search to speed up subsequence checking.

8. □ Test Cases

Test Case 1

```
s = "abpcplea"
dictionary = ["ale", "apple", "monkey", "plea"]
# Output: "apple"
```

Test Case 2

```
s = "abpcplea"
dictionary = ["a", "b", "c"]
# Output: "a"
```

Test Case 3

```
s = "abcd"
dictionary = ["abcde"]
# Output: ""
```

Test Case 4

```
s = "bab"
dictionary = ["ba", "ab", "a", "b"]
# Output: "ab" (both 'ab' and 'ba' are valid, but 'ab' is smaller)
```

9. □ Final Thoughts

- This problem teaches subsequence checking, sorting with custom keys, and greedy selection.
- Sorting the dictionary upfront saves extra logic for handling tiebreakers.
- Optimizations may be required for large-scale input, especially if *s* and dictionary words are near the upper bounds.