

# Documentation for the "Count The Repetitions" problem.

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### 1. Problem Statement

We are given two strings  $s1$  and  $s2$ , and two integers  $n1$  and  $n2$ .

- $s1$  is repeated  $n1$  times to form a long string  $str1$ .
- $s2$  is repeated  $n2$  times to form a long string  $str2$ .

The task is to determine the maximum number of times  $str2$  can be obtained by removing some characters from  $str1$ .

Example:

Input:

$s1 = \text{"acb"}, n1 = 4$

$s2 = \text{"ab"}, n2 = 2$

Output: 2

## 2. Intuition

To solve this problem efficiently, we need to simulate extracting  $s_2$  from  $str_1$ , which is  $s_1$  repeated  $n_1$  times. Given that both  $n_1$  and  $n_2$  can be as large as  $10^6$ , we must avoid directly constructing the strings.

The key is to keep track of how many times  $s_2$  can be matched within  $str_1$  using a cycle detection approach. Instead of iterating over all characters, we can speed up the process by recognizing repeating patterns.

## 3. Key Observations

- We are not interested in constructing the full strings  $str_1$  or  $str_2$ , as this would be inefficient given the constraints.
- The challenge is to find how many times  $s_2$  can be matched in  $str_1$  without explicitly building the large strings.
- Once a cycle in the matching process is detected, we can use it to quickly jump over repeated operations.

## 4. Approach

We will iterate through the repeated instances of  $s_1$ , matching characters with  $s_2$  as we go. Key steps include:

- **Simulate the Matching:** As we go through  $s_1$  (repeated  $n_1$  times), match characters from  $s_2$  by skipping non-matching characters.
- **Cycle Detection:** If the index of the character in  $s_2$  being matched repeats, it indicates a cycle. We can then calculate how many times we can skip the cycle to accelerate the matching process.
- **Final Count:** Once we know how many times  $s_2$  can be matched, divide it by  $n_2$  to get the final result.

The solution uses a dictionary to store the index of  $s_2$  after each iteration through  $s_1$ , and whenever we encounter the same index, we know a cycle has occurred.

## 5. Edge Cases

- Case 1:  $s_1$  and  $s_2$  are identical:
  - If  $s_1$  and  $s_2$  are identical, then each repetition of  $s_1$  counts as one occurrence of  $s_2$ .
- Case 2: One or both strings are extremely short or long:
  - The solution should handle small strings and large repetitions efficiently without constructing huge strings.
- Case 3:  $s_2$  is not a subsequence of  $s_1$ :
  - If  $s_2$  cannot be formed from  $s_1$ , then the result should be 0.
- Case 4:  $n_1$  or  $n_2$  is 1:
  - If either  $n_1$  or  $n_2$  is 1, we will need to handle this case where we don't have many repetitions of either string.

## 6. Complexity Analysis

### Time Complexity

- The time complexity is  $O(n_1 + \text{length of } s_1)$  due to the iteration over  $n_1$  repetitions of  $s_1$ . Each iteration checks each character of  $s_1$  once.
- The cycle detection mechanism ensures that we avoid reprocessing the same part of the string multiple times.

Thus, the time complexity is linear in terms of the total length of  $\text{str}_1$ .

### Space Complexity

- The space complexity is  $O(m)$ , where  $m$  is the length of  $s_2$ . We store the index of  $s_2$  positions as we process  $s_1$ , which requires only a small amount of additional memory.

## 7. Alternative Approaches

- i. Brute Force Approach:
  - a. A brute-force solution would involve constructing  $\text{str}_1$  and  $\text{str}_2$  explicitly, which is inefficient for large inputs (up to  $10^6$  repetitions).

- ii. Greedy Approach:
  - a. A greedy approach could match characters from  $s_1$  to  $s_2$  without explicitly checking for cycles, but would also suffer performance issues as  $n_1$  and  $n_2$  grow.

## 8. Test Cases

Test Case 1:

Input:

- $s_1 = \text{"acb"}, n_1 = 4$
- $s_2 = \text{"ab"}, n_2 = 2$

Output: 2

Test Case 2:

Input:

- $s_1 = \text{"acb"}, n_1 = 1$
- $s_2 = \text{"acb"}, n_2 = 1$

Output: 1

Test Case 3:

Input:

- $s_1 = \text{"abc"}, n_1 = 1000000$
- $s_2 = \text{"ab"}, n_2 = 500000$

Output: 500000

Test Case 4:

Input:

- $s1 = \text{"xyz"}, n1 = 1$
- $s2 = \text{"abc"}, n2 = 1$

Output: 0

## 9. Final Thoughts

This solution efficiently handles the problem using cycle detection to avoid brute-force string construction. It scales well even for large inputs ( $n1, n2$  up to  $10^6$ ), making it suitable for competitive programming and real-world applications.

The use of a cycle detection mechanism makes the solution both time and space-efficient, ensuring that we avoid unnecessary computations while maintaining clarity and correctness.