Lab 3 - Frequency Analysis

3.2

#

3.2.1 Synthesis of Periodic Signals

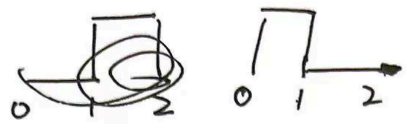
Recall:

i: Compute the Fourier series expansion

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t} = \sum_{k=-\infty}^{\infty} c_k e^{jk(2\pi/T)t}$$

Sig1:

T:



$$c_k = \frac{1}{T}\int_T x(t)\, e^{-jk\frac{2\pi}{T}t}\,dt$$

$$= \frac{1}{2}\int_0^2 x(t)\, e^{-jk\frac{2\pi}{2}t}\,dt$$

$$= \frac{1}{2}\int_0^1 e^{-jk\pi t}\,dt \qquad (\text{consider } k\neq 0)$$

$$= \frac{1}{2}\cdot\frac{1}{-jk\pi}\left[e^{-jk\pi}-1\right]$$

$$= \frac{1}{2}\cdot\frac{1}{jk\pi}\left[1-(-1)^k\right]$$

$$= \begin{cases} 0, & k \text{ even} \\ \frac{1}{jk\pi}, & k \text{ odd} \end{cases}, \quad \text{and} \quad c_0 = \frac{1}{2}$$

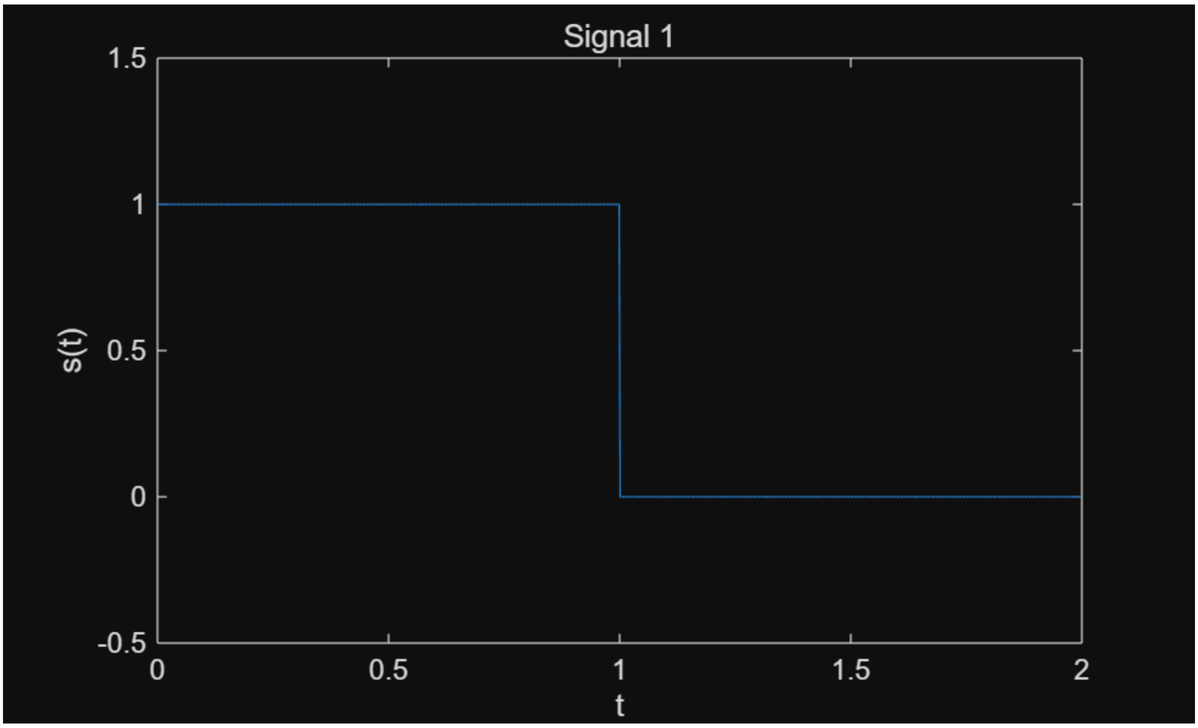
$$\therefore x(t) = \frac{1}{2} + \sum_{\substack{k \text{ odd},\\ k=-\infty}}^{\infty} \frac{1}{jk\pi} e^{jk\pi t}$$

$$= \frac{1}{2} + \sum_{k \text{ odd}} \frac{1}{jk\pi}\left[\cos(k\pi t) + j\sin(k\pi t)\right]$$

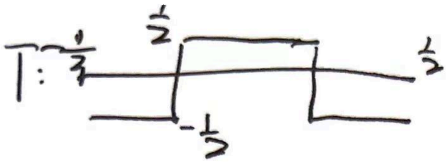
$\because x(t)$ real $\therefore x(t) = \frac{1}{2}\, x + x^*$

$$\therefore x(t) = \frac{1}{2} + \sum_{\substack{k \text{ odd}\\ k=-\infty}}^{\infty} \frac{\sin(k\pi t)}{k\pi} = \frac{1}{2} + \sum_{k=0}^{\infty} \frac{2\sin\left[(2k+1)\pi t\right]}{(2k+1)\pi}$$

```
t=linspace(0,2,1000);
x=[ones(1,500),zeros(1,500)];
plot(t,x), xlabel('t'),ylabel('s(t)')
title('Signal 1'), ylim([-0.5,1.5])
```



Sig2:



Even! And $a_0 = 0$.

$$a_n = \frac{2}{T}\int_{T_{\frac{1}{2}}} s(t)\cos\left(\frac{2\pi n}{T_0} t\right) dt$$

$$= 2\int_{-\frac{1}{2}} s(t)\cos(2\pi n t)\, dt$$

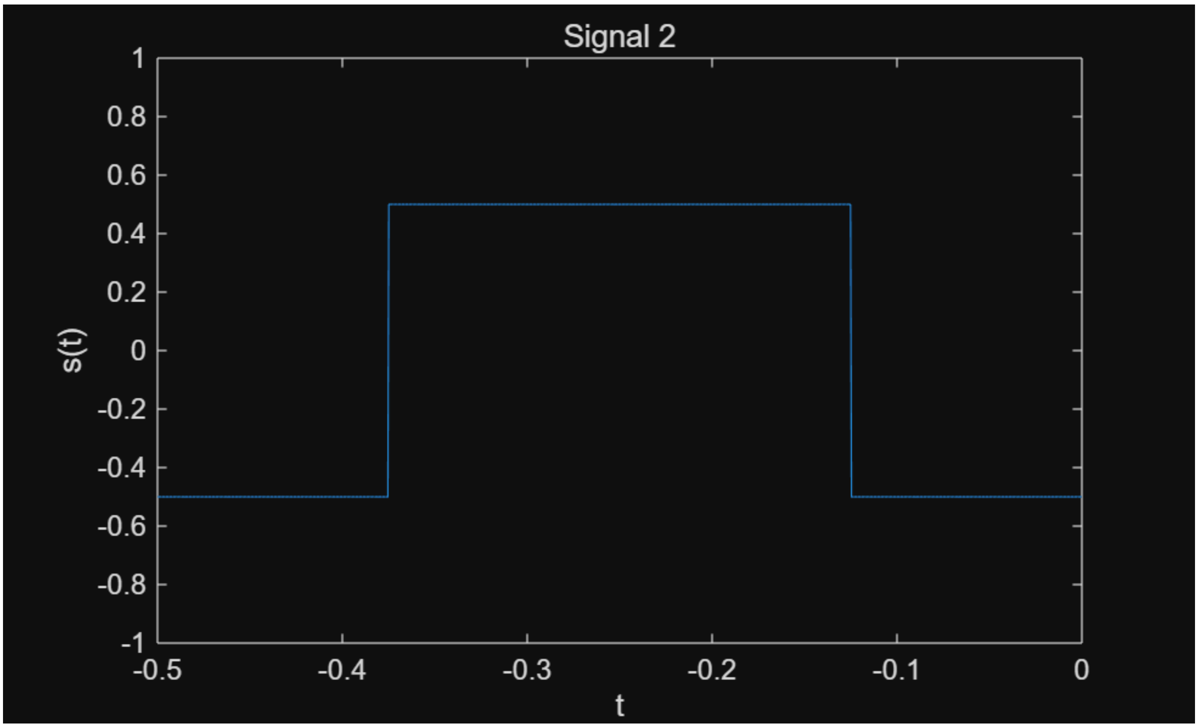
$$= 4\int_{0}^{\frac{1}{2}} s(t)\cos(2\pi n t)\, dt$$

$$= \cancel{\frac{4}{2\pi}}\, 4\cdot\frac{1}{2}\cdot\frac{1}{2\pi n}\sin(2\pi n t)\Big|_{0,\ \frac{1}{2}}^{\frac{1}{4},\ \frac{3}{4}}$$

$$= \frac{2}{\pi n}\sin\left(\frac{\pi}{2}n\right) = \begin{cases} \frac{2}{\pi n}, & k \equiv 1 \pmod 4 \\ -\frac{2}{\pi n}, & k \equiv 3 \pmod 4 \\ 0, & \text{else} \end{cases}$$

$$\therefore s(t) = 0 + \sum_{k=0}^{\infty} \frac{2}{\pi(2k+1)}(-1)^k \cos\left[2\pi(2k+1)t\right]$$

```
x(:)=-1/2;
x(251:750)=1/2;
t=t/2-1/2;
figure
plot(t,x), xlabel('t'),ylabel('s(t)')
title('Signal 2'), ylim([-1,1])
```

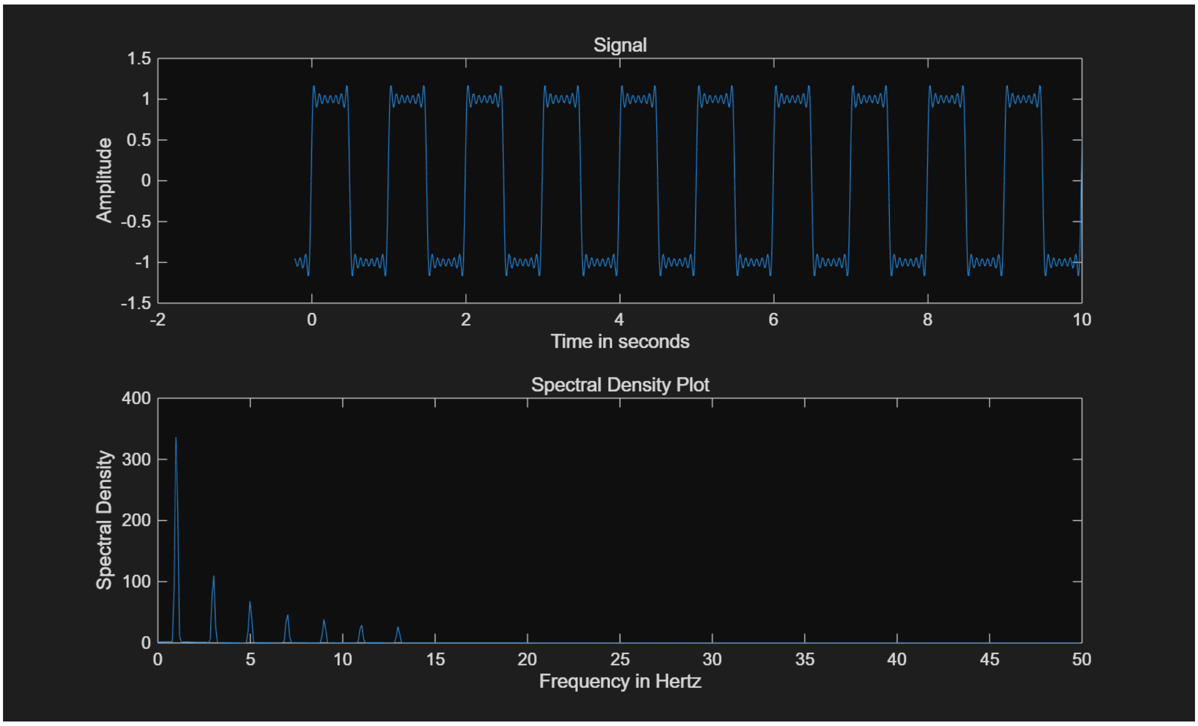


3.4 CT Frequency Analysis

3.4.1

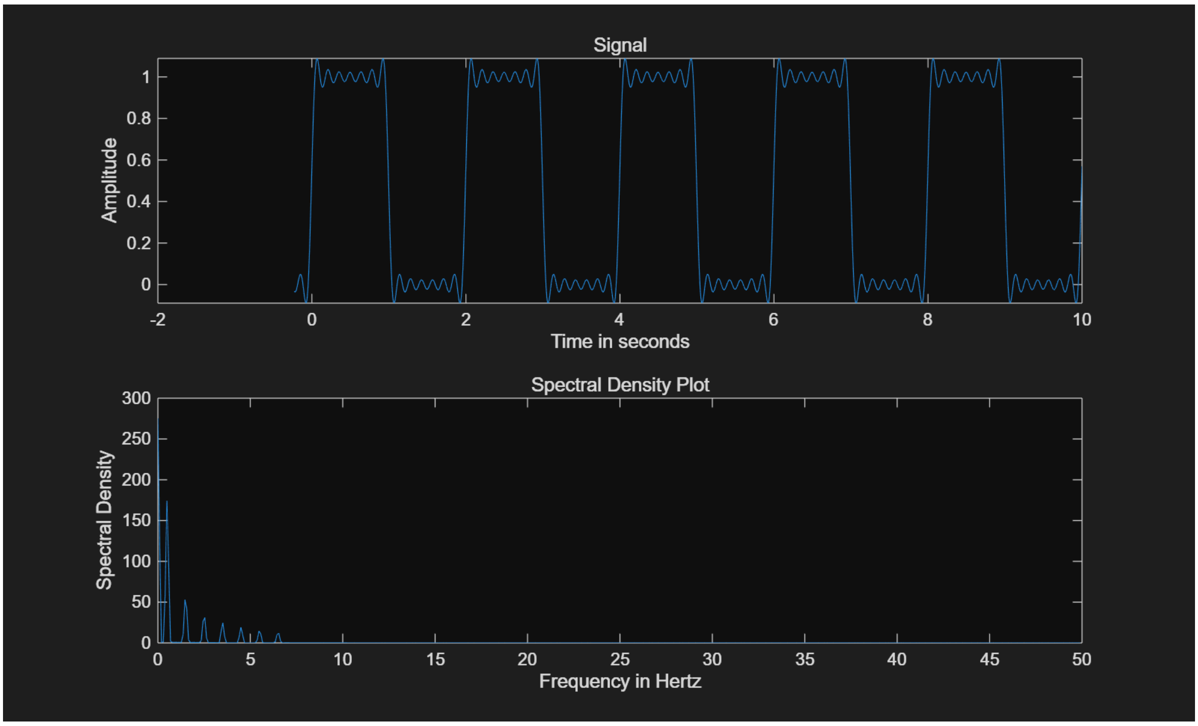
The default synthesized signal:

```
clear,figure
show_fig("34_1.fig")
```



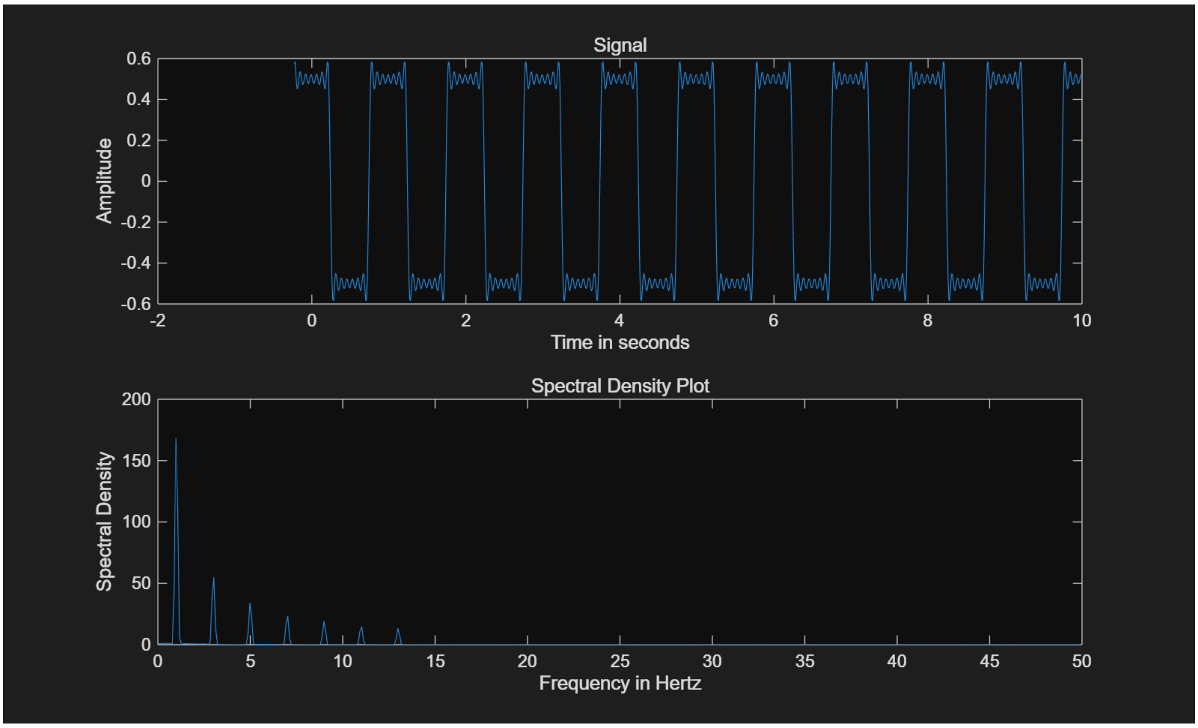
Sig 1:

```
show_fig("34_2.fig")
```



Sig 2:

```
show_fig("34_3.fig")
```

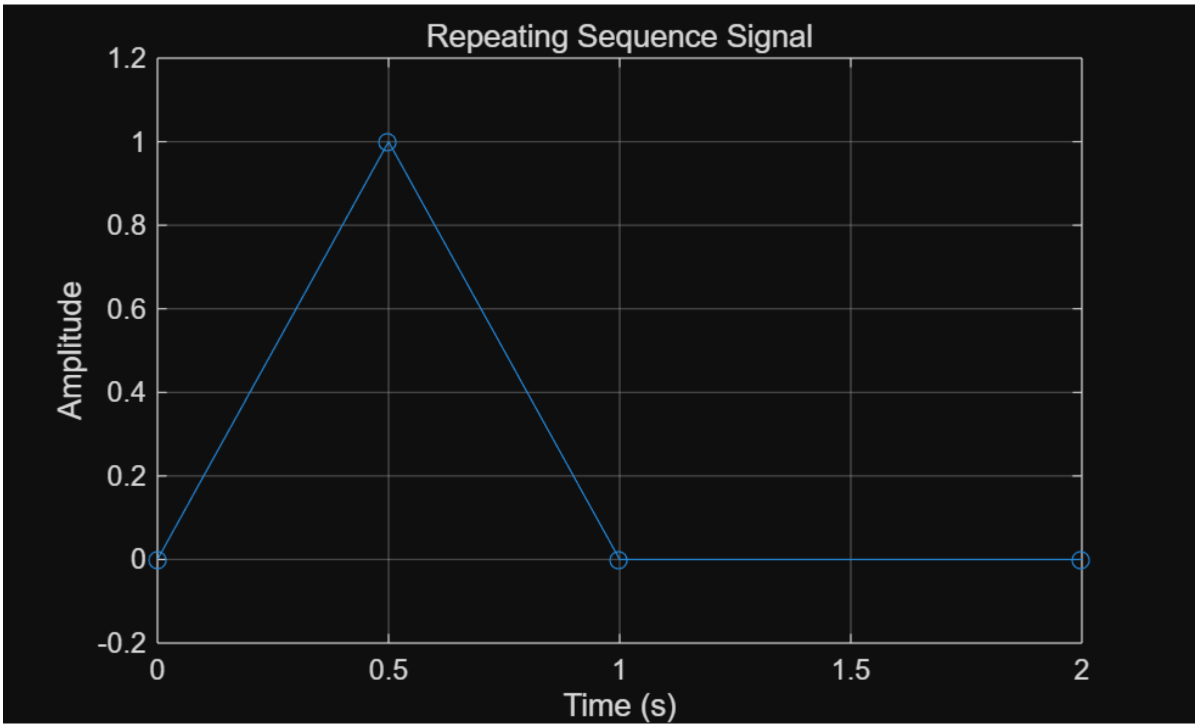


Comment:

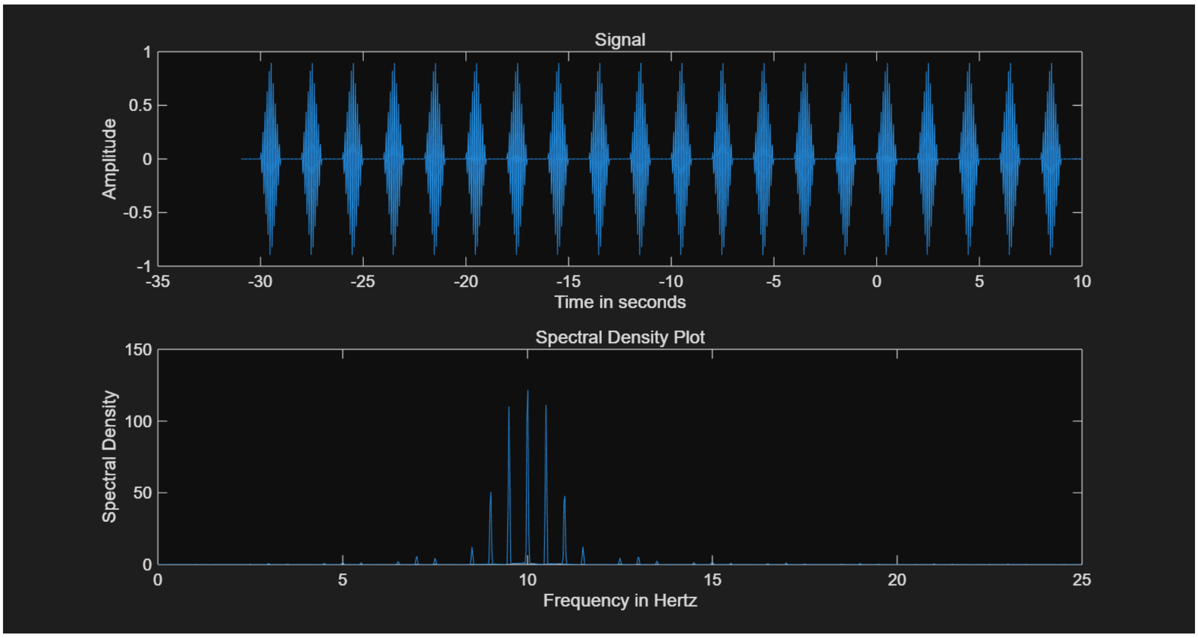
Gibbs' Phenomena can be found in all of the plots.

3.4.2 Modulation Property

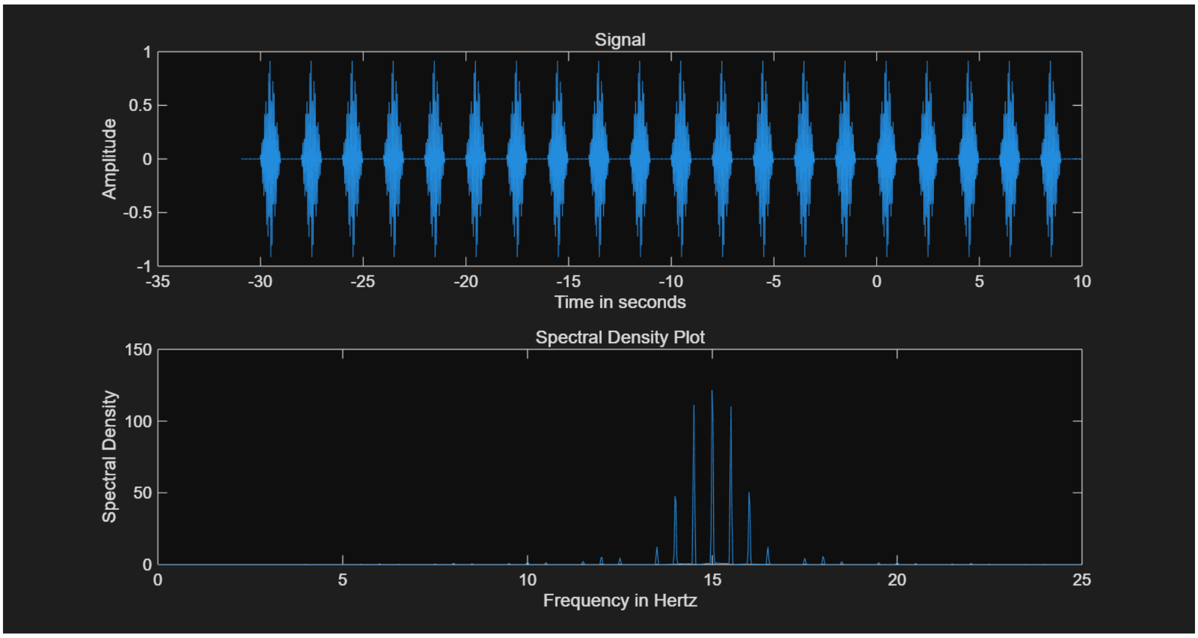
```
clear,figure
% 定义时间向量和输出向量
time_values = [0 0.5 1 2];
output_values = [0 1 0 0];

% 绘制信号
figure;
plot(time_values, output_values, '-o');
title('Repeating Sequence Signal');
xlabel('Time (s)');
ylabel('Amplitude');
grid on;
axis([0 2 -0.2 1.2]);
```



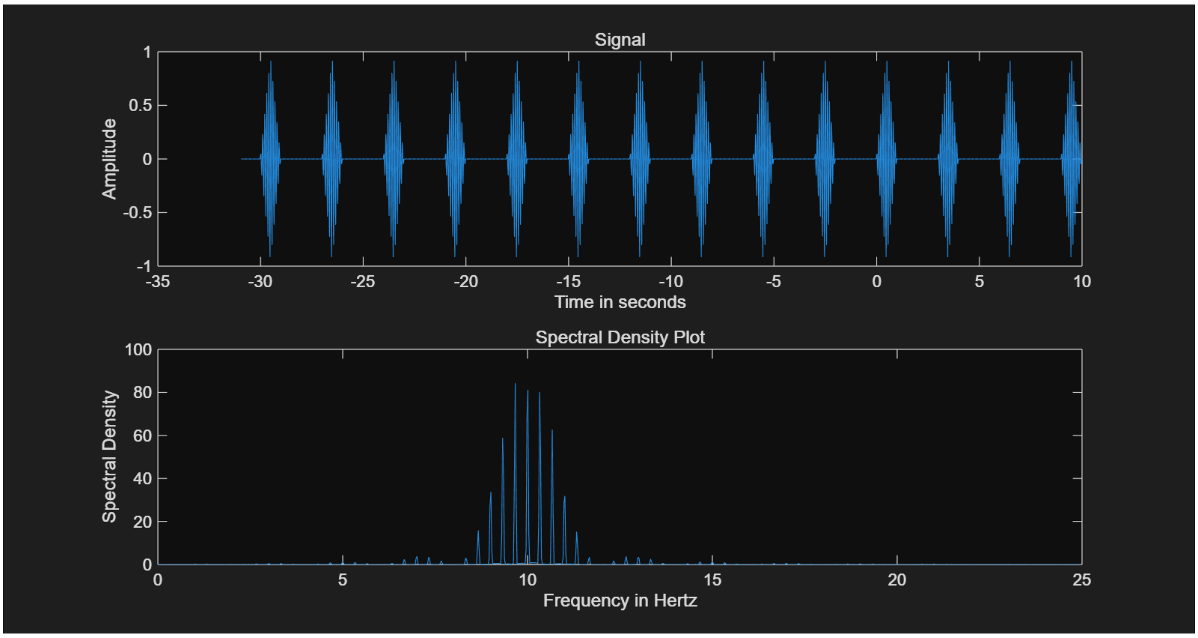

```
show_fig("342_1.fig")
```




```
show_fig("342_2.fig")
```




```
show_fig("342_3.fig")
```




```
show_fig("342_4.fig")
```

Comment:

1. The modulating frequency determines how the spectrum is moved horizontally.

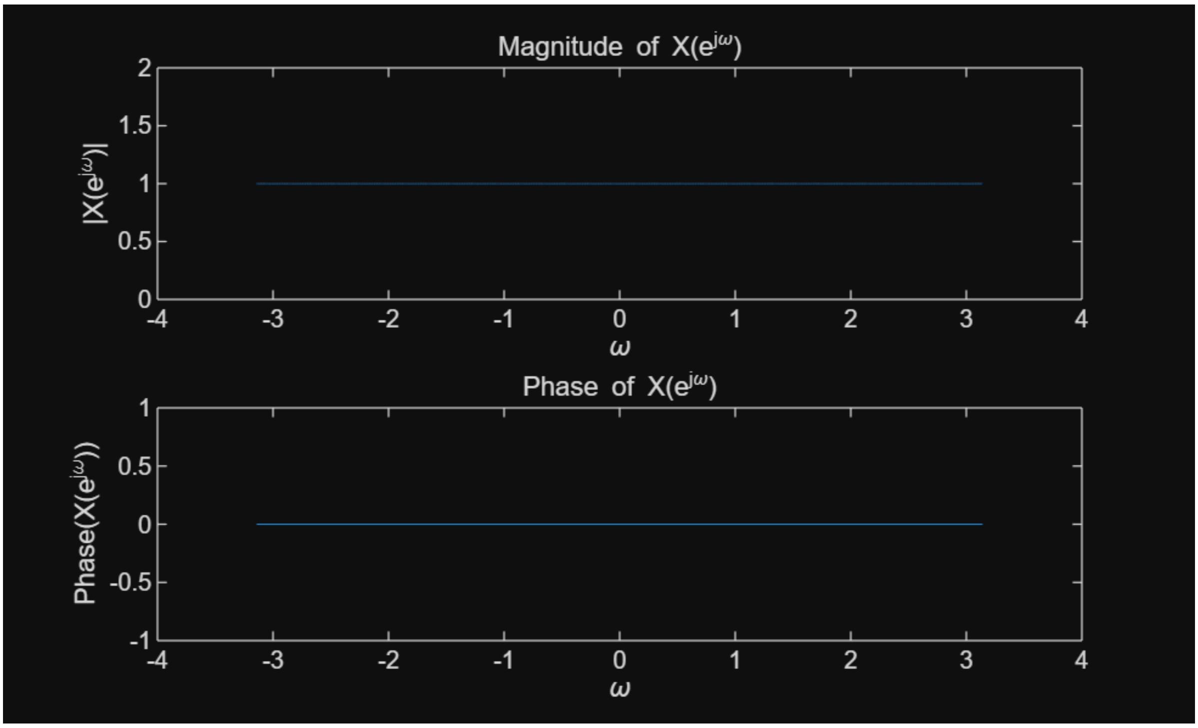
2. The original periodic signal's spectrum is discrete (Fourier Series), hence there is a comb structure.

The distance between impulses is f_0 , which is the fundamental frequency of the signal.

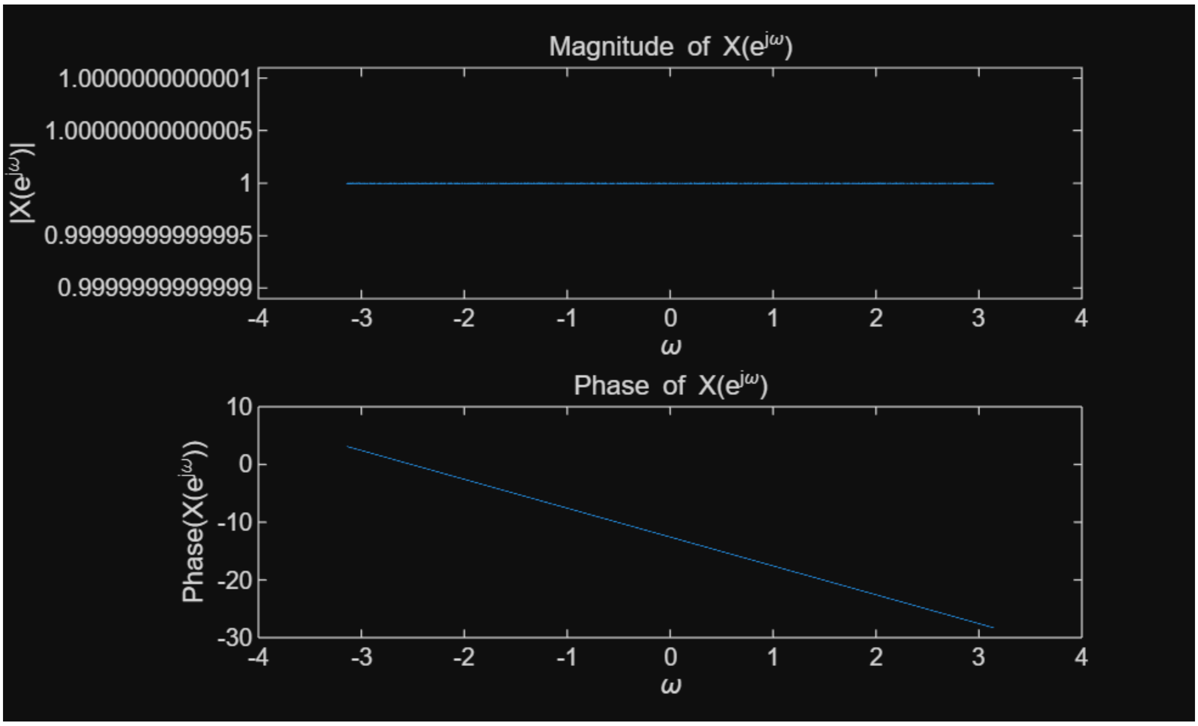
3. As the period increases, the spectrum becomes more compact, and converges to a continuous spectrum.

3.5 DT Frequency Analysis

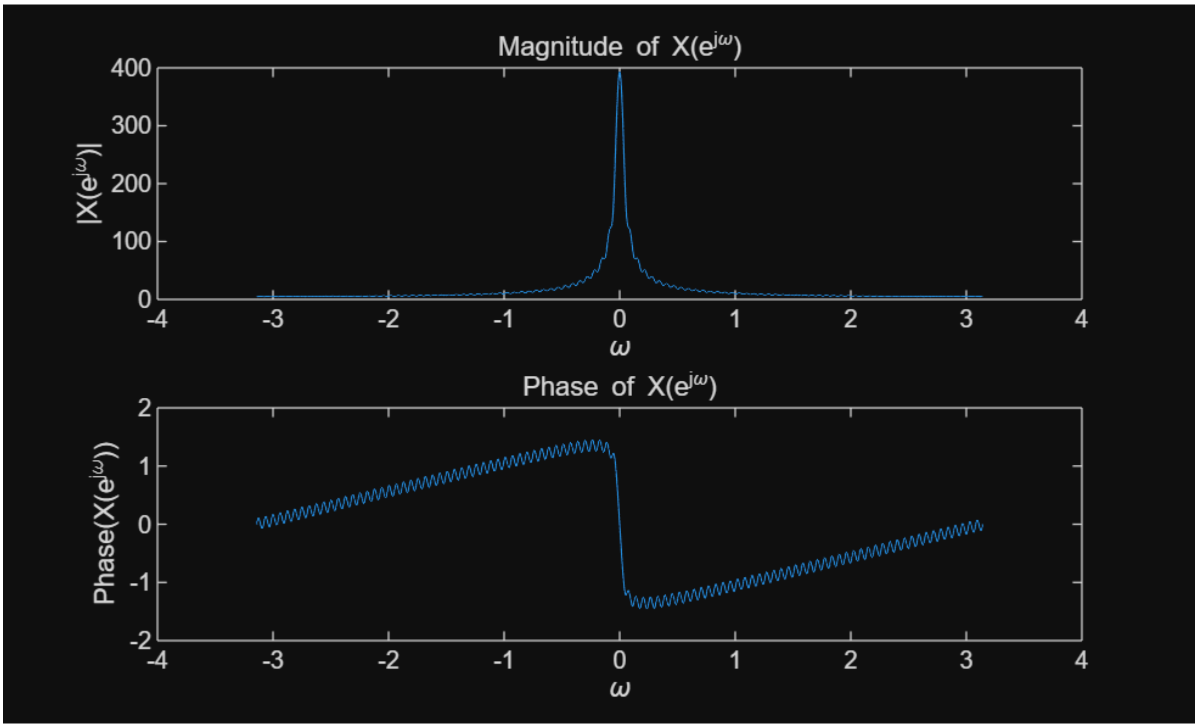
```
clear,figure

dw = 1/1000;
omega = -pi:dw:pi;

x1 = 1; n0_1 = 0;
X1=DTFT(x1,n0_1,dw);

x2 = 1; n0_2 = 5;
X2=DTFT(x2,n0_2,dw);

x3 = logspace(1, 0.5^99, 100);
n0_3 = 0;
X3=DTFT(x3,n0_3,dw);

plot_DTFT(omega,X1)
```




```
plot_DTFT(omega,X2)
```




```
plot_DTFT(omega,X3)
```




```
clear
```

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