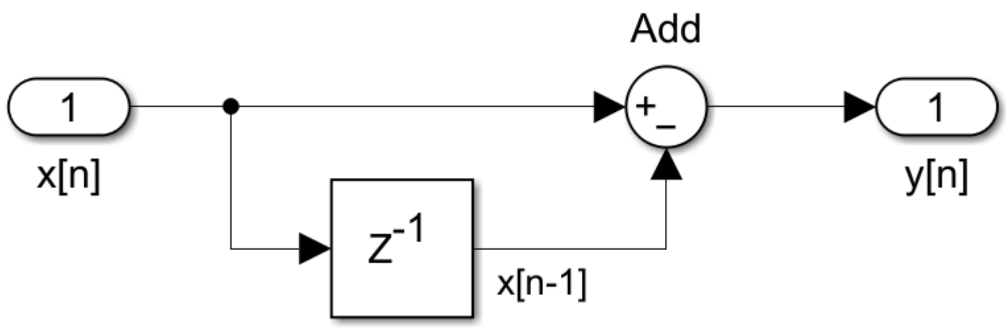
2.2.1

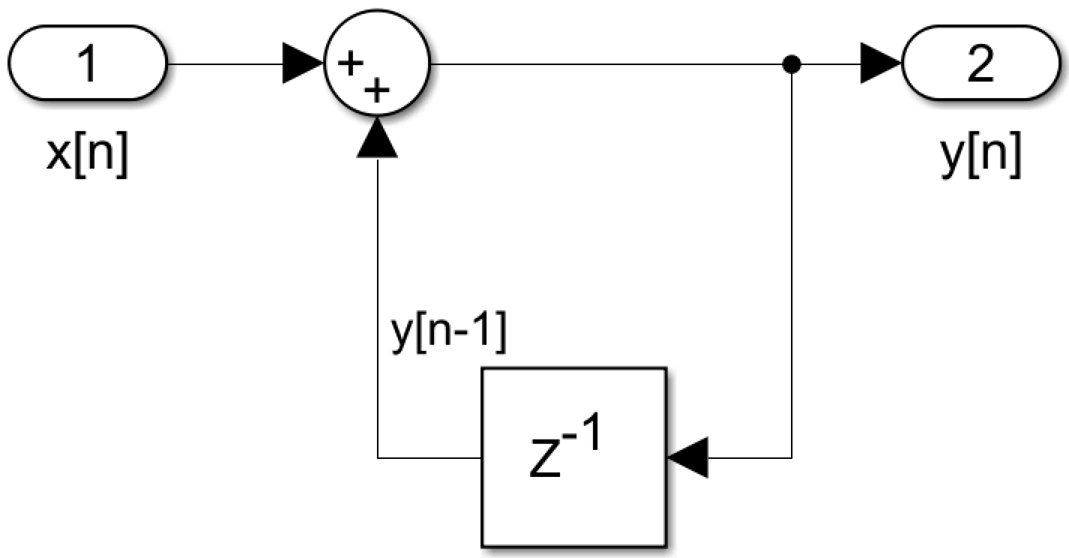
i. ii.

$y[n] = x[n] - x[n-1]$

$y[n] = y[n-1] + x[n]$

iii.

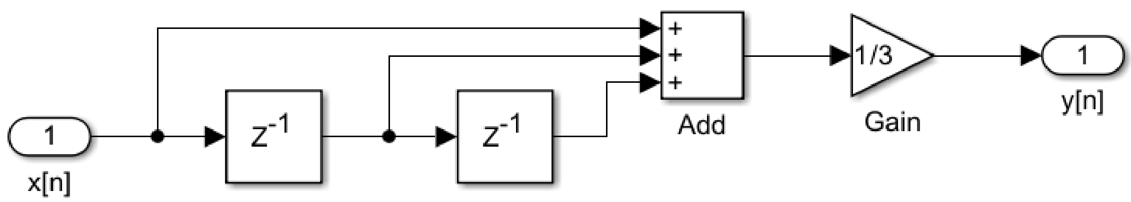




2.2.2

(2.3)

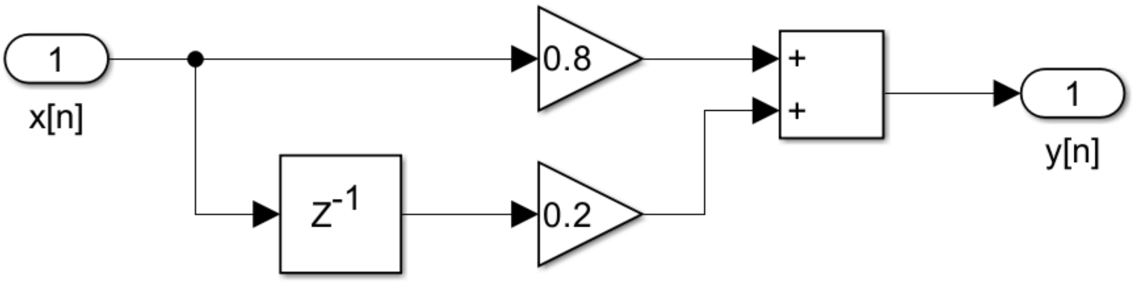
$y[n] = \frac{1}{3}(x[n] + x[n-1] + x[n-2])$



$h[n] = \frac{1}{3}(\delta[n] + \delta[n-1] + \delta[n-2])$

(2.4)

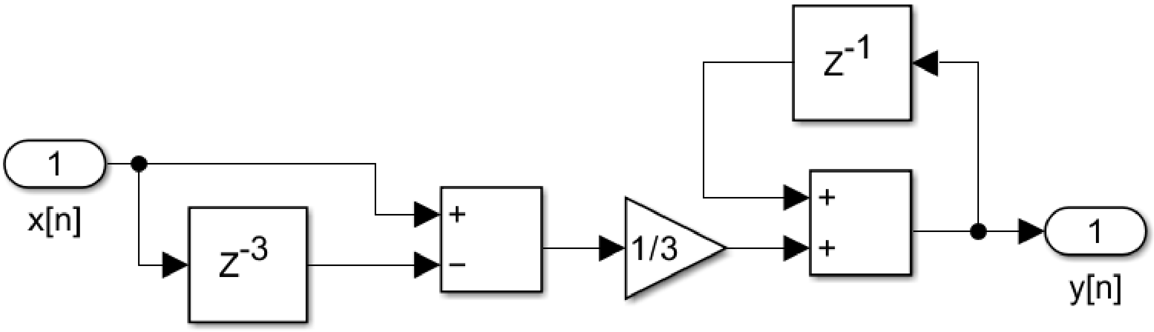
$y[n] = 0.8y[n-1] + 0.2x[n]$



$h[n] = 0.8h[n-1] + 0.2\delta[n]$

(2.5)

$y[n] = y[n-1] + \frac{1}{3}(x[n] - x[n-3])$



$h[n] = h[n-1] + \frac{1}{3}(\delta[n] - \delta[n-3])$

Note that this formulation is equivalent with (2.3), because:

When $y[n-1] = \frac{1}{3}(x[n-1] + x[n-2] + x[n-3])$ (from 2.3):

$$y[n] = \frac{1}{3}(x[n-1] + x[n-2] + x[n-3]) + \frac{1}{3}(x[n] - x[n-3]) = \frac{1}{3}(x[n] + x[n-1] + x[n-2])$$

,

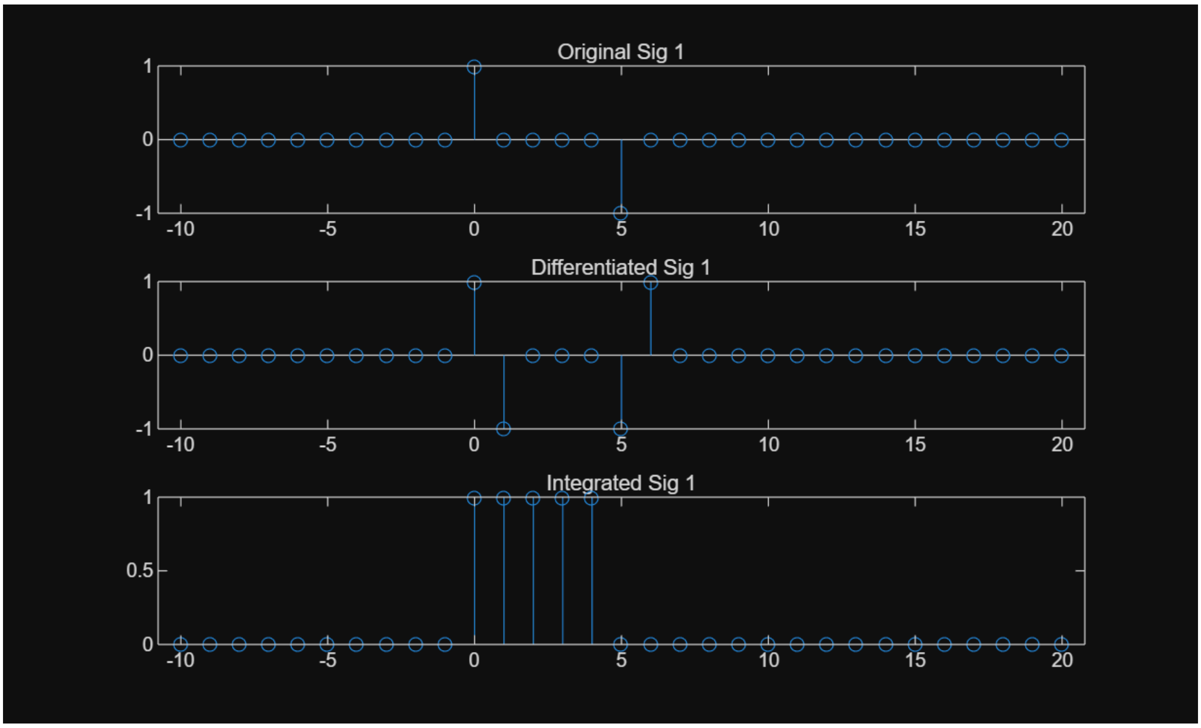
which is the same with (2.3).

Therefore, methods (2.3) and (2.5) are both known as moving averages because they computes the average value of the three most recent values. Method (2.4), however, seems to have a lagging characteristic, and does not compute average values.

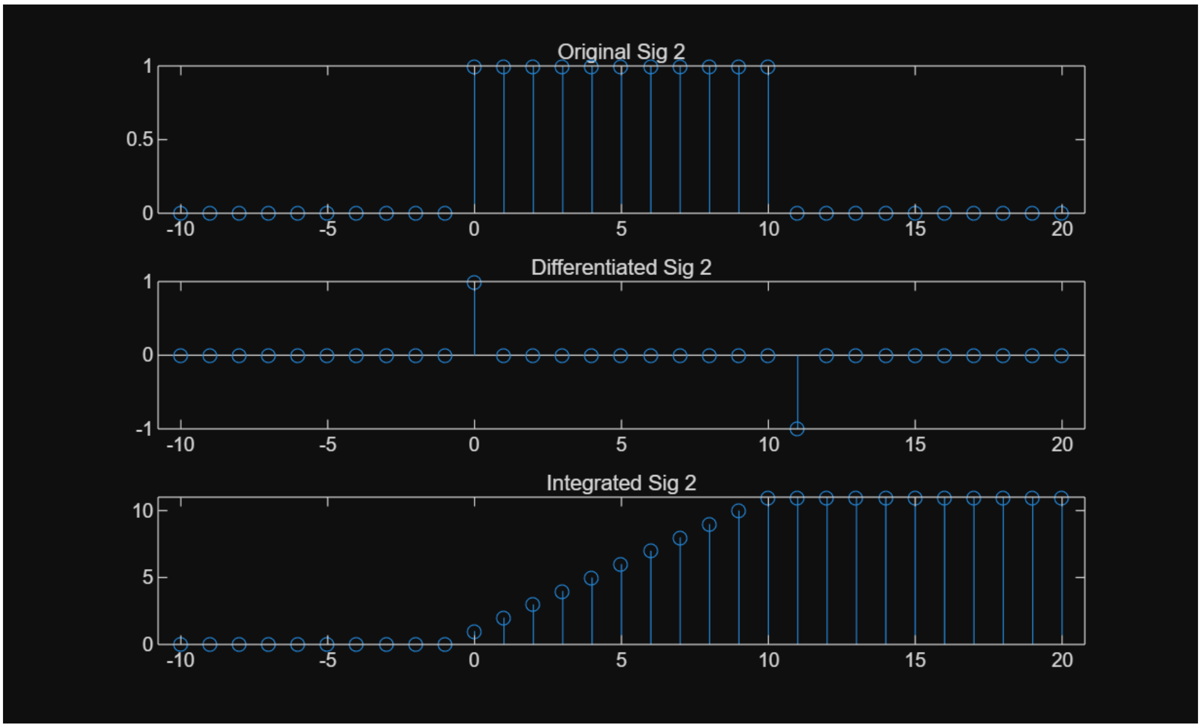
2.3

```
clear,figure
n=-10:20;
x1 = zeros(1,31);
x1(n==0) = 1;
x1(n==5) = -1;

subplot(311)
stem(n,x1), title('Original Sig 1')

subplot(312)
y1 = differentiate(x1);
stem(n,y1), title('Differentiated Sig 1')
```

```
subplot(313)
z1 = integrate(x1);
stem(n,z1), title('Integrated Sig 1')
```




```
figure
subplot(311)
x2 = (n>=0)-(n-11>=0);
stem(n, x2), title('Original Sig 2')
y2 = differentiate(x2);
z2 = integrate(x2);
subplot(312)
stem(n,y2), title('Differentiated Sig 2')
subplot(313)
stem(n,z2), title('Integrated Sig 2')
```



Differentiator is stable, because:

Assume $|y[n]| < B_y < \infty \quad \forall n$

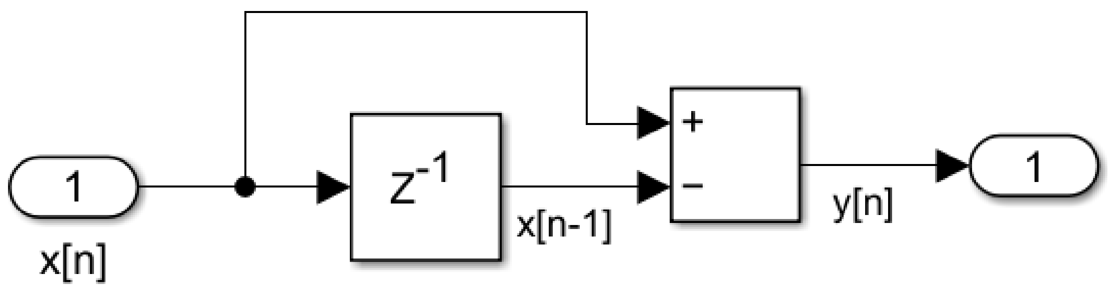
$y[n] - y[n-1] \leq |y[n]| + |y[n-1]| \leq 2B_y < \infty$

Integrator is unstable, because:

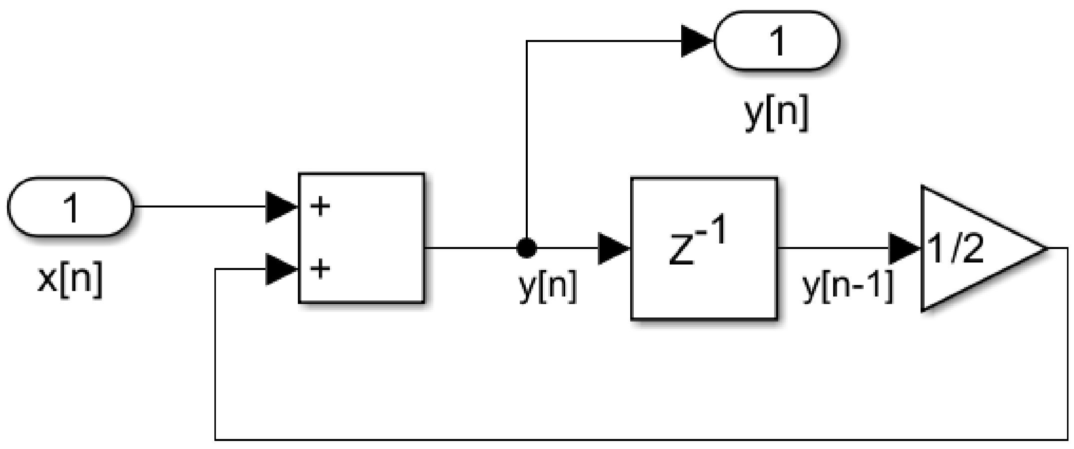
$\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=0}^{\infty} 1 = \infty$

2.4

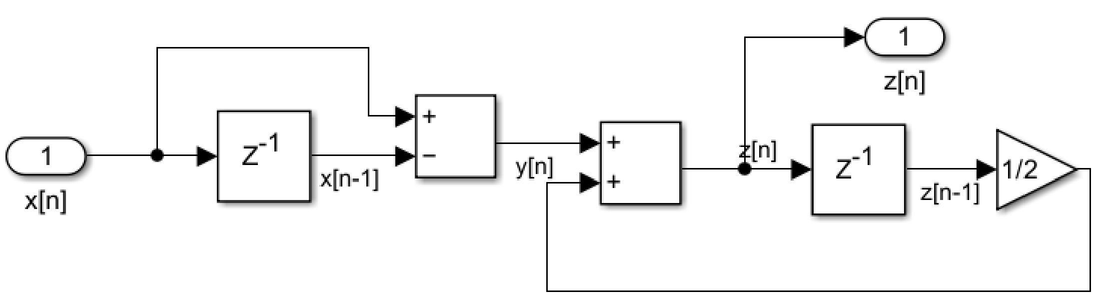
S1



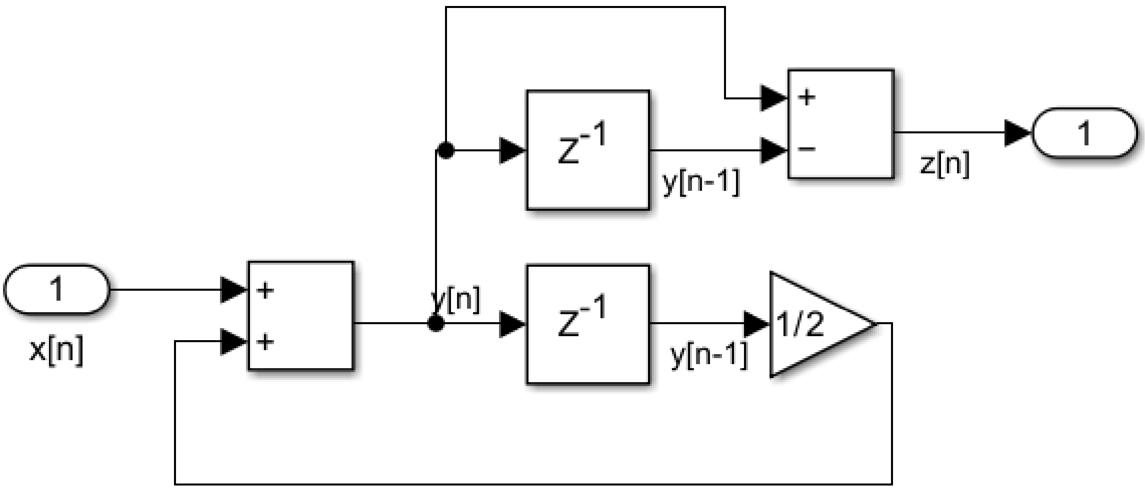
S2



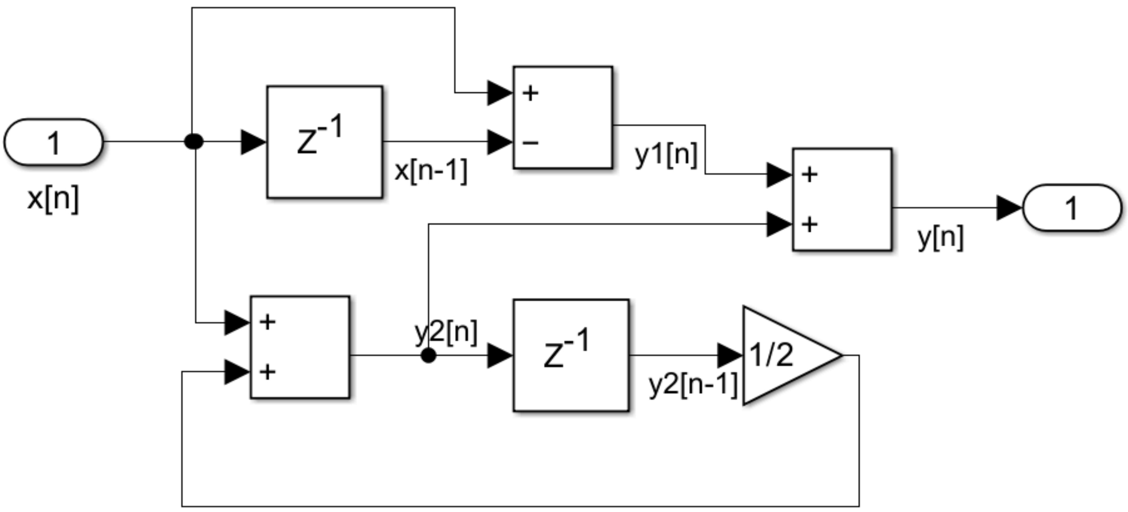
S3



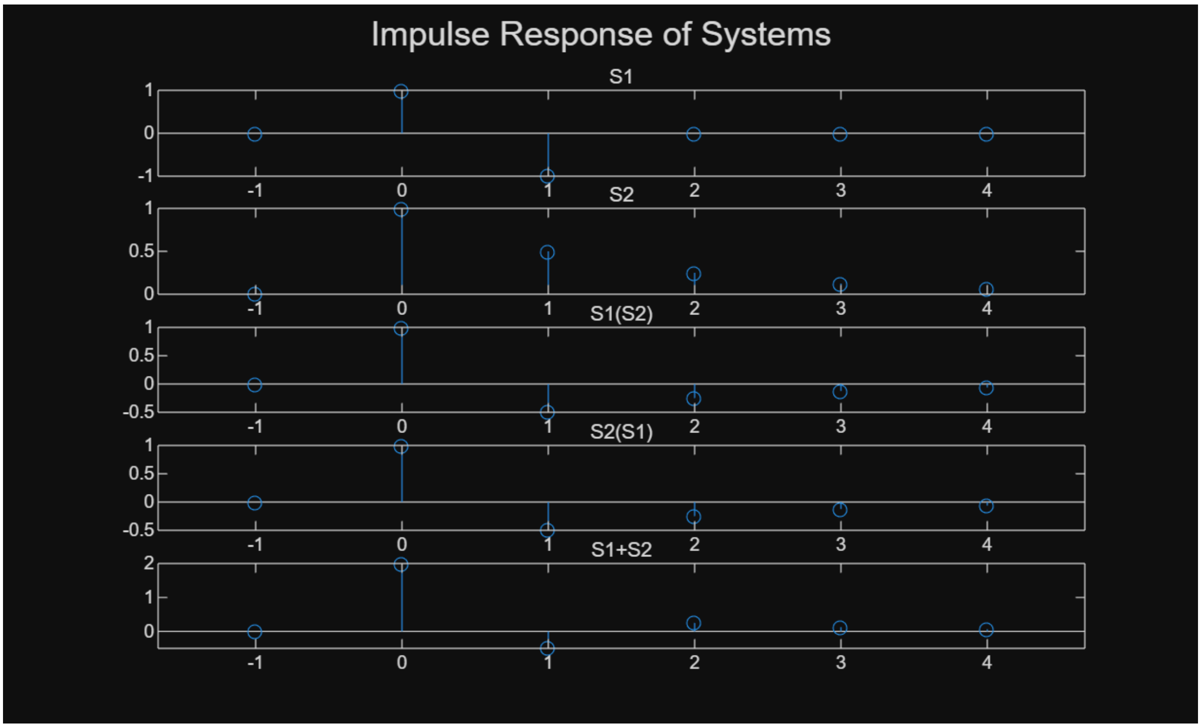
S4



S5



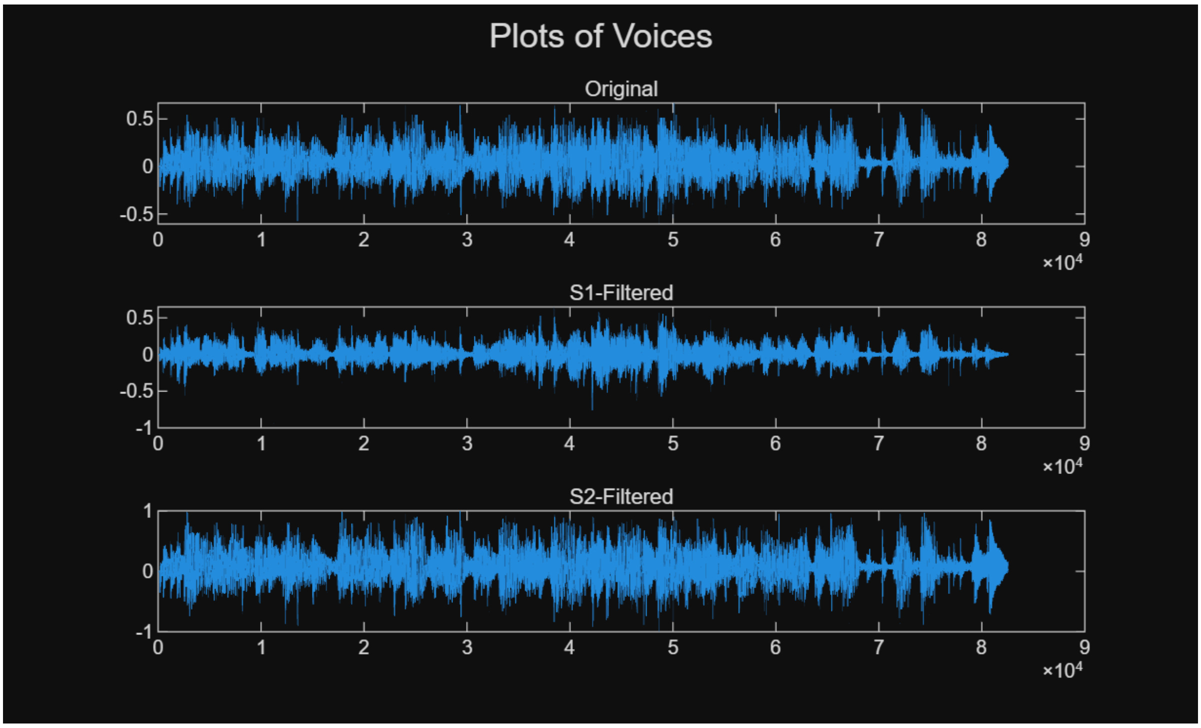

```
clear, figure

[n, delta]=impulse(-1,6);
% stem(n, delta)

y1 = S1(delta);
subplot(511)
sgtitle('Impulse Response of Systems')
stem(n,y1), title('S1')
y2 = S2(delta);
subplot(512), stem(n,y2), title('S2')
subplot(513), stem(n, S1(S2(delta))), title('S1(S2)')
subplot(514), stem(n, S2(S1(delta))), title('S2(S1)')
subplot(515), stem(n, S1(delta)+S2(delta)), title('S1+S2')
```



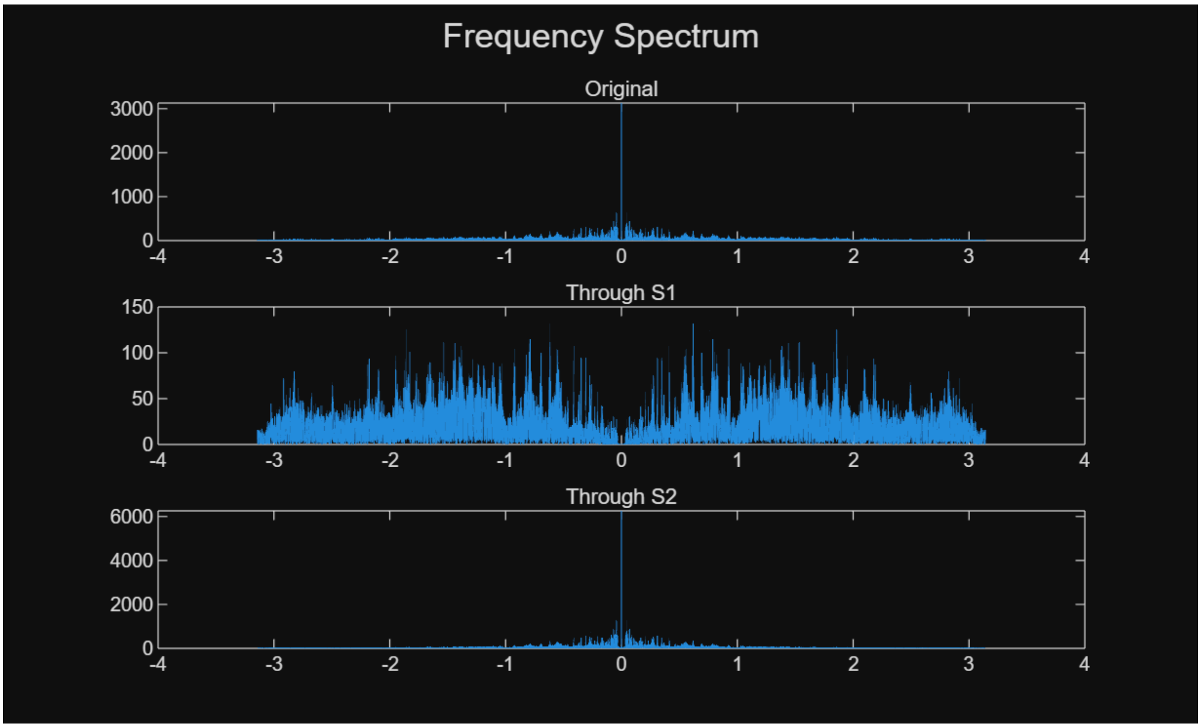
The plots show that commutivity, additivity is conserved for these two systems.

2.5

```
clear, figure
[audio_data, fs] = audioread('guidance/music.au');
% audio_data = audio_data(1:20000);

sound1 = S1(audio_data);
sound2 = S2(audio_data);

% sound(audio_data, fs);
% pause(3)
% sound(sound1, fs)
% pause(3)
% sound(sound2, fs)
subplot(311)
sgtitle('Plots of Voices')
plot(audio_data), title('Original')
subplot(312), plot(sound1), title('S1-Filtered')
subplot(313), plot(sound2), title('S2-Filtered')
```

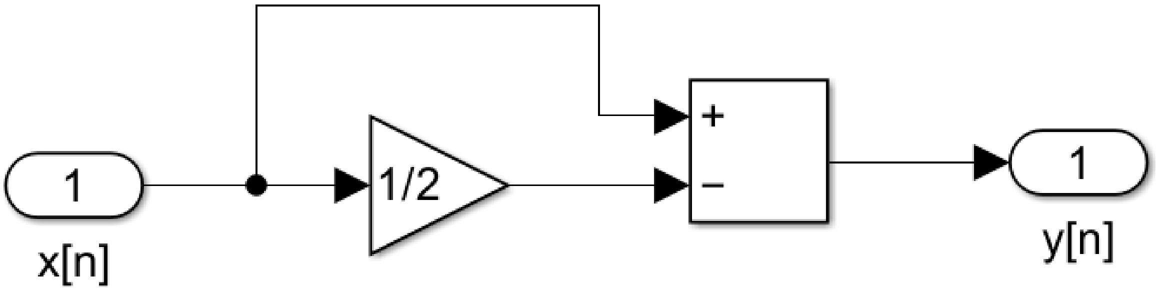



```
figure
omega = linspace(-pi,pi,length(audio_data));
freq = abs(fftshift(fft(audio_data)));
freq_1 = abs(fftshift(fft(sound1)));
freq_2 = abs(fftshift(fft(sound2)));
sgtitle('Frequency Spectrum')
subplot(311),plot(omega, freq), title('Original')
subplot(312),plot(omega, freq_1), title('Through S1')
subplot(313),plot(omega, freq_2), title('Through S2')
```

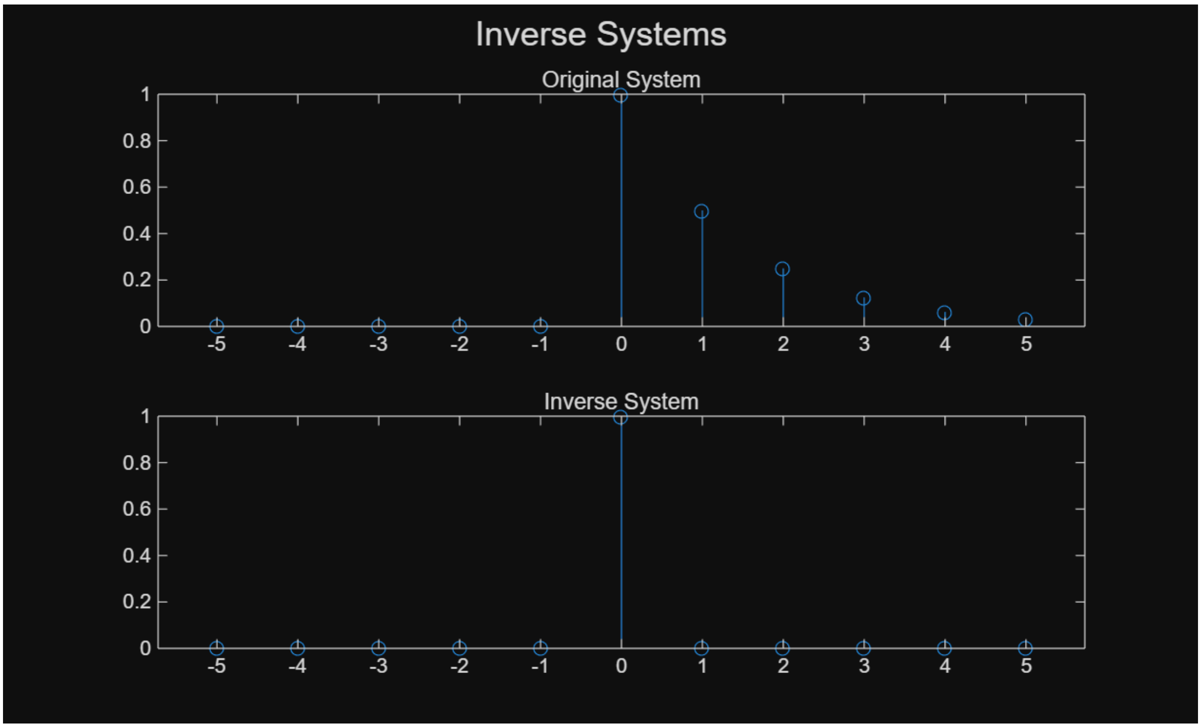



```
clear, figure
```

$y\,[n] = x\,[n] - \frac{1}{2}[n-1]$

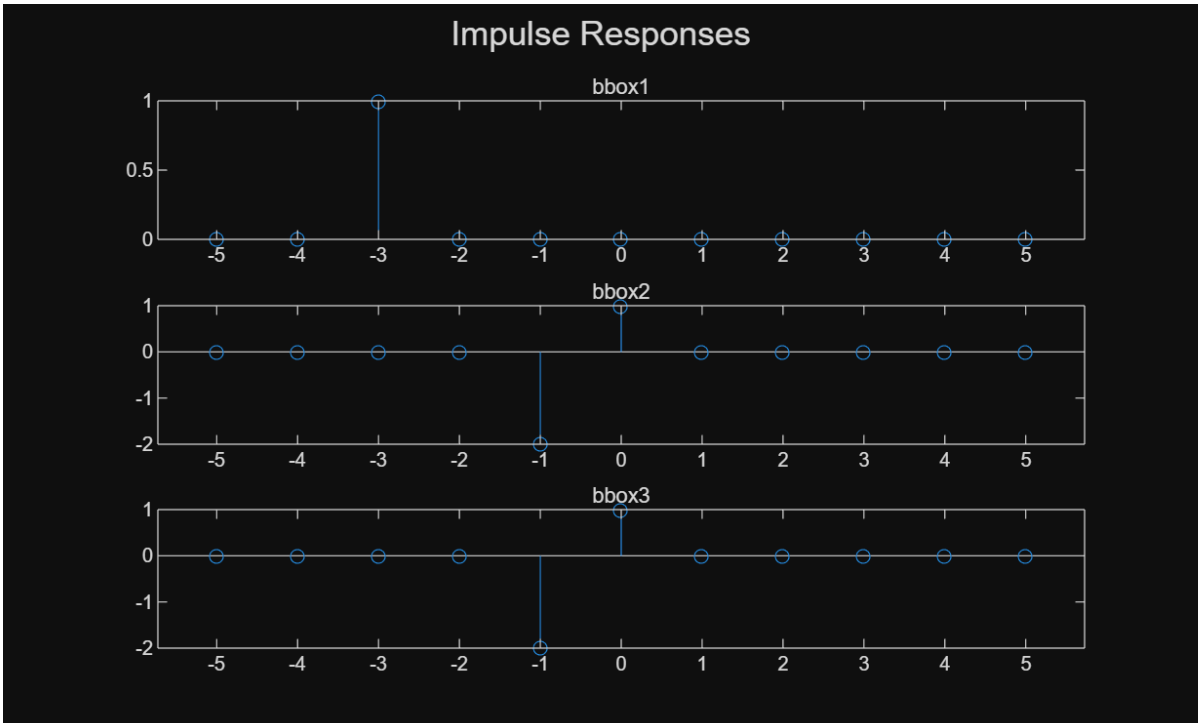



```
[n, delta]=impulse(-5,11);
y=S2(delta);
z=S3(y);
subplot(211), sgtitle('Inverse Systems')
stem(n,y), title('Original System')
subplot(212), stem(n,z), title('Inverse System')
```

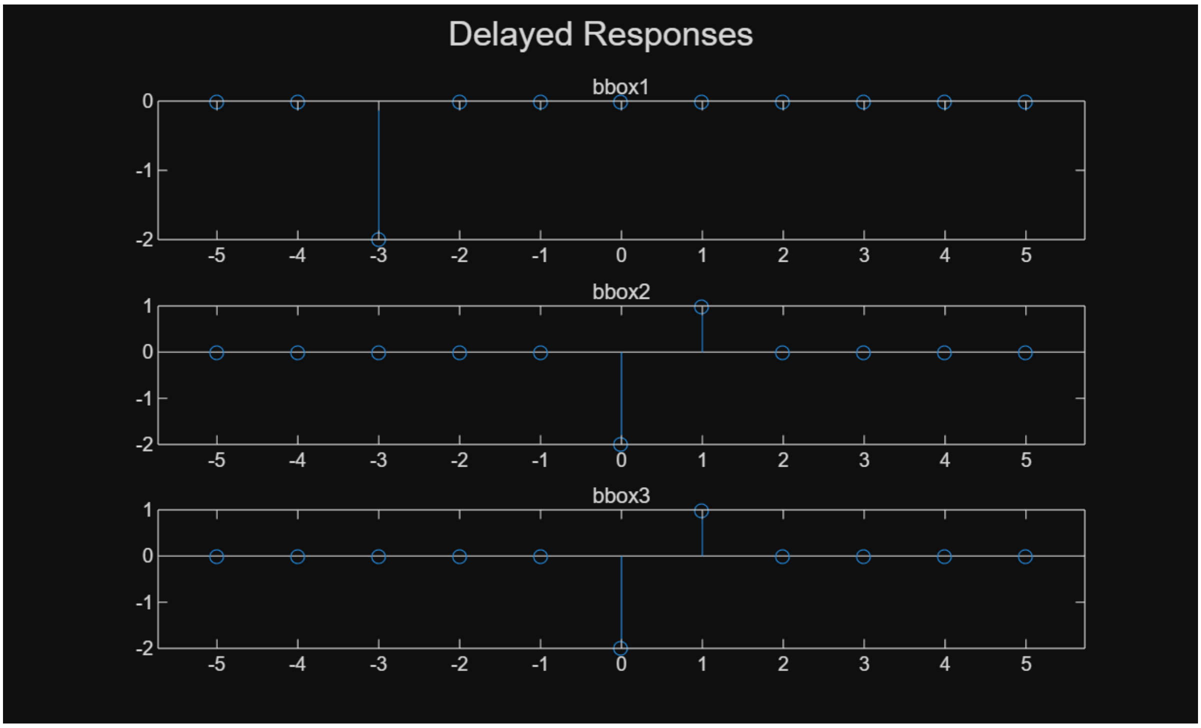


2.7

```
clear, figure

[n, x1] = impulse(-5,11);

[y11,y12,y13]=get_outputs(x1);

draw_outputs('Impulse Responses', n,y11,y12,y13)
```

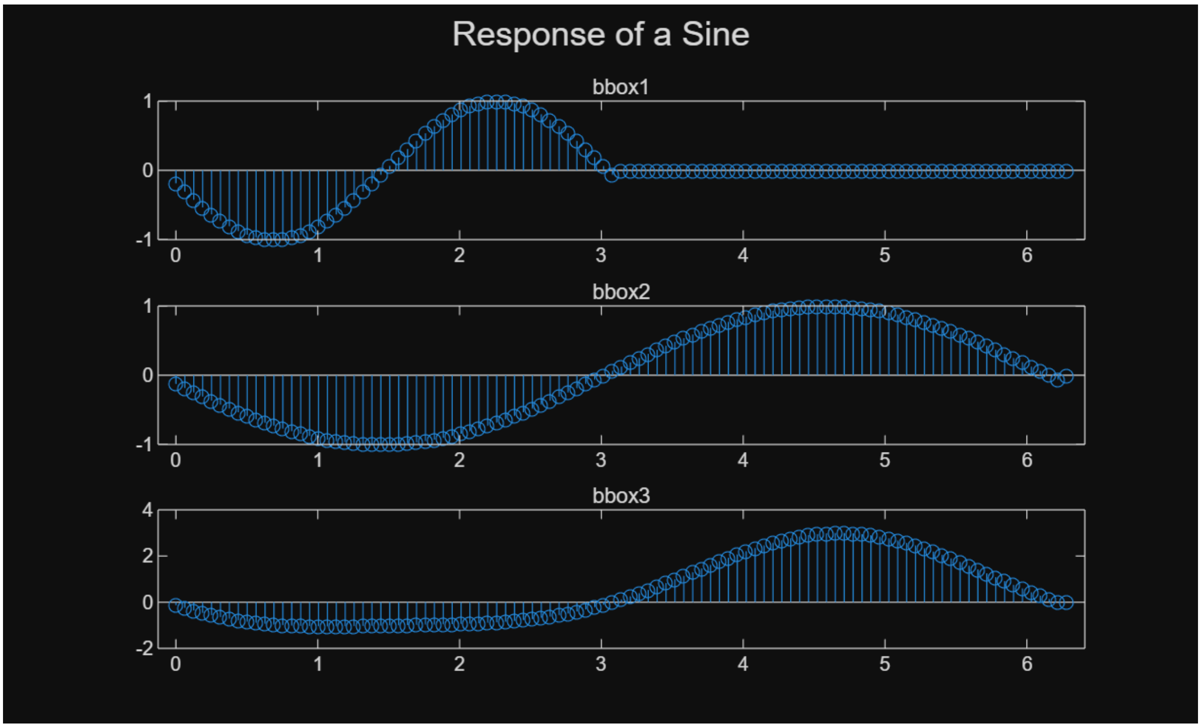



```
x1d = [0,x1(1:end-1)];
[y11d,y12d,y13d]=get_outputs(x1d);
draw_outputs('Delayed Responses',n,y11d,y12d,y13d)
```

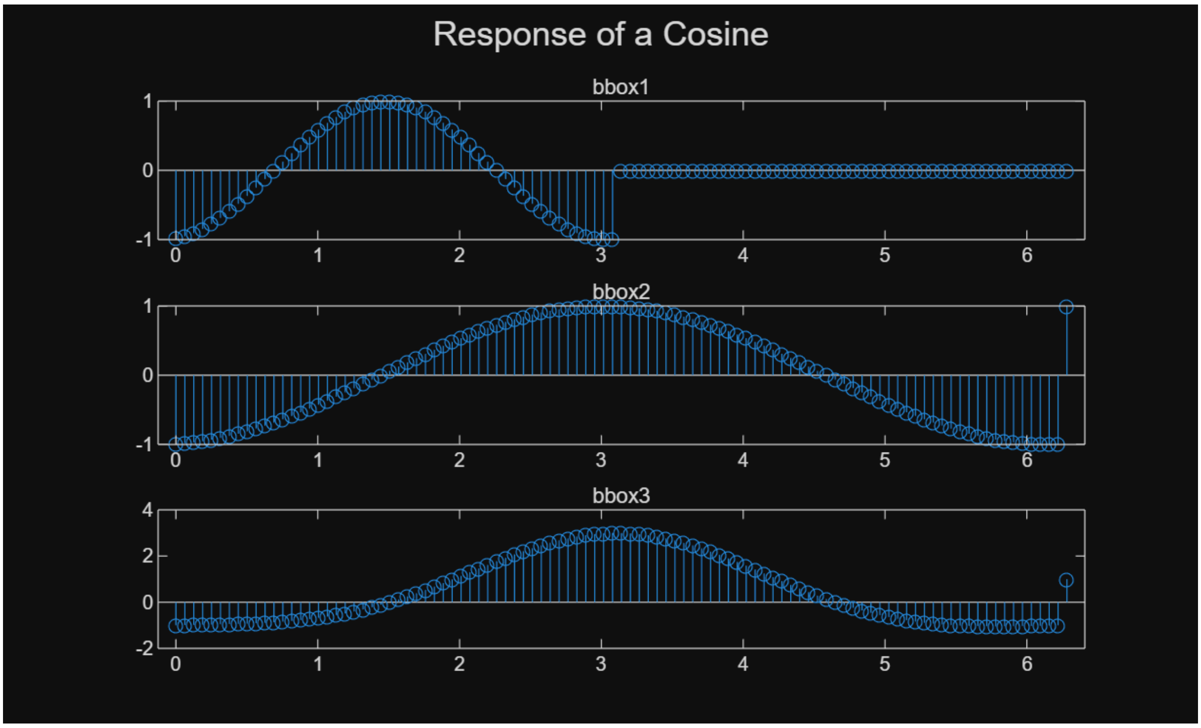


Therefore bbox1 is TI.

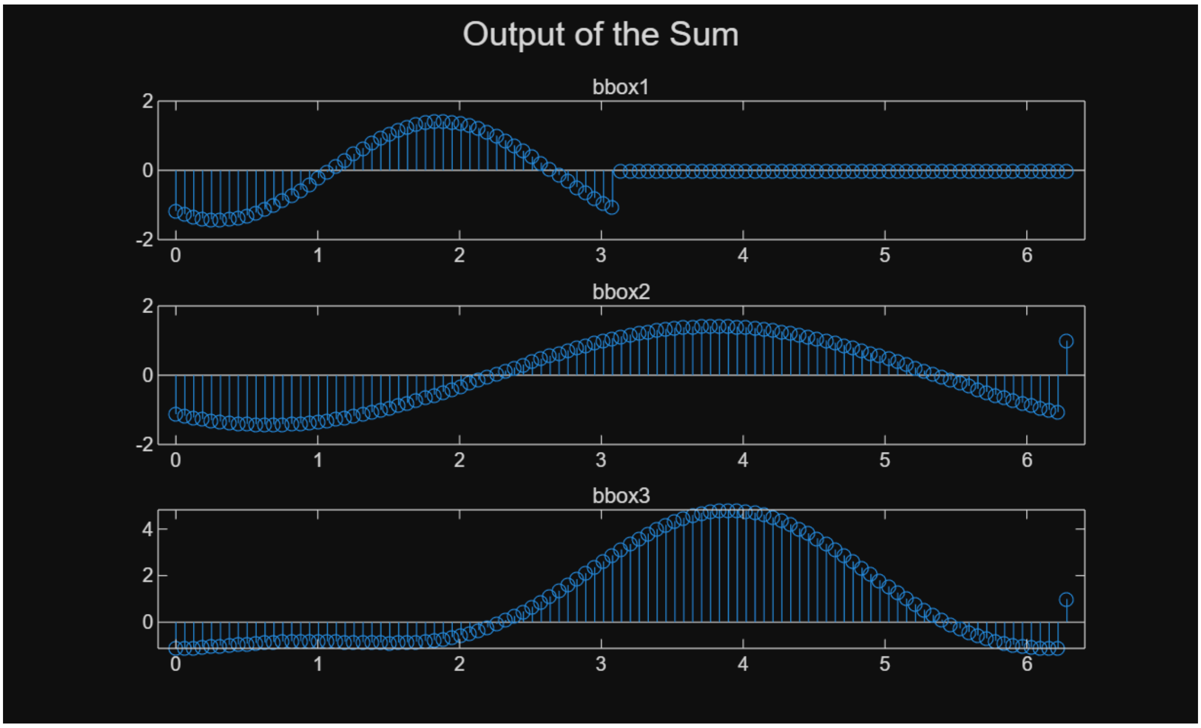
```
clear,figure

n=0:100;
n=n/100*2*pi;
x1=sin(n);x2=cos(n);
% subplot(211),stem(n,x1), title('Sine')
% subplot(212),stem(n,x2), title('Cosine')
% sgtitle('Input Functions: Sine and Cosine'), xlabel('n')
[y1,y2,y3]=get_outputs(x1);
draw_outputs('Response of a Sine', n, y1,y2,y3)
```



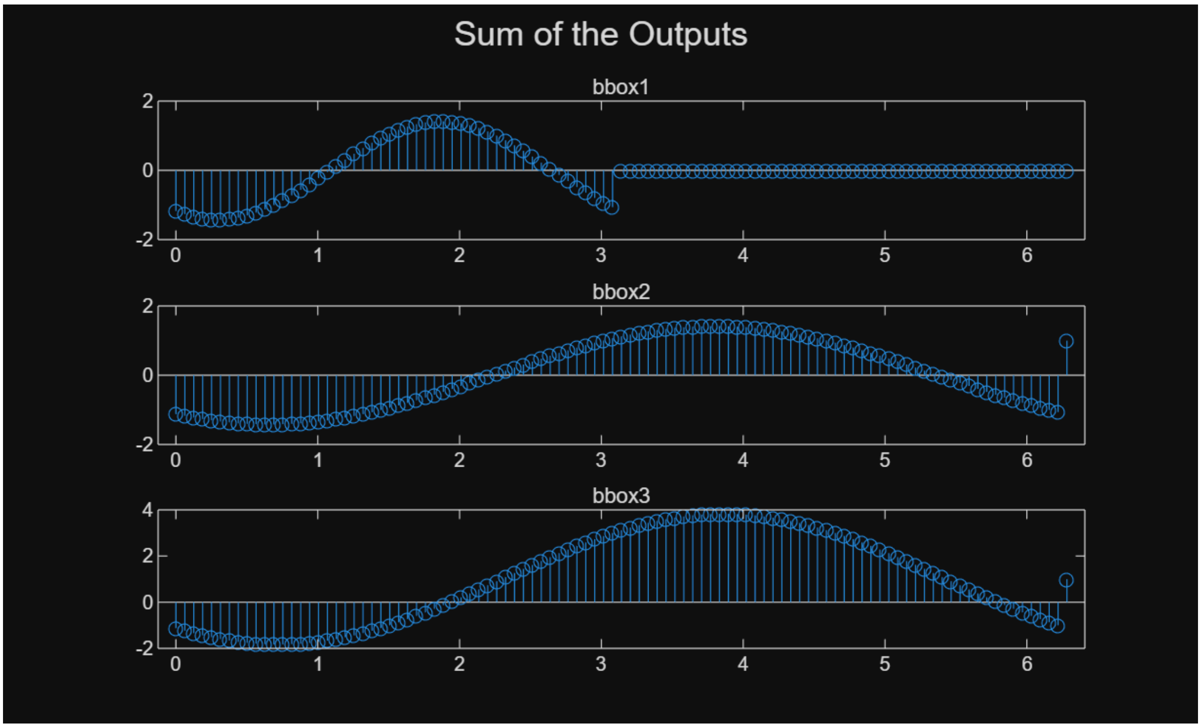

```
[z1,z2,z3]=get_outputs(x2);
draw_outputs('Response of a Cosine', n, z1,z2,z3)
```



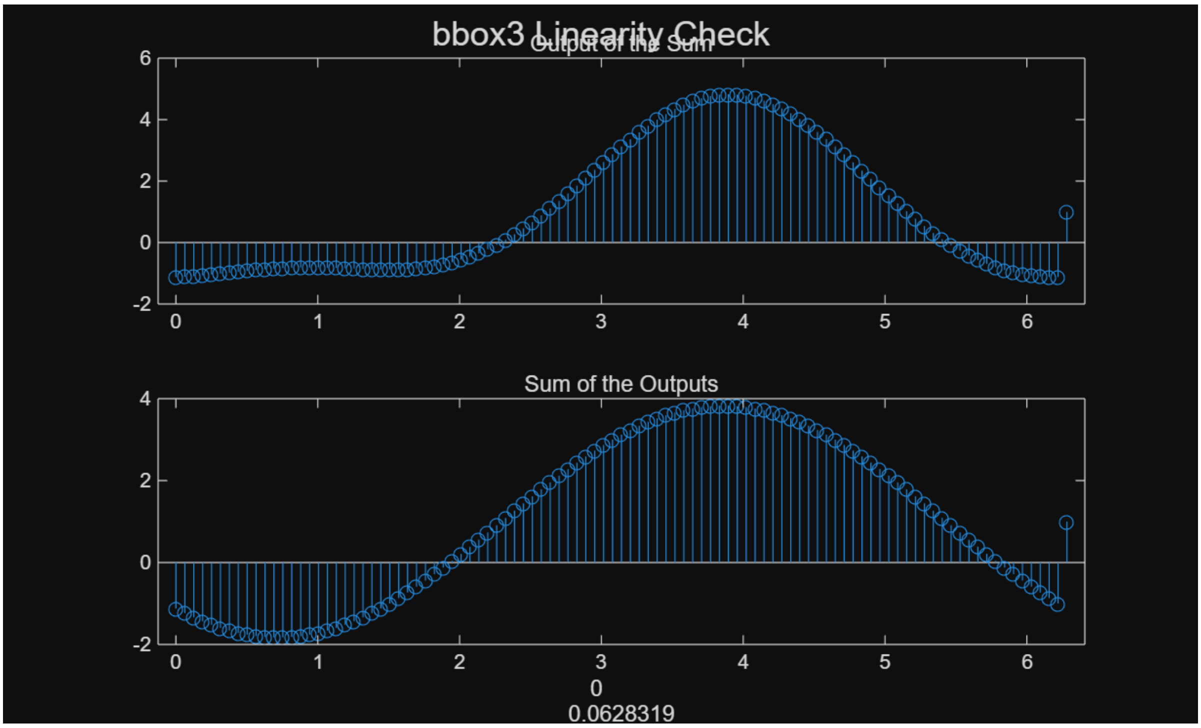
```
x3=x1+x2;
% figure,stem(n,x3),title('Sum of Inputs'),xlabel('n'),ylabel("x1+x2")
[a1,a2,a3]=get_outputs(x3);
draw_outputs("Output of the Sum", n, a1,a2,a3)
```




```
draw_outputs("Sum of the Outputs", n, y1+z1,y2+z2,y3+z3)
```




```
draw_pair(n,a3,y3+z3,"Output of the Sum", "Sum of the Outputs", "bbox3 Linearity
Check")
```



$f\left(\alpha x_1+\beta x_2\right) \neq f\left(\alpha x_1\right)+f\left(\beta x_2\right)$

Therefore, bbox3 is non-linear.

2.8

```
clear, figure
```

Recall:

(2.4)

$y\left[n\right]=0.8y\left[n-1\right]+0.2x\left[n\right]$

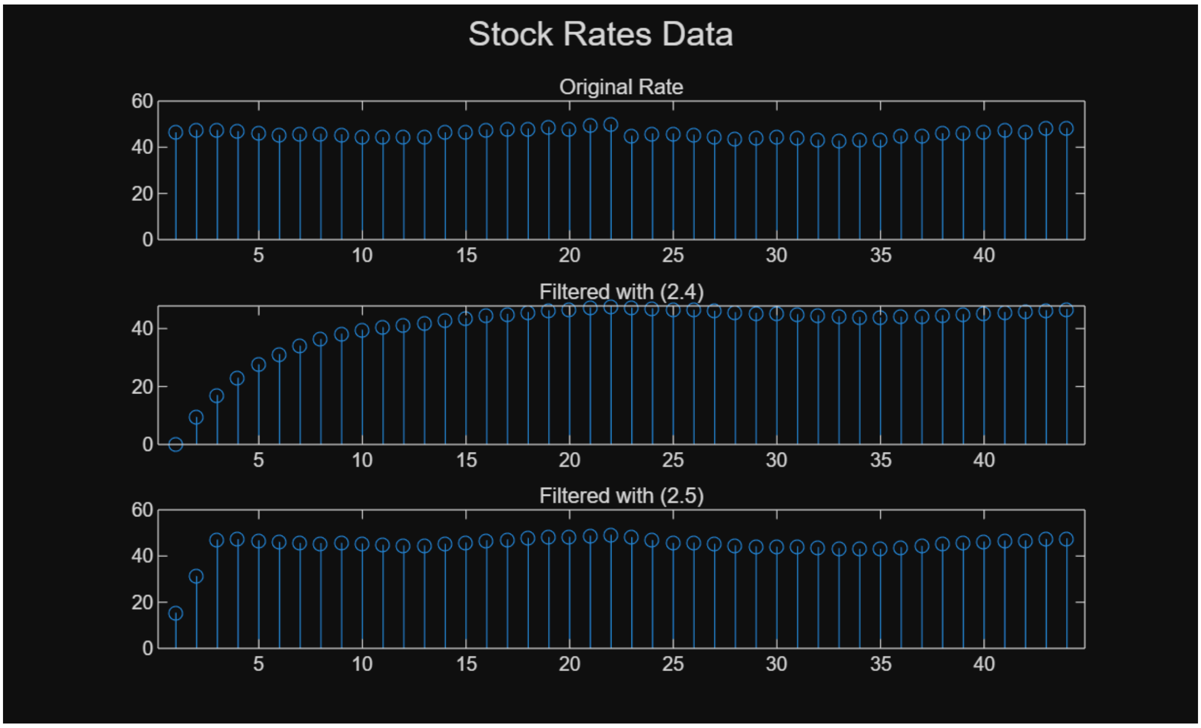
$h\left[n\right]=0.8h\left[n-1\right]+0.2\delta\left[n\right]$

(2.5)

$y\left[n\right]=y\left[n-1\right]+\frac{1}{3}(x\left[n\right]-x\left[n-3\right])$

$h\left[n\right]=h\left[n-1\right]+\frac{1}{3}(\delta\left[n\right]-\delta\left[n-3\right])$

```
rates = load('guidance/stockrates.mat').rate;

rates_1 = filtering24(rates);
rates_2 = filtering25(rates);

subplot(311)
sgtitle('Stock Rates Data')
stem(rates), title('Original Rate')
subplot(312)
stem(rates_1), title('Filtered with (2.4)')
subplot(313)
stem(rates_2), title('Filtered with (2.5)')
```




```
figure, subplot(311)
stem(filtering25_improved(rates))
```