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Critical bandwidth and consonance in relation to cochlear frequency-position coordinates

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A recent paper (Greenwood, 1990) has reviewed some of the data in the literature on the frequency-position coordinates of the cochlear partition in a number of species and the degree to which they are fitted by empirical functions developed in 1961 (Greenwood, 1961b, 1974b). Continued confirmation by physiological data makes this frequency-position function more independent of non-physiological data and provides a more secure means of testing possible relations of psychoacoustic data to cochlear coordinates. The present paper reviews various sets of critical band, or similar, data in humans and other species and finds that a considerable body of bandwidth estimates correspond to equal distances along the cochlear partition (on this assumption), conforming also to an exponential function of distance. As shown in 1961, such a function would imply that the same set of bandwidths is also a linear function of frequency. Some of the early critical bandwidth, and also 'consonant interval', estimates in man correspond to equal distances on the cochlear partition to a degree not generally recognized. Thus above about 300 to 500 Hz most of the critical band data (of Zwicker and Gässler collated by Zwicker et al., 1957), correspond quite well to equal distances on the Békésy-Skarstein cochlear map fitted by the frequency-position function, as opposed to the values published in the critical band table or curve (which do not do so above 3 kHz). Consonant interval data tend to correspond closely to equal distances, from below 100 Hz to about 3 kHz. Certain post-1961 'critical band' (ERB) estimates collated by Moore and Glasberg (1983) and extended by Moore et al. (1990) and Shailer et al. (1990) also correspond quite closely to constant distances calculated by the 1961 function. So too do some, but not all, of the frequency intervals shown by Plomp (1964) and Plomp and Mimpen (1968) to be required to resolve the components of a harmonic complex. Some critical bandwidth data from animal studies may also correspond approximately to equal distances. This survey of old and new results, plotted on a rational distance scale, may assist in explaining what potential mix of factors operates to determine the estimated bandwidths when the values differ across experiments or in different frequency ranges. The correspondence, in the preponderance of cases, of critical bandwidth to a constant distance may facilitate an understanding of the operational definitions of critical bandwidth in different experiments and of the common underlying mechanisms. The correspondence is consistent with the basic explanation of critical bandwidth and consonance offered in Greenwood (1971, 1972b, 1974b) and Greenwood et al. (1976). The critical bandwidths measured in pure-tone masking (the masker-notch interval), two-tone masking, two-tone dissonance-consonance judgments, AM and Quasi-FM detection, narrow-band masking, and two-band (notched-noise) masking reflect a common pattern of nonlinear effects, related to approximately constant 'dimensions' of the region of cochlear nonlinear input-output functions and accelerated phase accumulation (Greenwood, 1974b, 1977), and are codetermined by combination tones and an asymmetrical pattern of suppression (Greenwood and Goldberg, 1970), the latter being incorporated in a gain control mechanism (Greenwood, 1986a, b, c; Greenwood, 1988).

Critical bandwidth; Masking; Consonance; Cochlear coordinates; Nonlinearity; Combination tones; Gain control; Dominance; Suppression

Introduction

Physiologically based and accurate cochlear frequency-position functions facilitate interpretation of physiological and psychoacoustic data and

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facilitate modelling of cochlear mechanisms and spatio-temporal arrays of neural input provided to CNS mechanisms. A recent paper (Greenwood, 1990) has reviewed some of the data in the literature on the frequency-position coordinates of the cochlear partition in a number of species and the degree to which they are fitted by a family of empirical functions that were developed in 1961 (Greenwood, 1961b, 1974b). This family of almost-exponential functions was developed by integrating an exponential function fitted to a subset, from about 400 to 6500 Hz, of frequency resolution-integration estimates (critical bandwidths), defined operationally, for homo sapiens. The resulting frequency-position function was found to fit cochlear observations on human cadaver ears quite well and, with changes of constants, similar observations on other mammals–elephant, cow, guinea pig, rat, and mouse (Békésy, 1960) as well as in vivo (behavioral-anatomical) observations on cats (Schuknecht, 1953). The function essentially requires only the adjustment of a single parameter to set the upper frequency limit, while the 'slope' parameter can be left constant if cochlear partition length is normalized to 1 or can be scaled if distance is to be specified in physical units. Newer mechanical and other physiological data reported since 1961 on human, cat, guinea pig, chinchilla, monkey, and possibly gerbil have also been shown to be well fitted by the same basic frequency-position function (Kringlebotn et al., 1979; Liberman, 1982; and Greenwood, 1990).

In addition, Greenwood (1990) showed that certain post-1961 'critical band' estimates in man are well described by frequency intervals corresponding to constant distances, calculated by the function from 1961. In short, these empirical bandwidths were consistent with the equal-distance hypothesis, which will be discussed below. This illustration made the point that these physiologically supported coordinates and the functions that fit them can be used as originally envisaged, to plot psychoacoustic and physiological data on rational scales and to determine the relation of such data to cochlear coordinates or, potentially more directly, to haircell or innervation density or to hypothesized cochlear excitation patterns or mechanisms (Greenwood, 1961b, 1965, 1971, 1974b; Greenwood et al., 1976).

Nontheoretical reference to the 'critical' bandwidths estimated in different experiments may sometimes be made more difficult by the historical baggage carried by that expression. Use of the conventional name tends to suggest that there is agreement as to what it means while in fact calling up the variant meanings with which it may be invested by different users. Here, possibly divergent, or perhaps not fully comparable, 'bandwidth' data from several sources are to be reviewed without initial discussion of their meanings or any presumption of 'error'. Although data will be given their historical names to designate them, this practice does not imply agreement with conventional interpretations. What critical bandwidth is, what operational estimates of it really reflect, which sets of estimates should or should not agree, which should in the end be called critical bandwidth, and whether or not correspondence to equal distances should be a defining characteristic of critical bandwidth are all questions and issues either not addressed in this paper or postponed until a postsurvey discussion when matters may be clearer. However, there will be some outlining, as we proceed, of the implications for certain data, and for the hypotheses under examination, that have followed from the discovered differences between different sets of empirical bandwidths, but these implications are not believed to impugn any of the data.

As a play or novel requests a basic indulgence from its reader – 'the willing suspension of disbelief' – this paper requests the willing, but perhaps only temporary, acceptance of the frequency-position function of Fig. 1 as a cochlear map, on the basis of the evidence presented in the paper cited above (Greenwood, 1990). In short, the present paper is premised on the working assumption that the cochlear coordinate data are approximately correct and adequately fit by the function. After brief discussion of the cochlear map, this assumption will receive only occasional repetition. However, the intent of this paper, to examine in an approximately historical sequence some of the various bodies of 'critical band' data in man and other animals to determine their empirical relations to cochlear coordinates, requires that the assumption be kept in mind. Reference curves are employed to embody it, to represent those frequency intervals corresponding to various particular constant distances that differ from case to case, and to measure experimentally obtained frequency intervals to see if they correspond to a given reference distance. Readers may be unaccustomed to regard a curve, compared to data points, as a measuring device. Curves are usually expected to fit data points and deviant data points may themselves be subject to suspicion and coercion. Hence this reminder: Reference curves that are not predicting the bandwidth data, not designed to fit to them, and not in a circular dependence upon them – because physiologically supported and simply representative of equal distances on the cochlear map – are under no obligation to fit the data bandwidths. Neither are the data under duress to conform to the curves. Relationships will fall as they may, to provide, it is hoped, a clearer view, first, of the present status of an historical hypothesis, the equal-distance hypothesis – an empirical induction untested for 30 years – and, second, of what will remain to be considered after that. As noted in a previous paper, '...psychophysical estimates or measures of frequency resolution may not conform to equal distances, without any necessary adverse bearing on the accuracy of the frequency-position function or on their own repeatability and validity as data. Given the performance task in question and the nature of the system, the equal-distance hypothesis may simply not be correct in a given case' (Greenwood, 1990). However, when and if curves do fit the data they have measured, they can certainly have practical utility as descriptive functions, will support the hypothesis as applied to those data, and may carry implications. When they do not fit, that fact may still carry useful implications as to the data's dependence on other factors, as noted above. If the hypothesis is supported, it provides no explanation by itself but may well be an essential part of one.

The Frequency-position Function in Relation to Psychoacoustic Data

The development of the frequency-position function was reviewed recently (Greenwood, 1990) but requires a few words here. The emphasis of the earlier paper was on the frequency-position function itself to determine how well it described physiological data on cochlear coordinates. The emphasis here is shifted to the question of how various psychoacoustic data – of the general sort originally used to arrive at the function – are related to cochlear coordinates. As noted above, the function was initially derived by integrating a critical band function proposed to fit my critical band estimates in 1959–60 (Greenwood, 1961a, b). These bandwidths were fitted simply as an 'experiment' to see what the consequences would be. Those consequences, besides the derivation of a frequency-position function that fitted cochlear data (which attested that this set of bandwidths corresponded to equal distances) were the discovery (a) of the proportionality of other 'critical band' data to the function's derivative (implying their correspondence also to equal distances) and (b) of a lack of close overall agreement between the 'classical' critical band curve (Zwicker et al., 1957) and equal distances on Békésy's cochlear map, as fitted by the new frequency-position function.

The function described in 1961 (Greenwood, 1961b) hypothesized that critical bandwidth, in Hz, might follow an exponential function:

$$CB = 10^{ax+b},$$

of distance, x (in any physical units or normalized distance), along the cochlear partition, and correspond also to a constant distance on the basilar membrane. The latter hypothesis had been advanced and supported influentially by Fletcher (1940, 1953), and Zwicker et al. (1957). When the exponential critical-bandwidth function was integrated to obtain a frequency-position function, position on the membrane was initially expressed in critical-band units. But to the extent that the form of the function fits the cochlear coordinate data, the length of a critical-band unit in physical units can be determined by dividing the length of the membrane (in such units) by the number of critical bands end-to-end required to subtend the audible frequency range. By coincidence (again emphasized), the number proved to be the same as the length of the membrane in mm. Thus critical bandwidth was proportional to the derivative with a constant of 1. This would not, of course, have been true if the inch had been the unit of distance or if the frequency intervals had been larger or smaller. What was important was its proportionality to the derivative, not its constant of proportionality, an emphasis to be remembered in considering what critical bandwidth may be in another species, a point to which we will later return.The frequency-position function obtained as above (see Fig. 1), is:

$$F = A(10^{ax} - k), \qquad (1)$$

where F is in Hz and x is in mm and where suitable constants (for man) are: $A = 165$ and $a = 0.06$, the latter an empirical constant arising in the critical band function but found also to agree closely with the logarithmic slope of Békésy's volume compliance gradient for the human cochlear partition; and k, an integration constant left here at the original value 1, but that may sometimes be better replaced by a number from about 0.8 to 0.9 to set a lower frequency limit dictated by convention or by the best fit to data. Although the value $k = 0.88$ would yield the conventional lower frequency limit of 20 Hz for man, I will continue to use 1.0 for man and most of the other species in this paper, excepting the cat since Liberman (1982) has found that a k of 0.8 best adjusts this function to his low frequency data points in the cat.

As shown in Fig. 1, the frequency-position function quite closely agreed with Békésy's cochlear coordinates, and now also with Skarstein's (Kringlebotn et al. (1979). This major outcome was important in its own right, since such agreement, if Békésy's data applied also to living human cochleas, gave the frequency-position function the independent status earlier referred to and an immunity from the truth or falsity of the hypothesis that any particular set of critical bandwidth estimates corresponded to equal distances. With an independent status it could be used as a rational scale, with the added convenience of a mathematical expression. In addition, the agreement of the frequency-position function with Békésy's cochlear data was important for the reason that it supported the two ideas that the particular critical bandwidth estimates used might obey an exponential function and correspond to a constant distance. Since their close agreement with a nearly constant distance on Békésy's cochlear map was not disputable, the significance of that agreement was contingent only on whether or not the mechanical data accurately characterized living cochleas as well. The possibility the map might not do so was not commonly seen as a problem at that time. *

Critical Bandwidth Distances in Man

In 1961 the proposed exponential critical band function had not fit the low frequency values, below 500 Hz, of critical bandwidth as given by Zwicker et al. (1957) in their Fig. 12 and the accompanying table of values (the function that has come to be called the 'classical' critical band curve). However, the exponential function quite well fitted the values suggested by Zwicker et al. from about 500 to 3000 Hz or slightly more (comprising most of the region occupied by the set of Greenwood's data on which the exponential critical band function was based). Fig. 2 presents the

* Suggesting further in the 1960s that perhaps the applicability of Békésy's frequency-place data to living cochleas was not a problem was the demonstration in 1961 that the same function and a scalable (i.e. normalizable) slope provided a fairly good fit to Békésy's data from dead cochleas of other species and to data from the living cat. At the present time, the probability that the function and the expanded Békésy-Skarstein cochlear map may apply also to living cochleas, at least as regards slope, has been arguably increased by the extended observations that the same normalized slope has been shown to be closely consistent with cochlear coordinates determined more recently in both dead and several living preparations, encompassing a total of about 9 or 10 other species (Greenwood, 1990). As explained in a previous paper (Greenwood, 1990), it may be that the Békésy-Skarstein positions of maximum displacement-envelope amplitude shown in Fig. 1 have been displaced somewhat towards the base of the cochlea by the effects of death, as occurred – with preservation of relative position and hence slope - in the experiments of Kohllöffel (1972a,b,c), on successive days after death. However, within the first day after death this effect appears to be small. If the data points of Békésy-Skarstein were lowered on the ordinate by about 1 mm, they would be well, and possibly slightly better, fitted by a function of the same slope, a, and an A of 190, which yields an upper frequency limit of about 24 kHz. Such a function has been tested, and, since it maintains the same slope, it had no adverse effect on the comparisons of this paper, which have been made with the original function.

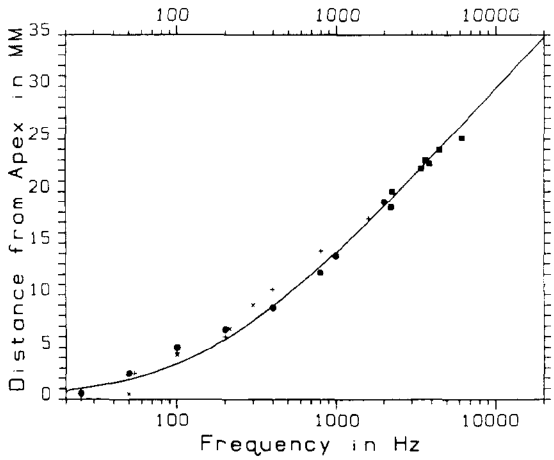


Fig. 1. Distance from stapes to the place of maximum displacement, as a function of frequency. Solid circles: Békésy (1960, 1942 series). Additional points obtained from two later series of Békésy's observations: Crosses: Békésy (1960, 1943 series); laterally displaced cross at 54 Hz actually falls at 50 Hz. X's: Békésy (1960, 1947 series); laterally displaced X at 215 Hz actually falls at 200 Hz. Solid squares: Skarstein (Kringlebotn et al., 1979); data points from 9.9 to 16.5 mm from the oval window. Curve: Greenwood (1961b, Function (1) - present paper); $A = 165$ (for Hz), $a = 0.06$, $k = 1$.

exponential critical band function and the critical band curve of Zwicker et al. (1957). In 1961 I did not compare the data bandwidths from the three kinds of experiments plotted by Zwicker et al. (in their Fig. 11) with the calculations from my own exponential critical band function, nor did I examine the data themselves in detail, since I assumed the 'classical' critical band values in their curve (their Fig. 12) and table were an average of those data. (All but the bottom set of the five data-curves in their Fig. 11 had been plotted on a relative log scale, and an accurate impression of the relationship between the data and the 'classical' curve itself in their Fig. 12 was not immediately apparent to the eye, as the reader can confirm. The data themselves will be considered explicitly later.)

It was then commented that those bandwidths not corresponding to equal distances on Békésy's coordinates, although they might not carry the same meaning as bandwidths that did, were not likely to be invalid as data but were more likely influenced by factors other than simply the tonotopic layout of frequency in the cochlea and that it would be of interest and importance to determine what those factors were. However, as regards the relation of critical band values to cochlear coordinates, the logical alternatives were, that for critical bandwidth estimates not corresponding to equal distances in the low frequencies, either (1) the cochlear coordinates or (2) the equal-distance hypothesis required rejection IF it were also maintained that the bandwidth values in the low frequencies represented exactly what they represented above 500 Hz. These conclusions (although necessary) seem not to have been generally drawn; neither Békésy's coordinates nor the equal-distance hypothesis were explicitly questioned although the hypothesis could not apply to all the 'classical' values if the coordinates were accepted. Conversely, if the correspondence to equal distances were relinquished as a non-essential characteristic of critical bandwidth, this cochlear map and the non-corresponding critical bandwidth estimates would not be in any necessary conflict, but neither would the latter provide a basis for an alternative cochlear map with physiological support.

Since the original subset of bandwidths to which the exponential function had been fitted did not extend below 400 Hz, some early data on 'beats' referenced by Licklider (1951) were examined to see how other possible indices of frequency 'resolution' might relate to cochlear coordinates. These data covered the frequency range below 400 Hz and they overlapped the frequency range above as well (Mayer, 1894). Mayer in 1894 had reported data on two-tone consonance ('the smallest consonant interval among simple tones') from experiments in which two tones were simultaneously sounded at about equal loudness and the subject sought to determine the smallest possible interval free of roughness or dissonance. He had also repeated earlier observations in 1874 and 1875 based on his determinations of the minimal interruption rate required to cause rather oddly amplitude modulated tones to blend into a uniform or continuous sensation. One can conclude from the data that he was effectively making the same psychophysical judgment in the two experiments. Although Mayer's purposes in the two kinds of experiment differed, he noted that the interruption (or modulation) rates (translated here into

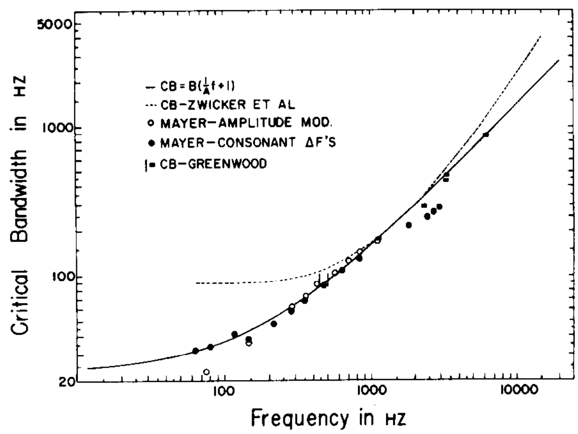


Fig. 2. From Greenwood (1961a). Squares and vertical bars are the critical band estimates from Greenwood (1961a). The solid curve was fitted only to these data. When integrated, it yielded the almost exponential frequency-position function shown in Fig. 1. The solid curve also represents all frequency intervals corresponding to 1 mm. The 'classical' critical band function (from Zwicker, 1957) appears here as the dashed line. Two sets of estimates of consonant frequency intervals, reported by Mayer (1894) also appear: 'the smallest consonant interval among simple tones') (closed circles) and determinations of the minimal interruption rate required to cause amplitude modulated tones to blend into sensations of a uniform intensity (open circles). Most of the two-tone consonant intervals correspond to equal distances (about 1 mm) and so also do most of the 'three-tone' intervals.

carrier-side-band separations) were quite consistent with his two-tone 'smallest consonant interval' data. As shown in Fig. 2 (from 1961), most of the two-tone consonant intervals clearly corresponded to equal distances (about 1 mm) and so also did the 'three-tone' intervals, if Békésy's coordinates were correct, whether or not consonant intervals should be considered as critical bandwidth estimates below 500 Hz or whether or not such correspondence should be considered as a defining characteristic of critical bandwidth. But it was clear that they agreed well with other critical band estimates (Greenwood, 1961a,b, Zwicker et al., 1957) up to about 1.5 kHz (leaving four somewhat deviant undersized intervals at and beyond about 1.8 kHz).

Except for Licklider's brief reference simply to Mayer's two-tone data, they appeared to have been unknown in modern times, and before 1961 no one had suggested that such data could be measures of critical bandwidth. There were and are a number of reasons for taking these data seriously. The two-tone consonance experiments were conducted with tuning forks, which could be made with considerable precision and purity. The experiments were performed in a laboratory where some of the first experiments on masking, sound localization of clicks, and other experiments on acoustics and hearing had been performed ingeniously over a period of years by a physicist whose work had been included in Helmholtz's book (1877), 'On the Sensations of Tone'. Moreover, the effects of unwanted harmonic structure, to the extent present, on subject judgments would be expected to be largely or entirely absent at the narrow separations (hence low ratios) found to be the 'smallest consonant intervals'. For the 5 determinations at lower tone frequencies from about 50 to 200 Hz one of the major researchers, Koernig, in France was the subject and the provider of forks noted for purity from one of the largest collections in Europe (Helmholtz, 1877, p. 528); for the 9 determinations at and above 250 Hz, 10 of the 12 subjects were professional musicians and the remaining two were experienced listeners, one of them Mayer.

Arguing against an importance of any harmonic complexity or a lack of close control of intensity in the two-tone experiments, there were Mayer's (1874, 1875) consistent results from the 'three-tone' amplitude modulation experiments 20 years earlier, repeated in 1894. In both experimental series the major side-band amplitudes would have been no more than half that of the carrier and there was considerable background noise from the earlier more primitive apparatus, two factors which did not preclude either making the judgments or their consistency as a function of frequency among the three subjects, who included one of Helmholtz's primary musician subjects, and with Mayer's two-tone results from 13 subjects. All of the results also agreed quite well with two-tone consonant intervals reported by Cross and Goodwin (1893), which had been obtained with more impure, and presumably more noisy, sound sources (air-driven pipes) than Mayer's forks.

In short, consistent independent determinations in three laboratories with experienced listeners and various apparatus had indicated the results were not much affected by harmonic complexity,

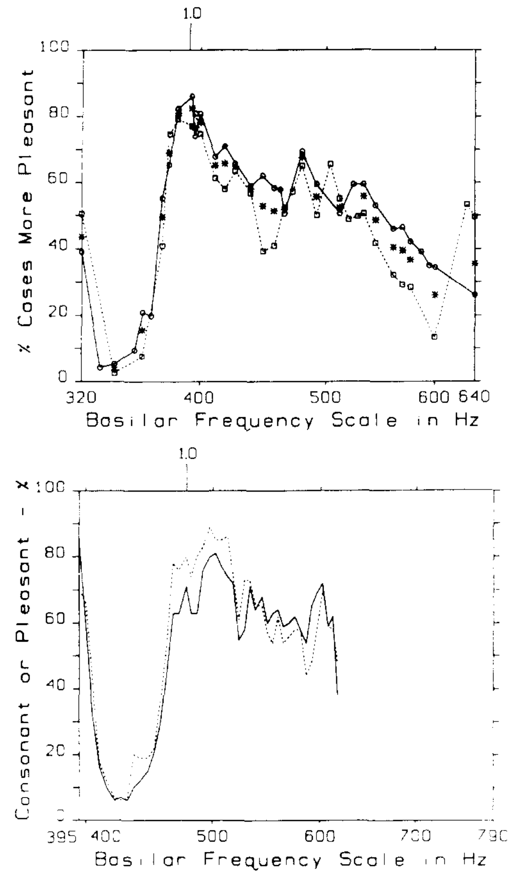


Fig. 3. (Top graph) Data of Kaestner (1909) (lower tone frequency of 320 Hz). Circles: means of three subjects in one series of judgments; squares: means of two subjects in a second series; asterisks: means of all five subjects at those frequency intervals common to both series. Kaestner used the method of paired comparisons, comparing each tone pair with every other pair and determining the percentage of times it was perceived as more pleasant. The upper frequency defined by the vertical line near the shoulder frequencies of these curves is about 392 Hz, representing a dyad geometrically centered at about 354 Hz and an interval of about 72 Hz. The tones were generated by tuning forks. (Bottom graph) Data of Guthrie and Morrill (1928) (lower tone frequency of 395 Hz). Solid curve presents data obtained using the criterion of 'consonance'; dashed curve was obtained with criterion of 'pleasantness'. Guthie and Morrill determined the percentage of subjects who judged a given pair either pleasant or consonant, depending on the experiment, which used 381 and 372 subjects, respectively. The upper frequency defined by the vertical line near the shoulder frequencies of these curves is about 478, representing a dyad geometrically centered at about 435 Hz and an interval of about 83 Hz. The tones were generated by Stern variators (nearly pure tones, see text). (Both graphs) The Δf corresponding to a basilar distance of about 1 mm is indicated by a vertical line at the top of the graphs. The sharpness of this shoulder would seem to be the basis for the single value characterizations of the smallest consonant intervals by and Mayer (1894) Cross and Goodwin (1893) in Figs. 2 and 8.

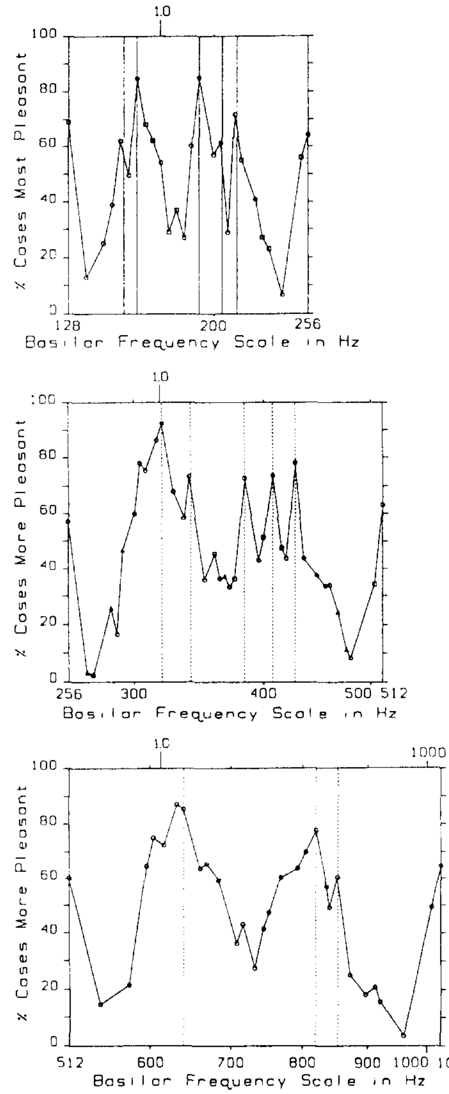


Fig. 4. Adapted from Kaestner (1909). Each tone was complex, with a rich harmonic structure (see text). Kaestner's data are based on 5, 5, and 2 subjects at the lower tone frequencies of 128, 256, 512 Hz, respectively. The consonant intervals defined by the shoulder frequencies in these studies are such that at those consonant intervals the dyads are geometrically centered at about 145, 290, and 570 Hz, respectively. The 1 mm distance is added to provide a common reference; the long vertical lines indicate various integral ratios. The abscissas, on a basilar or cochlear frequency scale, show the octave in different parts of the apical region of the cochlea. The octaves occupy about 2.6, 3.44, and 4.1 mm, respectively.

background noise, or unequal component levels and had argued strongly against coincidence as the explanation for the rather systematic results.

Other Psychophysically Significant Bandwidths in Man

Plomp and Levelt (1965) cited and partially republished consistent early data obtained by Kaestner (1909), Guthrie and Morrill (1928), and Cross and Goodwin (1893), none of whom appear to have been aware of the others, although Cross and Goodwin knew of Mayer's earliest results. The opportunity of readers to compare some of these data with cochlear distances has only been indirect and dependent upon juxtaposing figures from Greenwood (1961b, Fig. 10) and Plomp and Steeneken (1968, Fig. 2).

Figs. 3 and 4 are based on judgments of the consonance of many pairs of simultaneous tones from the work of Kaestner (1909) and from the work of Guthrie and Morrill (1928). In Fig. 3 Kaestner's data were obtained with tuning forks. Kaestner used the method of paired comparisons, comparing each tone pair with every other pair to determine the percentage of times a pair was perceived as more pleasant. Kaestner's data are based on 5 subjects. The lower tone was 320 Hz, and the upper frequency defined by the vertical line near the 'shoulder' frequencies of these curves is about 392 Hz, representing an interval of about 72 Hz. Guthrie and Morrill, who generated tones with Stern variators (whistles, found by a physicist collaborator to contain only faint traces of the third partial, others being inaudible), used the two criteria of 'pleasantness' and 'consonance'. They simply determined the percentage of a large number of subjects, 381 and 372 (in groups of about 50), who judged a given pair either pleasant or consonant, respectively. For a lower tone of 395 Hz, the consonant (shoulder) interval was about 75 to 85 Hz. The two curves may reasonably be suspected to represent population means. Their similarity to Kaestner's curves is striking.

Kaestner's data in Fig. 4 were obtained with the lower tone frequencies of 128, 256, and 512 Hz, based on 5, 5, and 2 subjects, respectively. All told, Kaestner's results in Figs. 3 and 4 were obtained from a total of 8 individuals. In Fig. 4 each tone of a pair was complex, with a rich harmonic structure generated by Appunn's Tonmesser – perhaps the conical resonators of G. Appunn which reproduced all partials of the fundamental (Helmholtz, 1877, pp. 373–374). Plomp and Levelt make the important point that the harmonic structure appears to make a difference only at intervals wider than the interval at the shoulder and conclude that the initial dissonance trough at narrow separations, and the rise to pleasantness (Annehmlichkeit), is due to the interaction of the fundamentals when they are very close on the cochlear partition. (Separation of their harmonics on the partition would be somewhat wider.) The basilar frequency scale shows the different distances occupied by an octave in different parts of the apical portion of the cochlea. Each panel shows one octave; the octaves from 128 to 256, from 256 to 512, and from 512 to 1024 Hz subtend about 2.6, 3.44, and 4.1 mm, respectively.

The point of interest here is that the 'shoulders' of the data curves at the top of the steep rise from dissonance occurred in a rather regular relationship to a Δf corresponding to a basilar distance near 1 mm, which is indicated in Figs. 3 and 4 by vertical lines across the tops of the graphs at the intervals corresponding to 1 mm. The sharpness of this shoulder would seem to suggest why single value characterizations of the smallest consonant intervals could be offered by Cross and Goodwin (1893) and Mayer (1894).

The work in the 19th and early 20th centuries had demonstrated that consonance was relatively high at very narrow separations, quickly reached a minimum of consonance, or a maximum of roughness or dissonance (as the upper tone was raised to a distance that is now realized to be about two-fifths of a critical band distance). Consonance then rapidly increased to a shoulder and local maximum (at an interval corresponding to a cochlear distance of about 1 mm) followed by an irregular and falling, 'plateau' at wider bandwidths, where the curve's form is influenced by intracochlear intermodulation distortion and by complexity of the external stimulus.

In the modern experiments by Plomp and Levelt (1962, 1965) each tone of a pair, widened geometrically about several center frequencies, was presented at about 65 dB SPL measured near the opening to the external ear canal. This constancy of external level seems to have presented problems for tones pairs centered at 125 Hz, where the descending lower tone would approach threshold while the upper became louder, owing to the sloping low frequency quiet thresholds. The subjects rated the pleasantness (or consonance) of a tone-pair on a 7-point scale.

The Plomp and Levelt data (preliminarily reported in 1962, not shown here) were included in an augmented and replotted form by Scharf (1970, his Fig. 7), in his review of the critical band. Scharf added arrows to indicate the degree of conformance of the curve shoulders to the critical bandwidth values published by Zwicker et al. (1957). Correspondence was excellent except for marked disagreement at the center frequency of 250 Hz, where the Plomp and Levelt curve shoulder occurred at 55 Hz, very similar to the values in the earlier work on consonance they cited by Mayer, Cross and Goodwin, and Kaestner (Figs. 2–4), rather than at 100 Hz, which would have conformed to the value sought by Scharf. The Plomp and Levelt data reported by Scharf (1970) are shown in Fig. 5, where the arrows are his and indicate the published values of 'classical' critical bandwidth.

Plomp and Levelt (1965) presented the more detailed results reproduced in Figs. 6 and 7. As in Figs. 3 to 5, the 'shoulders' of the data curves, at the top of the steep rise from dissonance, occurred in a strikingly regular relationship to a Δf corresponding to a basilar distance of about 1 mm (vertical arrows added), for the four center frequencies of 250, 500, 1000, and 2000 Hz. The curve at a center frequency of 125 Hz has the only shoulder interval that does not occur near 1 mm.

The curve minima (maximally dissonant intervals) occurred at smaller Δfs that correspond in all cases to distances of about 0.3 to 0.4 mm for center frequencies from 125 to 2000 Hz. Lines are added upwards from the abscissa in the region of maximum dissonance at narrower tone separations. Basilar distances of from 0.2 to 0.5 mm bracket roughly the trough centers (taking the upper quartile curve into account). The troughs seem to appear at somewhat narrower separations for higher frequency tone pairs, as indicated also in data of Cross and Goodwin (1893) considered later. The best comparison of shape would be to plot the trough and shoulder portion of these curves on an abscissa expressed in millimeters, or

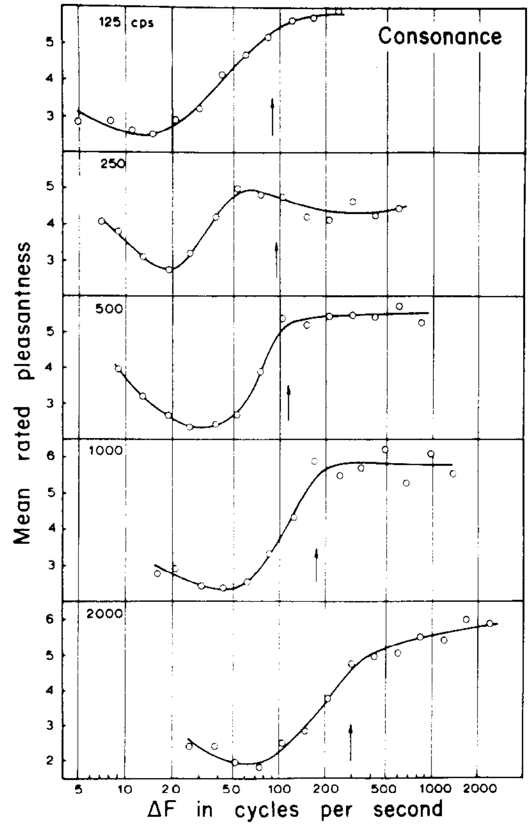
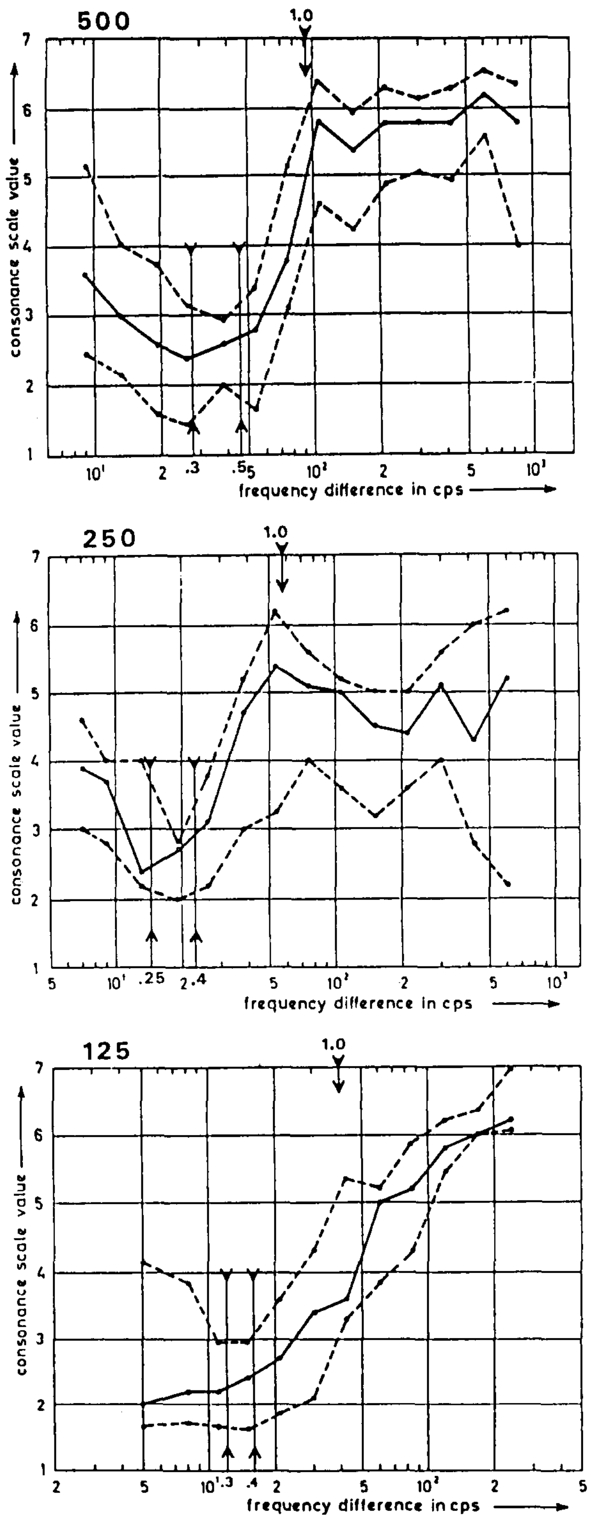


Fig. 5. Adapted from Scharf (1970, his Fig. 7). The Plomp and Levelt data originally reported in 1962 were presented in an augmented and replotted form by Scharf (1970). The arrows are Scharf's and indicate the values of critical bandwidth published by Zwicker et al. (1957). Each tone of a pair, widened geometrically about several center frequencies, was presented at about 65 dB SPL measured near the opening to the external ear canal. Note the gradualness of the 125 Hz curve and the disagreement of arrow and curve at 250 Hz, where the curve shoulder occurs at about 55 Hz (rather than 100 Hz) in agreement with earlier data cited by Plomp and Levelt (i.e. Mayer, Cross and Goodwin, and Kaestner, Figs. 2 to 4). Note also that the shoulders agree with the exponential critical band in Fig. 2 from 250 Hz to 2000 Hz.

on a basilar frequency scale as in Figs. 3 and 4. Maximum dissonance in the Guthrie and Morrill data occurred at a separation of about 0.4 mm, and in the five curves of Kaestner at about 0.3 mm, whose curve minina are comparable in breadth and shape. If replotted on a cochlear scale, Figs. 5 to 7 would more strongly resemble Figs. 3 and 4.



The one deviant curve at the lowest center frequency of 125 Hz in Figs. 5 and 6 seemed to indicate a consonant interval of about 100 Hz or more, about double the 55 Hz interval at 250 Hz. As noted above, these results are probably confounded by the tones' increasing disparity in sensation level owing to rising threshold in the low frequencies and the fact that both were held constant at about 65 dB SPL. The lower tone would have dropped to about half the sensation level of the upper by the time an octave separation (about 85 Hz) was reached (at the fourth point from the right). At the widest interval (tones of 53 and 293 Hz), the lower tone would (for an average subject) closely approach threshold, leaving a sensation of nearly a single tone. At much narrower separations, whether or not it has any significance, the median curve at 125 Hz in 1962 (Plomp and Levelt, 1962) showed a small sub-peak at 30 Hz similar to the blip at 40 Hz in the upper quartile curve in Fig. 6 below the 1 mm arrow. For comparison, the data of Mayer, Cross and Goodwin, and Kaestner in Figs. 2 to 5 indicate the frequency interval of about 40 Hz, corresponding to 1 mm at this center frequency.

In summary (except for the equivocal data at 125 Hz), the data of Plomp and Levelt (1965) – and the data they republished – well confirmed Mayer's two-tone intervals in the low frequency region. Mayer's intervals had corresponded closely to 1 mm, and to 'classical' critical bandwidth from 500 to 1500 Hz, and only 1 of Mayer's many points from 250 to 2000 Hz (at about 1800 Hz) was less than expected from the shoulder intervals

Fig. 6. Adapted from Plomp and Levelt (1965). Augmented and final summaries by Plomp and Levelt (1965) of data from experiments in Fig. 5. As in Figs. 3 to 5, the 'shoulders' of the data curves, at the top of the rise from dissonance, occur near a Δf corresponding to a basilar distance of 1 mm, for center frequencies of 250 and 500 Hz, as indicated by vertical arrow heads added at the Δfs corresponding to 1.0 mm. The curve at a center frequency of 125 Hz has the only shoulder interval that does not occur near 1 mm. The curve minima (maximally dissonant intervals) occurred at Δfs that correspond in all cases to comparable distances of about 0.3 to 0.4 mm from 125 to 2000 Hz. Lines are added upwards from the abscissa in the region of maximum dissonance. Basilar distances of from 0.2 to 0.4 mm or 0.3 to 0.5 mm bracket roughly the trough centers (taking the upper quartile curve into account).

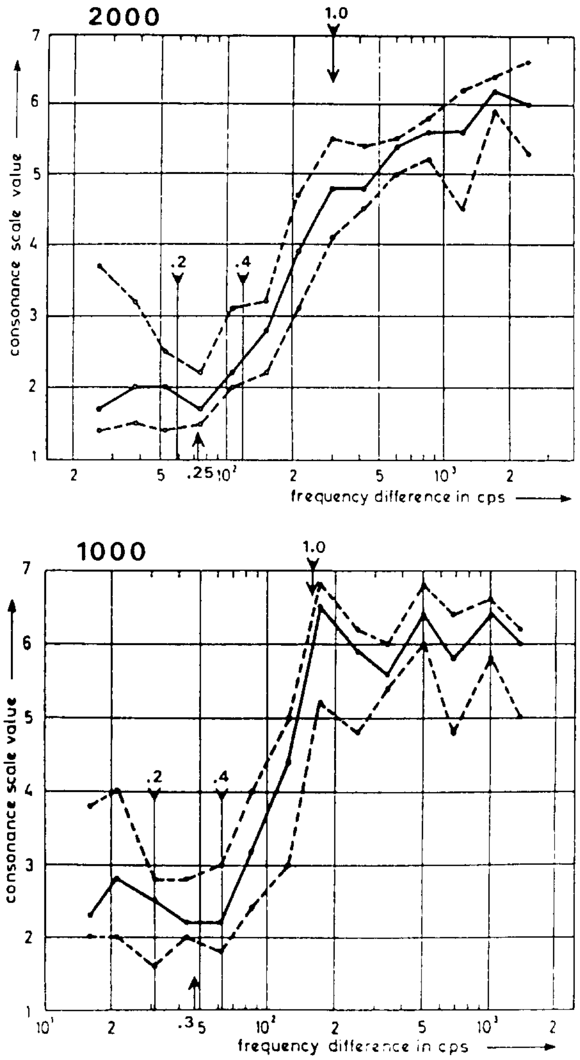


Fig. 7. Adapted from Plomp and Levelt (1965). As in Fig. 6, the 'shoulders' of the data curves, at the top of the rise from dissonance, occur near a Δf corresponding to a basilar distance of 1 mm, for center frequencies of 1000 and 2000 Hz, as indicated by vertical arrow heads added at the Δfs corresponding to 1.0 mm. The troughs seem to appear at somewhat narrower separations for higher frequency tone pairs, as indicated also in data of Cross and Goodwin (1893) considered later.

of Plomp and Levelt and the older data they had republished. Consequently, these data refute Scharf's (1970) statement that Mayer's consonant intervals were 'generally smaller' than those of Plomp and Levelt (which would require them to be smaller than the confirming results of Kaestner and of Guthrie and Morrill, results that Plomp and Levelt had shown were closely consistent between 250 and 500 Hz with their own curves). These points receive some emphasis here for their historical relevance to the odd and obscured relationship that ensued between consonant intervals and critical band intervals.

In a subsequent paper Plomp and Steeneken (1968) reported a further experiment on two-tone consonance using 20 subjects, again comparing their data to those of Mayer (1894), and Cross and Goodwin (1893). In these experiments they held the lower tone fixed in frequency and presented both at 60 phons rather than at approximately constant SPL near the canal entrance. In one series of observations the subjects were asked to adjust the higher frequency tone to the interval of maximum roughness and in a second series to the lowest frequency interval for which the two tones did not interfere. This latter criterion appears to have been equivalent to the determination of the 'smallest consonant interval' used by Mayer (1894). The median judgments and interquartile ranges associated with these estimates appear in the original publications; the medians will be used here.

In Fig. 8 the data of Plomp and Steeneken (large solid circles) are directly compared, on both a log frequency and basilar-membrane frequency scale, with Mayer's two-tone data from Fig. 2, the added data of Cross and Goodwin, and with a curve representing the Δf corresponding to a constant distance of 0.98 mm between the lower and higher tones. The upper graph of Fig. 8 is thus a near duplicate of Fig. 2 of Plomp and Steeneken, with the chief exceptions (a) that Zwicker's critical band curve (of 1957) is omitted (see Fig. 2 of the present paper for comparison) and (b) the 1961 constant-distance curve is drawn through the data for comparison.

The smallest consonant intervals of Plomp and Steeneken (1968) were closely consistent with the intervals in the experiments of Mayer (1894) and Cross and Goodwin (1893) and hence with a distance of about 1.0 mm, thus confirming also the shoulder frequencies of Plomp and Levelt from 250 to 2000 Hz, those of Kaestner from 128 to 512 Hz, and those of Guthrie and Morrill at 395 Hz. In contrast, the 1968 results were inconsistent with the interval of 100 Hz (or more) of Plomp and Levelt at a center frequency of 125 Hz. Smaller intervals for all above experiments were: Mayer – 41 and about 38 Hz intervals for pairs with geometric center frequencies at 115 Hz and 146 Hz; Cross and Goodwin – 46 Hz for a pair geometrically centered at 149 Hz; Plomp and Steeneken – 47 Hz at a geometric center frequency of 145 Hz. Consonant intervals at nearby lower and higher center frequencies, were consistent among experiments. Thus the lower end of the frequency range over which consonant intervals corresponded to an approximate distance of 1 mm began below 100 Hz and had been extended to 3 kHz.

Intervals yielding maximal dissonance in the results of Plomp and Steeneken (lower graph of Fig. 8) were in poorer agreement with Cross and Goodwin. Mayer (1874) comments that maximal dissonance judgements are more difficult and less precise than judgments of consonance, which seems borne out by the width of the curve troughs in Figs. 3 and 4 in contrast to the abruptness of their shoulders. However, Cross and Goodwin used only one subject and the dissonant troughs in Figs. 4 to 8 indicate both fairly good agreement with Plomp and Steeneken and a sufficient degree of broadness and variability to encompass most of the Cross and Goodwin intervals which corresponded to distances between 0.4 and 0.2 mm. The middle solid line (in lower graph of Fig. 8)

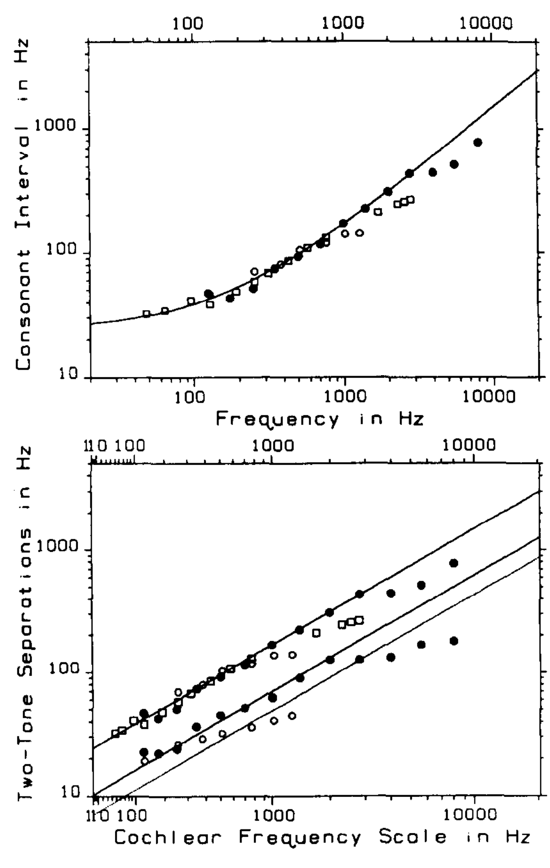


Fig. 8. Adapted from Plomp and Steeneken (1968, their Fig. 2) and Greenwood (1961b, Fig. 10). (Top panel) Solid circles: data of Plomp and Steeneken (1968) on two-tone consonance using 20 subjects. Both tones were presented at 60 phons rather than at approximately constant SPL. In one series of observations the subjects adjusted the higher frequency tone to the smallest frequency interval for which the two tones did not interfere and could be heard separately in that sense, appearantly equivalent to the determination of the 'smallest consonant interval' criterion used by Mayer (1894). The median judgments are presented (see original report for interquartile ranges). Open squares: two-tone data of Mayer (1894). Open circles: similar data of Cross and Goodwin (1893). Solid curve represents all frequency intervals corresponding to 0.98 mm. In all experiments the lower tone was fixed in frequency. The 1968 results are closely consistent with the Plomp and Levelt curve shoulders for tone pairs centered at 250 to 2000 Hz, those of Kaestner at lower tone frequencies from 128 to 512 Hz, and of Guthrie and Morrill at 395 Hz. Note that the intervals of Cross and Goodwin, Mayer, and Plomp and Steeneken all begin to correspond to lesser distances in a similar way at about 1, 1.5, and 3 kHz, respectively. (Bottom panel) Same data and solid curve, plotted on a cochlear frequency scale, which represents all frequency intervals corresponding to a constant distance as a straight line. Additional data are from a series of observations in which the subjects adjusted the higher frequency tone to the interval yielding maximum roughness or dissonance. These intervals are plotted in conjunction with the middle and lower lines. Cross and Goodwin used only one subject and the dissonant troughs correspond to distances between 0.4 and 0.2 mm (see Figs. 3 to 7). The middle solid curve indicates frequency intervals corresponding to a distance of about 0.43 mm; the lower line corresponds to a distance of 0.3 mm. Mayer reported that maximally dissonant Δfs for a given subject were a constant proportion of their smallest consonant intervals, a proportion varying from a minimum of 0.3 to a maximum of 0.6, with a norm of 0.4. Calculating from Mayer's smallest consonant intervals in Figs. 2 or 8, we find that the ratio of 0.4 yields frequency intervals corresponding closely to about 0.4 mm over the tested range, virtually coinciding with Plomp's and Steeneken's values from 100 to 2000 Hz and then decreasing as theirs do.

indicates frequency intervals corresponding to a distance of 0.43 mm; the dotted line corresponds to a distance of 0.3 mm. Mayer (1874) did not report dissonant intervals as Δfs, but stated that for a given subject maximally dissonant intervals were a nearly constant proportion of the smallest consonant intervals, a proportion that varied from a minimum of 0.3 (himself) to a maximum of 0.6, with a norm of 0.4. Applied to his 1874 data, the distance varied from 0.2 to 0.33 mm, with a mean at frequencies from 250 to 1000 Hz of 0.32 mm; applied to the data of Helmholtz's former subject (Mayer, 1875), the distance varied from 0.31 mm to 0.54 mm, with a mean above 250 Hz of 0.49 mm. Based finally on his data in Figs. 2 or 8 (Mayer, 1894), maximum dissonance occurs at intervals that correspond very closely to about 0.4 mm, virtually coinciding with Plomp's and Steeneken's values from 100 to 2000 Hz and then decreasing just as theirs did.

As noted, the consonant intervals in the different experiments also begin to correspond to lesser distances in a similar way at somewhat different higher frequencies. Cross and Goodwin's begin at about 1 kHz to fall below the line, Mayer's at about 1.5 kHz, Plomp's and Steeneken's at about 3 kHz. This is a potentially important point of approach to determine what mechanism underlies the determination of consonance and dissonance. The sources of the stimuli may be relevant to this difference. They were as follows: Cross and Goodwin used cylindrical resonators driven by blowing air across an embouchure; Mayer used tuning forks; Plomp and Steeneken used oscillators. One possibility is that the existence of extraneous noise leads to lesser values of consonant and dissonant intervals because it partially masks combination tones; and in the three experiments above there should have been progressively less admixture of extraneous noise. Combination tones may normally contribute to dissonance at narrow primary separations since they will contribute significantly to the excitation of the same population of eighth nerve fibers that are also excited by the primary tones, as suggested earlier (Greenwood 1971, 1972b; Greenwood et al., 1976). If this contribution were obscured by added noise, it is predictable that both smaller consonant and dissonant intervals would result.

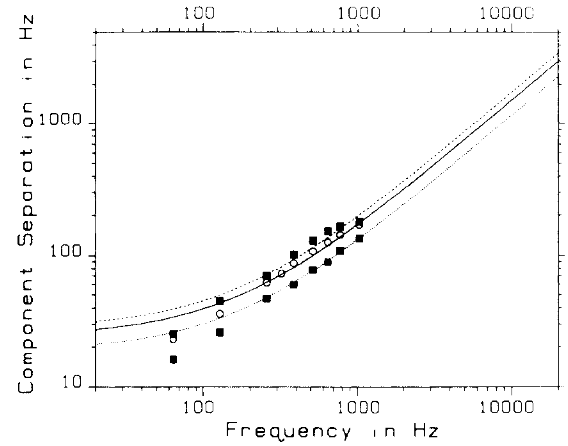


Fig. 9. Data of Mayer (1874, 1875) obtained by amplitude modulation in comparison with the 1894 modulation data given by open circles in Fig. 2. The lower set of solid squares are the mean intervals, corresponding to about 0.8 mm, of two subjects, one of them Mayer (Mayer, 1874). The upper set of solid squares were obtained by a musician and former subject of Helmholtz (Mayer, 1875) using the same apparatus. Her intervals corresponded to about 1.2 mm. Mayer felt that the subject was better able than he to cope with the noise of the apparatus and attend to the relevant cues. To reduce the noise level he later built an improved apparatus used in 1894. The open circle values of 1894 from the top graph were obtained with this less noisy apparatus, using Mayer as subject, and may be compared to his 1874 modulation intervals below.

Evidence that seems consistent with this suggestion appears in Fig. 9, which republishes data of Mayer (1874, 1875; Helmholtz, 1877) obtained by adjusting the rate of amplitude modulation, as described in relation to Fig. 2, to achieve, in effect, a consonant three-tone stimulus. The lower set of solid squares shows the mean intervals, corresponding to about 0.8 mm, of two subjects, a professional musician and Mayer, whose estimates were 5% less on average than the musician's (Mayer, 1874). The upper set of solid squares were obtained with the same apparatus by one of Helmholtz's musician subjects (Mayer, 1875), whose intervals corresponded to about 1.2 mm and followed nearly the same course of increase with frequency. Mayer felt that Helmholtz's former subject was better able than he or the other musician to cope with the noise of the apparatus and selectively attend to the relevant cues. To reduce the noise level he later built an improved apparatus used in 1894. The larger open circle values of

1894 (from Fig. 2) were obtained with this quieter apparatus, using Mayer as subject. The lesser noise level of the later apparatus, which Mayer emphasized, may well have produced lesser masking of combination tones and consequently required larger intervals to achieve consonance.

The foregoing figures have shown that below about 1500 Hz the agreement of the 1968 results of Plomp and Steeneken, especially as a function of frequency, with the data of Mayer (1894) and Cross and Goodwin (1893) is rather good, although these earlier experimenters presented little information on inter- or intra-subject variability. Nevertheless, overall, subjects of greatly varying sophistication gave congruent results. Among the total of 14 subjects used by these two experimenters, (Mayer, 12 at frequencies above 220 Hz and one below; Cross and Goodwin, one) – at least 10 of Mayer's were musicians and the three remaining were experienced listeners. Agreement with the more completely reported data of Kaestner and of Guthrie and Morrill is also good. Kaestner's 8 subjects were apparently musically sophisticated German professors. But of Plomp and Steeneken's 20 subjects, only 5 were familiar with tone perception experiments and their results did not differ from those of the others. The almost 400 subjects used by Guthrie and Morrill were university students not selected for musical talent.

Brief Discussion of Implications, as of 1961–1968, of Comparisons of Bandwidths to Cochlear Distances

In short, in the 1961–1968 period the 'classical' critical bandwidth curve (drawn here is a distinction between curve and data that will be explained in the next section) had been seen to agree – in the range from 500 to 3000 Hz – with equal cochlear distances on Békésy's map and with various sets of consonant interval data. Objectively viewed, the latter experiments, over a period of nearly a century, had by 1965 and 1968 consistently determined the size of consonant intervals in the low frequencies and had demonstrated the relation, of mixed disagreement and agreement, of two-tone consonance data with published values of 'classical' critical bandwidth. The deviation of the consonance data from the 'classical' critical values below 500 Hz was clearly systematic and unambiguous, based on a total series of 7 or more experiments, conducted differently in 5 laboratories and stretching over 94 years, collated and confirmed in the last two series of experiments in a modern laboratory using two different performance measures and numerous subjects. Hence, after 1968, these measured consonance intervals, whatever they meant, could not properly be characterized as agreeing with 'classical' critical bandwidth values below 500 Hz, but agreed with them well from 500 to 3000 Hz. Over a much wider range their considerable agreement with equal distances on the cochlear map provided in Fig. 1 was equally unambiguous, thereby demonstrating their closeness also to the suggested exponential critical band values in Fig. 2 from about 50 Hz to 3000 Hz. If one granted the consonance data themselves, any uncertainty about these agreements would lie then only in the physiological map, which was not explicitly questioned. If the map itself had been considered doubtful or unacceptable, the agreement of consonant intervals to spuriously equal distances would have had to be adjudged coincidental, conceivably defensible positions, if explicitly taken. To attribute their internal coherence also to coincidence might well be less so.

However, although the foregoing paragraphs may fairly be stated to be no more or less than a summary of the data available and collated in 1961, 1965, and 1968, the two conflicting views (a) that Mayer's consonant intervals (and by implication all that agreed with them) were too small below 500 Hz to be considered measures of critical bandwidth and were also generally smaller (Scharf, 1970) than those other measures of consonance that had in fact confirmed them and (b) that, nevertheless, consonance measures were among the bona fide and supporting manifestations of 'classical' critical bandwidth values, seem with a few exceptions to have coexisted without much comment, even in juxtaposition. Secondly, although the consonance data, in the aggregate (and from many more subjects), were in wider conformance with equal distances on a physiologically supported map (over nearly two-thirds of the cochlea if Békésy's cochlear map were accepted) the alternative hypothesis that 'classical' critical bandwidth values corresponded to equal distances has continued to be given more serious credence, judging from the preponderance of the post-1968 literature.

The implications of the above-cited differences, in 1968, were the same as in 1961, as earlier outlined, and by 1968 their consideration was more strongly warranted by the additional data, but these issues seem not to have been discussed in terms of logical alternatives. However, the tacit position seems to have been widely adopted that the 'classical' critical band table and the equal-distance hypothesis, taken together, were more likely to provide an accurate cochlear map than Békésy's cochlear map (conceivably, though unstated, because of the difficulties of the mechanical measurements and the use of cadaver cochleas) and that below 500 Hz the unanimous conformance of the smaller consonant interval bandwidths to erroneously equal distances on Békésy's map (on this view) was coincidental whereas from 500 to 3000 Hz their agreement with the equal distances represented by 'classical' critical bandwidths was genuine and expected. This position seems not to have been made explicit (and at times, as earlier noted, the agreement and smallness of consonant intervals below 500 Hz was denied or unrecognized), but a choice by default among the alternatives presented in 1968 nonetheless carries the same implications. A less than full acceptance of the consonance data for some years may have stemmed from a diminished consensus as to their importance (relative to earlier historical periods) despite exceptions (Greenwood, 1971, 1972b, 1974b; Greenwood et al., 1976; Plomp and Steeneken, 1968; Plomp, 1976).

In any case, the tacit solution does not address the need to explain the low frequency divergence of consonant and critical bandwidths and has for many years also implicitly dismissed, together with the consonance data, the existing direct observations of human and other cochleas as the basis for a human cochlear map, in favor of relying simply on other indirect psychoacoustic data – and the equal-distance hypothesis – as the basis for one or more psychoacoustically based and purely inferential 'cochlear maps'. If the latter such maps do not assume nor insist upon the correctness of the equal-distance hypothesis as it pertains to the psychoacoustically significant bandwidths on which they are based, then their units of measure and their status as maps are ambiguous.

Whatever the reason, since 1968 discussion of a moderately large and consistent body of data on two-tone consonance and some of the interesting questions they raise about their relation to critical bandwidth measurements and to the equal-distance hypothesis seem largely to have been postponed. But perhaps solutions exist that are not necessarily prejudicial to any experimental data, as data. One partial solution, applicable chiefly above 3 kHz, but to some lesser extent also below 500 Hz, consists in the discovery that some of the differences between the exponential critical band function of 1961 and the 'classical' critical band data may be much smaller than assumed, which is the topic of the next section. It is also true that the issue of critical bandwidth in the low frequencies has recently been raised again (Moore and Glasberg, 1983; Shorer, 1986; Fastl and Schorer, 1986; Moore et al., 1990), but only Fastl and Schorer include some consonance data among their comparisons, leaving out the older data.

Plotting the Original Critical Band Data Versus the Cochlear Map

We will compare the original critical band data themselves, summarized in Fig. 11 of the paper by Zwicker et al. (1957), with frequency intervals corresponding to equal distances as calculated by Function (1). In Fig. 2 only the 'classical' curve (as presented in their Fig. 12 by Zwicker et al. (1957) and in their table of critical bandwidth values) was compared with the exponential critical band function of 1961 (Greenwood, 1961b). A later (Zwicker, 1961) and still current (Zwicker and Terhardt, 1980) critical band function raised values slightly below 300 Hz. It will be used in most comparisons below. *

It was noticible in 1961 that the critical band estimates in the AM-QFM experiments (Zwicker, 1952; Zwicker et al., 1957, bottom set of data in their Fig. 11) indicated a bandwidth of about 60 Hz at a center frequency of about 60 Hz and only about 72 Hz at 125 Hz. However, not until mid-

1989 was a comparison made of this and the other curves to the exponential function of Fig. 2. All these data have now been carefully digitized from the original papers, and plotted in Figs. 10 to 14 relative to frequency intervals corresponding to equal distances, using Function (1), and also to the 'classical' function, to determine their relation to cochlear position and to the hypothesis that they may correspond to a constant distance (Zwicker et al., 1957). Note that in the following figures the sets of data differ somewhat in position on the ordinate. Consequently so do the reference curves that measure the degree to which the data correspond to differing constant distances in each case. However, the 'classical' critical band curve that is included for comparison does not, of course, shift on the ordinate. Thus it lies in different positions relative to the differing sets of data, coinciding here and non-coincident there, differently in each case. Since it is the shape or curvature of the 'classical' curve, relative to the data, to which attention should be directed, it will be useful for the reader to shift the data points conceptually to a canonical position of close registration to the 'classical' curve in the region of 1 to 3 kHz, very much as occurs naturally in Fig. 13. Any divergence of the curve from the data in low or high frequencies will then be most apparent, abstracted from the data's plotted ordinate position, which should not be allowed to obscure this shape comparison.

Fig. 10 presents Zwicker's (1954) measurements of the interval at which the masked threshold of a narrow band of noise centered between two tones of varying separation begins to decrease. There were just two subjects, Z (right panels) and G (left panels) in this experiment. For subject Z it can be seen that at and above 300 Hz the empirically measured frequency intervals agree fairly well with frequency intervals corresponding to 1.3 mm, using Function (1). In the lower right graph, which plots center frequency on a cochlear frequency scale, data above 300 Hz are thus fairly well fit by a straight line but would be better fit by one of slightly greater slope. Subject G's data appear in the left two panels. At and above 300 Hz they correspond very well to equal distances of 1.2 mm, as shown by the solid curves in both panels. Note that for subject G at most one point at 100 Hz and perhaps one of the two at 300 Hz could be considered to deviate from the solid line; with two deviant points at 100 Hz the pattern is similar for Z. This degree of agreement with equal distances was unexpected.

Neither set of data in Fig. 10 is well fit by the 'classical' critical band curve (Zwicker, 1961) given by the dashed line, which possess too much curvature. However, to the contrary, in the inset graph in the upper left panel, Zwicker's average curve of 1954, which he had drawn to fit these data specifically, agrees quite well above 400 Hz with a reference curve calculated from Function (1), which represents frequency intervals corresponding to 1.25 mm. Years later, a third subject's data points near 3.2 and 6.5 kHz (Greenwood, 1961a,b) corresponded to a reference curve representing a 1.0 mm distance. The later experiment also showed

* In view of the recent very unfortunate death of Dr. Eberhard Zwicker I wish to append this footnote. As will be argued in the following paragraphs, the hypothesis advanced successively by Fletcher (1940, 1953), Zwicker, Flottorp, and Stevens (1957), and Greenwood (1961) that critical bandwidths might correspond to equal cochlear distances is quite well supported above 400 Hz by the early data of Zwicker (1952, 1954) and Gässler (1954). This rather positive result, obscured earlier by reliance on the published curve, is an outcome whose emergence has required the 'negative' effort here to be very specific in distinguishing between the 'classical' critical band data and the 'classical' curve, since the latter curve does not agree well with the data (nor with the above hypothesis if we assume the cochlear map used in this paper) and diverts attention from the data. Thus it is hoped that the emphasis on the latter points is not seen as iconoclastic or destructive in its intent, but rather simply as needed both to extract full value from important and primary archival data and to address the inevitable resultant interest in the origin of the discrepant curve. After writing this paper, I composed a letter to Dr. Zwicker to make the points above, emphasizing my belief in the importance of the 'classical' data's consistency with the hypothesis he had urged. Dr. Zwicker's much regretted death has now occurred, and my letter to him about the present paper has remained unsent. But I think that he would have understood from my letter that I intended the 'good news' aspect of this paper's treatment of 'classical' data and hypothesis to be the positive and paramount outcome (rather than the discussion of the curve, and despite some views with which he might be expected to disagree). I trust readers will realize the high regard in which I hold him and his work, though I have found it necessary to make explicit the distinction between 'classical' curve and 'classical' data in testing the hypothesis in which our interest was mutual.

that the first reduction in threshold between the two masking tones being separated occurs below the center frequency. This (aside from any subject differences) could account for the somewhat smaller values obtained in the two experiments.

In 1954 Gässler published bandwidth estimates ('Kopplungsbreite') based on threshold measurements of tone complexes and noises as a function of their overall bandwidths. * These data for the same two subjects appear in Fig. 11. The data of subject Z appear in the right two panels. Above 300 Hz the bandwidths correspond quite closely to a distance of 1.4 mm, as calculated from Function (1). The 'classical' critical band function is given by the dashed line and again has too much curvature to fit the data. If, to compare shape better, subject Z's data were shifted downward on the ordinate into good correspondence with the 'classical' curve between 1 and 3 kHz (where they would conform like subject G's data to the 160 Hz value at 1 kHz from the 'classical' table), the flat part of the 'classical' curve would lie well above the data points below 400 Hz (as in Fig. 13), and above 3 kHz the curve would lie above 6 of the 7 points. For subject G, the left two panels indicate that above 500 Hz bandwidth corresponds fairly closely to 1 mm, using Function (1). For subject G the 'classical' critical band function from about 1 to 3 kHz happens to coincide in value with the data values, although its greater curvature is such that it lies above all but one point below 400 Hz and above 8 out of the 10 points above 3 kHz.

Again, neither set of threshold data is well fit by the more greatly curved form of the 'classical' critical band curve (Zwicker, 1961) given by the dashed line. In the inset graph in the upper left panel, Gässler's average curve, which he drew to represent the data of these two subjects, also possesses less curvature than the 'classical' curve. As expected, below 400 Hz the data exceed the distances represented by the reference curves, but also diverge downward from the flat portion of the 'classical' curve. Above 400 Hz Gässler's average curve conforms more nearly to a reference curve whose frequency intervals represent 1.2 mm. Gässler's average curve is wholly contained between the two reference curves corresponding to the distances of 1.0 and 1.4 mm. For both subjects, especially G, the differences in bandwidth size between the experiments in Figs. 10 and 11 indicate some task dependency, although the average curves in the two experiments are close and over most of the frequency range agree quite well with calculated curves representing distances differing by only 0.05 mm. A comparison of the two data averages published by Gässler and Zwicker with each other and with reference intervals corresponding to 1.25 mm appear in Fig. 12, where

* Secondly, it should be mentioned (with full agreement), that Cacace and Margolis (1983) have taken the position that the cited and familiar loudness-match data would in general look much different and could not be fit by two intersecting straight lines if bandwidth were not plotted on a log scale. They show this in a given case, in which the bandwidth scale is converted to the logarithm of the ratio of upper to lower band limits, which will be approximately proportional to a basilar distance scale in the part of the cochlea where frequency is logarithmically projected. In the original data, graphs plotting the log of bandwidth stretched out the part concerned with very narrow bandwidths and (short distances) and compressed larger bandwidths (longer cochlear distances). Use of an abscissa scale corresponding more closely to a distance scale (i.e. log of frequency ratio) does not afford clear bandwidth estimates by means of a two-line fit. The same criticism of the abscissa scale would apply to Gässler's experiment relating threshold to critical bandwidth in Fig. 11.

However, although the chief issue addressed here is the question of how well the bandwidths actually plotted as data points in 1957 correspond to equal distances, it can be pointed out that when all the loudness matches are plotted on a bandwidth scale that plots bandwidths as distances on the basilar membrane, Jeng (1989) has shown that the data from different parts of the cochlea follow such a similar course that they can be superimposed on a common scale of increasing cochlear bandwidth in mm, independent of their cochlear place (frequency range) of origin. From Jeng's plotting it is therefore clear that an ability to fit a common curve to the collection of data plots would imply that widening a stimulus complex to a common width has a common effect on loudness and that a single 'width' measure based on any interval selected (width reached) along the common curve would translate back to frequency intervals corresponding in each case to the same distance, i.e. frequency intervals that would still conform to equal-distance curves as in Fig. 14. Thus, the revised view of the proper way to plot and interpret the loudness-match data, which considers the original two-segment plots an unconvincing measure of a 'loudness critical band', does not diminish the data's continuing value. It instead emphasizes further the relation of loudness to cochlear distance or spatial extent.

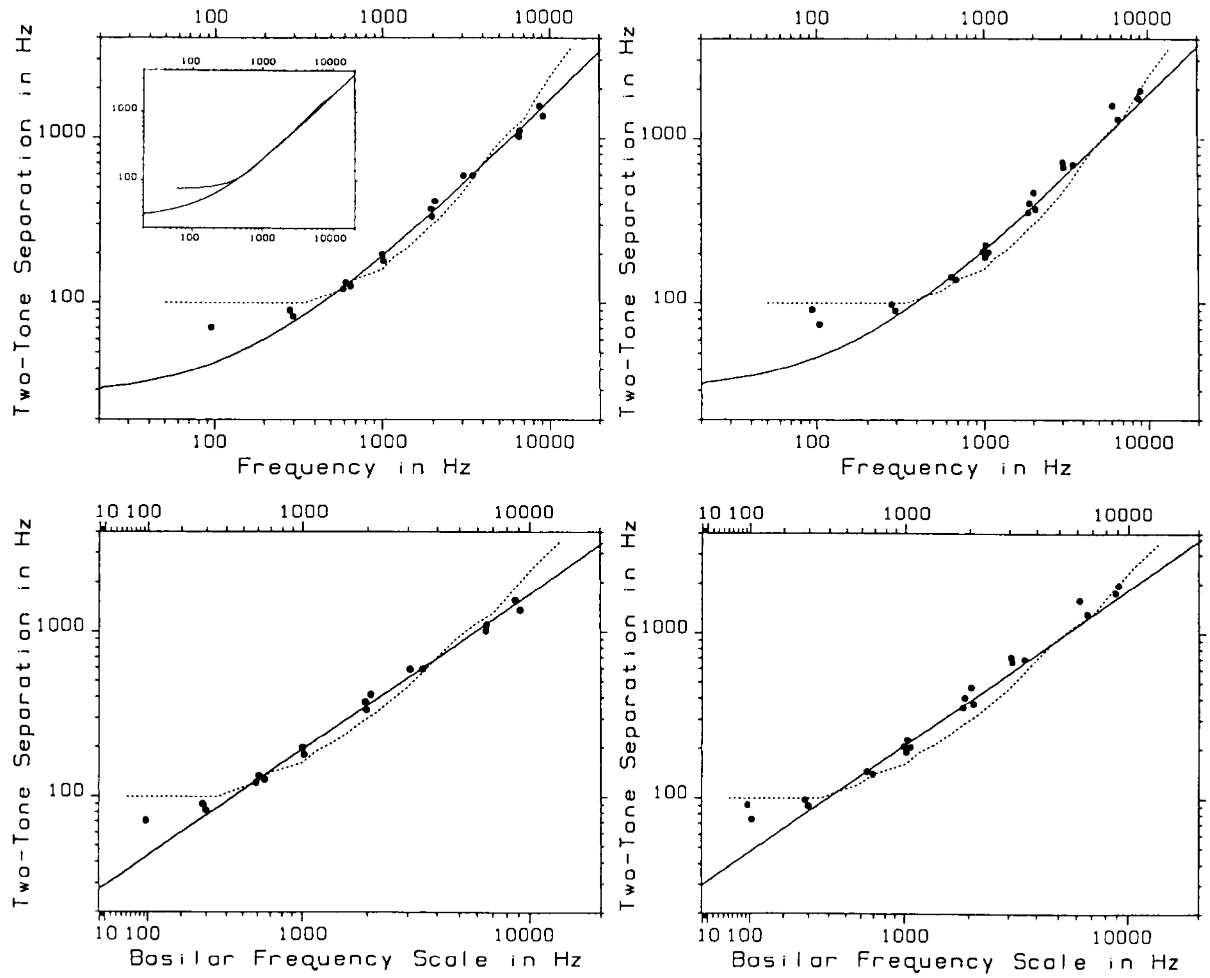


Fig. 10. Estimates of critical bandwidth based on thresholds of a narrow band of noise masked by two tones of varying separation centered on the noise. Dotted curve in all panels is 'classical' critical band curve (Zwicker, 1961). (Right panels) subject Z: Upper panel - above 400 Hz empirical frequency intervals are close to frequency intervals corresponding to 1.3 mm, solid curve as calculated from Function (1). Lower right graph plots same data and curve on a cochlear frequency scale. (Left panels) subject G: Upper and lower panels - above 400 or 500 Hz empirical intervals correspond very well to equal distances of 1.2 mm, as shown by the solid curves. (Inset panel) upper left: Zwicker's average curve, which he fitted to these data in 1954, is compared with the solid curves from both left and right panels and, above 400 Hz, agrees rather well with a curve representing frequency intervals corresponding to 1.25 mm (and much less well with the 'classical' critical band function plotted as the dotted line).

they demonstrate more clearly this good agreement with equal distances above 400 Hz.

A third type of experiment was included by Zwicker et al. (1957; their Fig. 11) on differential sensitivity to AM and QFM (Zwicker, 1952). These data were based on four subjects and appear in Fig. 13. Here the bandwidths correspond closely to 1.15 mm between 0.5 and 5 kHz. As in the other experiments, below 500 Hz the 'classical' critical band curve lies above the data. At 8 kHz the data point is close to the 'classical' curve, and apparently influenced the shape (Feldkeller and Zwicker, 1953) of the critical band curve represented by the dashed line in these figures, as will be explained later. However, recent replications of these experiments (Schorer, 1986) yield very similar values to the 1952 data from 1 to 4 kHz but a lesser value at 8 kHz of about 1.25 kHz as opposed to about 1.87 kHz in 1952; by comparison the values at 1 kHz of 160 Hz in 1986 and slightly less than 170 Hz in 1952 are much closer. The new estimates yield a straight line of increasing bandwidth from 1 to 8 kHz, extending good agreement to equal distances (of about 1.15 mm) to 8 kHz and failing to support the high-end curvature of the 'classical' critical band curve.

A fourth type of experiment of Zwicker et al. (1957) measured the changes in the loudness of tone complexes and noises as a function of bandwidth. Fig. 14 presents the earlier bandwidth estimates of Zwicker and Feldtkeller (1955), using bands of noise, with the data of 1957. It is clear in the top panel that the data are in close agreement with a distance of 1.15 mm from about 0.45 to 5 kHz and in almost equal agreement with the 'classical' critical band curve in the same region. However, there are no loudness data outside the 0.45 to 5 kHz region presented in these papers to

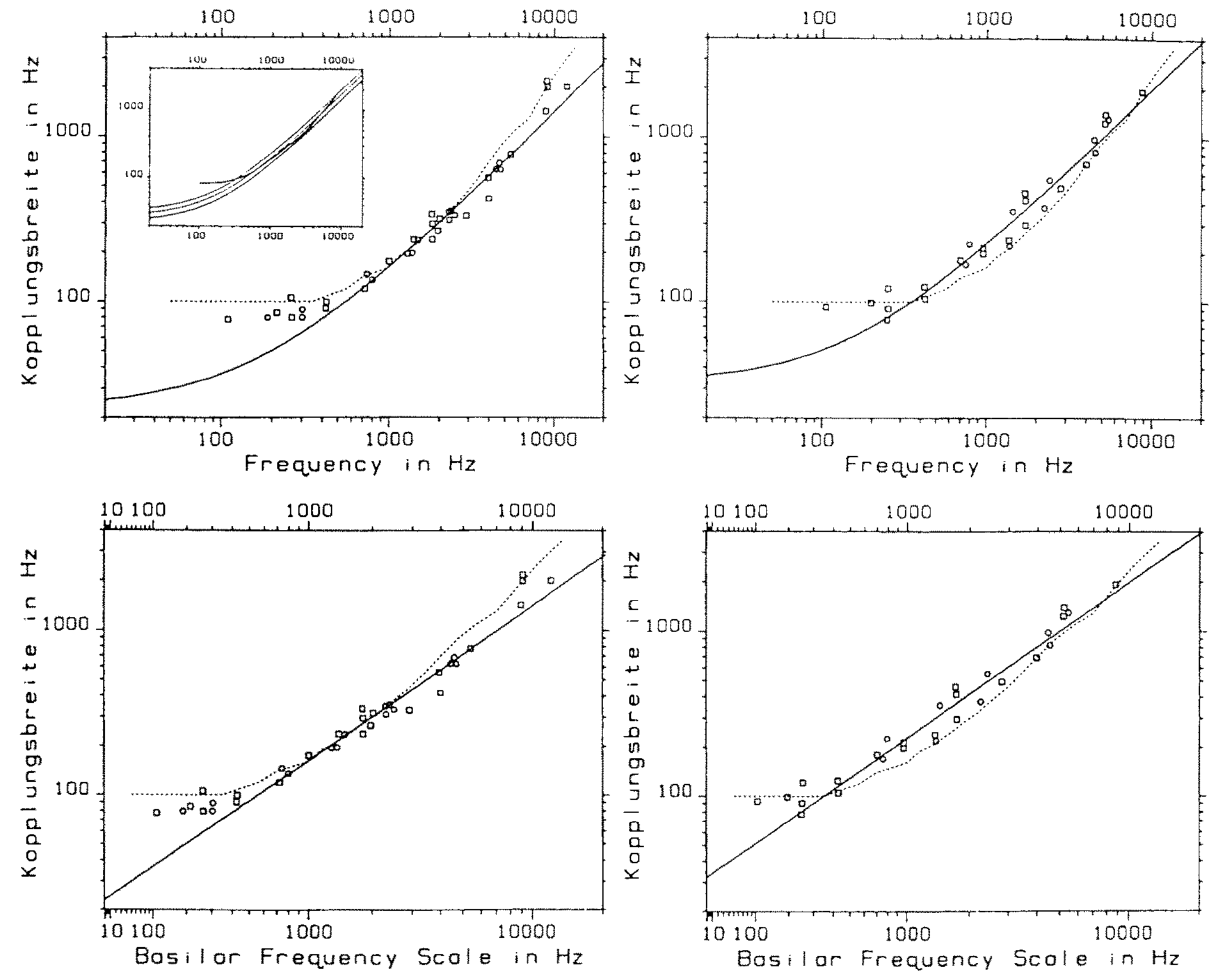


Fig. 11. Critical bandwidth estimates ('Kopplungsbreite') based on threshold measurements of tone complexes and noises as a function of their overall bandwidths (Gässler, 1954). Same two subjects as in Fig. 10. The 'classical' critical band function is given by the dashed lines (Zwicker, 1961). (Right panels) subject Z: Above 300 Hz the bandwidths correspond quite closely to a distance of 1.4 mm, as calculated from Function (1). (Left panels) subject G: Above about 500 Hz the bandwidths correspond fairly closely to a distance of 1.0 mm. (Inset graph) upper left panel: Gässler's average curve, which he fitted to these data in 1954, possesses less curvature than the 'classical' curve. Gässler's average curve agrees more closely above 500 Hz with a calculated curve whose frequency intervals correspond to 1.2 mm. The average curve above 400 Hz is wholly contained between the two calculated curves (dashed lines), corresponding to the distances of 1.0 and 1.4 mm, in contrast with the critical band function plotted as the dotted line in the other panels.

support the degree of curvature shown by the 'classical' curve (see footnote p. 180).

'Classical' Critical Band Data and Origin of the 'Classical' Critical Band Curve

The comparisons of the original critical band data to reference frequency intervals corresponding to equal distances has had the unexpected effect of demonstrating their agreement with the equal-distance hypothesis quite well in the frequency region above 3 kHz, with the exception of only a few points (see especially Fig. 12). Until now, that agreement with equal distances (relying on Békésy's map) was thought to extend from about 0.5 to only 3.0 kHz, when using the 'classical' critical band curve or tables as a basis for comparison.

As fully expected, however, below about 300 to 500 Hz, Figs. 10 to 13 have confirmed that the data bandwidths, like the tabular values in 1961, all exceed the calculated equal distance intervals

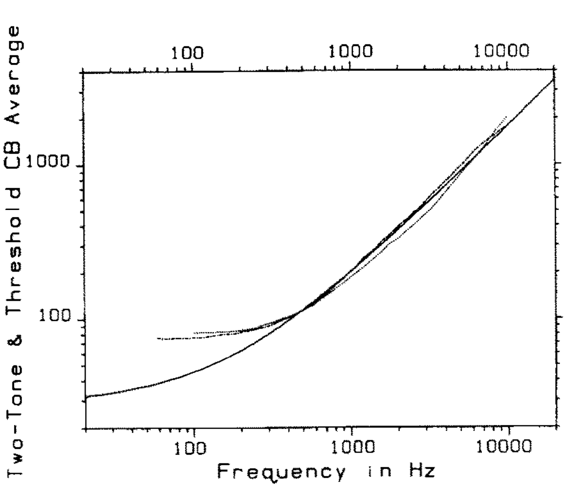


Fig. 12. The two average curves from the inset panels of Figs. 10 and 11, drawn by Zwicker (1954) (dot-dash curve) and Gässler (1954) (dotted curve) are compared with each other and calculated intervals (solid curve) corresponding to 1.25 mm. Gässler's average curve of 1954, which represents the data of these two subjects, possesses some curvature but appreciably less than the 'classical' curve. For both subjects, especially G, the differences in bandwidth size between the experiments in Figs. 10 and 11 indicate some task dependency, although the average curves in the two experiments are close and above 400 Hz agree quite well with equal distances, represented by reference curves corresponding to distances differing by only 0.05 mm.

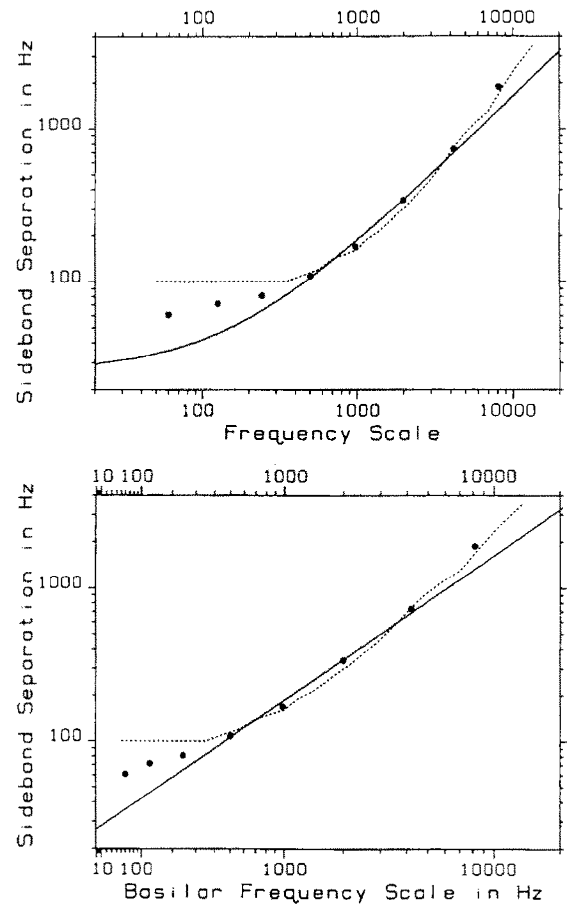


Fig. 13. Critical bandwidth estimates based on differential sensitivity to AM and QFM (Zwicker, 1952), using four subjects. Here the bandwidths correspond closely to 1.15 mm between 0.4 and 5 kHz, as shown by the solid curve. The 'classical' critical band curve is given by the dotted curve Zwicker (1961) and below 400 Hz lies above the data, a region of divergence that would be extended upward to about 600 Hz if the data were brought into exact agreement on the ordinate with the 'classical' curve between 1 and 3 kHz. Above 400 Hz, these data (among the four types of experiment considered in 1957) exhibit the closest agreement in form, due to the point at 8 kHz, with the 'classical' curve (and the frequency-position function derivative of 1953, see Fig. 15).

given by the reference curves, but only by about half as much. In other words, they have shown that the 'classical' curve, when in registration with the data from 1 to 3 kHz, diverged from the lower frequency data (and their averages in Fig. 12) to about the same degree, wherever data existed to permit comparison (the loudness data of Fig. 14 did not extend below 450 Hz). Thus, if the confor-

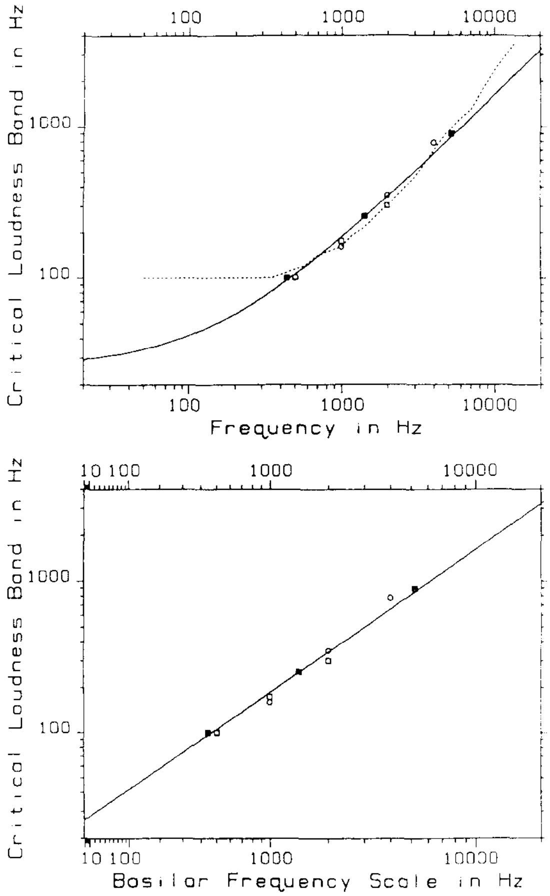


Fig. 14. Critical bandwidth estimates based on the changes in the loudness of tone complexes and noises with bandwidth (Zwicker and Feldtkeller, 1955; Zwicker et al., 1957). Solid squares: estimates based on bands of noise (Zwicker and Feldtkeller, 1955). Open circles: estimates based on tone complexes; open squares: estimates based on bands of noise (Zwicker et al., 1957). Data are in close agreement with a distance of 1.15 mm from about 0.4 to 5 kHz (solid curve) and in almost equal agreement with the 'classical' critical band (dotted) curve in the same region, published in Zwicker and Feldtkeller (1955). For plots of the loudness matches themselves on a scale that converts bandwidths to distances on the basilar membrane, see Jeng (1989).

mance of bandwidths to equal distances above 400 Hz carries a real (albeit unspecified) significance suggested by the equal-distance hypothesis, then whatever factor(s) lead to wider bandwidths at low frequencies have less deviation to account for than if the 'classical' curve had described the data.

If readers are surprised that the 'classical' critical band curve does not characterize the shape of the individual data plots or their averages in the original papers (see Fig. 12), nor appear to be a grand mean of means, it is not for lack of an apparent explanation in the literature. Two papers published in German by Feldtkeller and Zwicker (1953) and Gässler (1954) provide an understanding of the origin of the 'classical' critical band, or 'Frequenzgruppe', curve. As the reader has seen, average curves of two subjects in Fig. 12 (and an average curve of four subjects in Fig. 13) were used by Zwicker and Gässler, respectively, to characterize the actual data in their papers (Zwicker, 1954; Gässler, 1954; Zwicker, 1952). The differences between the 'classical' critical band curve later published by Zwicker et al., in 1957 (and by Zwicker and Feldtkeller, 1955) and the data themselves, arise from the fact that the 'classical' curve (above 300 Hz) duplicates the shape of an average derivative curve obtained earlier from two frequency-position functions, obtained by Steinberg in 1937 and independently by Stevens and Davis in 1936. These authors independently integrated the frequency DL data of Shower and Biddulph (1933). The paper by Feldtkeller and Zwicker (1953) displays two frequency-position functions, one Steinberg's and one after the manner of (nach) Stevens and Davis (1936 – but not identical to their publication of the function), together with the graphical derivatives ds/df (i.e. dx/df) of both parent functions and the average (Mittelwert) derivative (all obtained by Feldtkeller and Zwicker). Gässler (1954, in his Abb. 8, also using the symbol ds rather than dx) then presented the relationship between the average inverse derivative df/ds and his threshold critical band (Kopplungsbreite) data (see Fig. 11). Although the derivative was displaced from and had greater high-frequency curvature than Gässler's data, as shown in Figs. 11 and 12 here, its shape at higher frequencies conformed better to the 8 kHz point of the AM-QFM data, which antedated the 'Mittelwert' derivative (Zwicker, 1952; Feldtkeller and Zwicker, 1953).

In the present paper, this average derivative is replotted in Fig. 15 with the 'classical' critical band curve (Zwicker and Feldtkeller, 1955; Zwicker et al., 1957) lying above it as a dashed curve. To show that the 'classical' critical band curve has the same shape as the earlier derivative

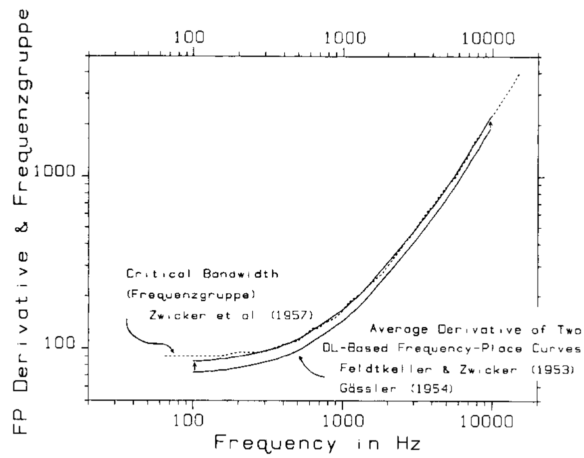


Fig. 15. Two papers (Feldtkeller and Zwicker, 1953; Gässler, 1954) provide an explanation of the origin of the Frequenzgruppe, or 'classical' critical band, curve and for the differences between the 'classical' critical band curve (Zwicker and Feldtkeller, 1955 and Zwicker et al., 1957) and the data themselves. Two frequency-position functions, obtained by Steinberg (1937) and Stevens and Davis (1936), had both been obtained by integrating frequency DL data of Shower and Biddulph (1933). The paper by Feldtkeller and Zwicker (1953) obtained the graphical derivatives dx/df of both parent functions and the average (Mittelwert) derivative lying between them. Gässler (1954) then displayed the relationship between his threshold critical bandwidths (Kopplungsbreite) and the average inverse derivative df/dx, citing Feldtkeller and Zwicker and the same two frequency-position functions. The lower solid curve displays this average derivative. The dashed line is the Frequenzgruppe or critical band curve published in 1955 and 1957. To demonstrate that the 'classical' critical band curve is the same curve above 300 Hz as the average derivative, the derivative curve at the bottom is shifted upward (higher solid curve) by 0.065 log units. The Frequenzgruppe curve is the same as the derivative with only a minor modification, a flattening, below 300 Hz. In 1961 (re-affirmed in 1980), the 'classical' curve below 300 Hz was flattened further, otherwise remaining the same shape as the frequency-position derivative of 1953.

above 300 Hz, the derivative curve at the bottom is simply shifted upward by 0.065 log units. The minor modification of the 1953 derivative curve which changed it into the 'classical' critical band curve consisted simply in raising and flattening it below 300 Hz, which had the effect also of reducing its similarity in shape to all of the individual data curves. In Zwicker (1961, re-affirmed in Zwicker and Terhardt, 1980) the 'classical' curve below 300 Hz was further flattened and raised, but above 300 Hz it remained the same in shape as the inverse Mittelwert frequency-position derivative of 1953. Hence the critical-band-rate scale (Zwicker and Terhardt, 1980), being the integral of the 'classical' critical band curve, must closely reproduce above 300 Hz the average of the two DL-based frequency-position functions of the 30's used by Feldtkeller and Zwicker (1953) to obtain the Mittelwert frequency-position derivative. A critical-band-rate-scale based on the critical band data themselves would have been closer to the Békésy-Skarstein cochlear map, especially if Schorer's (1986) AM-QFM high frequency data points had been available to replace Zwicker's (1952).

Summing Up: Extended Support for Originally Suggested Correspondence of 'Classical' Critical Bandwidths to Equal Cochlear Distances

The general consistency, more specifically in the mid-range, of the early critical band data (Figs. 10–14) with the shape of the frequency-position function derivative of 1953 based on DLs, tends to support the idea that the frequency DL and critical bandwidth might be closely related (Feldtkeller and Zwicker, 1953; Zwicker et al., 1957). It was also their influential view that 'Békésy's determinations ... result in a cochlear map that suggests an additional interesting hypothesis. Critical bands, equal mel intervals, and difference limens may correspond to equal distances along the basilar membrane. The precision with which some of these things can be measured does not yet permit a precise test of this possibility, but the general similarity of the several functions justifies our using it as a working hypothesis' (Zwicker et al., 1957, p. 557).

The only prior direct test of a part of this hypothesis, which included consonance data and then-new critical band data (Greenwood, 1961b; Fig. 2 of the present paper), made a precise comparison between the critical band curve and equal distances on Békésy's cochlear map, with the conflict noted between curve and hypothesis, or between curve and map, depending on interpretation. Since that time the human cochlear data have been extended (Kringlebotn et al., 1979) and new relevant data from living animal preparations have been reviewed (Greenwood, 1990). These are developments that arguably afford some increased confidence in the validity and accuracy of the cochlear map. Now the present comparison of the original critical band data themselves with cochlear distances has revealed that the data bandwidths actually agreed with the equal-distance hypothesis much better than realized in 1961. The old data's agreement with hypothesis is now seen to extend into the frequency region above 3 kHz, with the exception of only a few points at the highest frequencies (which new data by Schorer (1986) appears to replace). Thus the equal-distance hypothesis put forward in the paper by Zwicker et al., in 1957 appears quite well supported by nearly all the critical band data of that paper above about 400 Hz if the Békésy-Skarstein map is correct in slope. It appears that by consulting the data conflict between map and hypothesis is minimized. Below 400 Hz, however, 'classical' bandwidths, from the three relevant experiments (which do not include loudness), still lie above the equal-distance curves. As emphasized earlier, there is no presumption here that the bandwidths obtained in these experiments ought to correspond to equal distances. If other determining factors have become more influential or dominant in the low frequencies (as suggested for the critical ratio by Zwicker et al., 1957), equal distances should not be expected even if they have been found in the higher frequencies.

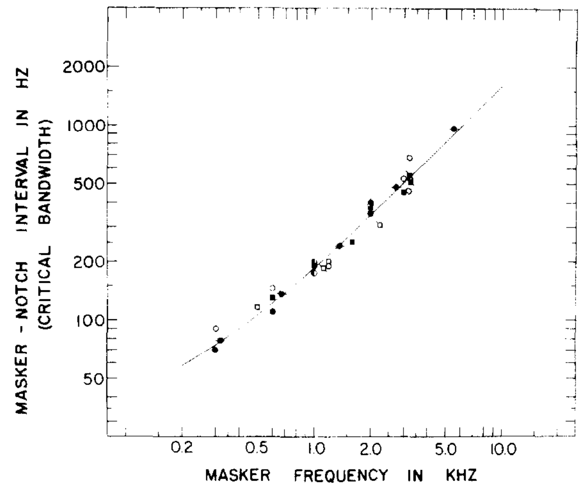


Fig. 16. From Greenwood (1971). Widths of the notch in pure tone masked audiograms, at low to moderate levels, on their high frequency side, which result from the determination of signal threshold by the detection of combination tones generated by signal and masker. The frequency interval plotted on the ordinate is the separation between the masking frequency and the low point of the notch on the high frequency side, when the masking tone was at levels at and below about 60 dB SL. These frequency intervals equal critical bandwidth and correspond to an approximately constant distance (Greenwood, 1961a,b, 1971). The solid circles with horizontal bars are based on Small's (1959) data and represent the means of six subjects. The remaining symbols represent individual subjects in 1971 (see original paper for explanation). The solid line indicates all frequency intervals that correspond to an equal distance of 1.065 mm, when the interval concerned is between masker and notch low point. The same frequency interval when reflected about the masker in the low frequency direction represents also 1.25 mm, if the frequency interval of interest is between the masker and the corresponding frequency of the cubic combination tone generated by the masker and a signal at the low point of the notch in the masked audiogram.

Comparing Other Frequency Resolution Data to the Cochlear Map

Pure tone masking of pure tone signals

Some later experiments provided evidence for a notch in pure tone masked audiograms on their high frequency sides (Chistovich, 1957; Small, 1959; Ehmer, 1959; Greenwood, 1961a), which appear related to critical bandwidth (Greenwood, 1961b) and provide a frequency interval that corresponds to an approximately constant distance (Greenwood, 1971). In Fig. 16 the frequency interval plotted on the ordinate is the separation between the masking frequency and the low point of the notch on the high frequency side, when the masking tone was at low to moderate levels (below about 60 dB SL). The solid line indicates all frequency intervals that represent an equal distance of 1.065 mm when the interval concerned is between masker and notch low point, but it corresponds to 1.25 mm when the interval designated lies between the masker and the frequency of the cubic ($2f_1$–f_2) combination tone whose detection is responsible for the notch in the masked audiogram. All of the plotted points reported in 1971 are from various individual subjects; note, however, that the values of Small are means from six subjects and are very close to the calculated curve.

At higher masker levels, another combination tone, often called the simple difference tone (masker – signal or f_2–f_1), is also detected at signal levels insufficient for the signal itself to be heard (Greenwood, 1971; Krammer and Greenwood, 1973), with the result that the detection of either or both combination tones mentioned determines a deeper and wider notch whose slope nearest the masker is still determined by the $2f_1-f_2$ combination tone. At lesser masking tone levels when the masked audiogram slope is monotonically decreasing (notchless), it still is determined by the detection of combination tones and tends to intersect quiet threshold when signal frequency is about 1.2, or slightly less, times masker frequency (see Fig. 6 in Greenwood, 1971 and Fig. 5 in Smoorenburg, 1972, for examples). This point of intersection tends to become the low-point of the notch at higher masker levels. These masker-notch experiments were suggested in 1971 to be one of the keys to the interpretation of other critical band data and will figure largely in later discussion.

Discriminability of AM from QFM

In experiments with AM and QFM signals, Goldstein (1967) replicated Zwicker's (1952) experiments discussed above (Fig. 13) in which AM and QFM stimuli were each discriminated from pure tones. In addition, he determined the ability of subjects to discriminate AM from QFM sounds, a different task entirely, over a wide range of center frequencies from 0.25 to 8 kHz at octave intervals. The maximum modulation rates that permitted discriminations to be made are well fitted by a curve corresponding to equal distances. If we fit such a curve to the maximum modulation rate at 0.5 kHz, it provides a fit from 0.5 to 8 kHz that is significantly superior to that provided by a curve proportional to the equivalent rectangular bandwidth (ERB) curve (ERB times 1.54) presented by Rosen (1986). The latter curve rises too steeply in relation to these data, as can be seen in Rosen's Fig. 1, yielding at 8.5 kHz about 2.35 kHz, which is 1.53 kHz (Shailer et al., 1990) times 1.54, as opposed to about 1.5 kHz at 8.5 kHz for an equal-distance curve fitted, as above, to Goldstein's data point at 0.5 kHz. Other evidence of excessive values of the ERB curve at 8 and 10 kHz (and below about 250 Hz) will be noted later.

If we use an equal-distance curve fitted to a modulation rate of about 115 Hz at 500 Hz, the frequency interval between side-bands required to discriminate AM from QFM corresponds on average, over the tested range from 0.25 to 8 kHz, to

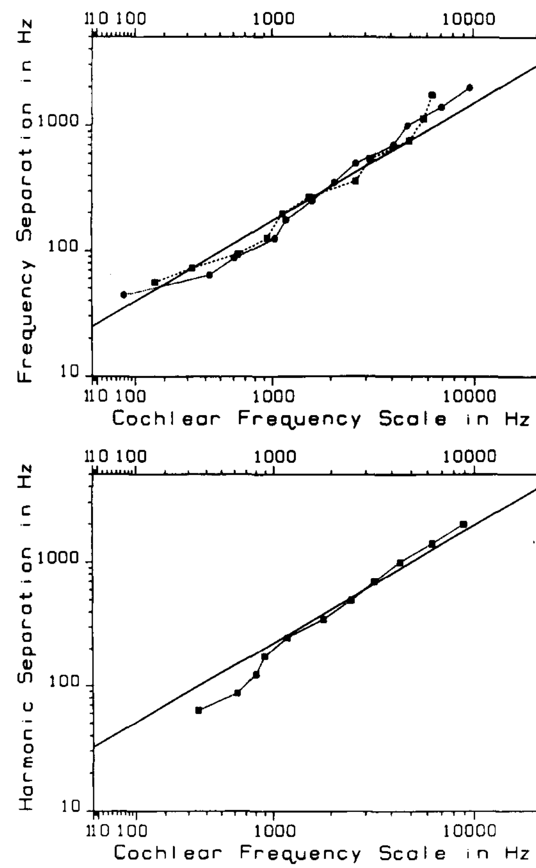


Fig. 17. (Top panel) Minimum frequency separation (circles) required between adjacent harmonics of a complex tone in order to match them correctly to single tones presented alone (Plomp, 1964). These results are consistent with those in a similar analysis of inharmonic complexes (squares). The solid comparison curve plots the frequency separation subtending a constant distance of 1.0 mm, which is close to the mean distance, obtained from the data, corresponding to the frequency separation between the highest identifiable component and the component just above it. There is both reasonable agreement with comparable distances in different parts of the frequency range and also apparently systematic curvilinearity. (Bottom panel) The experiments were later repeated by Plomp and Mimpen (1968) with the two original subjects plus four more. The latter series used only harmonic complexes and the data followed generally similar courses. The median data of the 1968 series appear in the bottom graph with a straight line indicating frequency intervals corresponding to 1.28 mm. Again there is a departure from constancy below 1.0 kHz but a slightly lesser departure in slope above it from the equal distance line.

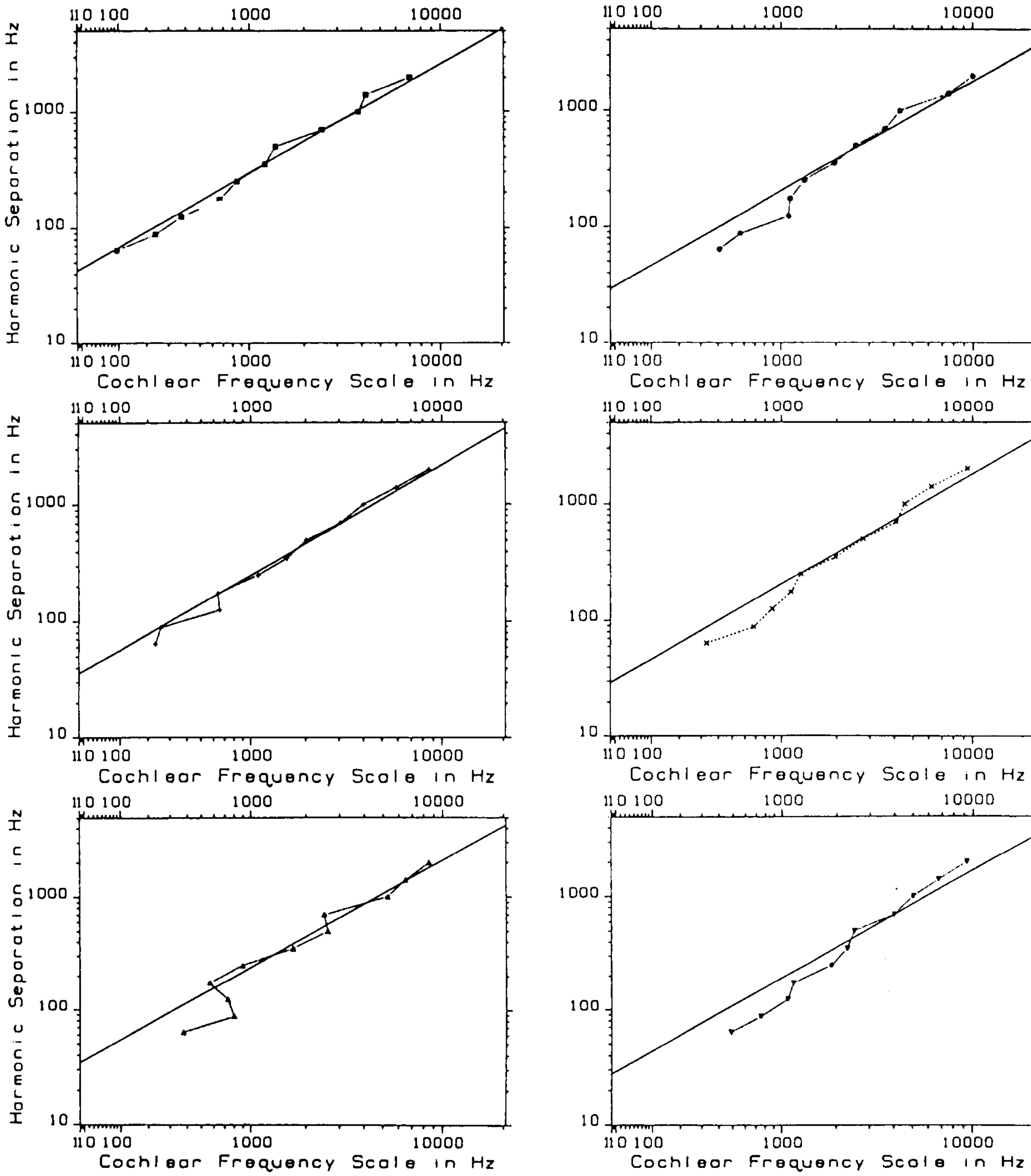


Fig. 18. To show the range of the data and subject differences, all six subjects are plotted separately. (Left panels) For these subjects the data conform more closely to constant distances (of 1.65, 1.41, 1.36 mm from top to bottom) and over a larger frequency range than do the data on the right. (Right panels) The slopes deviate increasingly from top to bottom panels from lines indicating constant distances of 1.17, 1.18, and 1.12 mm. For two subjects on the left (squares and pluses) the empirical frequency intervals conform fairly closely to constant distances over a frequency range of almost two decades, or approximately two-thirds of the cochlear partition. For subjects on the bottom left and top right (triangles and solid circles) the curves for a decade above 1000 Hz (or over about half the cochlea) largely conform to constant distances.

about 1.25 mm, which breaks down into a distance between lower side-band and carrier of 0.65 mm and between carrier and upper side-band of 0.6 mm. At the latter shorter distance the carrier and upper side-band have exceeded the maximum dissonance trough reached at about 0.4 mm and by a separation of 0.6 mm would, if presented alone, have entered upon the steep rise to the shoulder of the two-tone consonance curve (see Figs. 3 and 4) reached at about 1 mm.

Required separation to identify components within multi-tone complexes

In further 'frequency resolution' experiments, Plomp (1964) had determined the minimum frequency separation required between the harmonics of a 12-component complex tone in order for two skilled subjects to match the harmonics correctly to single tones presented alone, that is, to hear them separately in this sense. These results were quite consistent with those of a similar analysis using inharmonic complexes. Both sets of results are plotted together in the top graph of Fig. 17, where the frequency separation required between adjacent components is plotted versus the frequency of the highest discriminable component. The solid comparison curve plots the frequency separation subtending a constant distance, which defines a straight line on a cochlear frequency scale. This distance of 1.0 mm is near the mean distance obtained from the data and corresponds to the frequency separation between the highest identifiable component and the component just above it. There is both reasonably close agreement with this constant distance and also an apparently systematic curvilinearity. The experiments were later repeated by Plomp and Mimpen (1968) with the two original subjects plus four more. The median data of the 1968 experiments using only harmonic complexes appear in the bottom graph of Fig. 17 with a straight line indicating frequency intervals corresponding to 1.28 mm. Again there is a departure from constancy below 1.0 kHz and a slightly steeper median slope.

To show the range of the data, and differences among subjects (not adequately described simply as 'variability', and lost in plotting only medians), all six subjects are plotted separately in Fig. 18. In

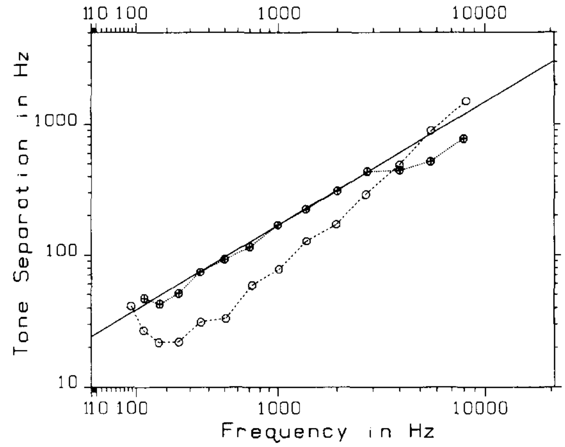


Fig. 19. Two-tone data obtained using basically the same method as with twelve harmonics in Fig. 17 (Plomp, 1964). Subjects were required to select which of two single tones was identical to one of a pair of simultaneous tones, when the alternative single tone was halfway between the pair (open circles and dashed line). These two-tone data are compared with the results of Fig. 8 of Plomp and Steeneken (1968), repeated here (partly filled circles and dotted line), and with a solid straight line representing intervals corresponding to about 1.0 mm. It is conceivable that the deviations below 1.0 kHz from the straight lines in Fig. 17 and Fig. 18 may be related to the generally steeper course of the two-tone results in the same 1964 experiment.

the three graphs on the left the data conform more closely to constant distances (of 1.65, 1.41, 1.36 mm from top to bottom) and over a larger frequency range than do the data on the right. On the right the slopes deviate mildly but increasingly from top to bottom panel from lines indicating constant distances of 1.17, 1.18, and 1.12 mm. For two subjects on the left (squares and crosses) the empirical frequency intervals conform fairly closely to constant distances over a frequency range of almost two decades, or approximately two-thirds of the cochlear partition. For subjects on the bottom left and top right (triangles and solid circles) the curves for a decade above 1000 Hz (or over about half the cochlea) largely conform to constant distances.

Plomp (1964) has also reported other results using basically the same method as with the twelve harmonics, but with only two tones, see Fig. 19. Subjects were required to select which of two single tones was identical to one of a pair of simultaneous tones, when the alternative single tone was halfway between the pair of simultaneous tones, when the alternative single tone was part-way between the pair. Whatever factors other than mere spatial separation of maxima along the cochlear partition determine the ability to identify which of two simultaneous components is the same as a single tone heard alternately, these two-tone experiments yielded a much steeper slope and smaller separations in the lower frequencies. The two-tone intervals thus disagreed markedly with frequency separations corresponding to constant distances. They differed also therefore from the results in Fig. 8 of Plomp and Steeneken (1968) and the other results of Mayer (1894) and Cross and Goodwin (1893), most of which closely agreed with constant distances and used the very different criterion of consonance, or the absense of interference. It is clear that factors related to the specific task and judgment required must clearly interact importantly with any basic spatial pattern information underlying 'resolution' performance. It is conceivable that the deviations below 1.0 kHz from the straight lines in Fig. 17 and Fig. 18 may be related to the generally steeper course of the two-tones results in the same 1964 paper. It is conceivable that temporal factors, more prominent in the low frequency phase-locking region, might underly the ability to identify components at a narrower separation than required to hear the pairs of tones as consonant or non-interfering.

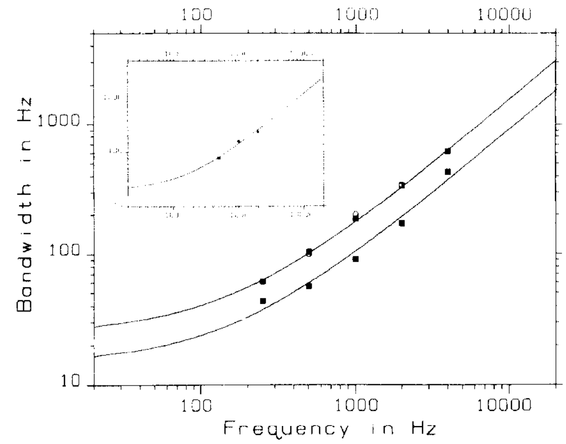


Fig. 20. Estimates of 'auditory filter bandwidths'. Main graph: Houtgast (1977) and Patterson (1976); Upper sets of estimates are of gaussian bandwidths derived from ripple-resolution data obtained in simultaneous masking experiments (squares) and from two-band noise (notched-noise) simultaneous masking experiments (circles), respectively. The upper calculated curve represents the frequency intervals corresponding to 1.1 mm, according to Function (1). The Patterson points at 500 and 2000 Hz nearly coincide with Houtgast's upper set of points at those frequencies, but Patterson's circles would be slightly more closely approximated by a line representing frequency intervals corresponding to 1.16 mm. Lower curve and points: Estimates of filter bandwidths derived from ripple-resolution data obtained in Houtgast's pulsation threshold measurements. The calculated curve represents the frequency interval corresponding to 0.65 mm as above. Inset graph: Patterson (1976); Estimates of equivalent rectangular bandwidths (ERB) derived from same data as in main graph. The calculated curve represents frequency intervals corresponding to 0.89 mm. Same data appear in Fig. 21 with other ERB data.

Estimated gaussian auditory filter bandwidths

Much more recently Houtgast (1977) and Patterson (1976) have performed entirely different experiments (on masking) leading to estimates of filter bandwidths that are closely proportional to the critical band function of 1961 and therefore also correspond closely with equal basilar distances. These data appear in Fig. 20 replotted in the same way as Fig. 2, and it is clear that they not only agree with the Δfs corresponding to a constant distance, but are precisely proportional to the masking and consonance data in Figs. 2 to 8 and to the two-tone masking, threshold, phase, and loudness data in Figs. 10 to 14, and to the masker-notch data in Fig. 16.

Estimated rectangular bandwidth (ERB)

These bandwidth estimates of Houtgast and of Patterson were among those used to develop a 'critical' (ERB) band function in papers by Shailer and Moore (1983) and Moore and Glasberg (1983), as discussed in a recent paper (Greenwood, 1990). Greenwood's (1961b) development of a critical band function had assumed critical bandwidth might be an exponential function of position (x) on the basilar membrane. This lead in 1961 to an almost-exponential frequency-position function and implied that critical bandwidth would also be a linear function of frequency, a function obtained when Function (1)'s inverse, (x as a function of frequency) was substituted for x in the exponential critical band function. Moore and Glasberg instead assumed, in effect, that ERB bandwidths might follow a second-order polynomial function of frequency. They fitted a function over the frequency range from 125 Hz to 6.5 kHz. The original ERB bandwidths had corresponded closely to 0.9 mm according to the 1961 frequency-position function (Moore, personal communication 1983), carrying the implication above. Subsequent bandwidth estimates (Peters and Moore, 1989; Shailer et al., 1990; Moore et al., 1990) have indeed proven more consistently proportional to the 1961 critical band function, with its linear relation to frequency, than to the quadratic polynomial. Additionally, the latter's integral, the ERB-rate function, does not agree with the cochlear map as well as does Function (1) in Fig. 1 (Greenwood, 1990, Fig. 9). The more recent ERB data are best introduced, however, by earlier data of Weber (1977), which demonstrate changes in ERB as a function of level.

Weber's data were obtained in notched-noise experiments (like those of Patterson) at five spectrum levels (10 to 50 dB SPL) and three frequencies (l, 2, and 4 kHz). At the three lowest levels (symbolized in Fig. 21 by open rhomboids (10 dB, 2 and 4 kHz), and by inverted triangles and triangles (20 and 30 dB SPL, 1 to 4 kHz) the frequency intervals correspond quite closely to a distance of 0.53 mm, as represented by the calculated dashed line. At the two higher levels of 40 and 50 dB SPL, the estimated ERB values increased, and with increasing frequency the functions cross each other. In Fig. 21 the uppermost squares and dotted line, represent the 50 dB SPL spectrum level, cross the dashed line connecting rhomboids representing the 40 dB SPL level. The solid 0.9 mm curve passes a little above the 50 dB SPL squares at 1000 and 4000 Hz by nearly the same amount.

Fig. 21 contains also the original data (circles) of Shailer and Moore (1983), (partially filled rhomboids) of Fidell et al. (1983) and (partially filled squares) of Patterson (1976), which were used in the construction of the ERB function. The ERB estimates of Peters and Moore (1989) (open squares) at notched noise spectrum levels between about 50 to 60 dB continue to agree fairly closely with 0.9 mm at lower frequencies from 100 to 800 Hz.

The sensation level at which data are gathered is potentially important (Greenwood, 1961a, 1983). At 8 and 10 kHz (Shailer et al., 1990) the results were obtained at different levels and from subjects with differing quiet thresholds. At 8 kHz and a spectrum level of 50 dB SPL, the mean bandwidth (hourglass) and the individual bandwidths for two out of three subjects (not shown) are in fairly good agreement with results at lower frequencies ob-

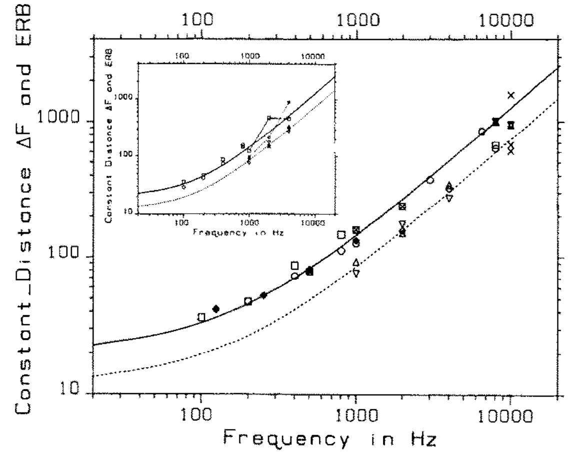


Fig. 21. Estimates of equivalent rectangular bandwidths (ERBs), reported by various authors. ERBs derived from two-band noise (notched-noise) simultaneous masking experiments: Patterson (1976), at 0.5, 1, and 2 kHz (partially filled squares). Weber (1977) - data at 1, 2, and 4 kHz, analogous to Patterson's and obtained at five spectrum levels from 10 to 50 dB SPL. Data at levels of 10, 20, and 30 dB SPL appear in both inset and main graphs. At levels of 40 (rhomboids and dashed line) and 50 dB SPL (squares and dotted line), the ERB values take larger values and display functions that cross each other; these data appear only in the inset graph. At the three lower levels, symbolized by open rhomboids (10 dB SPL, 2 and 4 kHz) and by inverted triangles and triangles (respectively, 20 and 30 dB SPL, 1 to 4 kHz), the frequency intervals correspond quite closely to a distance of 0.53 mm. Estimates by Peters and Moore (1989) at 100, 200, 400, and 800 Hz appear only in the inset graph (circles) together with later and similar estimates by Shailer et al. (1990) at the same frequencies (squares). The latter also appear in the main graph (squares). Estimates at 8 and 10 kHz by Moore, et al. (1990): means at 8 kHz - 35 (one circle), 20 (square), 50 dB SPL (hour-glass); at 10 kHz - 50 dB SPL the mean (hourglass) and individual subjects (operating at different sensation levels) (X) are plotted. ERBs based on temporal gap detection data are: Shailer and Moore (1983), (circles, at 200 Hz a circle is under a square); Fidell et al. (1983), (partially filled rhomboids at and below 1 kHz). As in earlier figures, the curves are not fitted to the data, but are rather proportional to the derivative of Function (1), as in Greenwood (1974b). The dashed curves represent frequency intervals corresponding to a distance of 0.53 mm; the solid curves represent 0.9 mm.

tained at corresponding levels. But for the third subject with higher thresholds at 8 to 16 kHz (thresholds at lower frequencies are not reported), threshold at 8 kHz is 8.4 dB higher, at 10 kHz it is 6.6 to 12.6 dB higher, and at 12 to 14 kHz it is about 21 dB higher (the latter region perhaps having little relevance) than for the two other subjects. Consequently for this subject the noise was at a lower sensation level (on the high-pass side at least), and the estimated ERB was the narrowest, about 73% of the other two. It is consistent that at 8 kHz and the lower levels of 20 and 35 dB SPL, lesser ERB estimates (square and circle) were obtained for all subjects. At these spectrum levels ERB estimates are less variable, moreover, and in good agreement with bandwidths obtained by Weber (1977) at comparable levels.

Again at 10 kHz, although the ERB estimates were obtained at a single spectrum level of 50 dB SPL, the quiet thresholds of one subject were in general considerably lower than the other two in the region of the notched noise. The 10 kHz signal itself for this subject was thus higher in sensation level by about 10 dB and 1 dB in respect to the other two subjects, while the sensation level of the low-pass noise was 10 dB greater at 8 kHz, and of the high-pass noise was about 20 dB greater at 12 to 14 kHz. Thus for this subject the notched noise on both sides of the signal was higher in sensation level and may have been comparable (certainly more nearly so) to the 50 dB level used at lower critical frequencies. In any case, this subject's ERB was comparable to the corresponding ERBs at lower frequencies and similar SPLs. In contrast, for the other two subjects tested at 10 kHz (and 50 dB SPL) whose quiet thresholds were 10 dB higher at 8 kHz and 20 dB higher at 12 and 14 kHz, the ERB estimates were instead similar to Weber's (1977) ratios at 10 to 30 dB SPL, and corresponded to nearly the same cochlear distance. The individual ERB values appear as X's and the mean ERB as an hourglass symbol in Fig. 21. As in earlier figures, each calculated curve is derived from Function (1) and represents frequency intervals corresponding to constant distances, in this case of 0.9 and 0.53 mm.

Shailer et al. (1990) and Moore (personal communication, 1989) have noted that the ERB function has too much curvature to fit ERB values at 8 and 10 kHz (see also references above to AM and QFM discrimination) and that a curve (as in Fig. 21) that is proportional to the exponential derivative of Function (1) fits their data better. Peters and Moore (1989) and Moore (personal communication, 1989) have also noted that the ERB curve does not fit low frequency ERB values (near CFs of 100 Hz) as well as the curve in Fig. 21, which simply calculates frequency intervals corresponding to a constant distance. Once more it should be said that there is no presumption that psychoacoustically significant bandwidths should necessarily correspond to equal distances. The purpose here is simply to determine, given Fig. 1 as a cochlear map, if they do or do not. Obviously, in the event that they do, it is a convenience that an 'equal-distance' curve thereby becomes an appropriate descriptive choice to summarize the data.

Critical Bandwidth and Other Δf Measures in Other Species

In the sections that follow we will consider bandwidth estimates for several non-human species. The survey does not intend to be exhaustive. The main purpose of the following comparisons is to illustrate more accurately how older data in the literature are related to frequency-position functions that now appear to be more securely determined (Greenwood, 1990).

Comments on the meaning of critical bandwidth relative to expectations of critical bandwidth values in other species

An additional purpose is to clarify the expectations that might be entertained in making such comparisons. Although among some mammalian species frequency-position functions appear to be normalizable (i.e. scaled) in respect to the coefficients of the exponentials (i.e. the slope constant, a, when position is plotted vs log frequency), although not with respect to frequency range, there is still only a hypothetical (albeit believed well founded) identification of a cochlear correlate of critical bandwidth in man to permit a possible calculated estimate of critical bandwidth in another species (Greenwood, 1974b). The correlate identified was the apical segment of the displacement envelope, which varies among species. If a salient characteristic of critical bandwidth in man were its relation to a constant distance and to some spatially determined properties of the displacement envelope (or to a spatially almost constant pattern of tonal interactions, including suppression), then we might expect it to correspond to a scaled distance in the other species (Greenwood, 1974b). This is to say that the ratio of lengths corresponding to a critical band in two species might equal the ratio of their cochlear lengths. In short, on the hypothesis of a scaled critical band, if the critical length were X mm in one species, it would obviously not be the same in another species with a cochlea of different length (and equality would also be implausible even if the distance did not scale).

To express the critical band distance as a frequency interval would then simply require that one determine the frequency interval to which the scaled, or in general different, length corresponded in the other species. This would amount to determining the appropriate constant of proportionality needed to relate critical bandwidth to the derivative of the frequency-position function in that species (Greenwood, 1974b), since the derivative itself, by definition the slope, will merely yield the frequency interval corresponding to the same unit distance (of 1 mm if the metric system is used). It was pointed out previously (Greenwood, 1961b, 1974b, 1990) and earlier in this paper that the apparent correspondence of the critical bandwidth distance in man to 1 physical unit, i.e. 1 mm, was coincidental. In short, any proportionality of critical bandwidth to the derivative will in general be with some other constant than 1, varying with the unit of measure employed and more importantly with a given species' cochlear length (meaning it will correspond to different distances). To proceed beyond these rather basic points and resume the earlier line of thought, it is important to note that although it may be plausible that critical bandwidth may be linked to some aspect(s) of the displacement envelope or in general be related to some spatial dimension of mechanical response, that aspect may not be related directly, and/or solely, to the envelope's dimensions. For an example, perhaps important also in the operation of the responsible mechanisms (together with the dimensions of displacement envelope peaks) may be the spacing within the 'telescoped' pattern of zero-crossings, which within the peak region exhibits a near constancy in the 'template' of decreasing inter-crossing distance over much of the cochlea, approximately 75% in guinea pig and quite surely in other species (Greenwood, 1977, 1980).

The question of why any particular Δf, such as critical bandwidth, that may be found to correspond to equal distances should correspond for a given species to a particular distance was not addressed until 1974 (Greenwood, 1974b). In 1961, no surmises as to what critical bandwidth in man might correspond to, either as an aspect of mechanical or neural response, were offered (Greenwood, 1961b), since no physical correlate of critical bandwidth was then readily apparent as a candidate, though my focus was on the apical segment of the displacement envelope (Greenwood, 1962). This was at a time when all dimensions of the displacement envelopes, based only on Békésy's data from cadaver cochleas (Békésy, 1960, Greenwood, 1962), were apparently much larger than than the approximately 1 mm distance to which critical bandwidth appeared to correspond. Hence, in addition, no surmises were offered in 1961 as to what distance, or constant of proportionality to the frequency-position function derivative, might be appropriate in any other species, for which in any case no critical bandwidth data were then available. However, in Greenwood (1971, 1974b) a direct association was suggested between critical bandwidth and the apical segment of the displacement envelope. The latter had appeared to be approximately scale-related among Békésy's cochleas (Greenwood, 1962) and was by 1967 to 1971 known to be much smaller in living cochleas (Johnstone and Boyle, 1967; Rhode, 1971). If it remained scaled among living species, in man it would appear to be in the neighborhood of 1.25 mm, when scaled from values of about 0.66 and at least 0.7 mm in guinea pig and squirrel monkey, respectively (Greenwood, 1974b, 1977, 1980).

This identification of critical bandwidth with the apical segment was further linked to the question of what physical distance and frequency interval was required (a) to shift combination tone psychophysical detection (Greenwood, 1971, 1972b) away from most of the apical-side masking influence of the lower of two primary tones, (b) to shift combination-tone primary-neuron driving (Greenwood et al., 1976) away from the excitatory effects of the lower of the primary tones, and (c) to shift a rate-suppressor tone basally away from a neuron's CF as far as the tip of the upper suppressive area as conventionally defined by average rate. Critical bandwidth was also linked (Greenwood, 1974b, 1977, 1980) to the distance in each direction from the position of maximum amplitude, which defined two points that subtended the total distance over which nonlinear input-output basilar amplitude functions were measured and over which the cumulative phase curve, having 'bent' at the point on the basal side of the maximum, was following a steeper and frequency-dispersive section (Rhode, 1971; Greenwood, 1974b, 1977, 1980). Thusly, a total, approximately two-critical-bandwidth, distance is considered to extend (in humans) about 1.065 mm basally and 1.25 mm apically from position of maximum amplitude.

However, real knowledge of the mechanisms involved, or detailed modeling of prospective mechanisms, together with knowledge of any task-specific determinants of the particular estimates obtained under a given set of psychoacoustic conditions, will have to be obtained before cross-species comparisons of the distances found to correspond to the psychophysical Δf measures actually obtained in another species will acquire much interpretability. Moreover, aside from differences (between animal experiments) in the task-specific determinants of results, the variability that characterizes some of the animal psychophysical data that we will consider will be itself an impediment to any easy interpretation of cross-species comparisons. And, although it is still worth addressing the simpler question as to whether bandwidths correspond to a constant distance (are proportional to the derivative of an animal's frequency-position function) – even without a capacity to interpret the significance of the actual distances, or other functional relations, revealed – the variability of some of the animal data compromises even this lesser goal. *

Relation between Critical Bandwidth and Critical Ratio in Other Species

In humans it has been found empirically that critical ratio measurements (signal threshold/spectrum level) in dB, when converted to bandwidths, yield frequency intervals smaller by about 2.5 than the 'classical' critical band values in much of the frequency range above 500 Hz (Scharf, 1970); in the low frequencies critical ratio was larger. What this relationship means in humans and how it applies to other species is not addressed in this paper. It has been assumed by others, in measuring critical ratios in other species, that the same relation might hold, but this is an assumption for which it might be useful to seek a more-than-empirical justification before expecting too firmly that it applies to another species. Be

* But in addressing the simpler question of correspondence to a constant distance, one cannot, to much useful effect, compare two species by plotting critical bandwidth for the two on the same log frequency scale. The better test and display of correspondence to a constant distance – that is, of proportionality of bandwidths to the derivative of the frequency-position function – is most easily accomplished by plotting critical bandwidth data on separate cochlear frequency scales, where any set of frequency intervals that happen to correspond to a constant distance will generate a straight line with the same slope as the value of the coefficient of the exponential in the frequency-position function used to plot the abscissa according to cochlear distance. If two such abscissas, or normalized distance abcissas, are made to occupy the same physical distance on the graph, bandwidths corresponding to constant distances will fall on lines both straight and parallel, although representing cochlear maps covering different frequency ranges. But if a log frequency abscissa is used, the bandwidth data of two species will not be properly compared (in regard to these matters) unless the two frequency ranges represented are taken between corresponding cochlear end-points (same normalized positions, hence corresponding to different frequencies) in the two cochleas and are made to subtend the same physical length on the graph, i.e. are in this way normalized. Bandwidths that, for each species, correspond to constant distances will then appear as parallel and curving lines, but this outcome is not a sufficient test since they must, as above, be shown to have the correct slope. A calculated curve plotting frequency intervals corresponding to a constant distance is still required to coincide with, or be parallel to, the empirical curves.

that as it may, in treating the critical ratio data in the comparisons to follow, the bandwidth values are merely converted here to distances for comparison to the equal-distance hypothesis, without any presumption as to their proper relation to critical bandwidth as measured by any of the 'direct' methods. It will be seen that critical bandwidth and critical ratio estimates are sufficiently variable within some experiments and between some experiments, that performance-task dependencies and methodological factors, not to mention differences in stimulus delivery and differences between species (or even subjects), may be quite large enough to preclude a secure conversion from critical ratio to critical bandwidth in animals – even if the approximate empirical relationship in human psychophysics were theoretically founded and even if its automatic extension to other species were also known to be justifiable theoretically or confirmed empirically in some instances.

Cat: Critical bandwidth and critical ratio

Although there are greater uncertainties as to the interpretation of critical bandwidth in other species, to examine again the comparison made by Pickles (1975) of his direct estimates of critical bandwidth (via band-widening) with equal distances on the membrane may be particularly useful, given that the precise determination of basilar membrane coordinates in the cat afforded by Liberman's (1982) data reduces one source of uncertainty. At four center frequencies from 1 to 16 kHz Pickles' estimates of critical bandwidths do not now correspond to distances differing greatly from the mean distance he had earlier calculated with the frequency-position function suggested in 1961 (which employed a 22 mm cochlear length and a 52 kHz upper frequency limit (Greenwood,

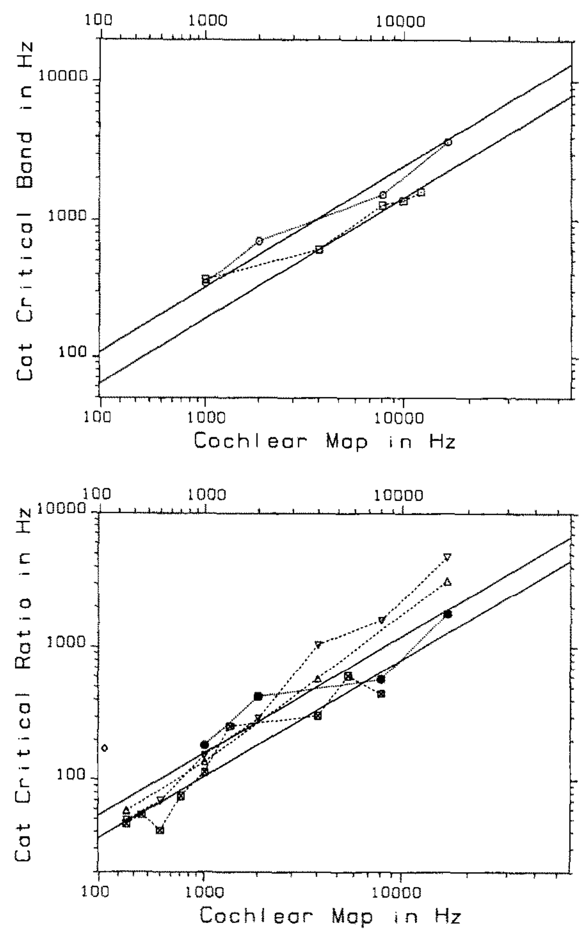


Fig. 22. (Top panel) Estimates of critical bandwidth (Pickles, 1975), compared with equal distances on the membrane, at four center frequencies of 1, 2, 8 and 16 kHz (circles). The mean distance is about 1.22 mm. Nienhuys and Clark (1979) report critical band estimates (squares), from similar experiments at frequencies of 1, 4, 8, 10, and 12 kHz in four cats. At 1 kHz their estimates, with a mean of 370 Hz, correspond to 1.37 mm, agreeing closely with Pickles, but at 4 to 12 kHz the bandwidths correspond to distances with a mean of 0.72 mm. This frequency range represents about 5.7 mm of the cochlea, or about 23%, with 4 kHz near the middle. (Bottom panel) Critical ratio measurements in the cat by Watson (1963) and Pickles (1975). Watson used bands of noise of one (inverted triangles), two (triangles), and three to five (filled squares) octaves in width, delivered through more than one type of speaker. Bandwidth estimates were obtained with directional (cone) speakers (both types of triangles) and more dispersive speakers (filled squares). Pickles, to compare his results with those critical ratios obtained by Watson using wide-band noise through more dispersive speakers (squares), used a free-field shuttle box similar to Watson's and took special care (a) that the sound field might promote a similar degree of uniformity at high frequencies by using similar dispersive speakers and (b) that the cat's head should be in a constant position at the moment the stimuli were presented. The Δfs represented by the two solid reference lines correspond to constant distances of 0.6 mm and 0.4 mm, close to the mean distances calculated from Pickles' bandwidths (solid circles, SD - about 0.2 mm) and from Watson's bandwidths (SD - about 0.14 mm) obtained with wide-band noises.

1961b)), but at 8 and 16 kHz they represent a somewhat shorter distance than at 1 and 2 kHz.

Because the cat frequency-position function is unchanged in form and its slope value has scaled exactly with the increase in the revised length of the cat basilar membrane (i.e. remained 2.1 in the proportional form), Pickles' original comparison remains the same in respect to the relative lengths of the distances corresponding to the bandwidths determined at the four center frequencies. The calculated distances merely increase somewhat owing to the revised cochlear length of 25 mm and extended upper frequency limit of about 57 kHz (Liberman, 1982). Their mean is about 1.22 mm; the mean at the 1 and 2 kHz center frequencies is about 1.4 mm and about 1.06 mm at 8 and 16 kHz. Fig. 22, top graph, plots Pickles' critical band estimates (circles), as Δf and as corresponding distance, versus basilar membrane coordinates. Subsequently, Pickles (1979) compared three direct methods (the original and two more) of estimating critical bandwidth at the center frequencies of 1 and 2 kHz. The mean corresponding distance at 1 and 2 kHz is 1.5 mm in this experimental series, without consistent differences across methods.

Nienhuys and Clark (1979) have also reported critical band estimates obtained in band widening experiments similar to Pickles' at frequencies of 1, 4, 8, 10, and 12 kHz in four cats. At 1 kHz their estimates, with a mean of 370 Hz, agree very closely with those of Pickles, corresponding to a distance of 1.37 mm at that frequency, but at 8 kHz their estimates are somewhat smaller. At the four higher frequencies of 4 to 12 kHz the bandwidths correspond to very similar distances with a mean of 0.72 mm. Their results (squares) are added to Fig. 22, top. This range of center frequencies represents about 5.7 mm along the basilar membrane, or about 23%, with 4 kHz near the middle.

How do the average critical band distances above compare to critical band distances scaled from man (Greenwood, 1974b), assuming basilar membrane lengths of 25 mm and 35 mm in cat and man, respectively? If the critical band in man corresponded to a 1.065 mm distance, the scaled critical band distance in cat would be 0.76 mm; a 1.25 mm distance in man would scale to about 0.89 mm in cat. The comparisons in hand are variable. The critical bandwidths measured by Pickles above correspond to distances which decrease somewhat with frequency, with a mean among the 1975 data of 1.22 mm. However, the estimates of Nienhuys and Clark at higher frequencies corresponded to the lesser distance of about 0.72 mm. There is also a small quantity of neurophysiological data from the cat (Greenwood et al., 1976) which has suggested a relation to critical bandwidth and which indicates an average distance of about 0.9 mm between a single neuron's CF and the low point of the rate-suppression side-band on the high frequency side of the tuning curve (a distance that has increased from the lesser value given in 1976 because of the revision of cat cochlear length from 22 to 25 mm by Liberman (1982)). Thus, by chance or otherwise, the 0.72 mm (Nienhuys and Clark) and 0.9 mm (Greenwood et. al) distances are close to the scaled basal and apical counterparts, 0.76 and 0.89 mm, of the corresponding 1.065 and 1.25 mm distances in man, but the larger values in the lower frequencies and obtained by Pickles at all frequencies remain divergent. How much precision is possible and what would satisfy? Before there is either satisfaction or undue interpretation of these cat critical distances, the other animal data should be considered.

In respect to the variation with frequency that is present in both sets of psychophysical data, it is perhaps relevant to recall that the frequencies of 1 and 2 kHz in the cat correspond, in respect to cochlear position, to the frequencies of about 0.33 and 0.7 kHz in human, which produce their major effects at fairly apical locations, whereas 4 kHz in the cat corresponds in position to about 1.45 kHz in man. Is this a fact relevant to the somewhat larger estimates, as distances, of Pickles at 1 and 2 kHz and of Nienhuys and Clark at 1 kHz? In the apical region in man, that is, below about 0.4 kHz, three sets of critical band measurements (Figs. 10–13) diverge from the distances to which they correspond above 0.4 kHz, whereas in the basal 70 percent of the human cochlea above 0.4 kHz each set of measurements shows a more nearly uniform correspondence to an approximately constant distance.

Critical ratio measurements in the cat were reported in 1963 by Watson and reviewed by Pickles (1975), along with his own estimates. Watson had used bands of noise of one, two, and three to five octaves in width, delivered through more than one type of speaker. Pickles noted that the critical ratios obtained by Watson using wide-band noise delivered through more dispersive speakers increased with frequency less than ratios obtained with the narrower widths of noise delivered through more directional speakers, and that the ratios obtained with wide-bands were the most similar to Pickles' ratios at higher frequencies. Although Watson had felt (for non-methodological reasons) that the measurements obtained with wide-band noise at three frequencies at and above 4 kHz were least likely to be reliable, Pickles suggested instead (for methodological reasons) that those three higher frequency data points, obtained with wider bands delivered through a loudspeaker that should have produced a more even sound field (similar to Pickles' own), may have been more accurate measurements than the other data points obtained with the more directional cone speakers used in the same frequency range for the narrower noises.

In support of his suggestion, Pickles' experimental test, when using a free-field shuttle box and cone speakers such as used by Watson with the one and two octave bands of noise, produced 'much higher' thresholds at the high frequencies in question. When special care was taken that the sound field, generated by more dispersive speakers, should be more uniform at high frequencies and, perhaps as important, that the cat's head should be in a constant position at the moment the stimuli were presented, Pickles' estimates at 8 and 16 kHz averaged 4 dB less than Watson's values obtained with narrow-band noise and cone speakers at the same frequencies. They were thus in closer agreement with Watson's wide-band estimates at 4, 5.6 and 8 kHz obtained with transducers similar to Pickles' dispersive speakers.

Fig. 22 (bottom) plots all of Watson's and Pickles' CR estimates, converted to Δfs, versus basilar membrane position. In these plots the two solid reference lines correspond to constant distances of 0.6 mm and 0.4 mm, close to the mean distances calculated from Pickles' bandwidths (solid circles) and from Watson's bandwidths obtained with wide-band noises. The bandwidths obtained by Watson with wide-band noises (three or more octaves) are indicated by the solid squares; with two-octave noises are shown by triangles; with one-octave noises are shown by inverted triangles. Whatever the explanations, methodological or otherwise, the critical ratios exhibit considerable variation among themselves and from a constant distance. For Pickles' ratios the standard deviation among the distances was about 0.2 mm. Like the critical band values, the corresponding distances were somewhat less at 8 and 16 kHz. The distances associated with ratios obtained by Watson with the wide-band noises exhibit the least overall slope and a standard deviation of about 0.14 mm. Whatever equivocality of conclusion accompanies these comparisons, presumably the major uncertainty derives from the data themselves, next from the not yet determined degree of conformance to equal distances or to some other relation, and least from the conversion of frequency intervals to corresponding distances, since the cochlear map for the cat appears at this juncture to be quite unambiguously specified by Liberman's correlations of cochlear place with primary fiber CF and is well corroborated in slope by other data (Liberman, 1982; Wilson and Evans, 1977; Greenwood, 1961b, 1990).

Since Pickles (1975) has also demonstrated what appear to be fairly parallel courses between his cat critical band measurements and effective tuning curve bandwidths and Q_{10s}, it would appear that the single unit measures may possess a similar relationship to cochlear position, but the scatter in such measures makes merely visual judgments ambiguous.

Monkey: Critical bandwidth

Gourevitch (1970) has reported direct estimates of critical bandwidth in a species of monkey (*M. nemestrina*), using two masking methods. In one method bands of noise were widened geometrically around a signal frequency whose masked threshold was measured. In the second two tones masked a third centered geometrically between them, as a function of their increasing separation. The data from two monkeys, circles for one and squares for the other, are plotted in Fig. 23. Estimates at 1 and 5 kHz were obtained with noise

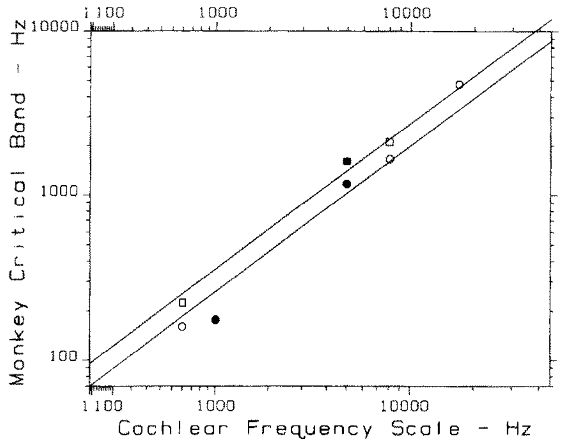


Fig. 23. Estimates of critical bandwidth in a species of monkey (*M. nemestrina*), using two masking methods (Gourevitch, 1970, his Fig. 15). In one method bands of noise were widened geometrically around signal frequencies at 1 and 5 kHz (filled symbols). In the second two tones masked a third centered geometrically between them at 0.6, 8, and 17 kHz (open symbols), as a function of their increasing separation. The data from two monkeys, circles for one and squares for the other, are plotted. The solid reference lines indicate bandwidths corresponding to constant distances of about 1 and 1.4 mm, which are close to the mean values for the two monkeys. As before, the lines are intended simply to indicate the relation of the bandwidths to position. The distances for at least one monkey seem to increase somewhat with frequency if the cochlear map is correct and the data are reliable. Existing data relating cochlear position to frequency cover the frequency range from 2 to 16 kHz (Stebbins and Moody, 1979). The frequencies of 1 and 2 kHz, for example, would fall at about 27% and 39% of the distance from apex to base if the function suggested by Greenwood (1990), and used here, describes *M. nemestrina's* cochlear map.

maskers, indicated by filled symbols; whereas those at 0.6, 8, and 17 kHz (open symbols) were obtained with two-tone maskers. These estimates were obtained from Gourevitch's Figure 15. It can be seen that one monkey's values (circles) seem lower than the other's at most frequencies. The solid reference lines indicate bandwidths corresponding to constant distances of about 1 and 1.4 mm, which are close to the mean values for the two monkeys.

As before, the lines are intended simply to indicate the relation of the bandwidths to cochlear distance versus position. More psychoacoustic data would be required to draw firm conclusions about the functional relation of these bandwidths to cochlear position or distances, even assuming there were physiological data to support the frequency-position function below 2 kHz in the cochlea, which is not the case. Existing data cover only the frequency range from 2 to 16 kHz (Stebbins and Moody, 1979). The frequencies of 1 and 2 kHz would fall at about 27 and 39% of the distance from apex to base if the function suggested by Greenwood (1990), and used here, describes M. Nemestrina's cochlear map.

Rat and Mouse: Critical ratio

Gourevitch (1965, 1970) has also reported critical ratio estimates from another species, the rat

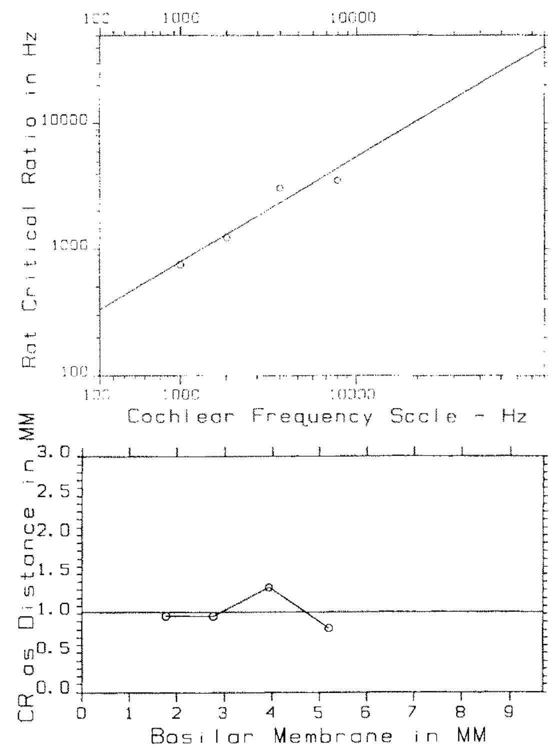


Fig. 24. (Top panel) Critical ratio estimates in the rat (five to seven animals), in the frequency region from 1 to 8 kHz (Gourevitch, 1965, 1970). Gourevitch had found in 1965, using an earlier frequency-position function (Greenwood, 1961b) that these critical ratios, when converted to bandwidths, appeared to correspond to fairly comparable distances. Given a revised function (Greenwood, 1990), fitting both Békésy's data and the upper frequency limit of 80 kHz suggested by Kelly and Masterson (1977), the estimates correspond somewhat more closely to equal distances. These data are plotted according to this newer function. (Bottom panel) The bandwidths are plotted as distances versus cochlear position. The distances they correspond to at the four frequencies of 1, 2, 4, and 8 kHz are 0.97, 0.97, 1.33, and 0.82 mm, with a mean of 1.02 mm.

(five to seven animals), in the frequency region from 1 to 8 kHz, that is, over a distance of about 3.4 mm starting at a point about 15% of the cochlea's length measured from the apex and ending at the approximate middle, an approximate third of the cochlea. In effect, he had found in 1965, using a frequency-position function suggested in 1961 as a possible fit to Békésy's data on the rat, that the critical ratios, when converted to bandwidths, corresponded to fairly comparable distances. When the function suggested in this paper, as a fit both to Békésy's data and to the upper frequency limit of 80 kHz suggested by Kelly and Masterson (1977), is used instead, the estimates correspond somewhat more closely to equal distances. These data are plotted according

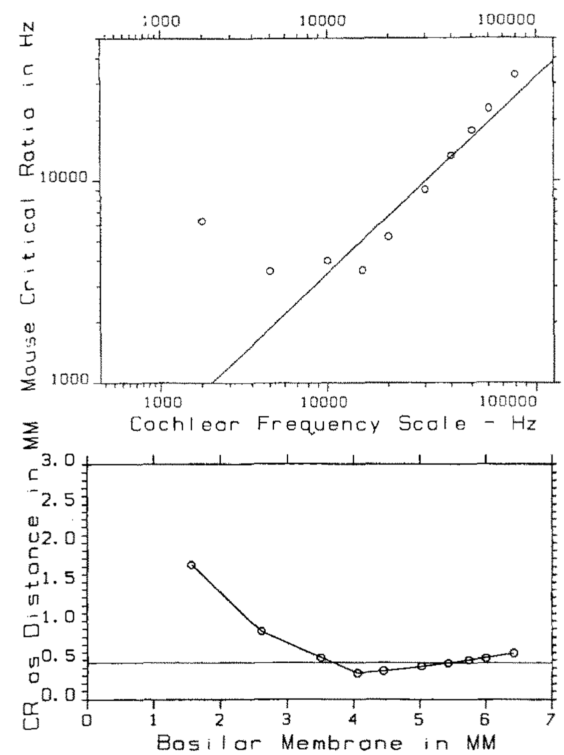


Fig. 25. (Top panel) Critical ratio estimates for thirteen house mice (Ehret, 1975), converted to bandwidths, using the frequency-position function suggested elsewhere (Greenwood, 1990). The converted CRs, depart systematically, from a constant distance in the low frequencies, in a way similar to the human CR data. Above 10 kHz the departure from a reference distance of about 0.465 mm is systematic and smaller. (Bottom panel) Same estimates plotted as distances versus cochlear position.

to this newer function in Fig. 24. The distances they correspond to at the four frequencies of I, 2, 4, and 8 kHz are 0.97, 0.97, 1.33, and 0.82 mm, with a mean of 1.02 mm. This is much more than a scaled distance from man or cat, and it would be greater still if critical bandwidth in the rat were 2.5 times critical ratio.

Critical ratio estimates for thirteen house mice have been reported by Ehret (1975). If we use the frequency-position function suggested here and elsewhere (Greenwood, 1990), which employs a scaled slope constant, is parallel to Békésy's data, and fits the upper frequency limit of 120 kHz suggested by Ehret (1975), the converted CRs in Fig. 25 depart systematically from a constant distance in the low frequencies in a way similar to the human CR data. Above 10 kHz the departure from a constant distance of about 0.465 mm is systematic, not large as a distance, but somewhat more as a percentage. Again for a short cochlea this is much more than a scaled distance from man or cat, yet by a lesser percentage than in the rat.

Chinchilla: Critical ratio and critical bandwidth

Three bodies of data provide estimates of critical ratios in the chinchilla. The mean estimates for 6 chinchillas provided by Miller (1964) are taken from Scharf (1970, Fig. 8) and appear in Fig. 26 as circles connected by a dotted line. The estimates of Seaton and Trahiotis (1975) are those of six 'well practiced' chinchillas and appear in Fig. 26 as squares with a dashed line. These estimates are converted to bandwidths and plotted against basilar membrane coordinates using Function (1) that has been shown in a previous paper (Greenwood, 1990) to fit the data of Eldredge et al. (1980) and Robles et al (1986); in this paper the value of k is changed from 0.85 to 1, which produces no significant difference at or above 500 Hz in either the frequency-position function or Fig. 26. The solid reference line corresponds to about 1 mm, an unscaled distance.

Critical bandwidth estimates provided by Seaton and Trahiotis (1975) are plotted in the bottom graph in Fig. 26. Added to the bottom graph are two estimates provided by Clark et al. (1975). Using bands of noise to mask geometrically centered signals of 1 kHz, they obtained for four

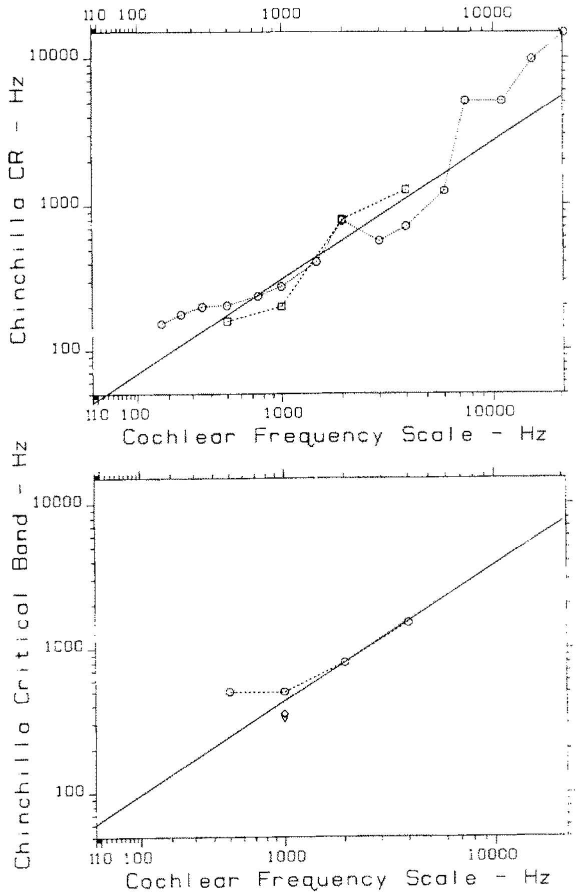


Fig. 26. (Top panel) Critical ratios in the chinchilla. The mean estimates for 6 chinchillas (Miller, 1964) are taken from Scharf (1970, Fig. 8) and appear as circles connected by a dotted line. Estimates by Seaton and Trahiotis (1975) are those of six 'well practiced' chinchillas and appear as squares with a dashed line. These estimates are converted to bandwidths and plotted on cochlear coordinates using Function (1) that was shown previously (Greenwood, 1990) to fit the data of Eldredge et al. (1980) and Robles et al. (1986). The solid reference line corresponds to a distance of about 1 mm. (Bottom panel) Critical bandwidth estimates by Seaton and Trahiotis (1975) are plotted as circles and a dotted line. Added are two estimates by Clark et al. (1975) who used bands of noise to mask geometrically centered signals at 1 kHz. For four chinchillas estimated critical bandwidths ranged from 205 to 500 Hz. Two tones were also used to mask a centered 1 kHz signal; the range of estimates was from 320 to 380 Hz. Geometric means of these two ranges of critical bandwidths, with noise (triangle) and with two tones (rhomboid), were 320 and 349 Hz, respectively. The solid reference line in the bottom graph corresponds to a distance of 1.4 mm.

chinchillas a range of estimated critical bandwidths from 205 to 500 Hz. Using two tones to mask a centered 1 kHz signal, they also obtained a range of estimates from 320 to 380 Hz. The geometric means of these ranges of critical bandwidths obtained with noise and with two tones were 320 and 349 Hz, respectively, and are plotted as the triangle and trapezoid. The solid reference line in the bottom graph corresponds to a distance of 1.4 mm.

Summary and Discussion

Initial basic purpose of the foregoing comparisons

Construction in 1961 of a potentially useful and possibly accurate empirical frequency-position function, supported since then by additional observations (Greenwood, 1990), created the opportunity to compare critical and other bandwidth estimates, obtained in various performance tasks, to distances on Bekesy's cochlear map as fitted by Function (1). This task, begun in 1961 and extended in intervening years by this author, has been reported here on the working, though still tentative, assumption that the map is basically correct. Certainly if a cochlear frequency-position function were accurate, it would have the capacity to test how any psychoacoustically significant bandwidths – such as critical bandwidth – are actually related to cochlear position and to what extent they may in fact correspond to a constant distance or to a varying distance with position. Clearly also, the fact that some sets of estimates differ systematically from each other means that some sets at least must correspond to a varying distance, an outcome that may depend on whatever other factors may be operative in the particular discriminatory or judgmental task set for the psychophysical observer, in addition to various possible physio-anatomical factors. The equal-distance hypothesis tested, it should be recalled, was and remains a tentative empirical induction, although it now indeed seems quite well supported over much of the frequency range in several experimental contexts.

But any Δf measure such as critical bandwidth, critical ratio, frequency DL, or other measure could conceivably be functionally dependent on more than (or other than) the spatial shift or separation of displacement patterns. For example, if perhaps there were dependencies on displacement pattern slopes, neural excitation pattern slopes, on cochlear innervation density, the spacing and telescoping pattern of zero-crossings, or on temporal factors, to name a few candidates, then plots of bandwidth values on cochlear coordinates could help to indicate the existence of such other dependencies.

Thus, the curves calculated from the map and compared with bandwidth data in the preceding figures simply represent frequency intervals corresponding to specific distances. They are neither predictions nor fits to the data. They are intended merely to assess the relation of the bandwidths to a constant distance or to show a changing relation to cochlear position, as the case may be. When the bandwidths do not fall on the line, they simply do not correspond to the distance represented by the line. When this is discovered, the measured bandwidths do not necessarily require either cutting or stretching to eliminate a presumptive 'error' nor is the assessing frequency-position function found thereby to be necessarily defective. Although the cochlear map may itself require modification, it presently seems to be reasonably well founded on physiological data and can be used provisionally and revocably.

Conditioned as we are to the expectation that a curve ought to fit data, it might be easier to keep in mind that the curves calculated from the frequency-position function are being used as measurement devices if data and calculations were compared differently. Most of the calculated reference curves used in the foregoing figures to assess the relation of bandwidth to distance do provide a reasonable measure of the distance to which the bandwidths correspond, when and if the data points fall accurately on or very near the line. But when points do not fall on the line, the juxtaposition of data and calculated curves obviously express an inequality; the empirical bandwidths correspond to distances greater than, or less than, the reference distance to which the calculated line corresponds. Nevertheless such reference curves were used because plotting the data in their original form as frequency intervals on a log scale is more recognizable and familiar, and because space was lacking to plot these bandwidths also, in companion figures, as the distances to which they correspond. However, the latter type of plots – better for measurement purposes – appeared in some of the figures plotting data of other species (Figs. 25 and 26).

Of course, data other than bandwidth estimates such as tuning curves, excitation pattern plots, spatial-analogues of Q_{10} (that is, normalized distance or D-10), slopes in dB/mm etc., may be usefully plotted on a cochlear frequency scale, as well as on conventional linear and log frequency scales that may be more appropriate for other purposes. This will further the task long urged by multiple authors to re-construct and visualize patterns of excitation on spatial coordinates (Allanson and Whitfield, 1955; Greenwood and Maruyama, 1965; Greenwood and Goldberg, 1970, Greenwood et al., 1976; Zwicker, 1969/70; Kohllöffel, 1971; Pfeiffer and Kim, 1975; Young and Sachs, 1979; Moore and Glasberg, 1983; Shamma, 1985).

Outcomes of the foregoing comparisons

Only the human data will be reviewed. Starting with the 'classical' critical bandwidths, we have found that above 300 to 500 Hz, they in fact agree quite well with equal distances on the Békésy-Skarstein cochlear map, despite the earlier erroneous conclusion that they diverged above 3 kHz, based on the 'classical' critical band curve rather than the data themselves (Greenwood, 1961b). For the two subjects in the threshold-critical band experiment (Fig. 11), the bandwidths conform reasonably well to 1.0 mm and 1.4 mm, with an average of about 1.2 mm. Such curvature as there is in the data does not introduce much variation from this average distance, which also agrees quite well with the average curve published by Gässler (1954). Below 400 Hz, the data bandwidths correspond to distances somewhat greater than the constant distance indicated by the calculated curve. Similarly, in the two-tone masking experiments (Zwicker, 1954) the bandwidths for the same two subjects correspond at and above about 300 Hz to distances of about 1.3 mm and 1.2 mm, and the mean of 1.25 mm agrees very closely with the average curve Zwicker offered in that paper.

Below 300 Hz the bandwidths correspond to larger distances and are similar to those in Gässler's paper. In the AM-QFM paper (Zwicker, 1952), from 500 to 5000 Hz the mean bandwidths of four subjects conform well to a distance of about 1.15 mm. At higher frequencies there is only a single point at 8 kHz that indicates a greater distance. As we have seen, the more recent data from Zwicker's laboratory (Schorer, 1986) would revise the bandwidth at 8 kHz and bring the upper curve into good agreement with equal distances. Below 500 Hz the bandwidths are again greater than equal distances and similar to those in the two experiments above. The data of Fig. 14 on the critical bandwidths obtained in loudness experiments agree very well with a distance of about 1.15 mm from 500 Hz to 5 kHz, but the data do not extend below or above those frequencies. In short, the 'classical' critical band data, as opposed to the critical band curve, correspond fairly closely to constant distances from about 300 or 500 Hz to about 5 or 10 kHz, depending on the experiment and tested frequency range – a distance of about 15 to 22 mm or about 42 to 64% of the cochlea. (However, see Footnote p. 180).

As shown in Fig. 15, the shape of the 'classical' critical band curve (and tables) originated (Feldtkeller and Zwicker, 1953; Gässler, 1954; Abb. 8) from an average graphical derivative of two frequency-position functions based on the psychoacoustical frequency-DL data of Shower and Biddulph (1933). To obtain its present position on the ordinate, this 'Mittelwert' derivative had been translated upward (about 0.065 log units) on the abscissa, bringing it into better general agreement with the critical band data, and then modified to a slightly flatter form than the derivative (or the data) below about 300 Hz. Although brought into general agreement with the data, the ordinate range of the 'classical' curve (between abscissa values of 0.5 and 10 kHz) is greater than the range of any of the five curves on which it later seemed to be based (Zwicker et al., 1957; Figs. 11 and 12).

Thus with this overdue second test of the equal-distance hypothesis, after the first in 1961, the comparison of equal distances on a cochlear map to critical bandwidth, as envisaged by Zwicker et al. (1957), seems to have become complete so far as the pre-1957 'classical' critical band data are concerned, since the earlier comparison in 1961 of the critical band curve was more nearly a comparison, in effect, of some multiple of the frequency DLs of 1933 with cochlear distances. The present comparison of the early critical band data is conclusive in showing that they correspond much more closely, above 300 to 400 Hz, to equal distances than did the curve and hence in this frequency region are more nearly proportional to the exponential critical band function of 1961 (see Figs. 12 and 14 which best summarize these comparisons).

Recently some of the low frequency estimates of critical bandwidth were summarized by Fastl and Schorer (1986) and by ensuing comments by various symposium participants on their paper, which take cognizance that newer data, and the 'classical' data themselves, depart from the 'classical' curve and from each other in the low frequencies to varying degrees for a variety of potential reasons. However, older data were still excluded. One comment (Greenwood, 1986d) pointed out that it had long been known (Greenwood, 1961b) that one of the earliest reported sets of consonant intervals (Mayer, 1894) systematically departed from 'classical' critical bandwidths in the low frequencies and corresponded to equal distances on the map of Fig. 1 in the present paper. These and other consistent data (Cross and Goodwin, 1893; Kaestner, 1909; Guthrie and Morrill, 1928; Plomp and Levelt, 1965) were not obtained using earphones. In considering, pro and con, the role of earphone equalization, or any other factors, in determining the departure of consonance data (specifically those of Plomp and Steeneken, 1968), or ERB values (Moore and Glasberg, 1983) from critical bandwidth estimates at frequencies below 500 Hz, the fact that all the early experimental data were precisely confirmed by Plomp and Steeneken (1968) should be taken into account.

The 1959-60 critical band estimates of Greenwood (1961a,b) obviously agreed fairly well with equal distances on the Békésy-Skarstein map, having provided the original empirical basis for obtaining the derivative of Function (1). These estimates were mostly based on shape changes of masked audiograms of narrow-band noise, with two estimates based on two-tone masking, and were obtained only from about 0.45 to 6.5 kHz.

Early estimates of the masker-notch frequency interval in pure tone masking of pure tone or narrow band signals (Greenwood, 1961b; Small, 1959; Ehmer, 1959) were later extended (Greenwood, 1971). They yielded a curve of frequency intervals that corresponded well in all tested parts of the frequency range, from about 400 to 8000 Hz, either to a distance of about 1 mm or one of 1.25 mm, depending upon whether the interval was seen as extending from the masker tone upward to the low-point of the notch (or to the masked audiogram's intersection at lower levels with quiet threshold) or instead from the masker tone downward to the basilar position of the frequency of the $2f_1-f_2$ combination tone (at the approximate foot of the masked audiogram) generated when a signal was at the low point of the notch.

In respect to two-tone consonance data, it was shown in 1961 that Mayer's (1894) intervals agreed well with a constant distance of about 1.0 mm with the exception of four intervals at frequencies between about 1800 and 2700 Hz, which had corresponded to lesser distances. We have seen that the replications by Plomp and Levelt (1965) at four out of five center frequencies from 250 to 2000 Hz very well confirmed Mayer's data and the 1.0 mm distance to which Mayer's data corresponded, and confirmed also the data they republished of the 19th and early 20th century. The latter were consistent with each other and Mayer's down to about 100 Hz. Plomp and Steeneken (1968) then extended this modern confirmation of the earlier data and the distance of about 1.0 mm to frequencies well below 250 Hz and up to 3 kHz. Above 3 kHz their data began to correspond to lesser distances, as had Mayer's (1894) and Cross's and Goodwin's (1893) at lower frequencies of 1.5 and 1 kHz, respectively. In the aggregate, and with the exceptions above, the two-tone consonance data corresponded quite closely to about a 1.0 mm distance, between frequencies from about 50 Hz to 3 kHz, corresponding to about 19.5 mm or about 55% of the cochlea, comprising a larger portion of the apical cochlea than was true for critical bandwidths.

In the experiments of Plomp (1964) and Plomp and Mimpen (1968), who sought to determine in complexes of 12 harmonics the closest harmonics that could be identified with a single tone aligned with a given harmonic, the median required harmonic separations corresponded to comparable though not equal distances of about 1 and 1.2 mm, respectively. However, among the six subjects in the second experiment, when the subjects were plotted separately, some sets of frequency intervals corresponded closely to equal distances, ranging from 1.65 mm to about 1.12 mm. In contrast, similar experiments on the ability of subjects to identify which of two single tones were aligned with one of two simultaneous tones indicated that the minimum intervals required to separate the simultaneous tones did not correspond to equal distances. The latter results stand in marked contrast to all the two-tone consonance results based on judgments of consonance (non-interference) and dissonance.

The gaussian bandwidths derived from rippled noise masking in simultaneous and pulsation threshold experiments fairly closely correspond to 1.1 and 0.65 mm (Houtgast, 1977). Finally, if we consider the equivalent rectangular bandwidths (ERBs) derived from notched-noise and temporal gap masking, we find a correspondence at moderate levels of the estimated bandwidths to a distance near 0.9 mm from 100 Hz to 8 or 10 kHz, or over about 76% of the cochlea. At lower sensation levels, and over a more limited tested range from 1 to 8 or 10 kHz, the smaller ERBs obtained correspond to distances nearer about 0.5 to 0.6 mm.

At the lowest 'level' of conclusion, the comparisons made here indicate that there is considerable evidence of correspondence of psychoacoustically significant bandwidths to constant or quite comparable distances, and there are departures from such correspondence – notably in the critical band data below 300 to 400 Hz and in the consonance data above 1, 1.5, or 3 kHz (depending on the experiment) – that might facilitate discovery of the factors that may be determining the results.

What May Be Concluded About the Determinants of (Empirical) Critical Bandwidth in the Experiments Considered?

The equai-distance hypothesis

By itself, plotting critical bandwidths or consonant intervals on a cochlear frequency scale and confirming that they mostly correspond to equal distances does not provide an answer to basic questions. What do the comparisons to distance mean or suggest? What is measured in one of these experiments? That is, what properties of the system determine the outcomes of the operational definitions employed to 'measure' 'critical bandwidth' or a 'consonant' interval? In the preceding comparisons these terms were unexamined for meaning; bandwidths were taken as they stood and merely plotted and compared. Our focus was upon the two hypotheses that were explicit for the testing: that 'critical bandwidth' might correspond to a constant distance and, more specifically, that critical bandwidth might obey an exponential function, and/or that some other psychoacoustically significant bandwidths might do the same. Since in extensive frequency regions these hypotheses seem supported for several sets of data, a direction in which to seek answers to the basic questions above is suggested.

The first hypothesis merely expresses an implicit conjecture that critical bandwidth, without defining it, may depend for its size simply on the physical separation in the cochlea of the effects exerted there by the spectral components of the stimulus. This is not likely to be so unless those mechanical effects, since they are not punctate, also possess similar spatial dimensions in different parts of the cochlea, i.e. diminish to comparable extents in comparable distances at different cochlear locations. If they did not do so, then for equal separations to be equivalent in their effects it would presumably be necessary to have compensatory mechanisms to redress, or preclude the consequences of, inequalities of shape and slope in different parts of the cochlea. In any case, the equal-distance hypothesis suggests a focus on the mechanical displacement of the cochlear partition in response to the ultimately simple stimulus, a pure tone – on its envelope dimensions, slopes, and, potentially, its zero-crossing pattern. A second focus is on the spatial aspects of the interaction of two such components, since critical bandwidth has been 'measured' using a minimally complex stimulus consisting of two tones, either equal in level or with one of them very weak in respect to the other.

The hypothesis and this more explicit focus need not entail a belief that the phenomena that lead in the end to an 'observation' or 'measurement' of the 'critical band' are exclusively cochlear and mechanical. It does assume that the starting point of an explanation of critical bandwidth must lie in the periphery, if only in establishing the basic similarities and differences in the patterns upon which later mechanisms operate. And if critical bandwidth corresponds to equal distances in the 'spacing' of cochlear events, that might depend, as noted above, upon a certain degree of constancy in the dimensions of displacement envelopes, in particular the apical segment and the region distributed about the peak.

An approach that seeks to relate critical bandwidth to cochlear data and to factors identified in psychoacoustical and physiological experiments

The following paper (Greenwood, 1991) will present a review of those dimensions and of evidence of a relation of critical bandwidth to mechanical data, before turning to an analysis of the main critical band experiments that have provided the estimates corresponding to equal distances. The latter account will proceed from the observation that the critical bandwidths measured in pure tone masking (the masker-notch interval), two-tone masking, two-tone dissonance-consonance judgments, AM and QFM detection, narrow-band masking, and notched-noise (two-band) masking reflect a common pattern of nonlinear effects of approximately constant cochlear dimensions. It has been suggested that these effects are related to the 'dimensions' of the region of cochlear nonlinear input-output functions and accelerated phase accumulation seen in mechanical data plots (Greenwood, 1974b, 1977). It is further argued that the bandwidths measured are co-determined by displacement envelope dimensions (and resulting interference and excitation pattern dimensions) and combination tones, as suggested initially, to which is added a gain control (Greenwood, 1986a,b,c; Greenwood, 1988) as the effector of a pattern of suppression long known from neural data to be asymmetrical (Galambos and Davis, 1944; Sachs and Kiang, 1968; Greenwood and Goldberg, 1970; Kiang and Moxon, 1974), which puts higher frequency tones at a disadvantage and confers on combination tones the critical role that they play in the experiments listed above.

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