Project 2: Passive Radar and Motion Detection

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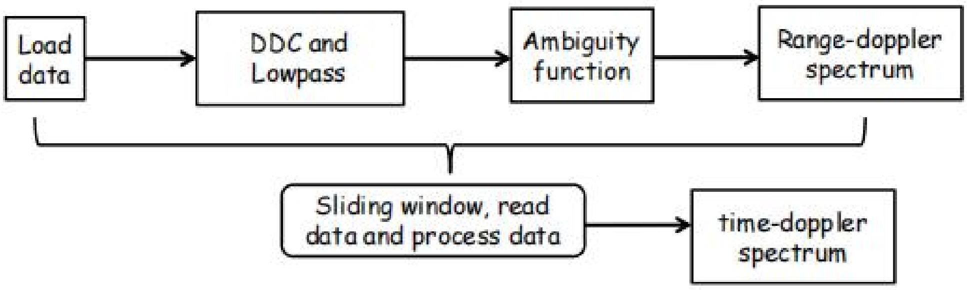
I. Introduction

(a) Principles of Passive Radar

1) Procedures



2) Relationship between TDOA and range

Range = TDOA × speed of light

TDOA is short for time difference of arrival. It reveals the difference of propagation delay between surveillance and reference signal.

3) Relationship between Doppler frequency shift and velocity

$$\mathrm{v} = \frac{\lambda * \mathrm{f_D}}{2\cos(\beta/2)}$$

β is the angle from surveillance channel to reference channel, f_D represents Doppler frequency shift, λ is the wavelength of the signal.

(b) Continuous-time Fourier Transform Realization in Matlab

$$\int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt = \int_{0}^{T} x(t)e^{-j\omega t}dt \approx \sum_{n=0}^{N-1} x(n\tau)e^{-j\omega n\tau}\tau,$$

where $T = N\tau$ and N is an integer. You can use the function `fft` to compute the sum in Eq. (4.7) for a discrete set of frequencies ω_k. If the N samples $x(n\tau)$ are stored in the vector `x` then the function call `X=tau*fft(x)` calculates

$$\begin{aligned} \mathtt{X}(k+1) &= \tau \sum_{n=0}^{N-1} x(n\tau)e^{-j\omega_k n\tau}, \quad 0 \le k < N, &(4.8)\\ &\approx X(j\omega_k), &(4.9)\end{aligned}$$

where

$$\omega_k = \begin{cases} \dfrac{2\pi k}{N\tau}, & 0 \le k \le \dfrac{N}{2}, \\ \dfrac{2\pi k}{N\tau} - \dfrac{2\pi}{\tau}, & \dfrac{N}{2}+1 \le k < N, \end{cases} \tag{4.10}$$

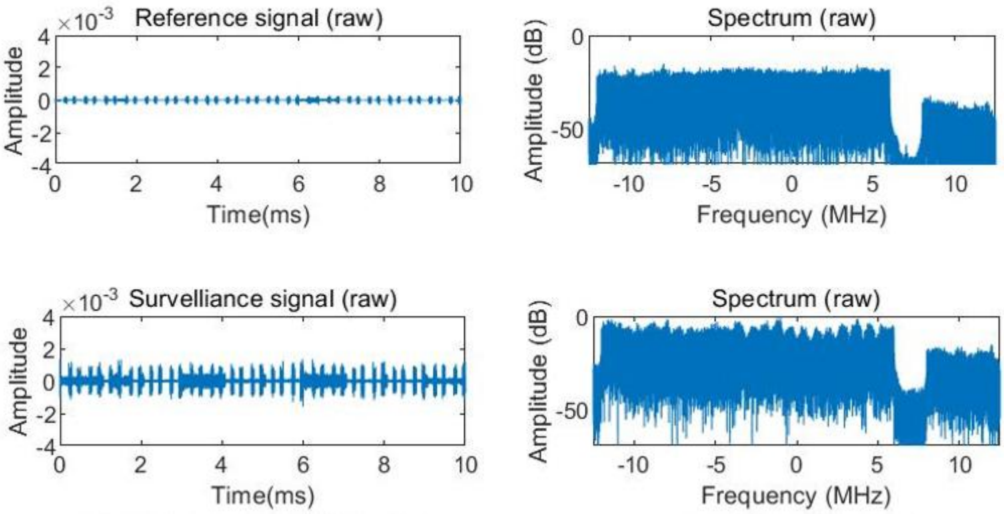
To emphasize, X is the samples of Fourier transform, Δf=Δω/2π=1/T

II. Task 1

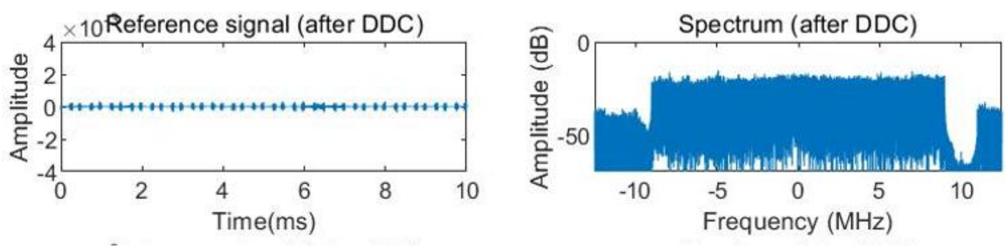
For the reference signal and the surveillance signal, draw their time domain waveform and their frequency domain waveform in their following processing process respectively: Original, After frequency conversion, After low-pass filtering

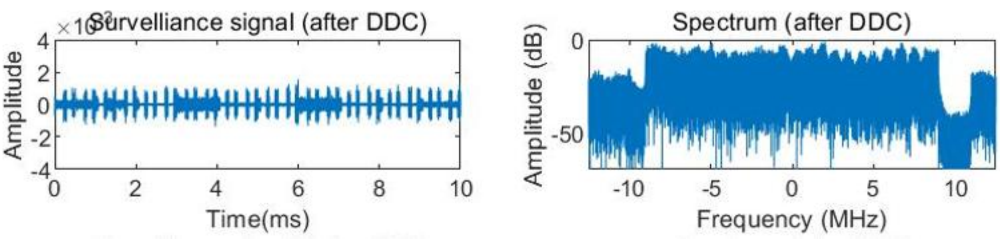
Before signal processing, we need to filter out the signal of 2130-2135MHz (5M bandwidth), and reserve the signal of 2110-2130MHz(20Mbandwidth). So we need to convert the signal 2110-2130MHz to the frequency centered on 0 Hz.Then filter out the high-frequency signal through the low-pass filter.

We take data_1.mat file as an example. This figure shows the original reference signal and the surveillance signals in the 0-0.5s required by the question. Only the first 0.01s is plotted below.

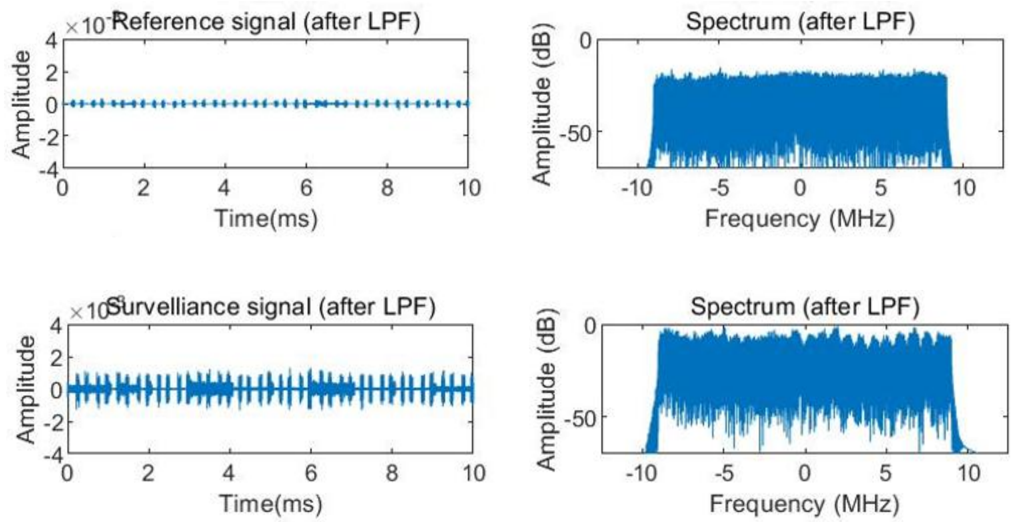


Then, to extract frequency components between 2110-2130MHz, we utilize DDC method to let 2120MHz to become the center. And since the carrier of the two original signal is 2123MHz, we need to left-shift 3MHz. The graph as followed shows the reference signal and the surveillance signals after DDC in the 0-0.5s required by the question.





Finally, to filter out the signal of 2130-2135MHz and reserve the signal of 2110-2130MHz, we used the LPF (Low pass filtering) to extract the signal spectrum between 2110MHz and 2130MHz. This figure shows the reference signal and the surveillance signals after LPF in the 0-0.5s required by the question.



As we can see, these signals are what we want.

Other 19 figures about the surveillance and reference signal between 0.5-10ss are on the appendix.

III. Task 2

Draw the Range Doppler spectrum of the signal located at 0 ~ 0.5s, 2 ~ 2.5s, 5 ~ 5.5s and 7 ~ 7.5s(data is provided), range=0:5, doppler_frequency_shift=-40:2:40

In Task2, we were asked to draw the range doppler spectrum. In task1, we obtain the signal of 2110-2130MHz (20Mbandwidth) after DDC and low-pass filter.

(a) Cross-Correlation

Now we have the reference signal and surveillance signal of 2110-2130Mhz. To compute the displacement and velocity of the object, we can find the time difference of arrival (TDOA) τ and Doppler frequency shift f_D from two processed signals with formulas provided in Introduction part.

To find τ and f_D, we can compute the cross-correlation of the two signals on frequency domain. The maximum value appears frequency domain on f_D.

(b) Comparison between Ambiguity function and FFT() method

(1) Ambiguity Function

Firstly, we calculate the ambiguity function of reference data and surveillance data by traveling all possible (τ, f_D), and select (τ', f') corresponding to the maximum value as the estimate value which τ' is the estimate of (τ_s - τ_r) and f' is the estimate of f_D. Use finite estimal method to calculate the Ambiguity Function

```
function [y] = AmbiguityFuntion(x1,x2,t,f,f_s)
    x2 = [zeros(1,ceil(t*f_s)) x2];
    y = 0;
    for i= 1:length(x1)
        y = y + x1(i).*conj(x2(i)).*exp(-1i*2*pi*f*i*(1/f_s));
    end
end
```

For continues time:

$$Cor(\tau, f_D) = \int_{t}^{t+T} y_{surv}(t) y_{ref}^{*}(t-\tau)\, e^{-j2\pi f_D t} dt$$

For discrete time:

$$Cor(\tau, f_D) = \sum_{n=0}^{N-1} y_{surv}[nT_s] y_{ref}^{*}[nT_s - \tau]\, e^{-j2\pi f_D n T_s}$$

(2) FFT Method

The ambiguity function is actually calculating the Fourier Transform of the cross-correlation. So we can directly compute the Correlation by fft() method. Since the duration of each data file is 0.5s, so the fundamental frequency is 1/0.5=2Hz. So we can fetch doppler frequency shift from -40Hz to 40Hz every 2 Hz as required. The realization code is as followed,

```
for t=1:length(tdoa)
    s_tau = [seq_sur_lpf(tdoa(t)+1:N),zeros(1,tdoa(t))];
    temp(t,:) = fftshift(fft(s_tau.*conj(seq_ref_lpf)));
end
A_TRD(idx_start_time,:,:) = temp(:, (-19+N/2:21+N/2));
```

To get the frequency components from -40Hz to 40Hz, the 41-medium components can be fetched after fftshift().

Using data_1.mat file as an example, we can get exactly the same Range-Doppler spectrum from two method. To further investigate two methods, we compare them in the next step.

(3) Complexity and Time

The calculate time of Ambiguity Function by fft() for each data. (t/s)

4.73036920000000	3.91924190000000	3.87951340000000	3.97828320000000	3.91132390000000
3.85358440000000	3.98467590000000	4.10776270000000	3.77962730000000	3.78376320000000
3.88077600000000	4.11397910000000	4.13412020000000	3.74849250000000	4.13251750000000
3.97507750000000	3.82223990000000	4.42361800000000	4.14010490000000	4.29882360000000

The calculate time of Ambiguity Function by integral for each data. (t/s)

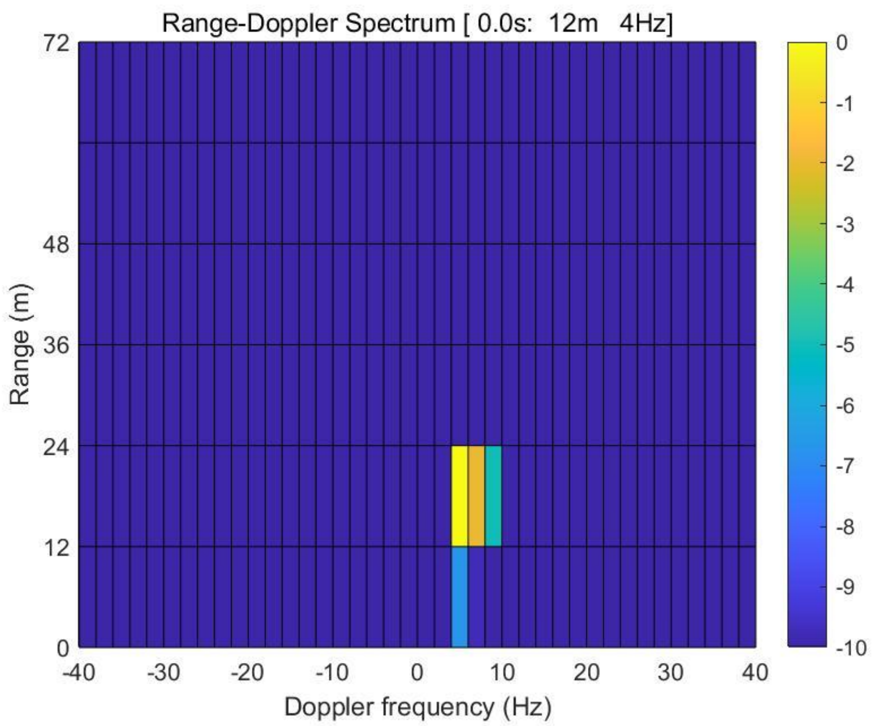
970.996146600000	964.537977000000	969.790953400000	979.566921200000	974.824273200000
967.250441500000	962.859640900000	950.646226600000	950.099349900000	949.165743100000
948.173874400000	950.717988700000	950.060159000000	948.684135400000	950.866340400000
950.984549000000	953.039819700000	952.023245300000	949.247048200000	948.405483000000

From the result, we can say that the fft() method is much more efficient than integration method.

(c) Range-Doppler Spectrum

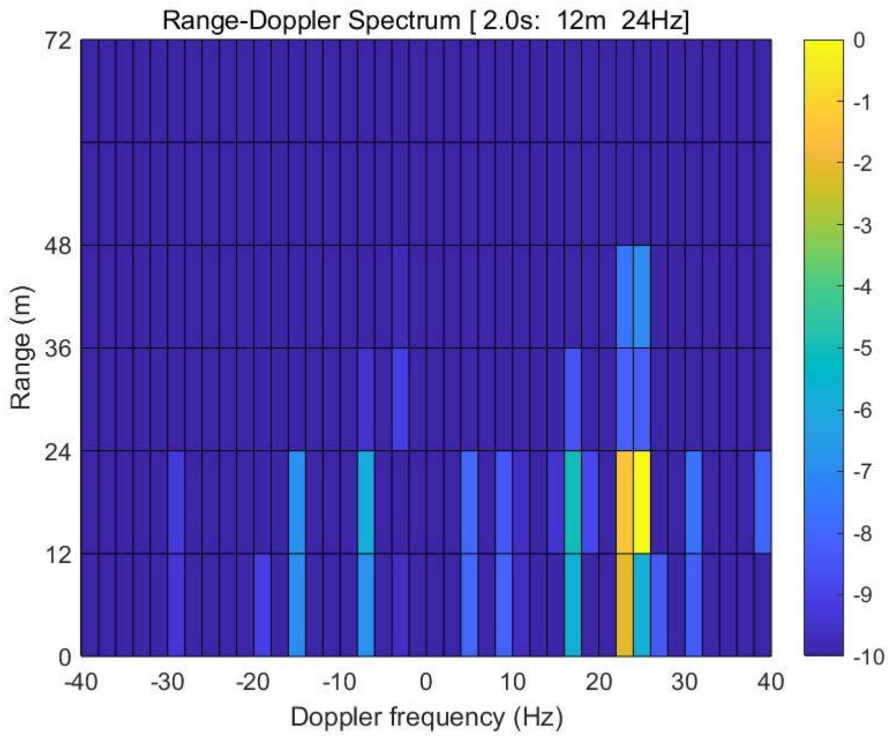
Considering the velocity of this moving object is limited, it is not necessary to traverse all the τ and f_D. Here we set the possible range of traversing—the travelling distance in 10 second is [0,72] (m), while the frequency is from -40 to 40 Hz.

1. 0-0.5 s



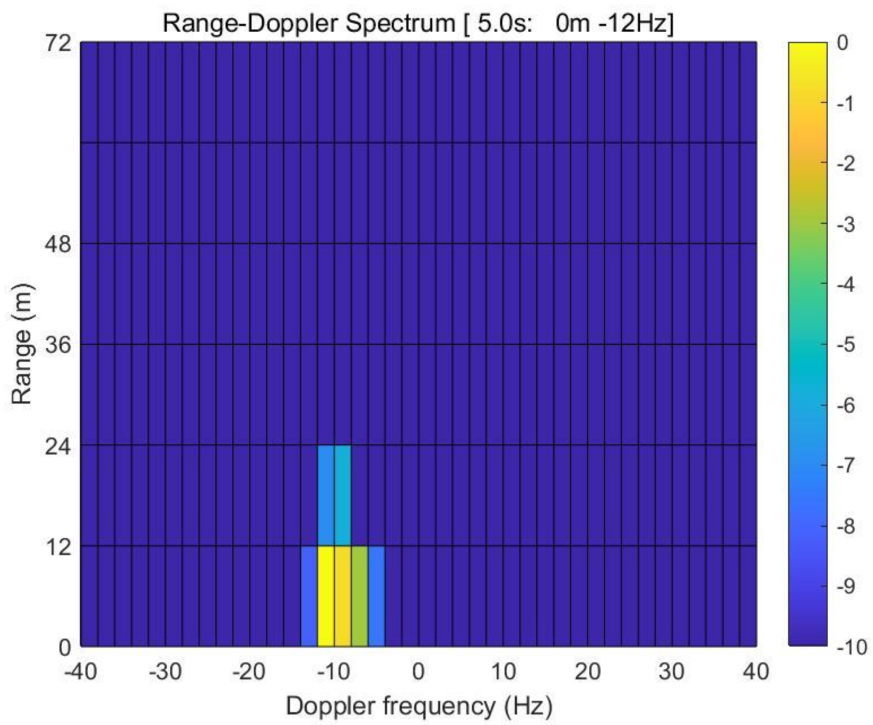
At time 0.0-0.5 s, the moving distance is around 12 m, the doppler frequency is around 4 Hz.

2. 2-2.5 s



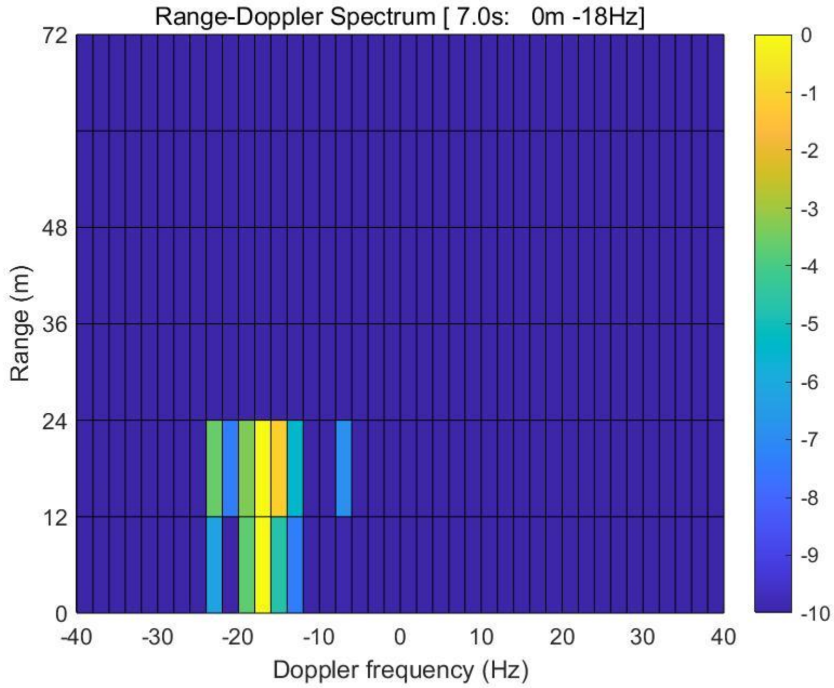
At time 2.0-2.5 s, the moving distance is around 12 m, the doppler frequency is around 24 Hz.

3. 5-5.5 s



At time 5.0-5.5 s, the moving distance is around 0m, the doppler frequency is around -12 Hz.

4. 7-7.5 s

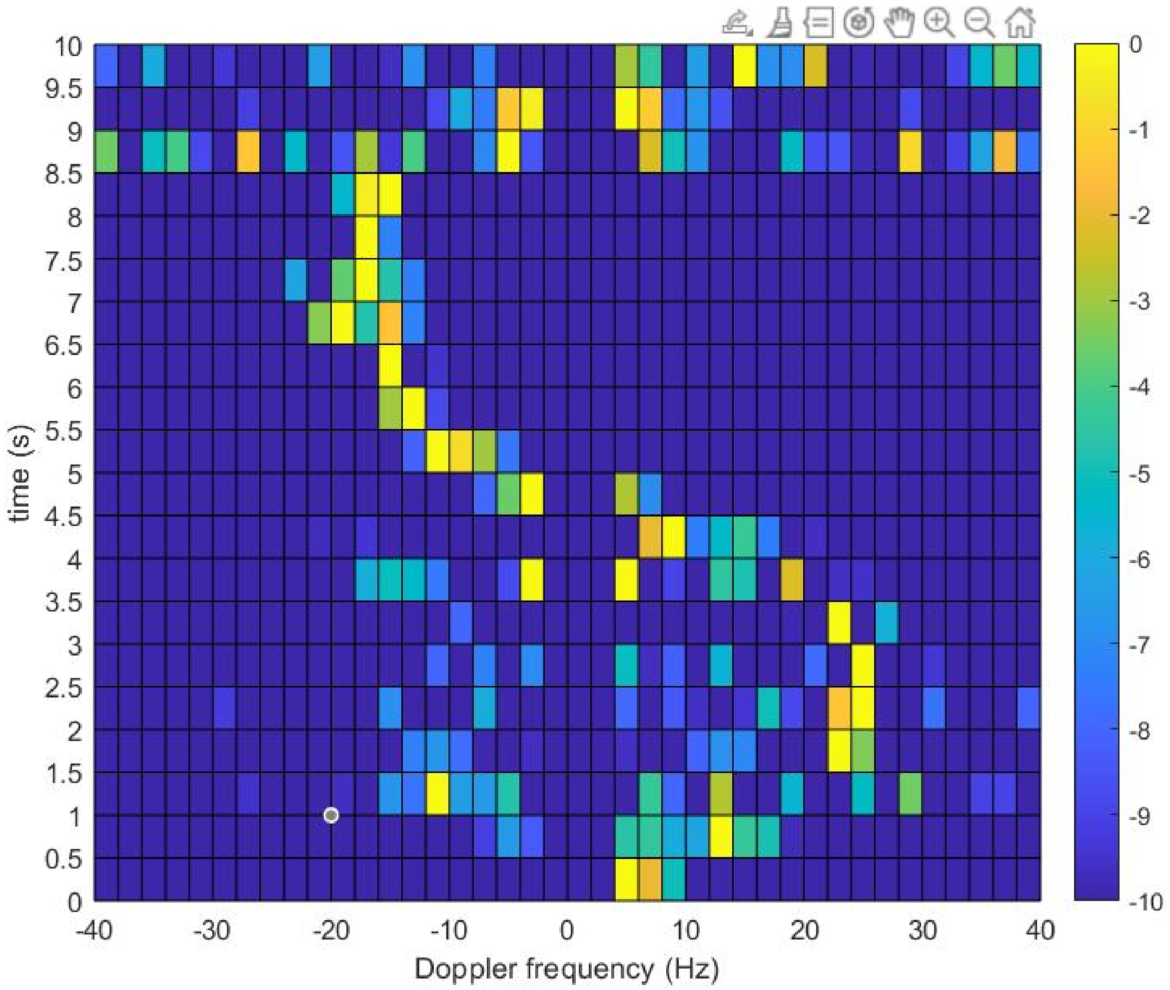


At time 7.0-7.5s, the moving distance is around 0 m, the doppler frequency is around -18 Hz.

IV. Task 3

Draw the Time-Doppler spectrum of the signal. We choose the maximum range of each data file, and put them together. CIT=0.5s.

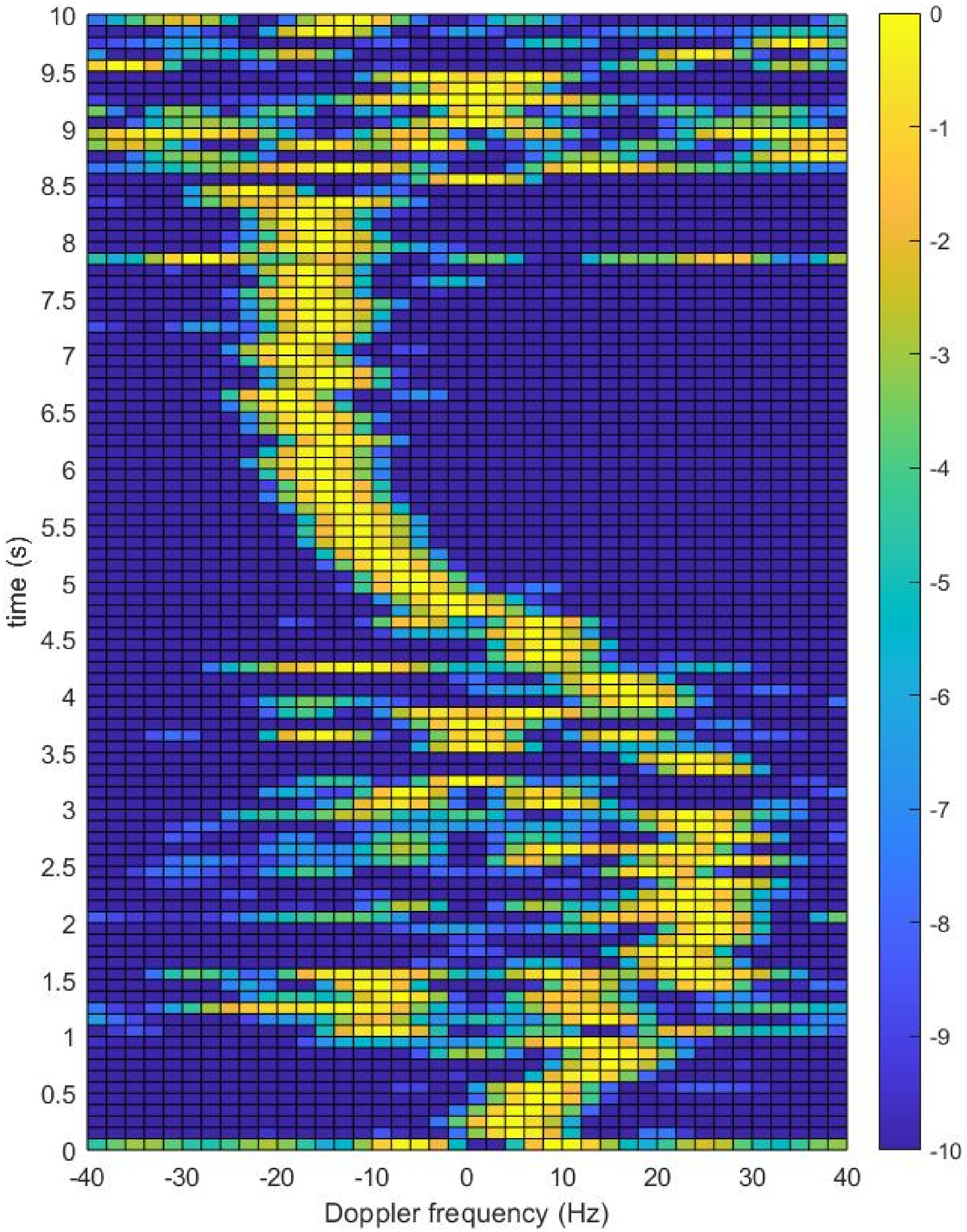
The Time-Doppler Spectrum(TD spectrum)



V. Bonus and further research

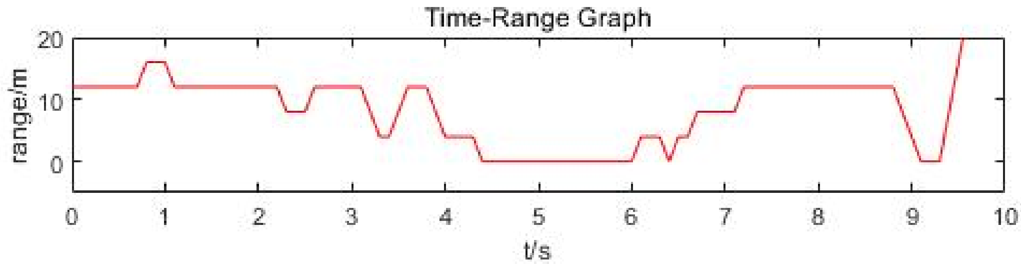
In Task 3, we draw the Time-Doppler spectrum of the signal with CIT=0.5s. To sketch a more accurate plot, we want CIT to be 0.1s. Then, with the relationship between Doppler frequency shift and velocity of the object, v-t and x-t graph can be created.

To be cautious, 0.5s corresponds to fundamental frequency of 2Hz in frequency domain, 0.1s corresponds to 10 Hz. So we can't directly use 0.1s signal to apply fft() method introduced in **Task2**. For CIT=0.1s, we extract 0.1s signal from data file first. After that, fill the rest of 0.4s with 0. Except this point, other procedures are exactly the same. The Time-Doppler spectrum is given below.

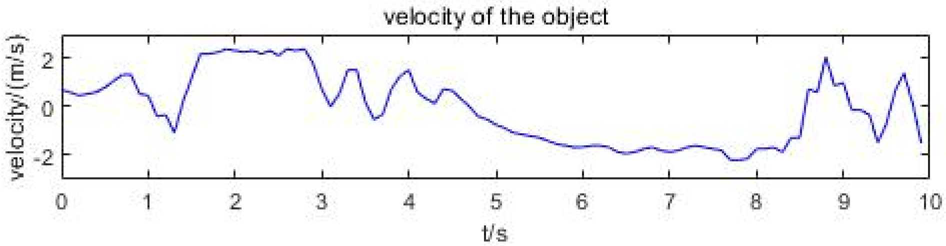


Compare TD spectrum of CIT=0.5s with that of 0.1s, 0.1s-figure is more accurate. However, the plot is still confusing after t=9s.

Then, as we have already stored the maximum-value ranges for each duration, we can plot the Time-Range plot below.

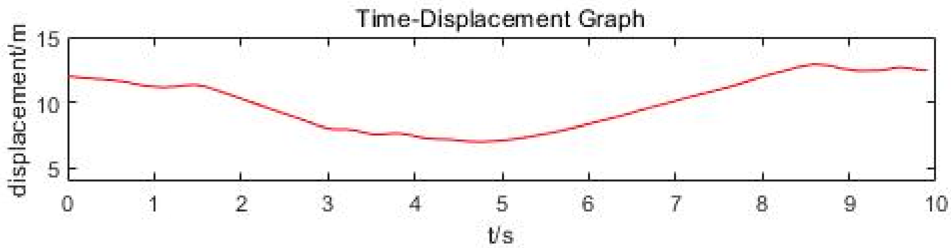


With the formula connecting velocity and Doppler frequency shift, we plot v-t as follows.



However, v-t graph doesn't match Time-range graph as expected. When t=2s-3s, the velocity is 2 meter per second, however, the range **doesn't change** much in total. The reason is that one unit of TDOA(time difference of arrival) is 4e-8, so 12 meter for one unit of range. The speed is less than 2.5 meter per second, so the displacement is more likely to lower than 12 meter. As a result, the Time-range plot is not reliable enough to show the movement.

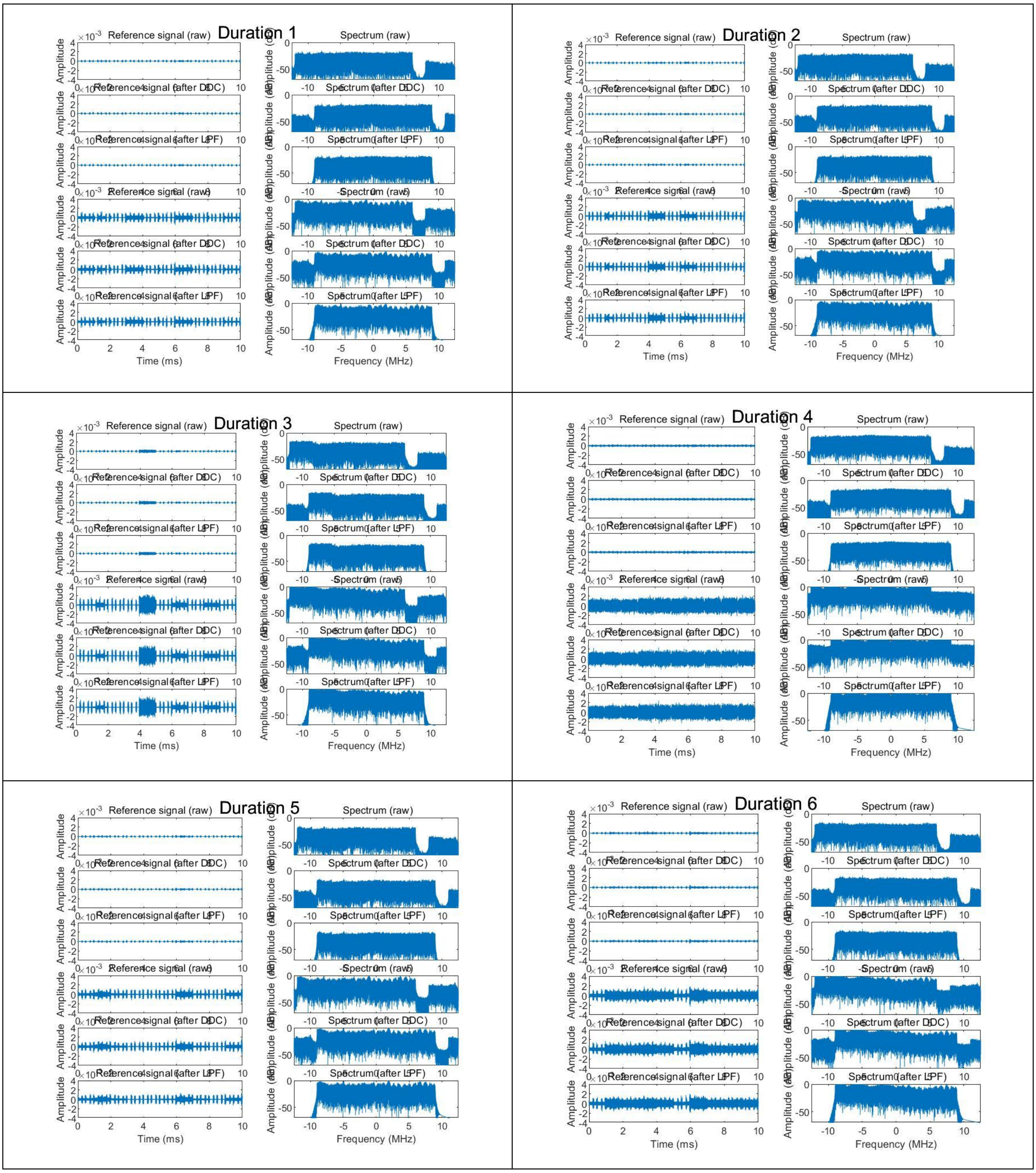
The velocity is more reliable. So we can calculate the exact displacement every moment by integrating velocity over time domain. The Displacement graph can be plotted below.



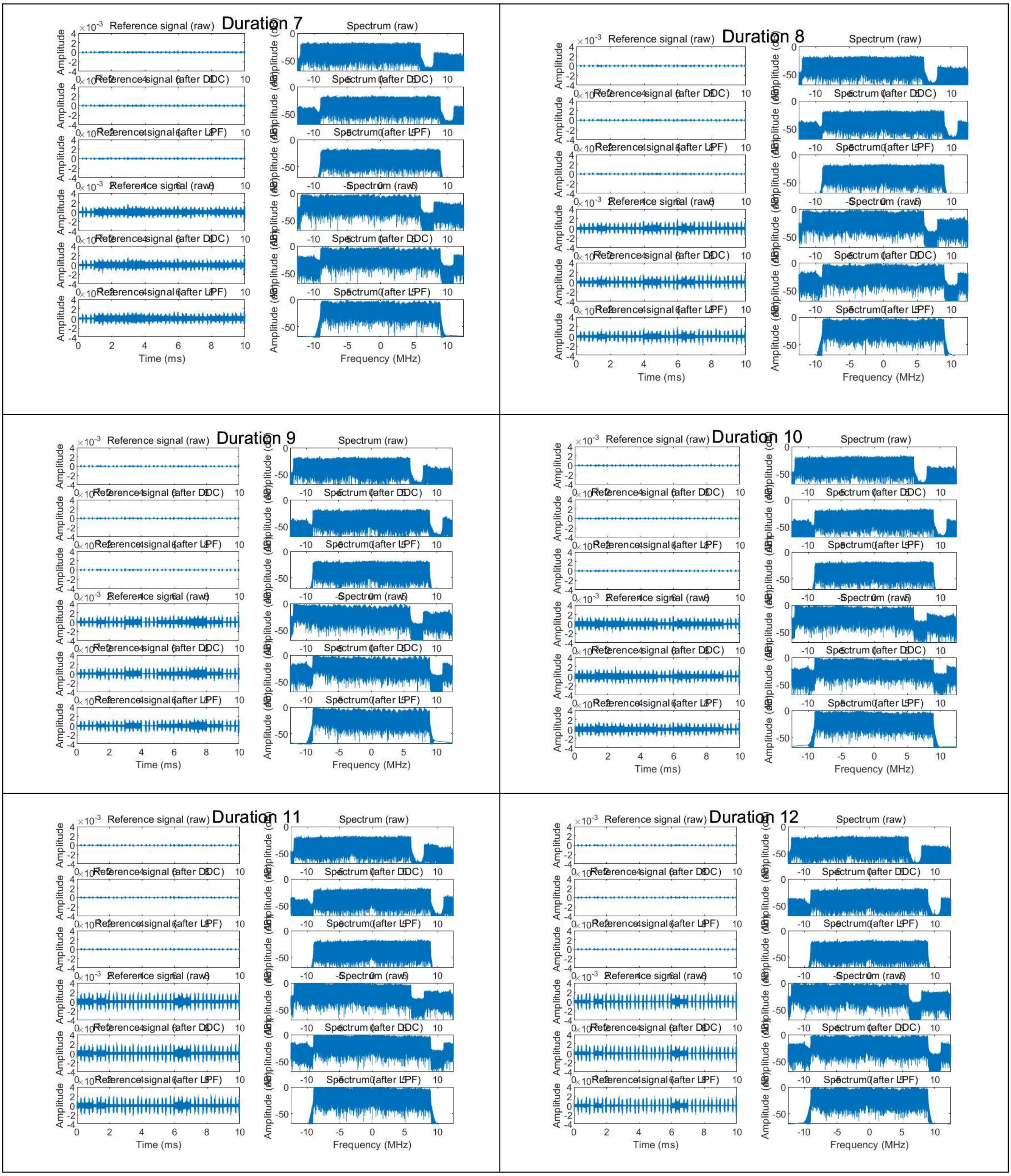
Experience	
Distribution	吴海涵、曾诗农、关馨语：30%；赵子昂：10%
Score	

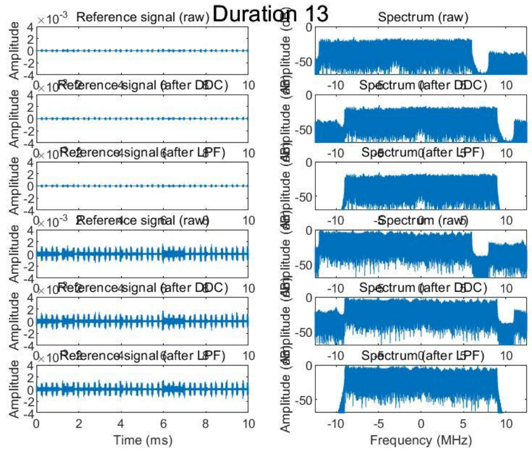
VI. Appendix

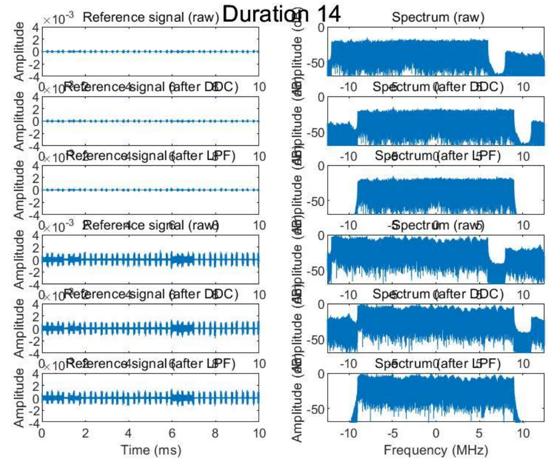
1. Task1 for 20 files

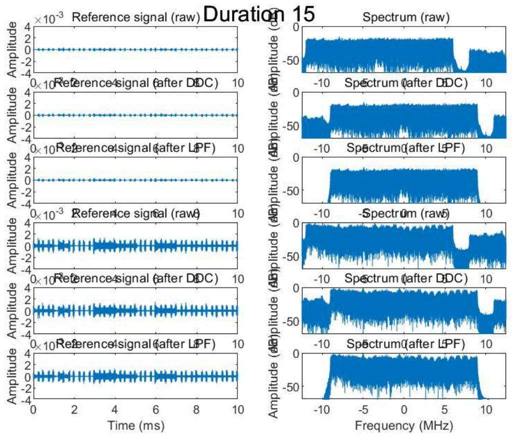


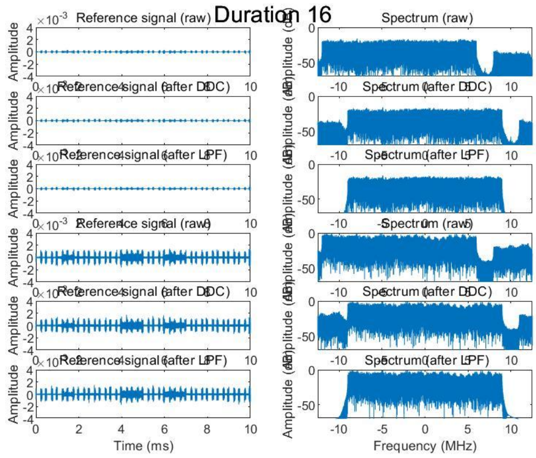
Duration 1
Duration 2
Duration 3
Duration 4
Duration 5
Duration 6
Reference signal (raw)
Spectrum (raw)
Amplitude
Time (ms)
Frequency (MHz)

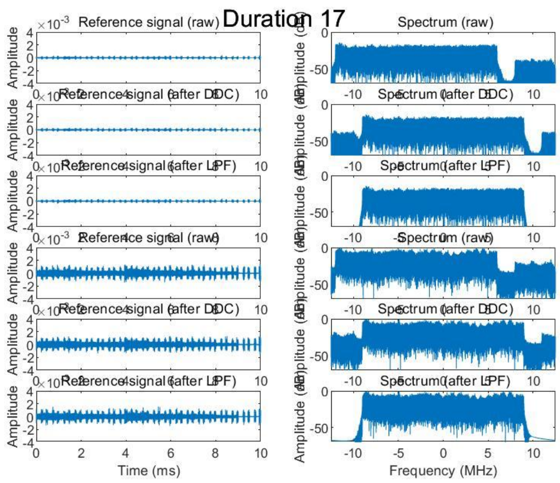

Duration 7
Duration 8
Duration 9
Duration 10
Duration 11
Duration 12
Reference signal (raw)
Reference signal (after DDC)
Reference signal (after LPF)
Spectrum (raw)
Spectrum (after DDC)
Spectrum (after LPF)
Amplitude
Amplitude (dB)
Time (ms)
Frequency (MHz)

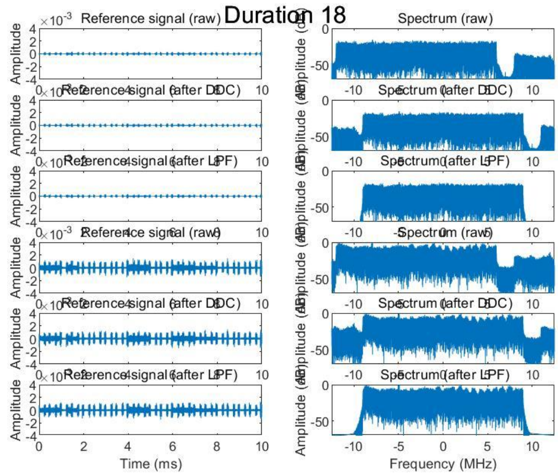

Duration 13
Reference signal (raw)
Spectrum (raw)
Time (ms)
Frequency (MHz)

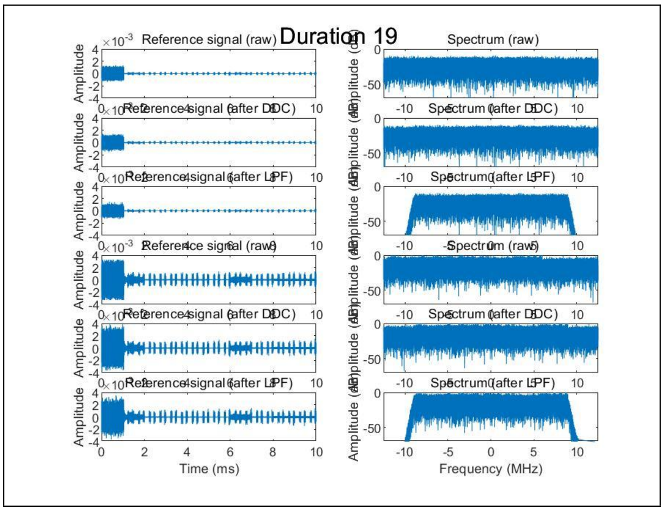

Duration 14
Reference signal (raw)
Spectrum (raw)
Time (ms)
Frequency (MHz)

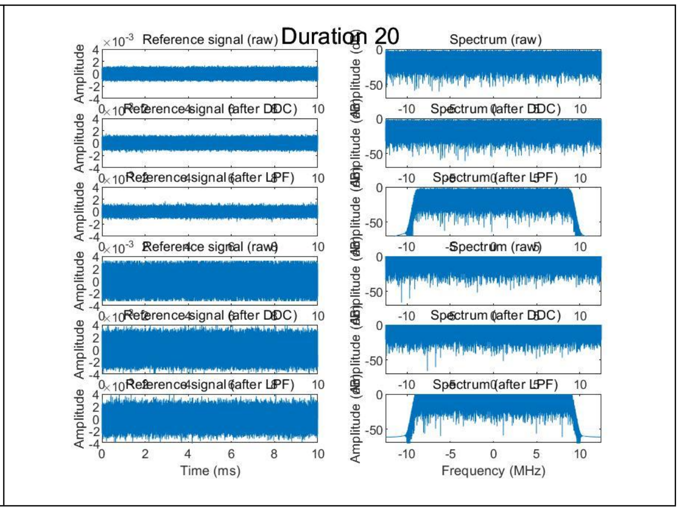

Duration 15
Reference signal (raw)
Spectrum (raw)
Time (ms)
Frequency (MHz)


Duration 16
Reference signal (raw)
Spectrum (raw)
Time (ms)
Frequency (MHz)

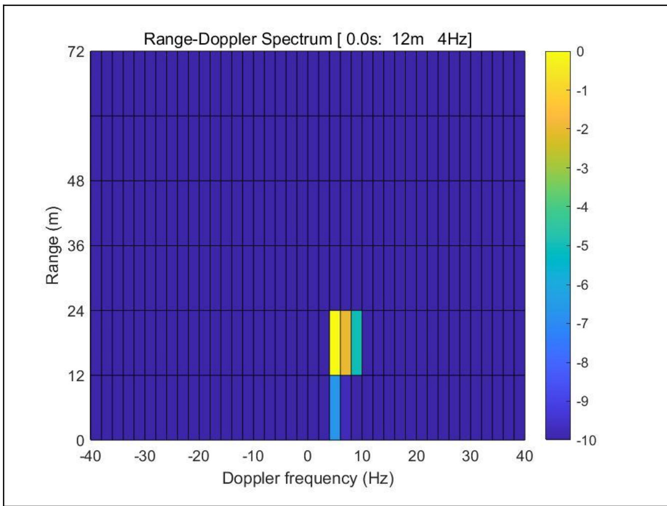

Duration 17
Reference signal (raw)
Spectrum (raw)
Time (ms)
Frequency (MHz)

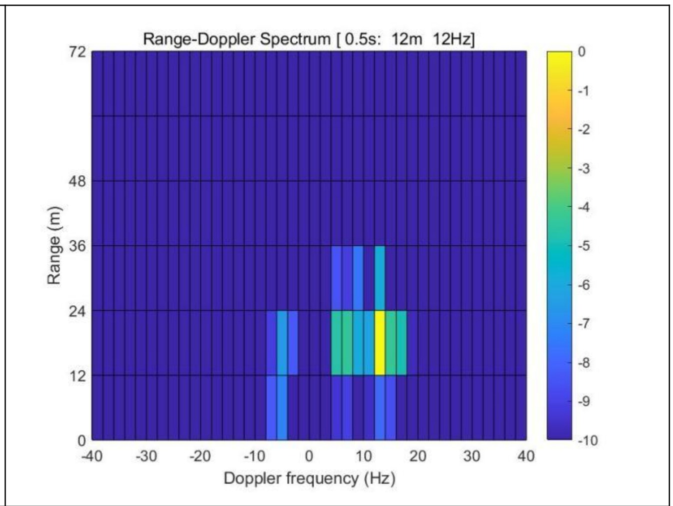

Duration 18
Reference signal (raw)
Spectrum (raw)
Time (ms)
Frequency (MHz)

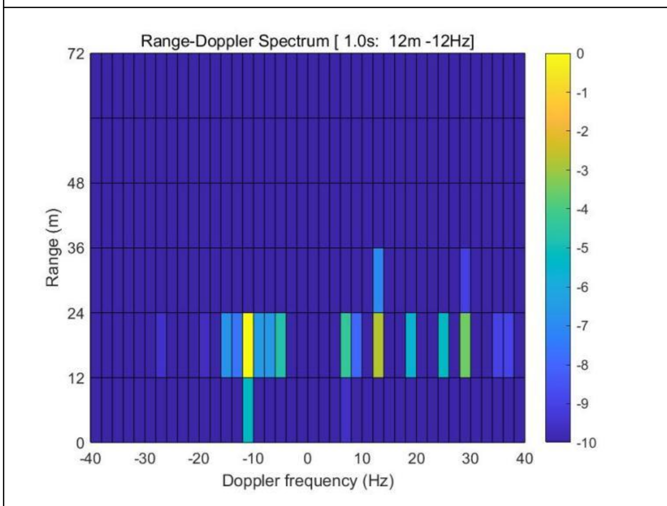


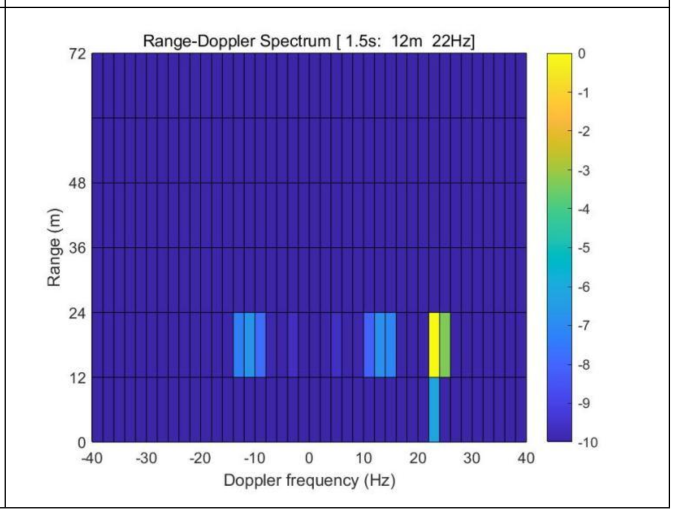


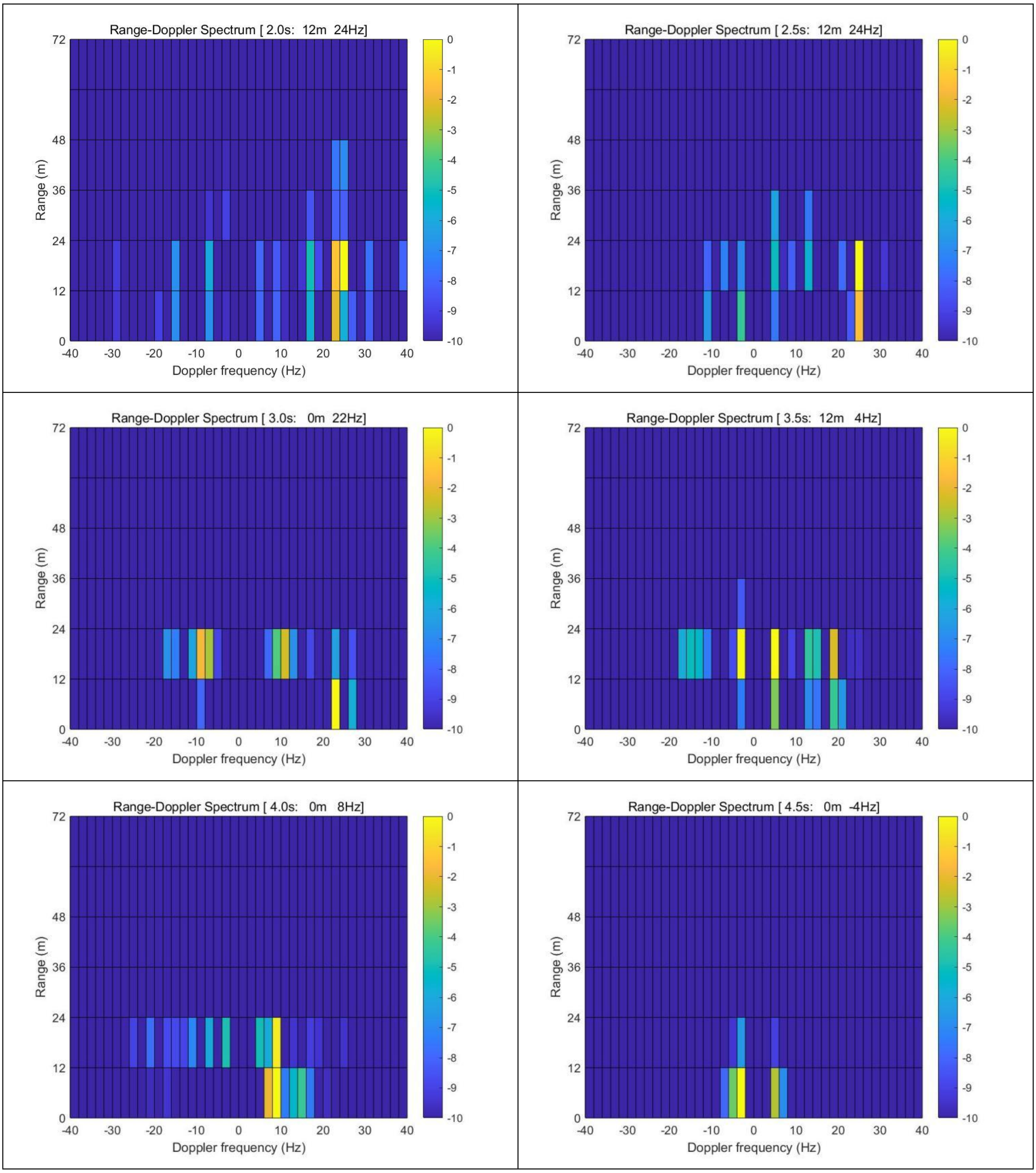
2. Task2 for 30 files

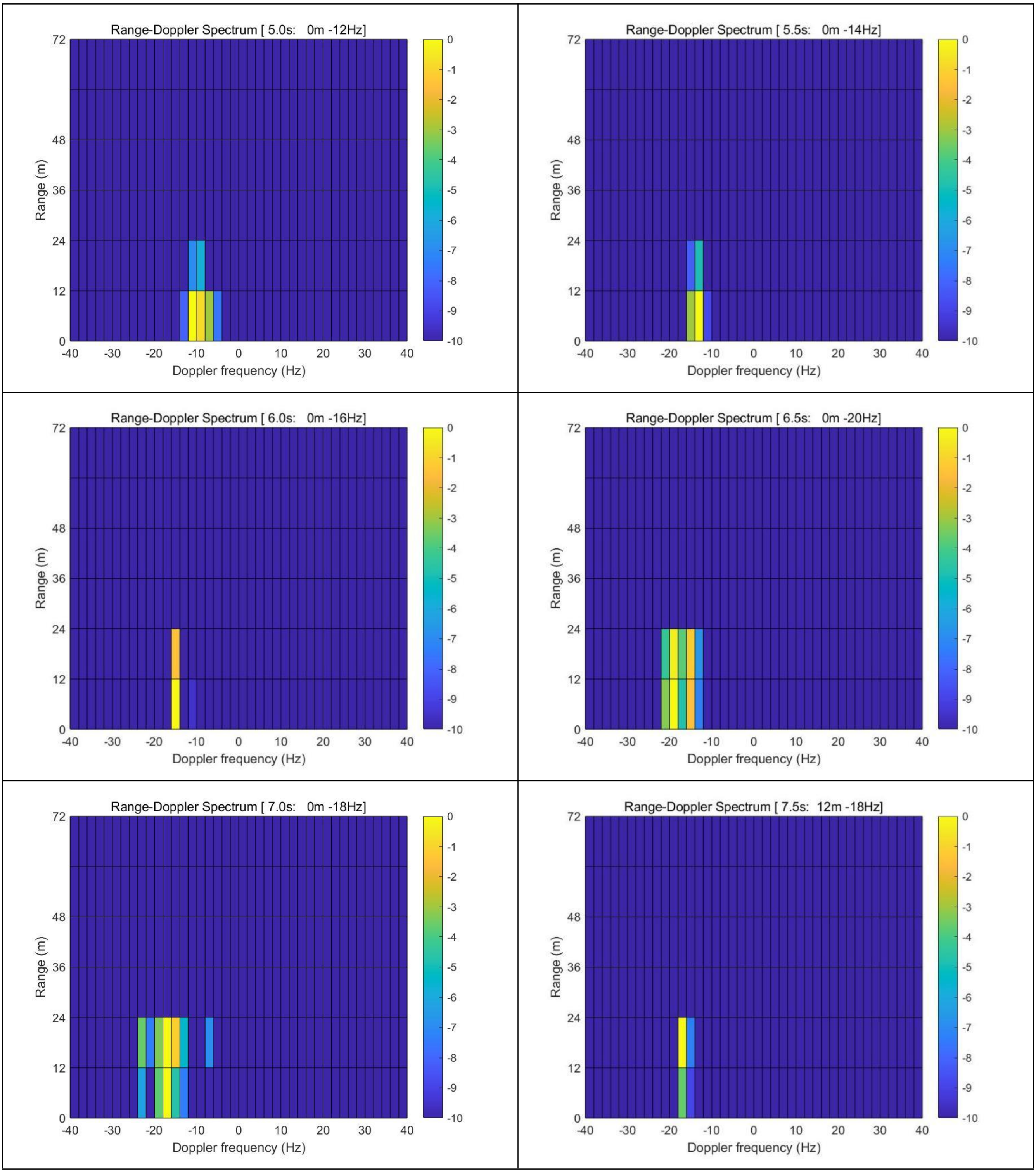


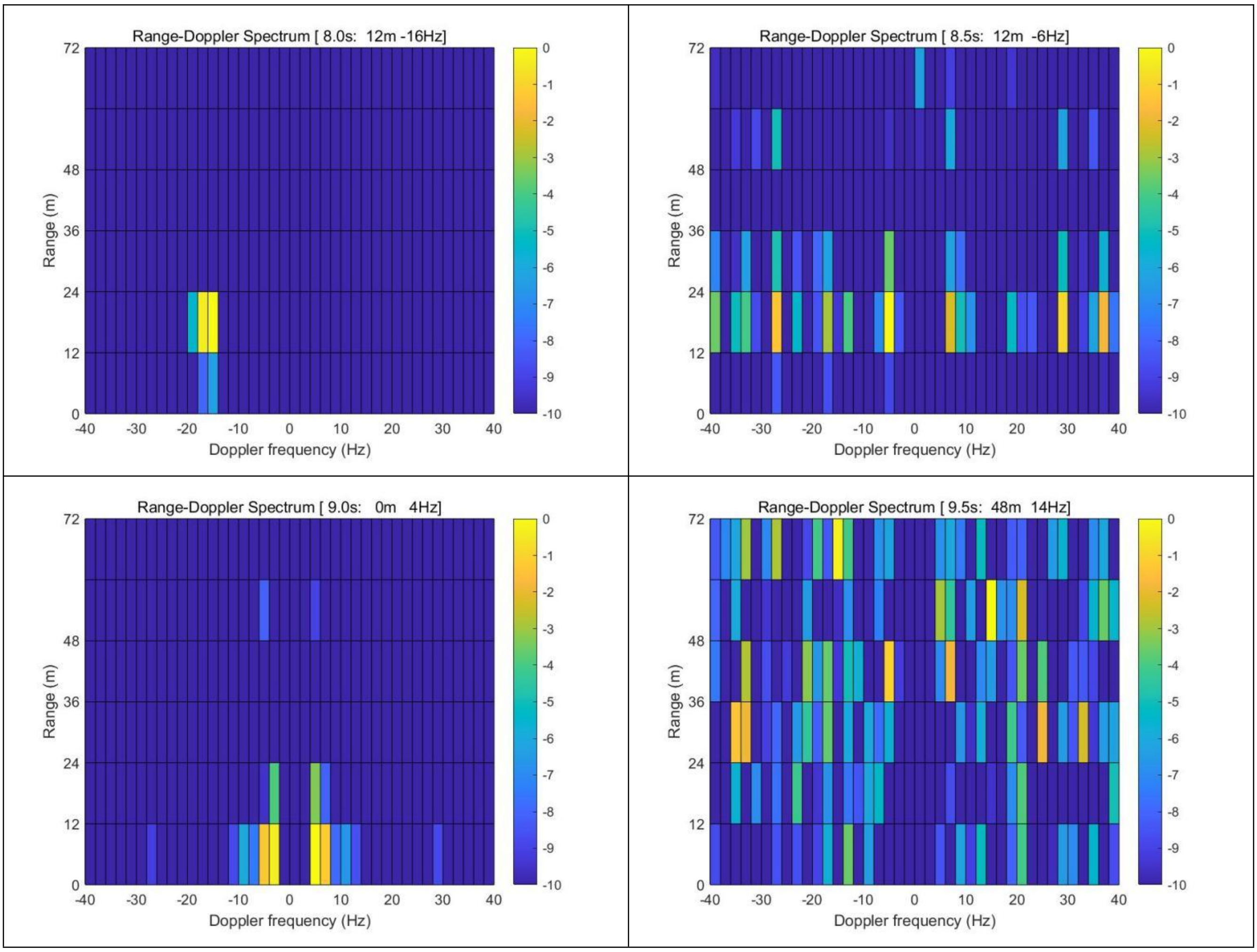







Range-Doppler Spectrum [2.0s: 12m 24Hz]
Range-Doppler Spectrum [2.5s: 12m 24Hz]
Range-Doppler Spectrum [3.0s: 0m 22Hz]
Range-Doppler Spectrum [3.5s: 12m 4Hz]
Range-Doppler Spectrum [4.0s: 0m 8Hz]
Range-Doppler Spectrum [4.5s: 0m -4Hz]
Range (m)
Doppler frequency (Hz)
0 12 24 36 48 72
-40 -30 -20 -10 0 10 20 30 40
0 -1 -2 -3 -4 -5 -6 -7 -8 -9 -10


Range-Doppler Spectrum [5.0s: 0m -12Hz]
Range-Doppler Spectrum [5.5s: 0m -14Hz]
Range-Doppler Spectrum [6.0s: 0m -16Hz]
Range-Doppler Spectrum [6.5s: 0m -20Hz]
Range-Doppler Spectrum [7.0s: 0m -18Hz]
Range-Doppler Spectrum [7.5s: 12m -18Hz]
Range (m)
Doppler frequency (Hz)
72
48
36
24
12
0
-40
-30
-20
-10
0
10
20
30
40
-1
-2
-3
-4
-5
-6
-7
-8
-9
-10



Experience

In this project, we learn how to calculate the velocity of moving object by signal from radar. The working principle is doppler principle and we apply it in practice by analysing the raw signal recorded by radar. After calculating the ambiguity function of the reference signal and surveillance signal, we can draw the doppler spectrum which clearly reflected the state of motion of the object. Furthermore, we were able to plot the v-t and x-t diagram.

From principle to practice, there were many details we should pay attention to. For example, how to remove the frequency we do not need? Here we need the Butterworth filter and DCC to pre process the signal. As for ambiguity function, we considered how to realize time shift for discrete signal and the range of traversing and interval of frequency and time are also critical. For bonus, we found that it is convenience to calculate ambiguity function by fft() in MATLAB, we have proved that in previous lab. After modified, the computation complexity significantly decrease.