

Fast Calculation for Cross Ambiguity Function in Passive Detection Systems

Ling Huang^{1,2,*}, Yu Zhang^{1,2}, Qingyu Li^{1,2}, Changyong Pan^{1,2} and Jian Song^{1,2}

¹ Tsinghua National Laboratory for Information Science and Technology (TNList), Tsinghua University

² Department of Electronics Engineering, Tsinghua University, Beijing, 100084, China

*Corresponding author

Abstract—Passive detection is an effective way to monitor the security of the airspace, in which cross ambiguity function is utilized to obtain the distance and velocity information about the targets. However, the calculation load of the conventional traversal method is tremendous, which cannot satisfy the real-time requirement of the system. In this paper, we propose a fast calculation algorithm in passive detection systems. Chicken swarm optimization (CSO) based heuristic search algorithm is exploited to lessen the computational burden. Location discretization, uniform initialization and random generation of the chicks' location are proposed to increase the computational efficiency further. The experimental results show that our proposed method can achieve the target detection with reduced complexity. Compared with the conventional traversal algorithm, the improved CSO algorithm can reduce the computational complexity for 4 orders of magnitude in general.

Keywords—Cross Ambiguity Function; Passive Detection; Chicken Swarm Optimization

I. INTRODUCTION

Passive detection is a kind of probing method that can be used to supervise the security of the airspace. The system itself does not emit signals, but passively receives the reflected electromagnetic waves from non-cooperative radiation source to locate and track the targets^[1]. This system definitely brings up many advantages of low cost, excellent concealment and strong anti-interference ability, etc. The study of passive detection began in 1922, Taylor detected a sailing wooden ship in a river using a bi-static continuous-wave radar in USA^[2]. By 1988, the Lockheed Martin developed the 'silent sentry'^[3], which was the representative of the detection system based on analog signal. Nowadays, more research concentrate on passive detection based on digital signals, like Wifi, GPS and so on^[4,5].

Fig. 1 shows the signal processing in passive detection systems. A correlation peak is generated by cross ambiguity function calculation between the received reference signal and the target signal, by which we can acquire the distance and velocity information about the target. In order to obtain satisfactory detection results, the accumulation time usually lasts long. Traversal method is a conventional way to search for the correlation peak. However, the calculation load of the traversal method is tremendous, because most of the time is wasted on calculating the point away from the peak point, which leads to low speed of peak search and poor real-time

performance^[6]. Therefore, how to reduce the calculation load in passive detection systems is worthy of research.

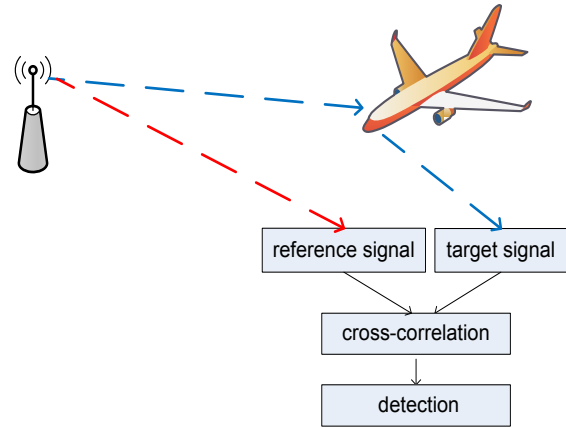


Fig. 1 Signal processing in passive detection systems.

In this paper, we propose a fast calculation algorithm to calculate the cross ambiguity function. Chicken swarm optimization (CSO) based heuristic search algorithm is exploited to lessen the computational burden^[7]. Location discretization, uniform initialization and random generation of the chicks' location are proposed to improve the computational efficiency further. The experimental results show that our proposed method can obtain the target detection with reduced complexity. Compared with the conventional traversal method, the improved CSO algorithm can reduce the computational complexity for 4 orders of magnitude in general.

The rest of the paper is organized as follows. In Section II, we describe the expression of the cross ambiguity function. Section III introduces the improved CSO algorithm and the complexity analysis. Simulation results are presented in Section IV. Finally, conclusions are drawn in Section V.

II. SYSTEM MODEL

In this section, the expression of the cross ambiguity function is introduced. In Fig. 1, the target signal $e(n)$ and the reference signal $r(n)$ can be expressed in Equation (1).

$$\begin{cases} e(n) = u(n - m_1) \exp(j2\pi k_1 n) + \omega_1(n), \\ r(n) = u(n - m_2) \exp(j2\pi k_2 n) + \omega_2(n) \end{cases} \quad (1)$$

In Equation (1), $u(n)$ is the source signal, n is the discrete time, m_1 and m_2 are time delay, k_1 and k_2 are Doppler shift, and $\omega_1(n)$ and $\omega_2(n)$ are the noise at the receiver, respectively. Accordingly, the expression of the cross ambiguity function can be written as Equation (2).

$$A(m, k) = \left| \sum_n r(n) e^*(n + m) \exp(-j2\pi \frac{nk}{N}) \right| \quad (2)$$

In Equation (2), m and k denotes the compensatory time delay and Doppler shift, respectively. Let f_s denotes signal sample frequency, N denotes signal sampling points, T denotes the accumulation time, $f_{d\max}$ denotes the maximum Doppler shift, accordingly we have $N = Tf_s$, $k = Tf_d$, $K = Tf_{d\max}$. Based on the principle of correlation, the maximum value of cross ambiguity function is obtained when the time delay m and the Doppler shift k are compensated. Let the maximum value of cross ambiguity function as the target, let the scope of time delay and Doppler shift as the limiting condition, the estimation can be equivalent to the problem, shown in Equation (3).

$$\begin{aligned} (\hat{m}, \hat{k}) &= \arg \max |A(m, k)| \\ m &\in [0, N-1] \\ k &\in [0, K-1] \end{aligned} \quad (3)$$

In Equation (3), \hat{m} and \hat{k} are the estimated value of the real m and k . The CSO algorithm does not need many constraints on the objective function or differentiable information, so is very suitable to solve the optimal search problem of discrete nonlinear objective [7]. Therefore, CSO algorithm is exploited to search for the time delay and Doppler shift information in this paper, which can reduce the computational complexity and improve the computational efficiency.

III. IMPROVED CHICKEN SWARM OPTIMIZATION ALGORITHM

A. Chicken Swarm Optimization

The theory of CSO is as follows: all chickens are identified by their own two attributes, i.e. 'location' and 'fitness value', which are written as Equation (4).

$$\begin{cases} x_i(j) = (m, k) \\ f_i(j) = -|A(x_i(j))| = -|A(m, k)| \end{cases} \quad (4)$$

In Equation (4), j denotes the iteration number, i denotes the i th chicken, f denotes the fitness value and x denotes the location, respectively. In the CSO model, the optimization problem is to find the minimum value, therefore we take the -

$|A(m, k)|$ as the fitness value. The chickens with the best fitness values are acted as roosters, while the individuals with the worst are regarded as chicks. The rest are the hens. Let RN , HN , MN and CN denote the number of the roosters, the hens, the mother hens and the chicks, respectively. Then we show the movement of the three kind of chickens.

The roosters show the strongest foraging ability; therefore, they have more advantages when searching for food. Their location update is Equation (5).

$$x_i(j+1) = x_i(j) \times (1 + \text{randn}(0, \sigma^2)) \quad (5)$$

$$\sigma^2 = \begin{cases} 1, f_i \leq f_r \\ \exp\left(\frac{f_r - f_i}{|f_i| + \varepsilon}\right) \end{cases} \quad (6)$$

In Equation (5) and (6), $\text{randn}(0, \sigma^2)$ is Gaussian distributed with mean 0 and variance σ^2 . ε is an infinitesimal constant to avoid zero division error. r is another rooster's index which is randomly selected.

As for the hens, they follow the roosters in the group to find food, but also randomly steal better food from other groups. The movement of the hens is shown in Equation (7)-(9).

$$\begin{aligned} x_i(j+1) &= x_i(j) + S1 \times \text{Rand} \times (x_{h1}(j) - x_i(j)) \\ &\quad + S2 \times \text{Rand} \times (x_{h2}(j) - x_i(j)) \end{aligned} \quad (7)$$

$$S1 = \exp\left(\frac{f_i - f_{h1}}{\text{abs}(f_i) + \varepsilon}\right) \quad (8)$$

$$S2 = \exp(f_i - f_{h1}) \quad (9)$$

In Equation (7)-(9) Rand is a random number uniformly distributed over $[0, 1]$. h_1 is the index value of the rooster, which represents the head of the i th hen group, while h_2 is the index value of any kind of chicken that is randomly selected, $h_1 \neq h_2$.

The chicks' foraging ability are the worst, they can only find food around their mother hens. The movement of the chicks is shown in Equation (10).

$$x_i(j+1) = x_i(j) + FL \times (x_h(j) - x_i(j)) \quad (10)$$

In Equation (10) $x_h(j)$ is the position of the i th chick's mother. FL is the coefficients which means how closely the chicks follow their mother hens to find food.

Let $Pbest$ and $Gbest$ denote the current best value of the individual's and the group's, respectively. After a couple of iterations denoted as G , the relationship of the chicken swam is

updated. Reclassify the kind of each chicken according to their fitness value, and find the global optimal solution by computing the above Equations.

B. Improved Chicken Swarm Optimization

Time delay m and Doppler shift k estimation based on cross ambiguity function is essentially a single-peak search problem, but its peak value is relatively sharp and the gradient range is quite narrow, which lead to long convergence time and low efficiency when applying the conventional CSO algorithm. To reduce the convergence time, we improve the CSO algorithm from three aspects. Details are described.

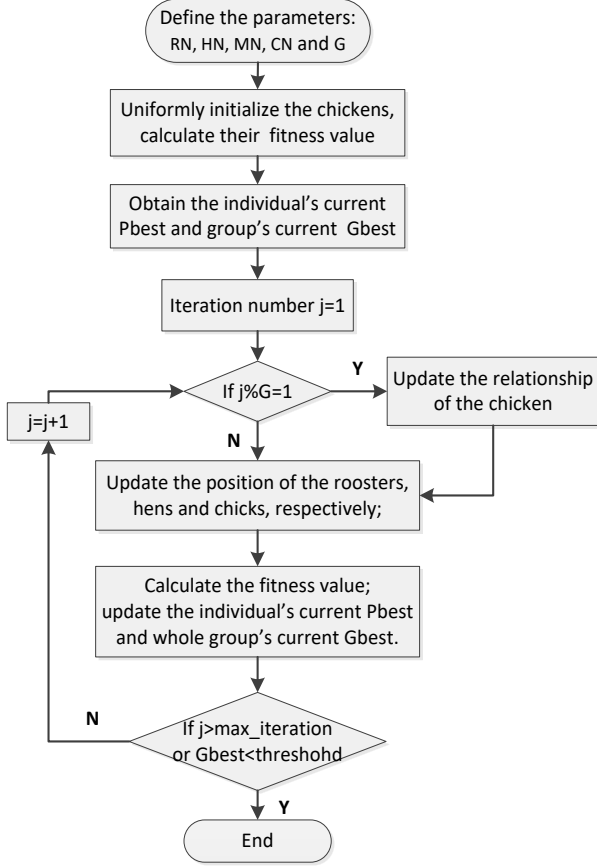


Fig. 2 Flow of the improved CSO algorithm.

- 1) The estimate value of the time delay \hat{m} and the Doppler shift \hat{k} are both discrete integer values, so in this paper, we need to discretize the CSO algorithm to integer estimation. After the locations of the swarm are updated, we use the rounding function to get a integer value, which is shown in Equation (11).

$$x_i^*(j+1) = \text{round}(x_i(j+1)) \quad (11)$$

- 2) In the conventional CSO algorithm, while the random initialization scheme is simple, it is not conducive to quickly solve the problem. In this paper, we propose a

uniform initialization scheme of the candidate solutions. This method could make the candidate solutions evenly distributed around the feasible solution, therefore effectively avoid the random initializations concentrated in a certain region, which may result in a local optimum. The search speed can be significantly improved. The initial locations are shown in Equation (12).

$$x_i(0) = \left((i-1) \times \left[\frac{N}{\text{group_size}} \right], (i-1) \times \left[\frac{K}{\text{group_size}} \right] \right) \quad (12)$$

In Equation (12), group_size denotes the number of the whole chickens, $i=1,2,\dots,\text{group_size}$, $[\cdot]$ denotes the integral function.

- 3) The foraging ability of the chicks is the weakest. In order to prevent the group trapped in local optimum and enhance the ability of finding the global optimal solution, we propose a scheme of randomly initializing the chicks' location in each iteration process. The movement of the chicks is rewritten as Equation (13).

$$x_i(j+1) = ([N \times \text{Rand}], [K / 2 \times \text{Rands}]) \quad (13)$$

In Equation (13), Rand is a random number uniformly distributed over $[0,1]$.

The flow of the improved CSO based cross ambiguity function estimation is shown in Fig. 2.

C. Computational Complexity Analysis and Comparison

The computational load of the improved CSO algorithm mainly comes from calculating the fitness value, i.e. the cross ambiguity function. The complexity of adopting improved CSO is shown in Equation (14).

$$C = \text{group_size} \times \text{num_iteration} \times N \quad (14)$$

In Equation (14), num_iteration denotes the ending number of iterations, and N is the complexity of computing (2) once. The complexity of improved CSO compared with the traversal method are shown in Table 1.

TABLE I. COMPUTATIONAL COMPLEXITY COMPARISON

Method	COMPLEXITY
Traversal method	$N \times K \times N$
Improved CSO	$\text{group_size} \times \text{num_iteration} \times N$

From Table 1 we can see that, the complexity of the improved CSO is much less than the traversal method as long as $\text{group_size} \times \text{num_iteration} \ll N$. By adjusting the parameters, $\text{group_size} \times \text{num_iteration} \ll N$ is easy to be realized.

IV. SIMULATION RESULTS

The CSO algorithm has no theoretical performance index, whose calculation accuracy and efficiency vary according to the different application scenarios. We use the Monte Carlo method to calculate the problem for a couple of times with a same set of parameters, and take the average results to represent the performance. The accumulation time T is set to be 1ms, 10ms, 50ms and 100ms, respectively. We carried out 800 independent experiments for the above 4 kinds. Let $\omega = 7.56\text{MHz}$. The ratio of RN, HN, MN and CN in improved CSO algorithm is set to be 0.2, 0.6, 0.5 and 0.2, respectively. Update the swarm relationship after 10 iterations, i.e. $G=10$. Let N denotes the length of the signal, denotes the real time delay, denotes the real Doppler shift, respectively, the parameters and part of the simulation results are given in Table 2.

TABLE II. PARAMETERS AND PART OF THE SIMULATION RESULTS BASED ON IMPROVED CSO ALGORITHM

parameters	1ms	10ms	50ms	100ms
N	7560	75600	378000	756000
m_0	1250	12500	125000	425000
k_0	5	15	30	50
<i>group_size</i>	50	100	200	300
<i>max_ieration</i>	100	400	700	1000
ratio of accuracy	99.01%	98.33%	97.33%	96.67%
average iterations	29.14	127.57	201.73	318.95

From Table 2, first, we can see that the ratio of accuracy of improved CSO based algorithm is extremely high, which shows the strong reliability of the proposed method. For example, the ratio of accuracy of 10ms is up to 98.33%. Second, the algorithm can get the final results after just a couple of iterations, which shows the fast convergence speed of the algorithm. As shown in Table 2, the average iterations of 10ms is 127.57. Third, for the very few cases that the correct results are not found, if we extend the maximum iteration number and carry on the algorithm, the correct results can be found finally.

Fig. 3 shows the cumulative statistics of the ending iteration numbers obtained by the 800 simulations. The cumulative probability is the ratio of termination iteration less than the iteration number shown in the horizontal axis. Take the situation of 100ms as an example, the ending iteration number less than 400 accounts for 70% of the total number of simulations, while the ending iteration number larger than 800 accounts for less than 5%. Fig. 3 indicates that the proposed algorithm converges very fast, and obtains the estimation results with high accuracy and efficiency.

Fig. 4 shows the complexity of the two methods along with the signal length. As shown in the figure, the traversal method has a higher complexity, and cannot satisfy the real-time requirements in practical applications. In this paper, the improved CSO based algorithm is exploited to solve the

problem, whose complexity is further decreased for about 4 orders of magnitude compared with traversal method. This is because the proposed method adopts intelligent search algorithm, which can avoid the whole space search and save a lot of unnecessary computation. The simulation results demonstrate the effectiveness of our proposed algorithm.

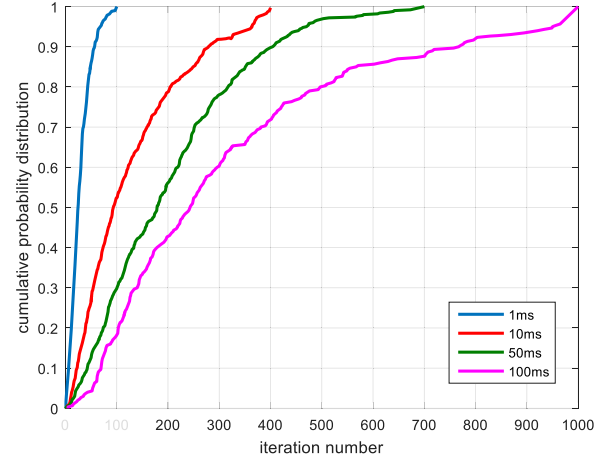


Fig. 3 Distribution curves of the ending iteration numbers.

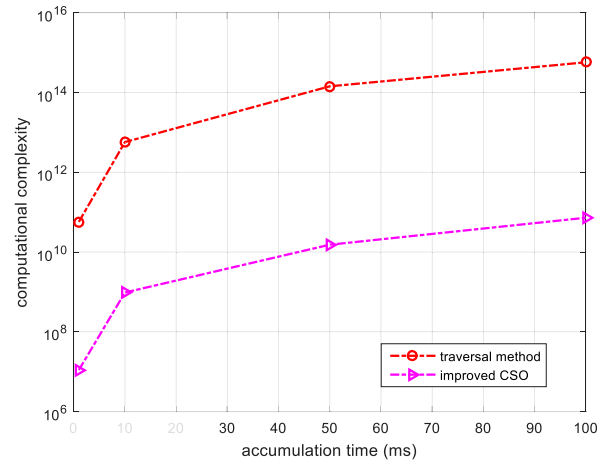


Fig. 4 Complexity Comparison of the two method.

V. CONCLUSIONS

This paper, proposes a fast calculation algorithm to calculate the cross ambiguity function in passive detection systems. Chicken swarm optimization (CSO) based heuristic search algorithm is exploited to lessen the computational burden. Location discretization, uniform initialization and random generation of the chicks' location are proposed to improve the computational efficiency further. The experimental results show that our proposed method can obtain the target detection with reduced complexity. Compared with the conventional traversal method, the improved CSO algorithm can reduce the computational complexity for 4

orders of magnitude in general. Future work will concentrate on the information analysis based on our results.

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