EE 368 机器人运动与控制方法（Robotic Motion and Control）
Reference Solutions to Assignment #5

Q-1. Given the nonlinear control equation for an α, β partitioned controller for the system:

$$\tau = 5\theta\dot{\theta} + 2\ddot{\theta} - 13\dot{\theta}^3 + 5.$$

Choose gains so that this system is always critically damped with the closed-loop stiffness $K_P = 10$.

Solution: Let $\tau = \alpha\tau' + \beta, \quad \alpha = 2\beta, \quad \beta = 5\theta\dot{\theta} - 12\dot{\theta}^3 + 5$ and

$\tau' = \ddot{\theta}_D + K_V\dot{e} + K_P e$ where $e = \theta_D - \theta$, and $K_P = 10$ and $K_V = 2\sqrt{10}$

Q-2. Draw a block diagram showing a Cartesian-space controller for the two-link robot arm with the Cartesian-space dynamic equation as follows:

$$\tau = J^T(\Theta)M_X(\Theta)\ddot{X} + B_X(\Theta)[\dot{\Theta}\dot{\Theta}] + C_X(\Theta)[\dot{\Theta}^2] + G(\Theta)$$

such that the robot arm is critically damped over its entire workspace. Assume that the forward kinematics of this robot arm is represented by $KIN(\Theta)$.

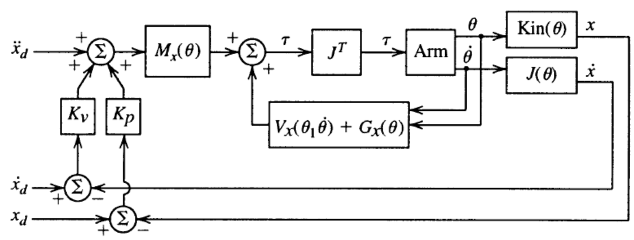
Solution:

Where $M_x(\theta)$, $V_x(\theta, \dot{\theta})$, $G_x(\theta)$ are as found in example 6.6. KIN(θ) is the kinematics (forward) for this simple two-link.

Also:

$$K_P = \begin{bmatrix} K_{P1} & 0 \\ 0 & K_{P2} \end{bmatrix} \quad K_V = \begin{bmatrix} K_{V1} & 0 \\ 0 & K_{V2} \end{bmatrix}$$

with $K_{Vi} = 2\sqrt{K_{Pi}}$



Q-3. Consider a position-regulation system with the open-loop dynamics as

$$\tau = M(\Theta)\ddot{\Theta} + V_M(\Theta, \dot{\Theta})\dot{\Theta} + G(\Theta)$$

that attempts to maintain $\Theta_d = 0$. Prove using Lyapunov stability that the control law

$$\tau = -K_P\Theta - \widehat{M}(\Theta)K_V\dot{\Theta} + G(\Theta)$$

with the Lyapunov function candidate

$$\mathcal{V} = \frac{1}{2}\dot{\Theta}^T M(\Theta)\dot{\Theta} + \frac{1}{2}\Theta^T K_P \Theta$$

yields an asymptotically stable nonlinear system. You may take K_V to be of the form $K_V = k_v I_n$, where k_v is a scalar and I_n is the $n \times n$ identity matrix. The matrix $\widehat{M}(\Theta)$ is a positive definite estimate of the manipulator mass matrix.

Solution:

Closed loop system, given by:

$$M(\theta)\ddot{\theta} + V_M(\theta, \dot{\theta})\dot{\theta} + G(\theta) = -K_p\theta - M(\theta)K_v\dot{\theta} + G(\theta)$$

$$V = \tfrac{1}{2}\dot{\theta}^T M(\theta)\dot{\theta} + \tfrac{1}{2}\theta^T K_p\theta \quad \text{(Lyapunov candidate)}$$

$$\dot{V} = \tfrac{1}{2}\dot{\theta}^T \dot{M}(\theta)\dot{\theta} + \dot{\theta}^T M(\theta)\ddot{\theta} + \dot{\theta}^T K_p\theta$$

$$\dot{V} = \tfrac{1}{2}\dot{\theta}^T \dot{M}(\theta)\dot{\theta} + \dot{\theta}^T[-V_M(\theta, \dot{\theta})\dot{\theta} - G(\theta) - K_p\theta - M(\theta)K_v\dot{\theta} + G(\theta)]$$

$$+ \dot{\theta}^T K_p\theta$$

$$\dot{V} = -\dot{\theta}^T M(\theta)K_v\dot{\theta}$$

This is non-positive if $M(\theta)K_v$ is positive definite. This matrix product is positive def. if $K_v = k_v I_N$, where K_v is positive scalar. Q.E.D.