EE 368 机器人运动与控制方法（Robotic Motion and Control）
Reference Solutions to Assignment #2

Q-1. Assign link frames to the RPR planar robot shown in Figure Q-1, give the D-H table (linkage parameters).

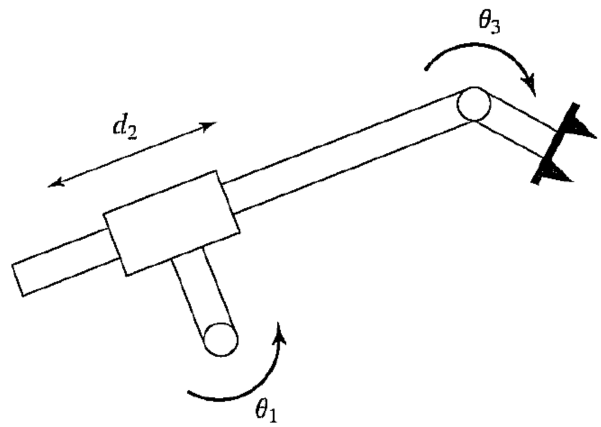
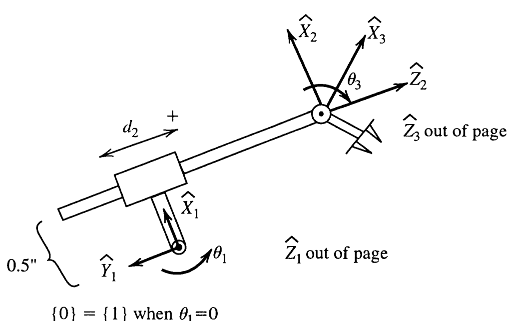


Figure Q-1

Solution: The frames are as shown, and the parameters are on the right-hand side below:



$$
\begin{array}{lll}
a_0 = 0 & a_1 = 0.5^\circ & a_2 = 0 \\
\alpha_0 = 0 & \alpha_1 = 90^\circ & \alpha_2 = -90^\circ \\
d_1 = 0 & & d_3 = 0 \\
& \theta_2 = 0^\circ &
\end{array}
$$

Q-2. For the two-link manipulator shown in Figure Q-2(a), the link-transformation matrices, ${}^0_1T$ and ${}^1_2T$, were constructed. Their product is

$$
{}^0_2T = \begin{bmatrix} c\theta_1 c\theta_2 & -c\theta_1 s\theta_2 & s\theta_1 & l_1 c\theta_1 \\ s\theta_1 c\theta_2 & -s\theta_1 s\theta_2 & -c\theta_1 & l_1 s\theta_1 \\ s\theta_2 & c\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

The link-frame assignments used are indicated in Figure Q-4(b). Note that frame {0} is coincident with frame {1} when θ_1= 0. The length of the second link is l_2. Find an expression for the vector ${}^0P_{tip}$, which locates the tip of the arm relative to the {0} frame.

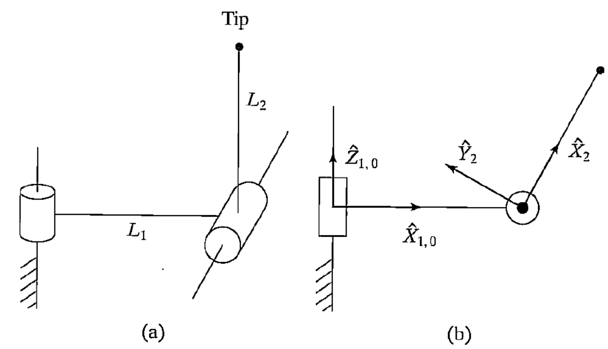


Figure Q-2

Solution:

$$\begin{bmatrix} {}^0P_{TIP} \\ 1 \end{bmatrix} = {}^0_2T \begin{bmatrix} {}^2P_{TIP} \\ 1 \end{bmatrix} = \begin{bmatrix} c_1c_2 & -c_1s_2 & s_1 & L_1c_1 \\ s_1c_2 & -s_1s_2 & -c_1 & L_1s_1 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} L_2 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} L_1c_1 + L_2c_1c_2 \\ L_1s_1 + L_2s_1c_2 \\ L_2s_2 \\ 1 \end{bmatrix} \Rightarrow {}^0P_{TIP} = \begin{bmatrix} L_1c_1 + L_2c_1c_2 \\ L_1s_1 + L_2s_1c_2 \\ L_2s_2 \end{bmatrix}$$

Q-3. Sketch the fingertip workspace of the three-link manipulator illustrated in Figure Q-3, for the case of $l_1 = 15$, $l_2 = 10$, and $l_3 = 3$.

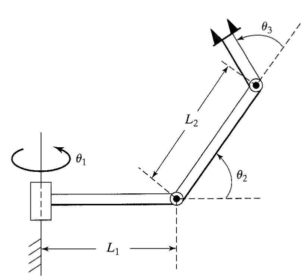
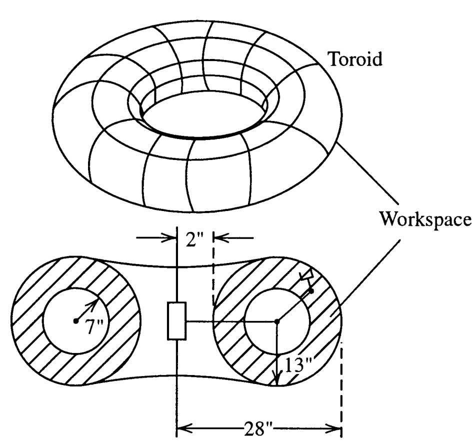


Figure Q-3

Solution:



Q-4. For the cases of (1) both the desired orientation and position of the tip frame with respect to the base frame are given as:

$$ {}^0_3T = \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

and (2) only the desired position of the tool frame is given as: ${}^3P_{tool} = [\ l_3 \quad 0 \quad 0\ ]^T$,

derive the inverse kinematics of the three-link manipulator in Figure Q-3, knowing that

$$ {}^0_3T = \begin{bmatrix} c_1c_{23} & -c_1s_{23} & s_1 & c_1(c_2l_2+l_1) \\ s_1c_{23} & -s_1s_{23} & -c_1 & s_1(c_2l_2+l_1) \\ s_{23} & c_{23} & 0 & s_2l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}. $$

Solution: **For Case (1)**,

${}^S_TT$ is given, so compute: ${}^B_WT = {}^B_ST\ {}^S_TT\ {}^W_TT^{-1}$. Now ${}^B_WT = {}^0_3T$ that we write out as:

$$ {}^0_3T = \begin{bmatrix} R_{11} & R_{12} & R_{13} & P_x \\ R_{21} & R_{22} & R_{23} & P_y \\ R_{31} & R_{32} & R_{33} & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C_1C_{23} & -C_1S_{23} & S_1 & C_1(C_2L_2+L_1) \\ S_1C_{23} & -S_1S_{23} & -C_1 & S_1(C_2L_2+L_1) \\ S_{23} & C_{23} & 0 & S_2L_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Equate elements (1, 3): $S_1 = R_{13}$, equate elements (2, 3): $-C_1 = R_{23}$, $\theta_1 = \text{atan2}(R_{13}, -R_{23})$.

If both $R_{13} = 0$ and $R_{23} = 0$, the goal is unattainable.

Equate elements (1, 4): $P_x = C_1(C_2L_2 + L_1)$, equate elements (2, 4): $P_y = S_1(C_2L_2 + L_1)$.

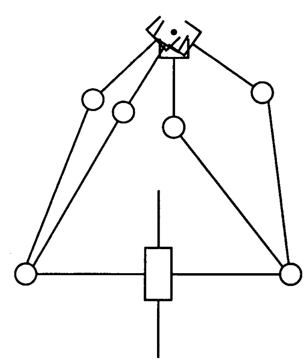
If $C_1 \neq 0$ then $C_2 = \frac{1}{L_2}(\frac{P_x}{C_1} - L_1)$, else $C_2 = \frac{1}{L_2}(\frac{P_y}{S_1} - L_1)$. Equate elements (3, 4): $P_z = S_2L_2$,

so, $\theta_2 = \text{atan2}(\frac{P_z}{L_2}, C_2)$.

Equate elements (3, 1): $S_{23} = R_{31}$, equate elements (3, 2): $C_{23} = R_{23}$,

so, $\theta_3 = \text{atan2}(R_{31}, R_{32}) - \theta_2$.

If both R_{31}, and R_{32} are zero, the goal is unattainable.



For Case (2), Assume ${}^3P_{tool} = L_3\hat{X}_3$, then

$$ {}^3P_{tool} = \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = \begin{bmatrix} L_1C_1 + L_2C_1C_2 + L_3C_1C_{23} \\ L_1S_1 + L_2S_1C_2 + L_3S_1C_{23} \\ L_2S_2 + L_3S_{23} \end{bmatrix} $$

First, $S_1 = \frac{P_y}{L_1+L_2C_2+L_3C_{23}}$, $C_1 = \frac{P_x}{L_1+L_2C_2+L_3C_{23}}$, so $\theta_1 = \text{atan2}(P_y, P_x)$ or $\theta_1 = \text{atan2}(-P_y, -P_x)$ since the sign of the "$L_1 + L_2C_2 + L_3C_{23}$" term may be + or -.

Next, define $\alpha = \begin{cases} \frac{P_x}{C_1} - L_1 & \text{if } C_1 \neq 0 \\ \frac{P_y}{S_1} - L_1 & \text{if } S_1 \neq 0 \end{cases}$. And we have $\begin{matrix} L_2C_2 + L_3C_{23} = \alpha \\ L_2S_2 + L_3S_{23} = P_z \end{matrix}$,

square and add the two equations to get:

$L_2^2 + L_3^2 + 2L_2L_3C_3 = \alpha^2 + P_z^2$, $C_3 = \frac{1}{2L_2L_3}(\alpha^2 + P_z^2 - L_2^2 - L_3^2)$, $S_3 = \pm\sqrt{1 - C_3^2}$,

$\theta_3 = \text{atan2}(S_3, C_3)$.

Finally, $L_3C_{23} = \alpha - L_2C_2$, $L_3S_{23} = P_z - L_2S_2$, so $\theta_2 = \text{atan2}(P_z - L_2S_2, \alpha - L_2C_2) - \theta_3$.

Q-5. (a) Describe a simple algorithm for choosing the nearest solution from a set of possible solutions.

(b) There exist 6-DOF robots for which the kinematics are NOT closed-form solvable. Does there exist any 3-DOF robot for which the (position) kinematics are NOT closed-form solvable?

Solution:

(a) To derive the "nearest" solution, we would like to minimize the rotation of each joint. Denote the starting angle of each joint as θ_{jo} (for *j*-th joint), and the final position of each joint as θ_{jF}.

For each proposed solution, compute: $S = \sum_{j=1}^{N} |\theta_{jF} - \theta_{jo}|$. And choose the "nearest" as the one that minimizes S. Sometimes a weighting factor is used (to penalize motion of "large" joints, for example) and so the score for each proposed solution is $S = \sum_{j=1}^{N} W_j |\theta_{jF} - \theta_{jo}|$.

(b) No. Pieper's method gives the closed form solution for any 3-DOF manipulator. (See Pieper's thesis for all the cases if interested).

Q-6. ${}^A_BR$ is a 3 x 3 matrix with eigenvalues 1, and e^{+ai}, and e^{-ai}, where $i = \sqrt{-1}$. What is the physical meaning of the eigenvector of ${}^A_BR$ associated with the eigenvalue 1?

Solution:

If V_i is an eigenvector of R corresponding to eigenvalue λ, then

$$RV_i = \lambda V_i$$

If the eigenvalue with V_i is $\lambda = 1$, then

$$RV_i = V_i$$

Hence the vector V_i is not changed by the rotation R. So V_i is the axis of rotation.