Fundamentals of Programming with C Language

NUMBER SYSTEMS

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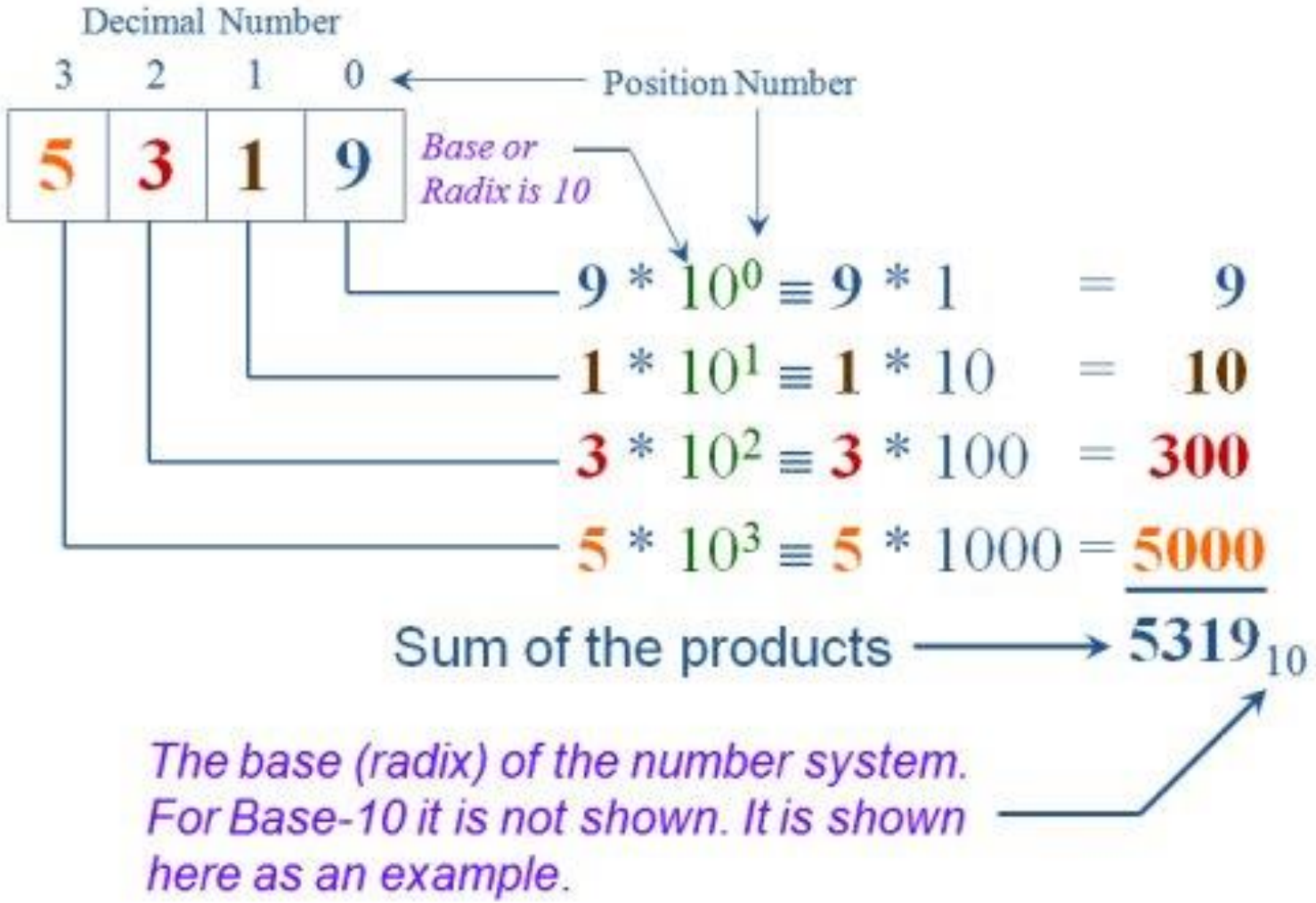
Content

- Binary Base 2
- Octal Base 8
- Decimal Base 10
- Hexadecimal Base 16

Decimal Numbers

- The decimal number system is the number system we use everyday, from counting to simple math like checking store receipts.
- The word 'deci' means 10, therefore there are ten numbers (digits) in the decimal number system.
- The largest symbol is the base – 1 i.e 10 – 1 = 9
- 9 is the largest symbol in Decimal numbers
- It has ten unique symbols
  - 0 1 2 3 4 5 6 7 8 9 (Total 10)
  - BASE-10

Meaning of Decimal Number



Meaning of Decimal Number

- The number 532.25 in decimal (base 10) means the following:

  = *(5 * 102) + (3 * 101) + (2 * 100) + (2 * 10-1) + (5 * 10-2)*

  = *500 + 30 + 2 + 2/10 + 5/100*

  = *532.25*

Binary Numbers

- Binary stands for 2
- Binary is the base 2 number system
- It has two unique symbols
  - 0 and 1
- The largest symbol in Binary is the base – 1 i.e 2 – 1 = 1
- 1 is the largest symbol in the Binary Number system

Octal Numbers

- Octal stands for Eight
- Octal is the base 8 Number System
- It has 8 unique symbols
  - 0 1 2 3 4 5 6 7
- The largest symbol is the base – 1 i.e 8 – 1 = 7
- 7 is the largest symbol in the Octal number system

Hexadecimal Numbers

- Hex = 6 and Decimal = 10 6+10 = 16
- Hex is the base 16 Number System
- It has 16 symbols
  - 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
- It must have 16 ***UNIQUE*** Symbols
  - 0 1 2 3 4 5 6 7 8 9 A B C D E F
- The largest symbol is 16 – 1 = 15 or F

Counting in Binary

	BINARY				HEX
Position:	8	4	2	1	
	0	**0**	**0**	**0**	**0**
	0	**0**	**0**	**1**	**1**
	0	**0**	**1**	**0**	**2**
	0	**0**	**1**	**1**	**3**
	0	**1**	**0**	**0**	**4**
	0	**1**	**0**	**1**	**5**
	0	**1**	**1**	**0**	**6**
	0	**1**	**1**	**1**	**7**
	1	**0**	**0**	**0**	**8**

Counting in Binary

	BINARY				HEX
Position:	8	4	2	1	
	1	**0**	**0**	**0**	**8**
	1	**0**	**0**	**1**	**9**
	1	**0**	**1**	**0**	**A**
	1	**0**	**1**	**1**	**B**
	1	**1**	**0**	**0**	**C**
	1	**1**	**0**	**1**	**D**
	1	**1**	**1**	**0**	**E**
	1	**1**	**1**	**1**	**F**

Counting in Binary – Examples

	BINARY								HEX
Position:	128	64	32	16	8	4	2	1	
	0	**0**	**1**	**1**	**0**	**1**	**0**	**0**	**52**
	1	**0**	**1**	**0**	**0**	**0**	**1**	**1**	**163**
	1	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**255**

Positional Notation – Meaning of Decimal

$$N = P_4P_3P_2P_1P_0$$

$$= P_4b^4 + P_3b^3 + P_2b^2 + P_1b^1 + P_0b^0$$

$$584_{10} = 5 \times 10^2 + 8 \times 10^1 + 4 \times 10^0$$
$$= 500 + 80 + 4$$
$$= 584$$

Positional Notation – Converting from Binary to Decimal

$$N = P_4P_3P_2P_1P_0$$

$$= P_4b^4 + P_3b^3 + P_2b^2 + P_1b^1 + P_0b^0$$

Binary

$$10110_2 = 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$$
$$= 16 + 0 + 4 + 2 + 0$$
$$= 22_{10}$$

Positional Notation – Converting from Hexadecimal to Decimal

$$N = P_4P_3P_2P_1P_0$$

$$= P_4b^4 + P_3b^3 + P_2b^2 + P_1b^1 + P_0b^0$$

Hex

$$3AF_{16} = 3 \times 16^2 + A \times 16^1 + F \times 16^0$$
$$= 3 \times 256 + 10 \times 16 + 15 \times 1$$
$$= 768 + 160 + 15$$
$$= 943_{10}$$

Positional Notation – Converting from Octal to Decimal

$$N = P_4P_3P_2P_1P_0$$

$$= P_4b^4 + P_3b^3 + P_2b^2 + P_1b^1 + P_0b^0$$

Octal

$$457_8 = 4 \times 8^2 + 5 \times 8^1 + 7 \times 8^0$$
$$= 4 \times 64 + 5 \times 8 + 7 \times 1$$
$$= 256 + 40 + 7$$
$$= 303_{10}$$

Converting from Octal to Decimal

- e.g 56564 OCT is Base 8

  $= 5 \times 8^4 + 6 \times 8^3 + 5 \times 8^2 + 6 \times 8^1 + 4 \times 8^0$

  $= 5 \times 4096 + 6 \times 512 + 5 \times 64 + 6 \times 8 + 4 \times 1$

  $= 20480 + 3072 + 320 + 48 + 4$

  $= 23924$

Fractional Numbers - Meaning of Decimal

$$N = ...P_3P_2P_1P_0 \,.\, P_{-1}P_{-2}P_{-3}...$$

$$= ... + P_3b^3 + P_2b^2 + P_1b^1 + P_0b^0 + P_{-1}b^{-1} + P_{-2}b^{-2} + P_{-3}b^{-3} + ...$$

$$375.17_{10} = 3 \times 10^2 + 7 \times 10^1 + 5 \times 10^0 + 1 \times 10^{-1} + 7 \times 10^{-2}$$
$$= 300 + 70 + 5 + 0.1 + 0.07$$
$$= 375.17$$

Fractional Numbers – Converting from Binary to Decimal

$$N = ...P_3P_2P_1P_0 \,.\, P_{-1}P_{-2}P_{-3}...$$

$$= ... + P_3b^3 + P_2b^2 + P_1b^1 + P_0b^0 + P_{-1}b^{-1} + P_{-2}b^{-2} + P_{-3}b^{-3} + ...$$

Binary

$$1101.11_2 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2}$$

$$= 8 + 2 + 0 + 1 + 1/2 + 1/4$$

$$= 11.75_{10}$$

Fractional Numbers – Converting from Binary to Decimal

- In the binary number system (base 2), each column represents a power of 2 instead of 10. For example, the number 101.01 means the following:

= (1 * 2^2) + (0 * 2^1) + (1 * 2^0) + (0 * 2^-1) + (1 * 2^-2)

= 4 + 0 + 1 + 0 + 1/4 = 5.25 Decimal

Fractional Numbers – Converting from Hex to Decimal

$$N = ...P_3P_2P_1P_0 \,.\, P_{-1}P_{-2}P_{-3}...$$

$$= ... + P_3b^3 + P_2b^2 + P_1b^1 + P_0b^0 + P_{-1}b^{-1} + P_{-2}b^{-2} + P_{-3}b^{-3} + ...$$

Hex

$$1AB.6_{16} = 1 \times 16^2 + A \times 16^1 + B \times 16^0 + 6 \times 16^{-1}$$

$$= 1 \times 256 + 10 \times 16 + 11 \times 1 + 6/16$$

$$= 256 + 160 + 11 + 0.375$$

$$= 427.375_{10}$$

Fractional Numbers – Converting from Octal to Decimal

$$N = ...P_3P_2P_1P_0 \,.\, P_{-1}P_{-2}P_{-3}...$$

$$= ... + P_3b^3 + P_2b^2 + P_1b^1 + P_0b^0 + P_{-1}b^{-1} + P_{-2}b^{-2} + P_{-3}b^{-3} + ...$$

Octal

$$173.25_8 = 1 \times 8^2 + 7 \times 8^1 + 3 \times 8^0 + 2 \times 8^{-1} + 5 \times 8^{-2}$$

$$= 1 \times 64 + 7 \times 8 + 3 \times 1 + 2/8 + 5/64$$

$$= 64 + 56 + 3 + 0.25 + 0.078125$$

$$= 123.328125_{10}$$

Converting from Hexadecimal to Decimal

- FF

  F in Hex is 15 so

  = $15 \times 16^1 + 15 \times 16^0$

  = $240 + 15$

  = 255

HEXADECIMAL NUMBERS!
0 1 2 3 4 5 6 7 8 9 a b c d e f

Converting from Decimal to Binary

2	100
2	50 – 0
2	25 – 0
2	12 – 1
2	6 – 0
2	3 – 0
	1 - 1

Answer: 1100100

Converting from Decimal to Hexadecimal

16	100
	6 - 4

64

16	222
	13(D) – 14 (E)

D E

16	1000
16	62 – 8
	3 - 14

3 14 8
3 E 8

Converting from Decimal to Octal

8	64
8	8 - 0
	1 – 0

So answer is 100 Octal equals 64 DECIMAL

Converting from Binary to Octal

- The largest symbol in Octal is 7
- It requires **THREE** binary bit to represent the number 7 ➔ 111_2
- To convert a binary number to Octal, start with the LSB and mark off groups of 3.
  - 101011010010_{LSB}
  - 101-011-010-010 Write Octal symbol below
  - 5 3 2 2_8

Converting from Binary to Octal

- Lets keep it very simple
  - Convert Binary to Decimal
  - And then Decimal to Octal

Converting from Octal to Binary

- To convert an Octal number to binary:
- Write the three bit binary equivalent below
  - 1 5 6 4_8
  - 001 101 110 100 = 1101110100_2
- Leading zeros can be left off.

Converting from Binary to Hexadecimal

- The largest symbol in Hex is 15 or F
- It requires **FOUR** binary bits to represent the number 15 or the symbol F ➔ 1111_2
- To convert a binary number to Hex, start with the LSB and mark off groups of 4.
  - 1011001110101110_{LSB}
  - 1011-0011-1010-1110 Write Hex symbols below
  - B 3 A E $_{16}$

Homework

- Convert in binary
  - 24581_{10}
  - 2475_{8}
  - $8DF5_{16}$
- Convert in Decimal
  - 4255_{8}
  - 1001110010110100_{2}
  - $8DA5_{16}$
- Convert in Octal
  - 24521_{10}
  - 1001110011110100_{2}
  - $8DF5_{16}$
- Convert in Hexadecimal
  - 4265_{8}
  - 1001110010110100_{2}
  - 863728_{10}