Assignment 1: Functional Programming Exercise

CSE271: Principles of Programming Languages
Mijung Kim

Out: September 16, 2021 (Thursday)
Due: September 30, 2021 (Thursday) at 23:59 (KST)

What to submit

Submit the `Hw1.scala` file on Blackboard.

Rules

- You must NOT use the `var`, `for`, or `while` keyword.

- You must NOT include any additional packages or libraries besides the ones that you already have.

Scala environment

1. Please install Java 11 and make sure you configure environment variables in your machine. You can download and install here.

   https://adoptopenjdk.net/

   Download and install OpenJDK 11(LTS), HotSpot

   You can check if you have Java installed by using the following command in a terminal (Windows: PowerShell, Mac/Linux: Terminal). You can also use the terminal through VS Code.

   ```
   Command Line
   java -version
   ```

   If you have Java 11 successfully installed, you'll see messages like these.

   ```
   Sample Output
   openjdk version "11.0.8" 2020-07-14
   OpenJDK Runtime Environment AdoptOpenJDK (build 11.0.8+10)
   OpenJDK 64-Bit Server VM AdoptOpenJDK (build 11.0.8+10, mixed mode)
   ```

   ```
   Sample Output
   openjdk version "11.0.2" 2019-01-15
   OpenJDK Runtime Environment 18.9 (build 11.0.2+9)
   OpenJDK 64-Bit Server VM 18.9 (build 11.0.2+9, mixed mode)
   ```

2. Now you can work with the assignment. Using a terminal, run sbt using these commands.

   On Windows,

On Mac/Linux,

It may take a long time to collect all necessary files from the Internet when you first run the command.

3. Once everything has been collected, you will enter the sbt shell. You can type `test` to test your submission. It will test the submission using accompanied test cases in `src/test/scala/Hw1Test.scala`.

> **Command Line**
>
> ```
> sbt:hw0-cse271> test
> ```

Problems

Problem 1 (10 points)

GCD (greatest common divisor) of two integers is the largest positive integer that divides each of the integers.

For example,

```
gcd(10, 25)
```

evaluates to 5.

Write a function gcd that evaluates to the greatest common divisor:

```
def gcd (n: Int): Int
```

Problem 2 (10 points)

Write a function

```
def oddSum(f: Int=>Int, n: Int): Int
```

that takes another function f and an integer n as arguments and computes the sum of the numbers obtained by applying the odd numbers smaller than n (i.e., $1, 3, 5...n$) to f.

In other words,

```
oddSum(f,n)
```

computes

$$\sum_{k=1}^{\lceil n/2 \rceil} f(2k-1) \tag{1}$$

For example,

```
sum((x: Int) => x, 10)
```

evaluates to 25.

Problem 3 (10 points)

Take this from the lecture slide as the definition of a list data structure.

```scala
sealed trait IntList
case object Nil extends IntList
case class Cons(v: Int, t: IntList) extends IntList
```

Write a function

```scala
def foldRight(init: Int, ftn: Int=>Int, list: IntList): Int
```

that takes a function from integers to integers and a list as an input, and folds to right. For example,

```scala
foldRight(100, (x: Int, y:Int) => x % y, Cons(5,Cons(3,Nil())))
```

evaluates to $(100 \% 3) \% 5 = 1$ while

```scala
foldRight(100, (x: Int, y:Int) => x % y, Cons(3,Cons(5,Nil())))
```

evaluates to $(100 \% 5) \% 3 = 0$

Problem 4 (10 points)

Write a function with type:

```scala
def map(f: Int => Int, list: IntList): IntList
```

that construct a list from another list by applying function.
For example,

```scala
map((x: Int) => x * 3, Cons(5,Cons(3,Cons(6,Nil()))))
```

evaluates to `Cons(15,Cons(9,Cons(18,Nil())))`.

Problem 5 (10 points)

Write a function with type:

```scala
def iter[A](f: A => A, n: Int): A => A
```

that computes a composition of another function by the given number of times, like this.

$$
\begin{aligned}
\text{iter}(f, 1) &= f \\
\text{iter}(f, 2) &= f \circ f = f^2 \\
&\dots \\
\text{iter}(f, n) &= f^n
\end{aligned}
\tag{2}
$$

Problem 6 (20 points)

Write a function that inserts a new integer to a binary search tree. Binary search tree is a binary tree in which all left children are smaller than the parent and all right children are larger than the parent. Use the following as the definition of the tree:

```
sealed trait BTree
case object Leaf extends BTree
case class IntNode(v: Int, left: BTree, right: BTree)
extends BTree
```

And, write a function with type:

```
def insert(t: BTree, a: Int): BTree
```

Assume that the input tree (i.e.,t) does not have any duplicate nodes and the number to be inserted (i.e., n) does not exist in the input tree (t).

Problem 7 (30 points)

Write a function that evaluates a formula into a truth value. The set of formula is defined as follows:

$$
\begin{aligned}
f \rightarrow & T|F \\
& | \ !f \\
& | \ f \ \&\& \ f \\
& | \ f \ || \ f \\
& | \ f \ \text{->} \ f
\end{aligned}
\tag{3}
$$

The following code snippet defines the types representing the formular.

```
sealed trait Formula
case object True extends Formula
case object False extends Formula
case class Not(f: Formula) extends Formula
case class Andalso(left: Formula, right: Formula) extends Formula
case class Orelse(left: Formula, right: Formula)  extends Formula
case class Implies(left: Formula, right: Formula) extends Formula
```

Thus, the function will have the type:

```
def eval(f: Formula): Boolean
```

For example,

```
eval(True())
```

evaluates to true.

```
eval(Andalso(Not(True()),OrElse(Not(True()),True())))
```

evaluates to false.