ASSIGNMENT-2

AI22BTECH11022 Ruvva Suraj Kumar

MISCELLINIOUS EXERCISE 16 : Q9

Question : If 4-digit numbers greater than 5,000 are randomly formed from the digits 0,1,3,5 and 7, what is the probability of forming a number divisible by 5 when,

(i) the digits are repeated?
(ii) the repetition of digits is not allowed?

Solution:

Let i represents digits of the four digit numbers where $i \in \{1, 2, 3, 4\}$

X_i=represets occurence of 0,1,3,5 and 7 in the ith digit where $i \in \{1, 2, 3, 4\}$

EVENT	DESCRIPTION
X_1	occurence of 0,1,3,5 and 7 in the units place
X_2	occurence of 0,1,3,5,and 7 in the tens place
X_3	occurence of 0,1,3,5, and 7 in the hundreds place
X_4	occurence of 0,1,3,5 and 7 in the thousands place

TABLE II

(i) For $i \neq j$ where $i, j \in \{1, 2, 3, 4\}$

$$\Pr\left(X_i = X_j\right) \tag{1}$$

Conditons:

a) Greater than 5000 $\Rightarrow X_4 \in \{5, 7\}$
b) Divisible by 5 $\Rightarrow X_1 \in \{0, 5\}$
c) Repetition allowed $\Rightarrow$ X_2 and X_3 $\in \{0, 1, 3, 5, 7\}$

$$\Rightarrow \Pr\left(X_i = X_j\right) = \frac{(2 \times 5 \times 5 \times 2 - 1)}{(2 \times 5 \times 5 \times 5 - 1)} \tag{2}$$

$$= \frac{99}{249}. \tag{3}$$

(ii) For $i \neq j \ \forall$ i,j $\in \{1, 2, 3, 4\}$

$$\Pr\left(X_i \neq X_j\right) \tag{4}$$

There are three possible cases :

a) $X_4 = 5$
   $X_1 = 0$

b) $X_4 = 7$
   $X_1 = 0$

c) $X_4 = 7$
   $X_1 = 5$

d) repetition not allowed $\Rightarrow$ X_2 and X_3 = two digits from the three left

$$\Rightarrow \Pr\left(X_i \neq X_j\right) = \frac{(6 + 6 + 6)}{(2 \times 4 \times 3 \times 2)} \tag{5}$$

$$= \frac{18}{48} \tag{6}$$

$$= \frac{3}{8}. \tag{7}$$