ASSIGNMENT-2

AI22BTECH11022 Ruvva Suraj Kumar

MISCELLINIOUS EXERCISE 16 : Q9

Question : If 4-digit numbers greater than 5,000 are randomly formed from the digits 0,1,3,5 and 7, what is the probability of forming a number divisible by 5 when,

(i) the digits are repeated?

(ii) the repetition of digits is not allowed?

Solution:

Let i represents digits of the four digit numbers where $i \in \{1, 2, 3, 4\}$

X_i=represets occurence of 0,1,3,5 and 7 in the ith digit where $i \in \{1, 2, 3, 4\}$

EVENT	DESCRIPTION
X_1	occurence of 0,1,3,5 and 7 in the units place
X_2	occurence of 0,1,3,5,and 7 in the tens place
X_3	occurence of 0,1,3,5, and 7 in the hundreds place
X_4	occurence of 0,1,3,5 and 7 in the thousands place

TABLE II

(i) For $i \neq j$ where $i, j \in \{1, 2, 3, 4\}$

$$\Pr\left(X_i = X_j\right) \tag{1}$$

a) Greater than 5,000 $\Rightarrow$ first digit = 5 or 7.

b) Divisible by 5 $\Rightarrow$ last digit = 0 or 5.

c) repetition allowed $\Rightarrow$ second and third digits = 0 or 1 or 3 or 5 or 7.

$$\Rightarrow \Pr\left(X_i = X_j\right) = \frac{(2 \times 5 \times 5 \times 2 - 1)}{(2 \times 5 \times 5 \times 5 - 1)} \tag{2}$$

$$= \frac{99}{249}. \tag{3}$$

(ii) For $i \neq j$ for all $i, j \in \{1, 2, 3, 4\}$

$$\Pr\left(X_i \neq X_j\right) \tag{4}$$

There are three possible cases :

a) first digit = 5
last digit = 0

b) first digit = 7
last digit = 0

c) first digit = 7
last digit = 5

d) repetition not allowed $\Rightarrow$ second and third digits = two digits from the three left

$$\Rightarrow \Pr\left(X_i \neq X_j\right) = \frac{6 + 6 + 6}{2 \times 4 \times 3 \times 2} \tag{5}$$

$$= \frac{18}{48} \tag{6}$$

$$= \frac{3}{8}. \tag{7}$$