

ASSIGNMENT-2

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MISCELLINIOUS EXERCISE 16 : Q9

Question : If 4-digit numbers greater than 5,000 are randomly formed from the digits 0,1,3,5 and 7, what is the probability of forming a number divisible by 5 when,

- (i) the digits are repeated?
- (ii) the repetition of digits is not allowed?

Solution:

Let i represents digits of the four digit numbers where $i \in \{1, 2, 3, 4\}$

X_i =represents occurrence of 0,1,3,5 and 7 in the i th digit where $i \in \{1, 2, 3, 4\}$

| EVENT | DESCRIPTION |
|-------|----------------------------------------------------|
| X_1 | occurrence of 0,1,3,5 and 7 in the units place |
| X_2 | occurrence of 0,1,3,5, and 7 in the tens place |
| X_3 | occurrence of 0,1,3,5, and 7 in the hundreds place |
| X_4 | occurrence of 0,1,3,5 and 7 in the thousands place |

TABLE II

To form a number divisible by 5, X_4 must be either 0 or 5. There are 2 favorable outcomes for X_4 out of the 5 possible values.

- (i) For $i \neq j$ where $i, j \in \{1, 2, 3, 4\}$

$$\Pr(X_i = X_j) \quad (1)$$

The $X_4 \in \{5, 7\}$

The remaining 3 places can be filled with any of the remaining 4 digits, that is in 5 ways except 5000

$$\Rightarrow \text{Total number of 4-digit numbers greater than 5000} = 2 \times 5 \times 5 \times 5 - 1$$

A number is divisible by 5 $\Rightarrow X_1 \in \{0, 5\}$

$$\Rightarrow \text{Total number of 4-digit numbers greater than 5000 that are divisible by 5} = 2 \times 5 \times 5 \times 2 - 1$$

$$\Rightarrow \Pr(X_i = X_j) = \frac{(2 \times 5 \times 5 \times 2 - 1)}{(2 \times 5 \times 5 \times 5 - 1)} \quad (2)$$

$$= \frac{99}{249}. \quad (3)$$

- (ii) For $i \neq j \forall i, j \in \{1, 2, 3, 4\}$

$$\Pr(X_i \neq X_j) \quad (4)$$

The thousands place can be filled with either of the two digits 5 or 7, i.e., in 2 ways.

The remaining 3 places can be filled with any of the remaining 4 digits.

$$\Rightarrow \text{Total number of 4-digit numbers greater than 5000} = 2 \times 4 \times 3 \times 2 = 48$$

For total number of 4-digit numbers greater than 5000 that are divisible by 5 there are three possible cases :

$$\begin{aligned} \text{a) } X_4 &= 5 \\ X_1 &= 0 \end{aligned}$$

$$\begin{aligned} \text{b) } X_4 &= 7 \\ X_1 &= 0 \end{aligned}$$

$$\begin{aligned} \text{c) } X_4 &= 7 \\ X_1 &= 5 \end{aligned}$$

d) repetition not allowed $\Rightarrow X_2$ and X_3 = two digits from the three left

$$\Rightarrow \text{Total number of 4-digit numbers greater than 5000 that are divisible by 5} = 2 \times 3 + 2 \times 3 + 2 \times 3$$

$$\Rightarrow \Pr(X_i \neq X_j) = \frac{(6 + 6 + 6)}{(2 \times 4 \times 3 \times 2)} \quad (5)$$

$$= \frac{18}{48} \quad (6)$$

$$= \frac{3}{8}. \quad (7)$$