

ASSIGNMENT-2

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MISCELLINIOUS EXERCISE 16 : Q9

Question : If 4-digit numbers greater than 5,000 are randomly formed from the digits 0,1,3,5 and 7, what is the probability of forming a number divisible by 5 when,

- (i) the digits are repeated?
- (ii) the repetition of digits is not allowed?

Solution:

Let i represents digits of the four digit numbers where $i \in \{1, 2, 3, 4\}$

X_i =represents occurence of 0,1,3,5,7 in i th digit where $i \in \{1, 2, 3, 4\}$

$\Rightarrow X_i = \{0, 1, 3, 5, 7, \}$ where $i \in \{1, 2, 3, 4\}$

- (i) for $i \neq j$ where $i, j \in \{1, 2, 3, 4\}$

$$\Pr(X_i = X_j) \quad (1)$$

- a) Greater than 5,000 \Rightarrow first digit = 5 or 7.
- b) Divisible by 5 \Rightarrow last digit = 0 or 5.
- c) repetition allowed \Rightarrow second and third digits = 0 or 1 or 3 or 5 or 7.

$$\Rightarrow \Pr(X_i = X_j) = \frac{(2 \times 5 \times 5 \times 2 - 1)}{(2 \times 5 \times 5 \times 5 - 1)} \quad (2)$$

$$= \frac{99}{249}. \quad (3)$$

- (ii) for $i \neq j$ for all $i, j \in \{1, 2, 3, 4\}$

$$\Pr(X_i \neq X_j) \quad (4)$$

There are three possible cases :

- a) first digit = 5
last digit = 0

- b) first digit = 7
last digit = 0

- c) first digit = 7
last digit = 5

- d) repetition not allowed \Rightarrow second and third digits = two digits from the three left

$$\Rightarrow \Pr(X_i \neq X_j) = \frac{6 + 6 + 6}{2 \times 4 \times 3 \times 2} \quad (5)$$

$$= \frac{18}{48} \quad (6)$$

$$= \frac{3}{8}. \quad (7)$$