ASSIGNMENT-2

AI22BTECH11022

MISCELLINIOUS EXERCISE 16 : Q9

Question:

If 4-digit numbers greater than 5,000 are randomly formed from the digits 0,1,3,5 and 7, what is the probability of forming a number divisible by 5 when

(i) the digits are repeated?

(ii) the repetition of digits is not allowed?

Solution:

Let X={0, 1} be a random variable which represents the 4-digit number formed from the digits 0,1,3,5 and 7.

let Y={0, 1} be another random variable representing the repetition of digits in the 4-digit number.

(i)

$$\Pr\left(\frac{(X=0)\,(X=1)}{(Y=0)}\right) \tag{1}$$

a) Greater than 5,000 $\Rightarrow$ first digit = 5 or 7.

b) Divisible by 5 $\Rightarrow$ last digit = 0 or 5.

c) repetition allowed $\Rightarrow$ second and third digits = 0 or 1 or 3 or 5 or 7.

$$\Rightarrow \Pr\left(\frac{(X=0)\,(X=1)}{(Y=0)}\right) = \frac{(2\times5\times5\times2-1)}{(2\times5\times5\times5-1)} \tag{2}$$

$$= \frac{99}{249}. \tag{3}$$

(ii)

$$\Pr\left(\frac{(X=0)\,(X=1)}{(Y=1)}\right) \tag{4}$$

There are three possible cases :

a) first digit = 5
last digit = 0

b) first digit = 7
last digit = 0

c) first digit = 7
last digit = 5

d) repetition not allowed $\Rightarrow$ second and third digits = two digits from the three left

$$\Rightarrow \Pr\left(\frac{(X=0)\,(X=1)}{(Y=1)}\right) = \frac{6+6+6}{2\times4\times3\times2} \tag{5}$$

$$= \frac{18}{48} \tag{6}$$

$$= \frac{3}{8}. \tag{7}$$