Dynamic radioastronomic image reconstruction using Kalman filter under ionospheric perturbations

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Contents

Part I
Introduction

Part II
Notations and problem

1 Notations

1.1 General notations

1.2 Hadamard product

Hadamard product, denoted $\odot$, is the element-wise product of same sized matrices :

$$\forall (\mathrm{A},\mathrm{B}) \in \left(\mathbb{C}^{m\times n}\right)^2, \quad \mathrm{A}\odot\mathrm{B} \in \mathbb{C}^{m\times n} \quad \text{and} \quad \forall (i,j) \in [\![1;m]\!] \times [\![1;n]\!], (\mathrm{A}\odot\mathrm{B})_{i,j} = \mathrm{A}_{i,j} \times \mathrm{B}_{i,j}$$

We can further introduce Hadamard inversion :

$$\forall \mathrm{A} \in (\mathbb{C}\backslash\{0\})^{m\times n}, \quad \mathrm{A}^{\circ -1} \in (\mathbb{C}\backslash\{0\})^{m\times n} \quad \text{and} \quad \forall (i,j) \in [\![1;m]\!] \times [\![1;n]\!], \left(\mathrm{A}^{\circ -1}\right)_{i,j} = \left(\mathrm{A}_{i,j}\right)^{-1}$$

And notation $\oslash$ being Hadamard division :

$$\forall (\mathrm{A},\mathrm{B}) \in \mathbb{C}^{m\times n} \times (\mathbb{C}\backslash 0)^{m\times n}, \quad \mathrm{A}\oslash\mathrm{B} \in \mathbb{C}^{m\times n} \quad \text{and} \quad \forall (i,j) \in [\![1;m]\!] \times [\![1;n]\!], (\mathrm{A}\oslash\mathrm{B})_{i,j} = \frac{\mathrm{A}_{i,j}}{\mathrm{B}_{i,j}}$$

1.3 Norms and distances

In order to compute distances between matrices, we note $||\bullet||_\mathrm{F}$ Frobenius norm :

$$\mathrm{A} \in \mathcal{M}_{m,n}(\mathbb{K}) \quad ||\mathrm{A}||_\mathrm{F} := \sqrt{\mathrm{tr}\left\{\mathrm{A}\mathrm{A}^\mathrm{H}\right\}} = \sqrt{\sum_{\substack{1\le i\le m\\ 1\le j\le n}} \left|\mathrm{A}_{ij}\right|^2}$$

Inducing the distance d_1 :

$$\forall (\mathrm{A},\mathrm{B}) \in \left(\mathcal{M}_{m,n}(\mathbb{K})\right)^2 \quad d_1(\mathrm{A},\mathrm{B}) = \frac{||\mathrm{A}-\mathrm{B}||_\mathrm{F}}{mn}$$

In order to measure influence of a specific element, the ??? distance d_2 is the following :

$$\forall (\mathrm{A},\mathrm{B}) \in \left(\mathcal{M}_{m,n}(\mathbb{K})\right)^2 \quad d_2(\mathrm{A},\mathrm{B}) = \sum_{\substack{1\le i\le m\\ 1\le j\le n}} \left|\mathrm{A}_{ij} - \mathrm{B}_{i,j}\right|$$

2 Evolution model and Kalman filtering

We consider a set $\{x_k\}$ of n vector as theoretical images taken at a regular time interval. Those images are measured by an antenna mesh which UV plane will be written H, more generally written as observation model. Measurements vectors will be designated by n measures $\{y_k\}$.
We will consider the following model to describe time evolution and observation equation :

$$\begin{cases} x_k = \mathrm{A}x_{k-1} + w_k \\ y_k = \mathrm{H}x_k + v_k \end{cases}$$

Where :

- A is the linear state-transition model between time $k-1$ and k, supposing evolution is time invariant
- $\{w_k\}$ is the state noise sequence
- $\{v_k\}$ is the measurements noise sequence

In order to build images dynamically with better precision, a Kalman filter has been implemented.

This filter requires more inputs to estimate the state of the system :

- $\{\mathrm{Q}_k\}$, the covariance of the process noise sequence
- $\{\mathrm{R}_k\}$, the covariance of the observation noise sequence

Which can be rewritten as

$$\begin{cases} \mathrm{Q}_k = \left\langle w_{k-1} w_{k-1}^{\mathrm{H}} \right\rangle - \langle w_{k-1} \rangle \langle w_{k-1} \rangle^{\mathrm{H}} \\ \mathrm{R}_k = \left\langle v_k v_k^{\mathrm{H}} \right\rangle - \langle v_k \rangle \langle v_k \rangle^{\mathrm{H}} \\ \langle x x \rangle \end{cases}$$

Part III
Reliability of Kalman Filter under perturbations

1 Perturbations sources

2 Effects

2.1 Position

2.2 Direction

Part IV
Experimental correction

1 implementation

2 Measures