

Dynamic radioastronomic image reconstruction using Kalman filter under ionospheric perturbations

G. Robert-Dautun

Supervised by I. Vin

Satie
France
July 2022

Contents

I	Introduction	2
II	Notations and problem	3
1	Notations	3
1.1	Hadamard product	3
1.2	Norms and distances	3
2	Evolution model and Kalman filtering	3
III	Reliability of Kalman Filter under perturbations	4
1	Perturbations sources	4
2	Effects	4
2.1	Position	4
2.2	Direction	4
IV	Experimental correction	5
1	implementation	5
2	Measures	5

Part I

Introduction

Part II

Notations and problem

1 Notations

1.1 Hadamard product

Hadamard product, denoted \odot , is the element-wise product of same sized matrices :

$$\forall (A, B) \in (\mathbb{C}^{m \times n})^2, \quad A \odot B \in \mathbb{C}^{m \times n} \quad \text{and} \quad \forall (i, j) \in \llbracket 1; m \rrbracket \times \llbracket 1; n \rrbracket, (A \odot B)_{i,j} = A_{i,j} \times B_{i,j}$$

We can further introduce Hadamard inversion :

$$\forall A \in (\mathbb{C} \setminus \{0\})^{m \times n}, \quad A^{\circ-1} \in (\mathbb{C} \setminus \{0\})^{m \times n} \quad \text{and} \quad \forall (i, j) \in \llbracket 1; m \rrbracket \times \llbracket 1; n \rrbracket, (A^{\circ-1})_{i,j} = (A_{i,j})^{-1}$$

And notation \oslash being Hadamard division :

$$\forall (A, B) \in \mathbb{C}^{m \times n} \times (\mathbb{C} \setminus \{0\})^{m \times n}, \quad A \oslash B \in \mathbb{C}^{m \times n} \quad \text{and} \quad \forall (i, j) \in \llbracket 1; m \rrbracket \times \llbracket 1; n \rrbracket, (A \oslash B)_{i,j} = \frac{A_{i,j}}{B_{i,j}}$$

1.2 Norms and distances

In order to compute distances between matrices, we note $\|\bullet\|_F$ Frobenius norm :

$$A \in \mathcal{M}_{m,n}(\mathbb{K}) \quad \|A\|_F := \sqrt{\text{tr}\{AA^H\}} = \sqrt{\sum_{\substack{1 \leq i \leq m \\ 1 \leq j \leq n}} |A_{i,j}|^2}$$

Inducing the distance d_1 :

$$\forall (A, B) \in (\mathcal{M}_{m,n}(\mathbb{K}))^2 \quad d_1(A, B) = \frac{\|A - B\|_F}{mn}$$

In order to measure influence of a specific element, the d_2 distance is the following :

$$\forall (A, B) \in (\mathcal{M}_{m,n}(\mathbb{K}))^2 \quad d_2(A, B) = \sum_{\substack{1 \leq i \leq m \\ 1 \leq j \leq n}} |A_{i,j} - B_{i,j}|$$

2 Evolution model and Kalman filtering

We consider a set $\{x_k\}$ of n vector as theoretical images taken at a regular time interval. Those images are measured by an antenna mesh which UV plane will be written H . Measurements vectors will be designated by n measures $\{y_k\}$.

We will consider the following model to describe time related evolutions and measure equation :

$$\begin{cases} x_k = Ax_{k-1} + w_{k-1} \\ y_k = Hx_k + v_k \end{cases}$$

Where :

- A is the linear evolutionary model between time $k-1$ and k , supposing evolution is time invariant
- $\{w_k\}$ are the additive evolutionary errors
- $\{v_k\}$ are the measurements noises

In order to build images dynamically with better precision, a Kalman filter has been implemented.

This filter requires assumptions about

Part III**Reliability of Kalman Filter under perturbations****1 Perturbations sources****2 Effects****2.1 Position****2.2 Direction**

Part IV**Experimental correction****1 implementation****2 Measures**