Semiring FSA Practice

Kevin Liang
Ling 185a
August 30, 2024

1 Semirings

We define a semiring in Haskell as a type class that has the four following items:

```
class Semiring a where
  (&&&) :: a -> a -> a
  (|||) :: a -> a -> a
  gtrue :: a
  gfalse :: a
```

By defining these functions (and ensuring they satisfy semiring properties), we can now write general/abstract functions on any semiring FSA, regardless of what values are associated with the FSA. We can also write functions that don't operate on FSAs, but just the semiring type itself, such as `gen_and` and `gen_or`.

Caution! There are four types of "and"/"or" we have looked at so far.

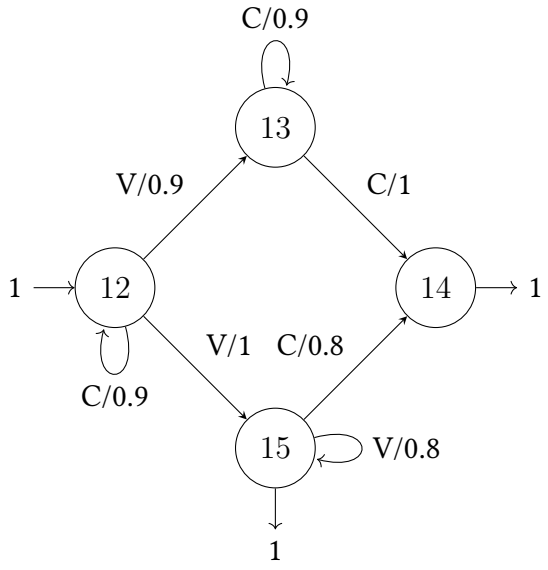
```
&&, || :: Bool -> Bool -> Bool
and, or :: [Bool] -> Bool
&&&, ||| :: Semiring a => a -> a -> a
gen_and, gen_or :: Semiring a => [a] -> a
```

The term "generalized" has been used to characterize all but the first, but they generalize our logical operators in different ways. When in doubt, check types!

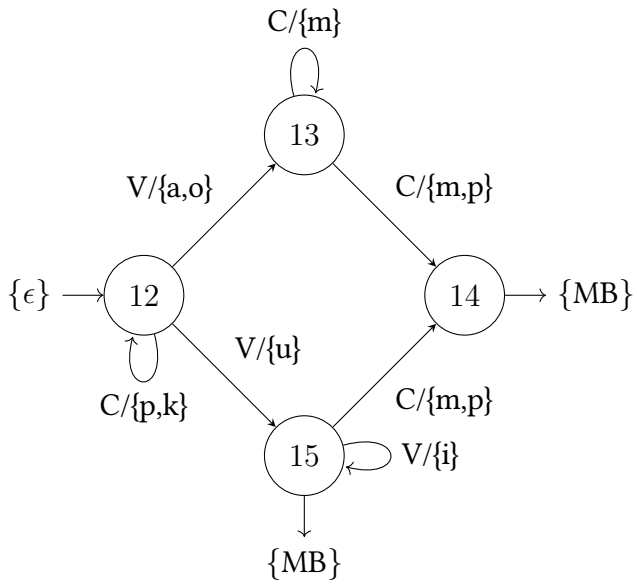
2 Semiring forward values

Calculate the forward values for these semiring FSAs on the string CVC. What are the overall values assigned to the string CVC?

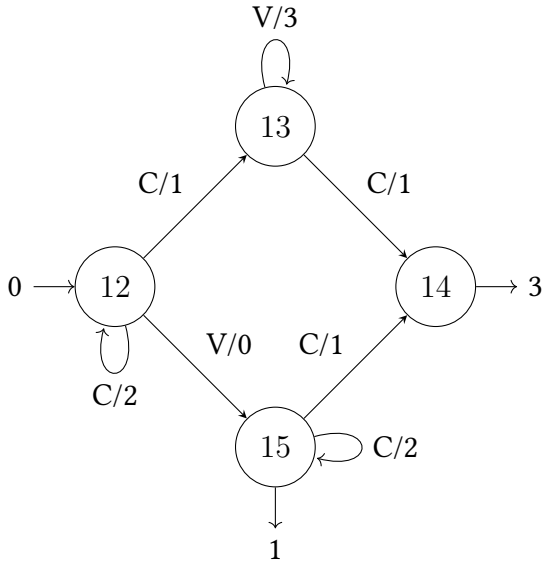
Weighted FSA



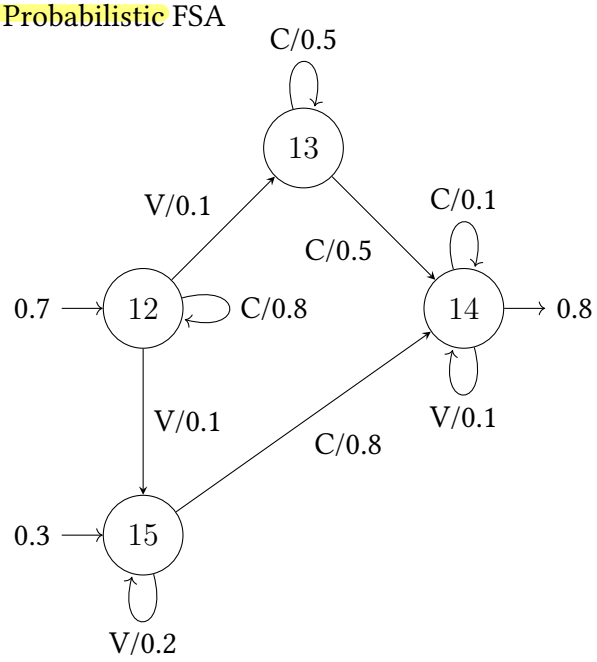
Finite State Transducer



Cost FSA



Probabilistic FSA



state	c	v	c
12	$\{t\}$		
13	$\{\}$		
14	$\{\}$		
15	$\{\}$		

$\cup \{ fwd(t)(12) \cdot del(12, c, 1\cancel{2}), \; fwd(t)(13) \cdot del(13, c, 1\cancel{2}), \; fwd(t)(14) \cdot del(14, c, 1\cancel{2}),$
$fwd(t)(15) \cdot del(15, c, 1\cancel{2}) \}$

(red annotations above each "1$\cancel{2}$": 15, $\cancel{14}$, $\cancel{13}$; below the last: $\cancel{13}$, $\cancel{14}$, 15)

$\cup \{ \{t\} \cdot \{p, k\}, \; \{\} \cdot \{\}, \; \{\} \cdot \{\}, \; \{\} \cdot \{\}, \; \{\} \cdot \{\} \}$

$\cup \{ \{p, k\}$

$\cup \{ \{t\} \cdot \{\}, \; \{\} \cdot$

$\cup \{ \{t\} \cdot \{\}, \; \{$

$\cup \{ \{t\} \cdot \{\}, \; \{\}$

$\cup\{fwd(c)(l2)\cdot del(l2,V,l3),\ fwd(c)(l3)\cdot del(l3,V,l3),\ fwd(c)(l4)\cdot del(l4,V,l3)$

$fwd(c)(l5)\cdot del(l5,V,l3)$

$\cup\{\{p,k\}\cdot\{q,v\},\{\}$

$\downarrow$

$\cup\{\{pq,pv,kq,kv\}$

$\cup\{fwd(cv)(l2)\cdot del(l2,c,l4),\ fwd(cv)(l3)\cdot del(l3,c,l4),\ fwd(cv)(l4)\cdot del(l4,c,l5),$

$fwd(cv)(l5)\cdot del(l5,c,l4)$

$\cup\{\{\}\cdot\ ,\{pq,pv,kq,kv\}\cdot\{\},\{\}\ ,\{pv,kv\}\cdot\{\}$

$\cup\{\{\}\cdot\ ,\{pq,pv,kq,kv\}\cdot\{m\},\{\}\ ,\{pv,kv\}\cdot\{\}$

$\vee\{\{\}\ ,\{pq,pv,kq,kv\}\cdot\{m,p\},\{\}\ ,\{pv,kv\}\cdot\{m,p\}$

$\vee\{\{\}\ ,\{pq,pv,kq,kv\}\cdot\{\},\{\}\ ,\{pv,kv\}\cdot\{\}$

$\{pqm,pvm,kqm,kvm,pqp,pvp,kqp,kvp,\ pvm,kvm,pvp,kvp$

Using the WFSA:

State		C	V	C
12	1.0	0.9	0.0	0.0
13	0.0	0.6	0.81	0.729
14	0.0	0.0	0.6	0.81
15	0.0	0.0	0.9	0

$\max[fwd(t)(12) \times dcl(12,c,12), fwd(t)(13) \times dcl(13,c,13), fwd(t)(14) \times dcl(14,c,14)$

$0.81 \times 13,c,13\}$

0.81×0.9

Using the cost FSA:

State	C	V	C
12	0		
13	∞		
14	∞		
15	∞		

$$\min[fwd(t)(12) + dc|(12,c,12), fwd(t)(13) + dc|(13,c,12), fwd(t)(14) + dc|(14,c,12), fwd(t)(15) + dc|(15,c,12)]$$

$$\min[\quad 0 + 2 \quad , \quad \infty + \infty \quad , \quad \infty + \infty \quad , \quad \infty + \infty$$