Assignment questions on Laplace transform

1. Laplace transform of $cos^2\ 2t =$ _____.
2. If the Laplace transform of $f(t)$ is $log\left(\frac{s-1}{s}\right)$, then the Laplace transform of $e^{-t}f(2t)$ is _____.
3. Evaluate $\int_0^\infty e^{-\sqrt{2}t} \sinh 3t\ dt$ by using Laplace transform.
4. If $L[f(t)] = \frac{5}{s\ (s^2+7)}$ then $L[f(2t)] =$ ___.
5. The region of convergence for the Laplace transform of $\sin 2t$ is _____.
6. $L\left[e^{-\frac{t}{2}}\sqrt{t}\right]$=_____.
7. Evaluate $L\left[\int_0^t t(t+2)^2\ dt\right]$
8. Find the Laplace transform of $\int_0^t \frac{\sin t}{t} dt + te^{-2t}\ cos^2\ 2t + \frac{e^{-at}-\cos bt}{t}$
9. $L[\cos(2t+6)]$ +$L\left[\int_0^t e^{2t}t^3\ dt\right]$
10. Find the Laplace transform of $2^t + \frac{\cos 2t-\cos 3t}{t} + tsin\ t$
11. Find the Laplace transform of $f(t) = e^{3t}\left(\frac{1-\cos t}{t}\right)$.
12. Evaluate $\int_0^\infty e^{-6t}t \sin t\ dt$ by using Laplace transform.
13. Evaluate $\int_0^\infty e^{-4t}\frac{sin^2\ t}{t} dt$ by using Laplace transform
14. Compute the Laplace transform of the function $f(t) = \begin{cases} t, & 0 < t < \pi \\ \pi - t, & \pi < t < 2\pi \end{cases}$ such that $f(t+2\pi) = f(t)$.
15. Find the Laplace transform of periodic function $f(t) = \begin{cases} \cos t\,, & 0 < t \le \pi \\ -1, & \pi \le t \le 2\pi \end{cases}$ where $f(t+2\pi) = f(t)$.
16. Find the Laplace transform of periodic function $f(t) = \begin{cases} t, & 0 < t \le a \\ 2a - t, & a \le t \le 2a \end{cases}$ where $f(t+2a) = f(t)$. Sketch the graph of function.
17. A half wave rectifier function of period $\frac{2\pi}{\omega}$ is defined by $f(t) = \begin{cases} E\ sin\ \omega\ t, 0 \le t < \frac{\pi}{\omega} \\ 0, \frac{\pi}{\omega} \le t \le \frac{2\pi}{\omega} \end{cases}$ where E and ω are positive constants. Prove that $L\{f(t)\} = \frac{E\omega}{(s^2+\omega^2)(1-e^{-\pi s/\omega})}$
18. Show that the Laplace transform of the periodic function $f(t)$ is $\frac{a}{s}\tanh\left(\frac{as}{2}\right)$, where $f(t) = \begin{cases} k & 0 \le t \le a \\ -k, & a \le t \le 2a \end{cases}$, $f(t+2a) = f(t)$. Also sketch the graph of f(t).
19. Find the Laplace transform of the functions: (i) $t^2 \sin 4t$ (ii) $(e^{t-2} + 3t - 6)H(t-2)$
20. Express the following function in terms of unit step function and hence find the Laplace transform of $f(t) = \begin{cases} \frac{t}{a} & 0 < t < a \\ 1, & t \ge a \end{cases}$
21. Express the following function as a Heaviside step function and hence find the Laplace transform of $f(t) = \begin{cases} 1, & 0 \le t < 1 \\ t, & 1 \le t < 2 \\ t^2, & t > 2 \end{cases}$

22. Express the following function as a Heaviside step function and hence find the Laplace transform of (i) $f(t) = \begin{cases} 2, & 0 \le t < 1 \\ 3t+1, & 1 \le t < 2 \\ t^2, & t > 2 \end{cases}$

23. Express the following in terms of unit step function and hence find the Laplace transform of $f(t) = \begin{cases} \cos t, & 0 < t \le \pi \\ 1, & \pi < t \le 2\pi \\ sint & t \ge 2\pi \end{cases}$

24. Express the following function as a Heaviside step function and hence calculate the Laplace transform of $f(t) = \begin{cases} t^2, & 0 \le t < 2 \\ 4t, & 2 \le t < 4 \\ 8, & t > 4 \end{cases}$

25. Express the following function as a Heaviside unit step function and hence find the Laplace transform of $f(t) = \begin{cases} \sin t, & 0 \le t < \pi \\ \sin 2t, & \pi \le t < 2\pi \\ \sin 3t, & t > 2\pi \end{cases}$

26. Express the following function as a Heaviside unit step function and hence find the Laplace transform of $f(t) = \begin{cases} \cos t, & 0 \le t < \pi \\ \cos 2t, & \pi \le t < 2\pi \\ \cos 3t, & t > 2\pi \end{cases}$