Linear Regression and the CAPM project

Giada Martini, Carmen de Cabo: Writing Report and Analysis
Riccardo Caruso: Coding and Checking Report

UNIPD 2023/2024

Introduction

Linear regression and the Capital Asset Pricing Model (CAPM) provide valuable insights into variable relationships and asset pricing assessments. This report is about applications of linear regression — a model that evaluate the relationship between a dependent variable and one or more independent variables, simultaneously, with the CAPM model. We aim on the analysis of ten distinct banks within the STOXX EURO 600 index, examining their excess market returns through the lens of alpha, beta, and r-squared parameters, in order to derive considerations about the effectiveness of the model. To derive the parameters of the linear model, we use the Ordinary Least Squares (OLS) function. In this way, we can identify coefficients that minimize the sum of squared residuals, thereby optimizing the fit of the model to the observed data.

In order to capture more of the variability in the excess bank returns, we run a multifactor model introducing additional explanatory variables to the linear model. Therefore, we compare the correlations across residuals of the two models.

Furthermore, to assess the model's validity, we conduct specific diagnostic tests: Reset test, White test, Breusch-Godfrey test, Durbin-Watson test and Chow test.

Finally, we use another approach to estimate the CAPM, i.e. the rolling window approach, as an alternative to the Chow test.

Analysis

First of all, we take the data from Refinitiv searching for the corresponding constituent list and we select the information of our interest. We initiate the process by defining a monthly date-time index using the *date_range* function. We decide to start from November 1st, 2009, onward, as there has been a recovery in stock values, marking a rebound from the 2007-2008 financial crisis, extending through to October 31st, 2023.

We import data from an Excel file named $'Data\_Banks\_EuroStoxx600\_Euribor.xlsx'$. This includes information on the STOXX EUROPE 600 Total Return Index, 3-month EURIBOR rates, and the banks constituents.

In the STOXX EUROPE 600 we find 44 banks, we extract only the 10 banks with the highest market value, which is the price an asset would fetch in the marketplace, or the value that the investment community gives to a particular equity or business. In addition, a company's market value is a good indication of investors' perceptions about its business prospects. Market value is also dependent on numerous other factors, such as the sector in which the company operates, its profitability, debt load, and the broad market environment.

Market value can give an indication of whether a company's shares are over - or undervalued, depending on the difference between market value and the fair value. Traders and investors will often buy and sell stocks based on their findings. This allows them to take advantage of the disconnect between the two prices when the market corrects itself. It can also be quite an objective measure, as share prices are determined by fluctuations in supply and demand. This means that the market value of an asset only represents what someone is willing to pay for it, rather than its intrinsic value.

In summary, we decide to adopt it as our guideline due to its concrete methodology, which eliminates ambiguity and uncertainty when determining the value of an asset.

In our approach to determine interest rates, we choose to leverage the 3-month EURIBOR rates. This selection is based on the fact that these rates are derived from the interest rates at which a diverse panel of European banks engage in interbank borrowing. By opting for the

3-month EURIBOR rates, we aim to capture a comprehensive and dynamic reflection of the current lending landscape among European financial institutions.

We calculate the monthly returns for both the market (STOXX EUROPE 600) and the selected Banks, converting them into percentages and adjusting for logarithmic differences.

Finally, excess returns are calculated by subtracting the risk-free rate (monthly EURIBOR rate) from the returns of both the market and the selected banks, creating erBanks and erMkt arrays for further analysis. Then, we've created scatter plots comparing the market index with individual equity time series. We saved the plots as images ("Equity vs Mkt" folder), which are reported and commented below.

The OLS function is used to perform Ordinary Least Squares regression on excess market returns (erMkt) and excess bank returns (erBanks). This approach allows us to determine coefficients that minimize the sum of squared residuals, providing a detailed analysis of the relationship between market and bank excess returns. Then, the regression results are stored.

The scatter plots (Fig.1-10) presented in our analysis reveal an upward trend, indicative of a linear relationship following a straight line. This could suggest that changes in the market index returns (STOXX EUROPE 600) are associated with proportional changes in the banks excess returns. Comparing the different scatter plots, we see that they are following approximately the overall trend. However, some of them exhibit a less consistent pattern, i.e. the HSBC Holdings (Fig.5), the Danske Bank (Fig.3) and the Santander Bank Polska (Fig.7), characterized by a greater dispersion of values when compared to their counterparts. This divergence implies that, despite the general linear trend, these particular banks may have unique characteristics or external factors influencing their returns.

Figure 1: Banco Santander

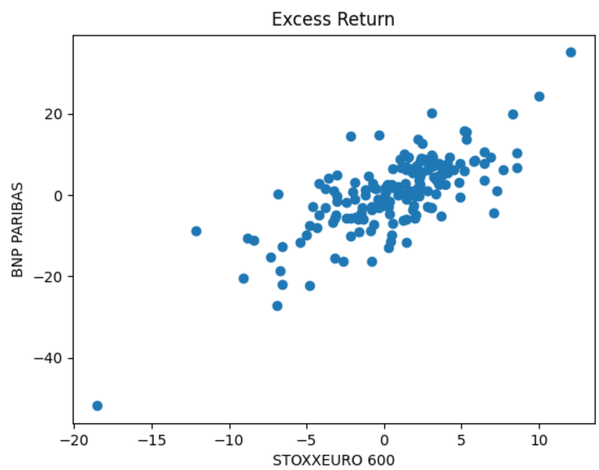


Figure 2: BNP Paribas

Figure 3: Danske Bank

Figure 4: DNB Bank

Figure 5: HSBC Holdings

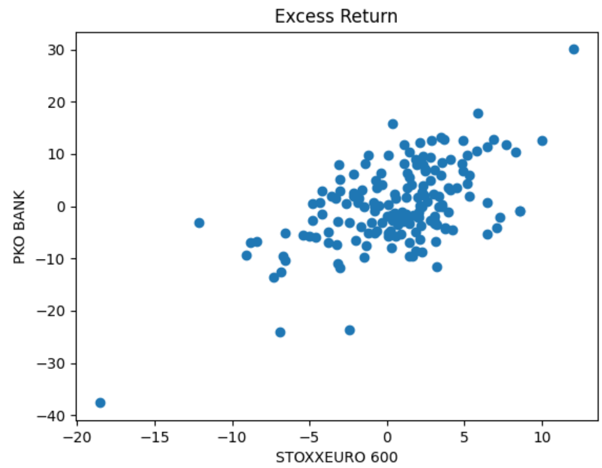


Figure 6: PKO Bank

Figure 7: Santander Bank Polska

Figure 8: Skandinaviska Enskilda BankenA

Figure 9: Swenska Handelsbanken A

Figure 10: Swedbank A

For the CAPM estimation, we run the OLS function, that is defined inside the 'Function.py' file. We selected the quantities of interest (alpha, beta, p-value, r-squared) and reported them in the table below (1). For the analysis, we consider that alpha represents the excess return of an asset or portfolio beyond what would be predicted by the CAPM, beta measures the sensitivity of an asset's excess returns to changes in the market excess returns. It indicates the asset's systematic risk. The p-value test the significance of an observed result, it represents its probability. Finally, the r-squared measures the proportion of the variance in the dependent variable (bank excess returns) that is explained by changes in the independent variable (market excess returns).

banks	**alpha**	**beta**	**p-value**	**r-squared**
DANSKE BANK	-0.3081	1.0833	0	0.2931
HSBC HOLDINGS	-0.4557	0.9659	0	0.3083
SKANDINAVISKA ENSKILDA BANKEN A	0.2785	1.1719	0	0.4686
SVENSKA HANDELSBANKEN A	0.1481	0.8394	0	0.3686
BANCO SANTANDER	-1.1139	1.3970	0	0.3969
DNB BANK	0.3707	1.1004	0	0.4489
BNP PARIBAS	-0.6881	1.7379	0	0.5566
SWEDBANK A	0.4740	1.0007	0	0.3369
SANTANDER BANK POLSKA	-0.0428	1.0531	0	0.2738
PKO BANK	-0.5373	1.0903	0	0.3248

Table 1: Quantities of interest of the CAPM estimation for each bank.

For each bank we analyse what follow.

If alpha is negative, i.e. for Banco Santander, HSBC Holdings, BNP Paribas , PKO bank, Danske Bank, Santander Bank Polska, that means the asset is underperforming the expected return. It implies that the asset is not providing returns commensurate with its beta. On the other hand, if alpha is positive, i.e. for Skandinaviska Enskilda Banken, Swedbank A, Svenska Handelsbanken A, DNB Bank, the asset is outperforming the expected return given its beta. It suggests the asset is providing higher returns than the market.

Considering the beta, we can say that the asset is more volatile than the market if beta is greater than 1, i.e. for Skandinaviska Enskilda Banken , Banco Santander, BNP Paribas, Danske Bank, DNB Bank, PKO Bank, and Santander Bank Pollska. In particular, this last one has a volatility grater than the 70% of the market index. If beta is less than 1, i.e. for Svenska Handelsbanken A, the asset is less volatile than the market. Otherwise, if beta equal to 1, i.e. for Swedbank A, HSBC Holdings, then the asset tends to move in line with the market.

Taking in consideration r-squared, we can say that if r-squared is closed to zero, i.e. for Danske Bank, HSBC Holdings or Santander Bank Polska, the model does not effectively explain the variability, indicating that other factors may influence the asset's returns. This explain what we have discovered in the second assignments for these three banks, as we can see also in their plots.

On the other hand, if r-squared is closed to 1, i.e. for BNP Paribas, Skandinaviska Enskilda Banken, DNB Bank, a percentage of the variability in the asset's returns is explained by the market and the model effectively explain the variability.

Finally, we can state that, as the p-value is zero, the parameters are statistically significant.

From the evaluation of the returns of an equally weighted portfolio (table 2), we can observe that is has a higher volatility than most banks taken individually, that's because some of the banks have a volatility significantly greater than the market index (i.e. Banco Santander and BNP Paribas).

Finally, we can say that the equally weighted portfolio is coherent with the market index, as its r-squared has a value extremely closed to 1 compared with the r-squared of each bank.

Therefore, a larger proportion of the investment's variability can be explained by market movements.

	alpha	**beta**	**p-value**	**r-squared**
Equally Weighted Portfolio	-0.1878	1.1440	0	0.6125

Table 2: Quantities of interest of the CAPM estimation for the Equally Weighted Portfolio.

In order to verify the linear restriction on parameters, we perform a battery of diagnostic tests: Reset test, White test, Breusch-Godfrey test and Durbin-Watson test. The purpose of running these test is to verify the functional form of the model challenging the linearity hypothesis, the existence of homoskedasticity and independence on the residuals. In the table below (3), for each test is reported a particular value used to determine if the null hypothesis has to be rejected or not.

The Reset test has the purpose of evaluating the functional form of the model, so we have to estimate the null hypothesis of linearity. For each bank we estimate the auxiliary regression and compute the p-value of the F-test distribution. As the p-value is greater then the level of significance (0.05), we obtain that we do not reject the null at the 5% confidence level and the linear model is appropriate. That is there exist a linear relation between parameters.

One of the assumptions made about residuals in OLS regression is that the errors have the same but unknown variance. This is known as constant variance or homoskedasticity. When this assumption is violated, the problem is known as heteroskedasticity, that it's one of the Gauss-Markov assumptions and it is tested by the White test.

The White test compare OLS estimates in presence and absence of heteroskedasticity. It's the most common test in the case innovation variances changing across subjects. Considering the regression

$$y_i = \alpha + \beta x_i + \delta z_i + \epsilon_i,$$

to detect heteroskedasticity it's taken the residuals of the fitted model $\hat{\epsilon}_i$ and use them to build an auxiliary regression. In this way, the squared residuals on covariates, their squares and their cross-products has been regressed. In addiction, we do have heteroskedasticity if the coefficients of covariates, squared covariates and cross-products are not null, so the null hypothesis evaluate that these coefficients are null. For that reason, in order to verify the absence of the heteroskedasticity, we need to accept the null.

Analyzing the p-value of the chi-square distribution, we observe that only three banks (HSBC Holdings, Svenska Handelsbanken A and Swedbank A) out of the ten have a p-value over the level of confidence (0.05), therefore we accept the null hypothesis, signifying the presence of homoskedasticity for the three specified banks. For the rest of the banks, we observe a p-value under the level of confidence, so we have to reject the null meaning we have heteroskedasticity.

As there is an significance presence of heteroskedasticity, using the standard errors may not be appropriate. In such case, robust standard errors may provide a more accurate estimate of the standard error and regression parameters, as they take into account the heteroskedasticity present in the data. Therefore, we evaluate and compare the standard error with the robust standard error, concluding that using the robust standard error is more appropriate for this analysis.

Another assumption on the residuals in OLS regression is that there is no serial correlation between the errors. The Breusch-Godfrey Test is a test for auto-correlation in the errors, where the null hypothesis is independence on residuals and the test statistic is evaluated by means of an auxiliary regression. In this sense, in order to be coherent with the assumption of the model, we need to accept the null. We evaluated the p-value of the chi-square distribution and we accept the null hypothesis at the 5% confidence level.

Finally, the Durbin-Watson Test is also a test for independence on residuals, where the null hypothesis is that there is no correlation between residuals, as the alternative one is that the residuals are correlated. The test statistic value is included between 0 and 4, with the following interpretation: if the test statistic is equal to 2 there is no serial correlation (we accept the null), if the test statistic is closer to 0, there is proof of positive serial correlation, otherwise, if the test statistic is closed to 4 that means there is a negative serial correlation. As we observe the all the value estimated are between 1.9 and 2.2, so closed to 2, we can accept the null and say there is independence on residuals.

In summary, these tests contribute to the verification process of the utilized model.

Banks	**RESET-test**	**White-test**	**BG-test**	**DW-test**
DANSKE BANK	0.8436	0.0186	0.4094	2.2125
HSBC HOLDINGS	0.8712	0.3648	0.9652	2.0161
SKANDINAVISKA ENSKILDA B.	0.7203	0.0000	0.0641	2.3693
SVENSKA HANDELSBANKEN A	0.8956	0.7991	0.2243	2.2401
BANCO SANTANDER	0.7486	0.0000	0.2234	2.0010
DNB BANK	0.7471	0.0088	0.0499	2.3719
BNP PARIBAS	0.7060	0.0000	0.0723	2.1284
SWEDBANK A	0.8521	0.2916	0.6385	1.9994
SANTANDER BANK POLSKA	0.6692	0.0000	0.7867	1.9013
PKO BANK	0.7295	0.0000	0.7700	1.9052

Table 3: Diagnostic tests.

In order to capture more of the variability in market returns, we introduce additional explanatory variables to our linear model. We download the Fama/French European 5 Factors [1]. We choose the European data as we're working on the European Market. The 5 factors are Rm-Rf, SMB, HML, RMW and CMA, but we're not taking into account the risk free return, since we already have it (the EURIBOR).

Now, we're going to explain the meaning of these factors:

- The SMB (Small Minus Big) is the average return on the nine small stock portfolios minus the average return on the nine big stock portfolios.
- The HML (High Minus Low) is the average return on the two value portfolios minus the average return on the two growth portfolios.
- The RMW (Robust Minus Weak) is the average return on the two robust operating profitability portfolios minus the average return on the two weak operating profitability portfolios.
- The CMA (Conservative Minus Aggressive) is the average return on the two conservative investment portfolios minus the average return on the two aggressive investment portfolios.

There is no specific transformation that has been applied to the downloaded data, we just take the data in the period that we're studying (from November 1st 2009 to October 31st 2023) as, in the Kennet Fench website, the data are reported from 1991.

Incorporating these additional factors into the analysis can help to achieve a more precise representation of the risk factors that impact excess bank returns. Consequently, we employ a multifactor model, an extension of the Capital Asset Pricing Model (CAPM), to integrate supplementary factors and increase our understanding of the variability in excess bank returns. While the CAPM focuses on the correlation between excess bank's expected return and its systematic risk (beta), multifactor models consider various risk sources that can affect an asset's overall performance.

To assess whether the multifactor model offers a better fit or captures additional dimensions of risk compared to the simpler CAPM, we conduct a comparative analysis of the alpha, beta, and r-squared parameters for both models. The values found for the parameters are stored in the "Parameters in Multifactor" file.

Comparing what we obtain with the multifactor model (table 4) to the values of the parameters found with the CAPM model (table 1), we can say that the parameters are approximately the same between the two models. It can suggests that, in terms of the specific factors considered, the additional complexities of the multifactor model are not significantly improving the predictive accuracy over the simpler CAPM.

banks	**alpha**	**beta**	**p-value**	**r-squared**
DANSKE BANK	-0.2888	1.0922	0	0.2948
HSBC HOLDINGS	-0.5735	0.9840	0	0.3172
SKANDINAVISKA ENSKILDA BANKEN A	0.2386	1.1663	0	0.4733
SVENSKA HANDELSBANKEN A	-0.0644	0.8310	0	0.3890
BANCO SANTANDER	-1.3162	1.3961	0	0.4077
DNB BANK	0.4055	1.1271	0	0.4645
BNP PARIBAS	-0.7974	1.7492	0	0.5673
SWEDBANK A	0.2697	1.0079	0	0.3457
SANTANDER BANK POLSKA	-0.3021	1.0904	0	0.2983
PKO BANK	-0.7180	1.1210	0	0.3436

Table 4: Quantities of interest of the multifactor model estimation for each bank.

The bank's performance is well-captured by market risk alone, and the inclusion of other factors in the multifactor model does not alter the evaluation of the bank's risk and excess return characteristics. Therefore, the market factor remains the dominant driver of the asset's performance, and the other factors included in the multifactor model may not be contributing significantly. In conclusion, there may be a level of consistency in explaining the bank's excess returns between the two models. The multifactor model doesn't offer a better fit or captures additional dimensions of risk compared to the simpler CAPM.

Furthermore, we analyze the correlation across the residuals of the CAPM model and the correlation across the residuals of the multifactor model (all the plots are reporteded in "Residuals with Market and 4 factor of Fama-French" file). The correlation across residuals is a valuable metric for comparing the effectiveness of the CAPM and multifactor models, it's helpful to understand whether the additional factors in the multifactor model contribute significantly to explaining excess bank return variability beyond what is captured by the CAPM.

As another check, we also analyze the frequencies (see "Histogram of residue frequencies" file) across the residuals for both models and the results are coherent with what we found above. Therefore, the frequencies across residuals of the multifactor model are concordant with the ones of the CAPM for almost all of the banks. In addition, we observe that the cases more frequent are the one in which the residuals are closed to zero. This behaviour derives from the fact that the banks exhibiting this characteristic, such as Skandinaviska Enskilda Banken A, DNB Bank, or BNP Paribas, tend to have higher r-squared values compared to those that do not show it.

Knowing that residuals are the differences between the actual observed values and the predicted values, the correlation across residuals measures the degree of similarity or dissimilarity between the unexplained variations of the two models. We can observe from the plots that the correlation is high, so both models are explaining similar aspects of the variability in asset returns. This may indicate that the multifactor model is not significantly improving upon the CAPM in terms of capturing the unexplained variations. From the plots, we can observe that there is no particular differences in their level and in their distribution.

The analysis of the correlation across the residuals is coherent with what we found comparing the parameters of both models.

In order to verify for structural change in some or all of the parameters of the model in cases where the disturbance term is assumed to be the same in both periods, we run a Chow test.

The Chow test is typically used in the field related with time series data to determine if there is a structural break in the data at some point. The null hypothesis for the test asserts that there is no alteration in the regression coefficients over time. Consequently, if we reject this null hypothesis, it provides substantial evidence to affirm the presence of a structural break point in the data.

In order to see if we have a structural break, we run the test and reported the results in graphs (see "Chow_Testing" file). If the value of the chow test goes under the level of confidence 0.01 (orange line), we reject the null and confirm that there is a structural break point and the regression coefficients are changing after that particular point over time.

We observed from the graphs that some banks present a structural break point around a certain period of time, i.e Banco Santander (2018), BNP Paribas (2020, 2023), Skandinaviska Enskilda Bank A (2016, 2017), Svedbank A (2022).

Yet, only two of the banks experienced a structural break from which they have not been recoveder. These are PKO Bank and Santander Bank Polska.

We can see from the graph that PKO Bank (Fig.11) suffers a break point around 2019 due to the COVID-19 pandemic. This increase the level of uncertainty, its consequences affect the Group's financial results and position, including, among others, on the expected credit losses or goodwill recognized [2]. Instead, for Santander Bank Polska (Fig.12) 2018 was a year of major changes, including, the rebranding and acquisition of a demerged business of Deutsche Bank Polska S.A. [3].

In addition, we can say that, since the structural break points are in different years the break dates aren't concordant across equities.

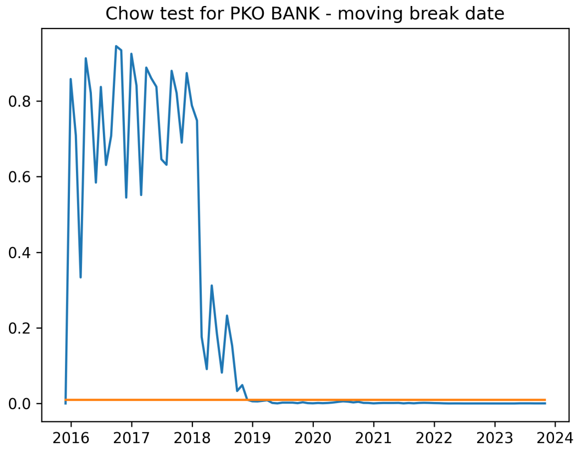


Figure 11: PKO BANK

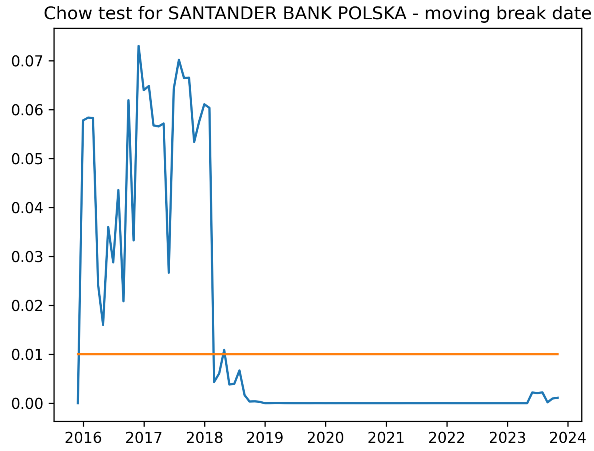
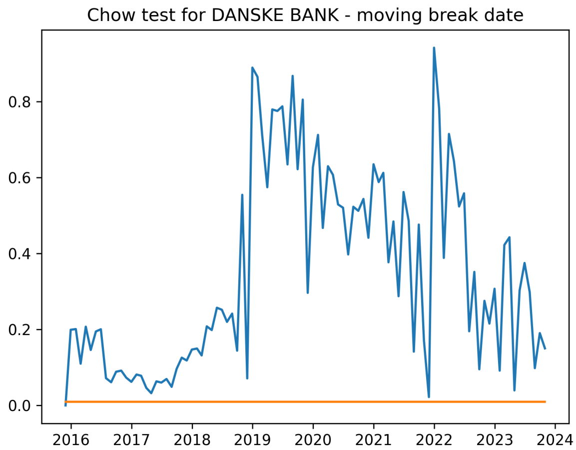
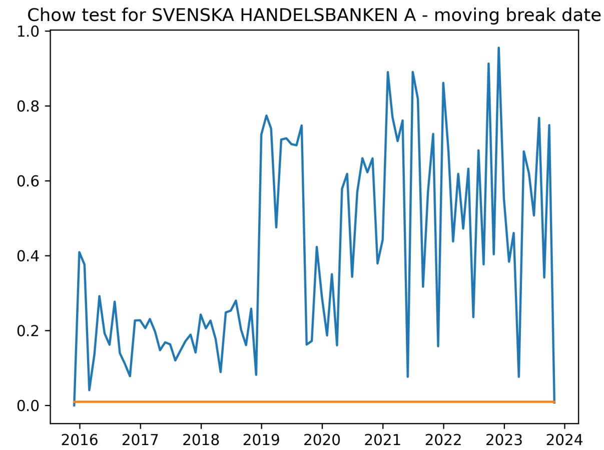


Figure 12: SANTANDER BANK POLSKA

To see how the other banks without a structural break point behave, we report two examples (figures below). In these cases, we accept the null hypothesis as the value of the chow test is over the confidence level (0.01). Therefore, there is no change in the regression coefficients over time, meaning there is not a structural break point.





Another approach to estimate the the Capital Asset Pricing Model (CAPM) is by using a rolling window approach, where the model is re-estimated for each window of data. This allows us to examine how the estimated parameters (alpha, beta and r-squared value) change over time.

In this analysis, we employ a 5-year window and recalibrate the CAPM for each window within our sample. The recalibration involves shifting the estimation sample by one month at a time. The results, including the estimated parameters along with their confidence intervals, are presented in plots (see the "Rolling" file).

Upon examining the plots, it can be concluded that they exhibit minimal fluctuations, remaining stable within their respective confidence intervals throughout the observed time period.

As we saw before, there are two banks from our sample that suffers a structural break point (the PKO Bank and the Santander Bank Polska). Using the rolling window approach, we can make some considerations, based on the result plots, about alpha, beta and r-squared around the break point.

Starting with PKO Bank, alpha value was on the rise until reaching a peak of 0 in 2018. From 2019 onwards, it starts to decline, reaching -1 by 2023. This could indicate a sensitivity of PKO Bank's to factors post the pandemic break point like with economic uncertainty and shifts in investor expectations.

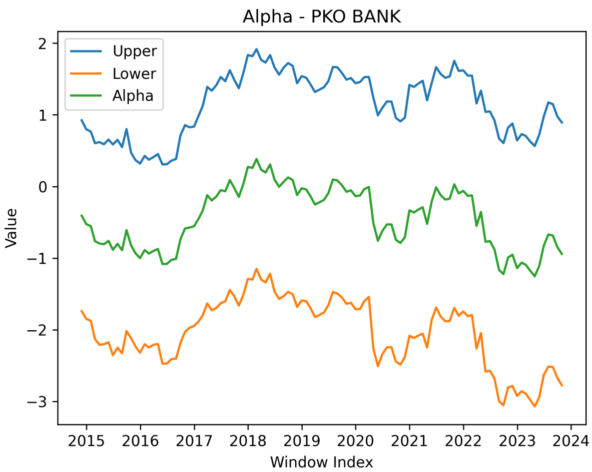
As for PKO Bank's beta, it descended from 1 to 0 between 2015 and 2020, until in 2020, it rapidly increases, taking values of 1 and even 1.5. The sudden spike in 2020 suggests that PKO Bank's response to market movements became more pronounced, possibly due to the volatility and exceptional economic conditions caused by the pandemic.

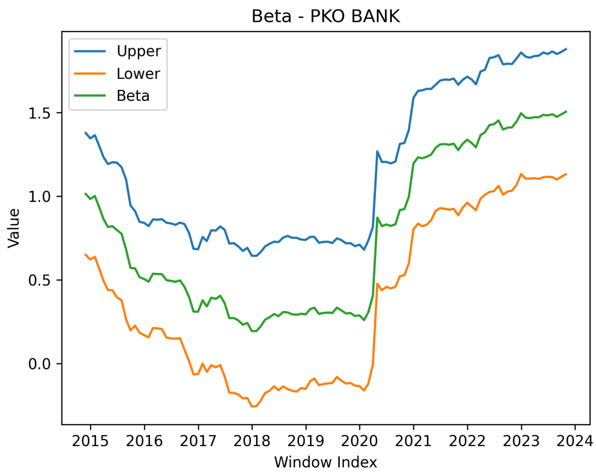
PKO Bank's r-squared was practically at 0 between 2017 and 2020, until it rapidly grows in 2020, reaching up to 0.5 in 2021. From 2021 onwards, it remains at 0.5. The lower r-squared before 2020 and the swift increase in that year indicate a shift in the model's ability to explain the variability of PKO Bank's returns. This change might reflect the CAPM's difficulty in modeling extreme events like the pandemic.

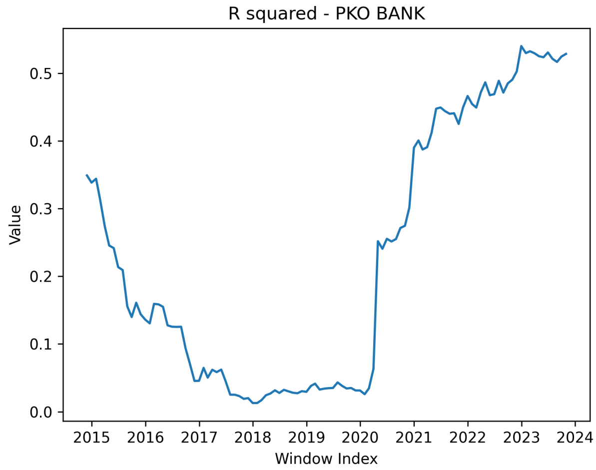
Now let's delve into Santander Bank Polska. The alpha remains stable at 1 until 2018, followed by a steady decline until reaching -1 in 2021. This change could be related to the acquisition of Deutsche Bank, as this have impacted the perception of risk and performance.

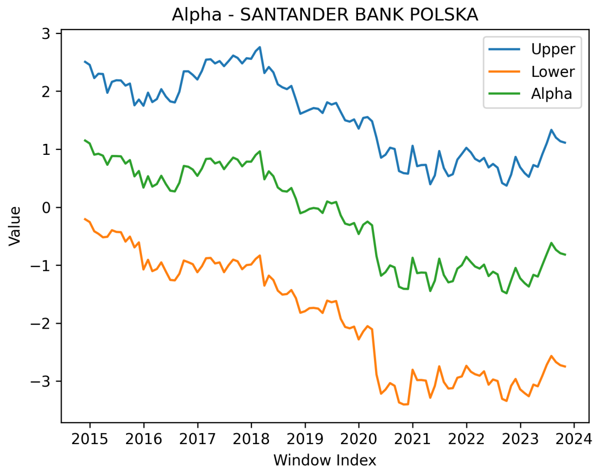
As for beta, it stayed between 0 and 0.5 until 2020, suggesting a moderate sensitivity to market movements. The rapid increase in beta in 2020, reaching values of 1 and even 1.5, could be linked to the pandemic. Global events significantly affect beta, reflecting the sensitivity of an asset to the market.

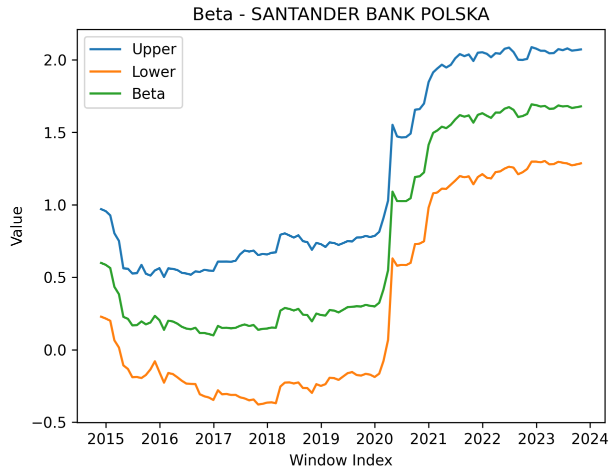
R-squared is low until 2020, indicating that the model explains a minimal portion of the variance. A rapid growth in 2020, reaching 0.3 and up to 0.5 in 2021, suggests an improvement in the model's explanatory power, capturing more of the variance.

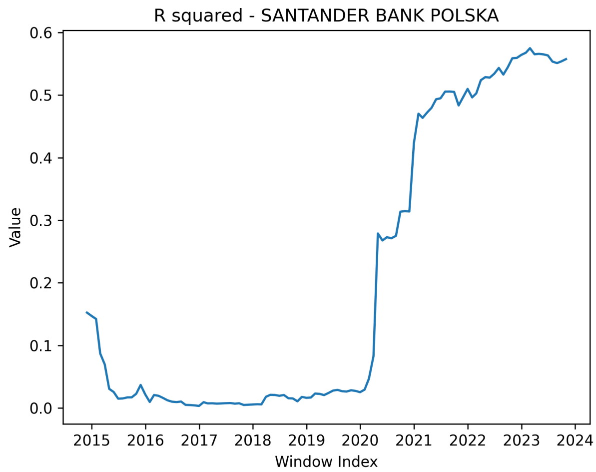












Considering all banks, we observe a trend in the behavior of each parameter over time. Notably, in the period between (2018)2019 and 2020(2021), one of the primary concerns was associated with the COVID-19 pandemic, where the markets experienced extreme volatility as news about the spread of the virus and its potential economic impact unfolded. This impacts also the banks. In this way, beta, a measure of systematic risk, becomes crucial for understanding how a bank's stock reacts to market movements. In that period, we find a beta less than 1 (or around 1) for every bank. This implies that the bank's stock is theoretically less volatile than the market, providing some degree of insulation during extreme market swings.

It's important to note that market conditions during the COVID-19 pandemic were dynamic and influenced by a combination of health, economic, and geopolitical factors. Investors faced unprecedented challenges, and market reactions were often swift and unpredictable.

The market's decline aligns with the downturn in the banking sector, as indicated by the r-squared analysis, demonstrating a correlation between the two. Due to the correlation and knowing that the market returns are similar to those of the company, the stability of alpha is notable, as any significant variation would indicate higher volatility for the company.

Conclusion

Through the application of linear regression, we examined the relationships between variables, particularly the dependencies between market and bank returns. The CAPM model allowed us to estimate the expected returns of the banks in relation to their risk and wide market conditions.

The examination of ten diverse banks within the STOXX EURO 600 index revealed different trends and fluctuations in alpha, beta, and r-squared parameters. Particularly noteworthy were the scatter plots, which exposed distinct patterns. Certain banks exhibited unique behaviors, suggesting the presence of potential distinctive factors influencing their dynamics.

Furthermore, running the multifactor models, show us that adding more explanatory variables to the linear model is not significantly improving the CAPM model in terms of capturing the unexplained variations.

Diagnostic tests, including the Reset test, White test, Breusch-Godfrey test, Durbin-Watson test, and Chow test, were instrumental in assessing the robustness of the models. These tests provided insights into the statistical significance and structural stability of our analyses.

Moreover, the introduction of the rolling window approach as an alternative to the Chow test added depth to our examination, offering a perspective on how the models performed over time.

In summary, the findings and methodologies presented contribute to a more informed understanding of market dynamics and investment strategies on the banks.

Bibliography

[1] *Dartmouth - Kennet R. French*. Description of Fama/French Factors.

[2] *PKO Bank website*. Annual Report 2019.

[3] *Santander Bank Polska website*. Corporate Social Responsibility Report 2018.