Astronomy & Astrophysics

Lab Report

Limb Darkening of the Sun

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1.1 Introduction

A study on Limb darkening effect in the Sun and observing its intensity profile by observing an image through Image processing Technique. The RGB values of a particular pixel is taken and their sum is divided by 3 to get its intensity [1].

1.2 Theory

1.21 Limb darkening

When the Sun is seen through a telescope using a Solar filter [2] (polymers which allows only 1/1,00,000 of total light to pass through it and prevent Eye damage), we see a perfect sphere of gas with a bright yellow center and dark edges. A natural gradient is observed in the Sun which is similar to different layers placed on each other. This is ‘Limb Darkening effect of the Sun’.

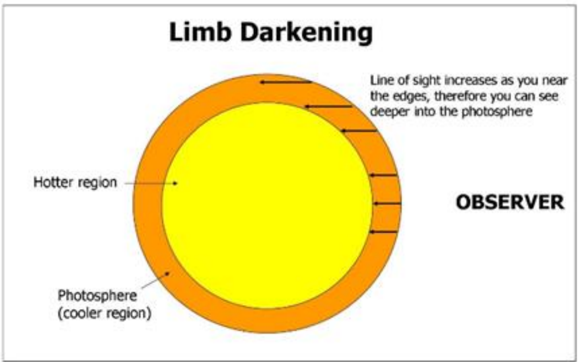


Fig. 1.2.1

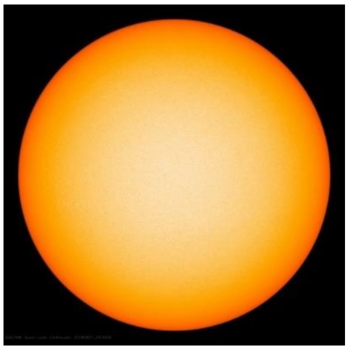


Fig 1.2.2

Limb darkening is a phenomenon where the Sun’s layers become darker as we move away from its centre to one of its corners. In scientific words, it’s an optical effect where the line of sight increases as we move to the edges of the sun, the deeper we can see into the Suns layers. The word ‘Limb’ or ‘Limbus’ stands for corner/edges and their darkening has termed this phenomenon as Limb darkening.

The corners are red in colour while the Center is Bright yellow. The primary reason for this effect to occur is due to various layers of the Sun's atmosphere having different temperatures. The Sun is a dense Ball of fire which can be considered as a Perfect black body made of different layers.

The Center refers to the Sun's core is the brightest region and has the maximum temperature while the limbs or the outer edges of the atmosphere have the lowest temperature.

Limb darkening is commonly found in many places in our daily life. A good example is a Burning flame of a candle. The Center is the brightest part of the flame while the edges/corners appear to be red or dark orange.

The temperature at the corners (top) is comparatively high due to high amount of oxygen available for combustion while the middle section corresponds to partial combustion and thus lower temperature.

Fig. 1.2.2

But according to our hypothesis for the Sun then the edges should be brighter (high temp.) and the center should be dark. This occurs because of our line of sight, The more layers we see, the brighter and section appears.

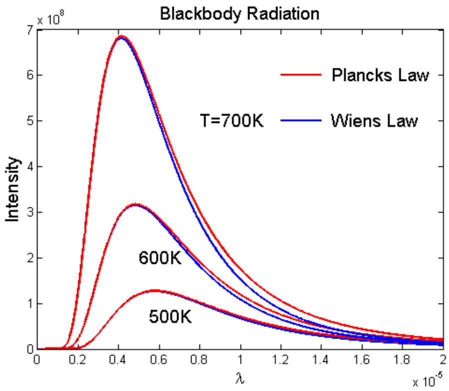


Fig. 1.2.3

According to Planck's law, hotter objects glow the brightest. This is explained by the below graph

The Sun is made of Gases at very High temperature and as we move closer to its centre, the optical depth increases and the inner layers of the sun become visible.

1.3 Analysis

1.31 The Intensity profile of the Sun

To study the various features of the Sun's inner layers, an image of the sun is processed in python and the Intensity I value of each pixel is extracted (in python code). Make a table corresponding to these values starting from the center to one of its edges along the radius R_o.

Find the maximum value of Intensity (At the Center) as I_{max}. The Values of intensity at any point in the image is directly proportional to the radial distance from its center.

$$\Rightarrow I \propto R \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(1)}$$

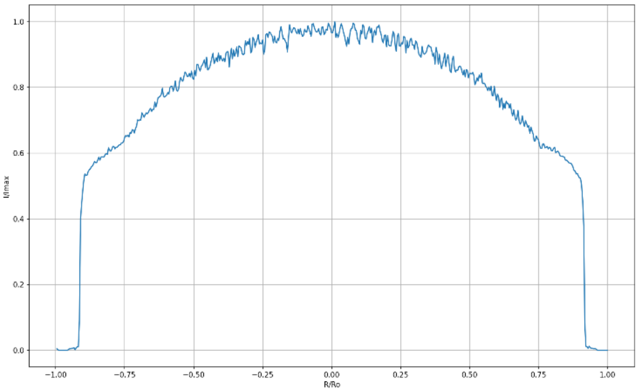


Fig. 1.3.1

R is the distance from the center

R_o is the radius of the Sun

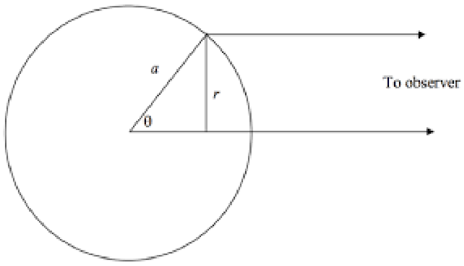
I_{max} is the maximum intensity of the sun

The same graph can be made by changing the unit of measurement on the x-axis to $cos\theta$, where θ is the angle subtended by the line of sight at the surface of the sun to the direction of the normal radial vector passing through the point. It is shown in the figure below.

Make a table of the intensities and their corresponding values of θ. When $\theta = \frac{\pi}{2}$, the value of $cos\theta$ becomes 0 which means the line of sight is at the limb or edge and when $\theta = 0$, the corresponding the line of sight is at the center of the disc.

$$\frac{I}{I_{max}} = \alpha + \beta \cos\theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Comparing (2) with the general form of linear equation, i.e., $\boldsymbol{y = mx + c}$



where m is the slope of the curve and c being a constant function

on comparing, we get

$$\boldsymbol{\alpha = c} \text{ \& } \boldsymbol{\beta = m} \text{ \& }$$

$$\boldsymbol{\alpha + \beta = 1}$$

The values of $\boldsymbol{c}$ and $\boldsymbol{m}$ are yet to be found from the graph between $\boldsymbol{I/I_{max}}$ **vs** $\boldsymbol{cos\theta}$ below,

Method 1

The graph shows the values of Intensity of all the values of θ ranging from $\frac{\pi}{2}$ to 0 degrees.

The slope of the curve (orange) $\boldsymbol{m}$ **= 0.5985 =** $\boldsymbol{\beta}$

The constant $\boldsymbol{c}$ (Limb Darkening constant) =

1 – m = $\boldsymbol{\alpha}$ **= 1 – 0.5985 = 0.4015**

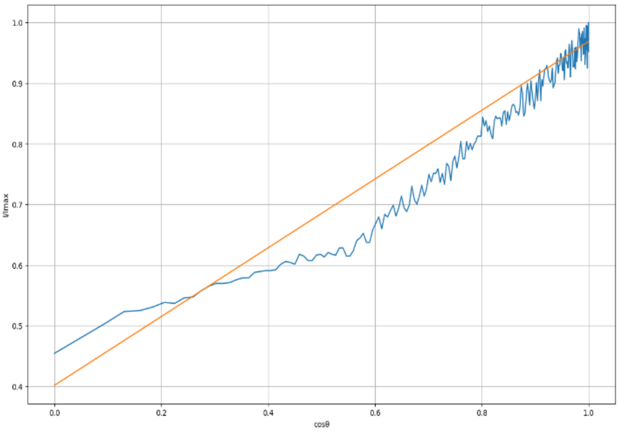


Fig. 1.3.2

Method 2

The other method is finding the values by substitution method in eq. (2) i.e., the blue curve

The value of the y-axis at x = 0 is equal to the constant c i.e., the value of α = **0.4543**

The corresponding value if β is **0.5457**

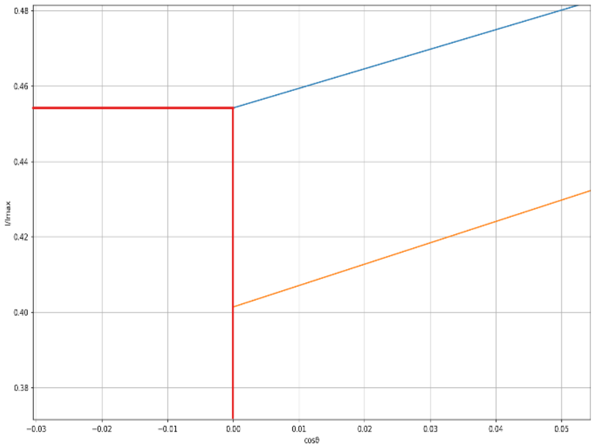


Fig 1.3.3

1.32 Temperature and Intensity

According to the Stefan-Boltzmann law, the total energy radiated from a black body is directly proportional to the fourth power of its absolute temperature. It is given as

$$I = \epsilon\, \sigma\, T^4 \text{ ....(2)}$$

I is the intensity of the radiation (P/A)

ϵ is the emissivity and,

T is the absolute temperature.

Higher the temperature of the black body, the higher the intensity of light coming from it. Thus, it can be concluded that the Intensity and temperature are proportional to each other and the corresponding relation is

$$I \propto T^4 \text{ .... (3)}$$

Now substituting the relation in (1) gives $\Rightarrow T^4 \propto R$ .... (4)

The plot for ***T/T***

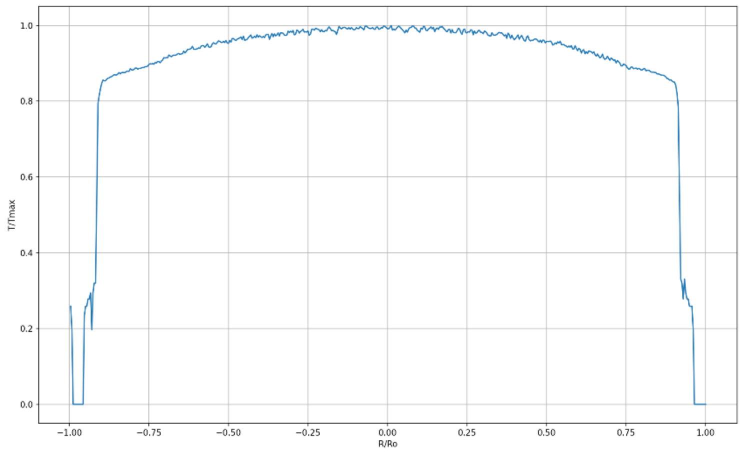


Fig. 1.3.4

The maximum temperature of the Sun is estimated as 5000^0C and the radius R_o is 696340 km . On plugging these values at various points will provide exact data on the temperature of the Sun.

1.4 Error Analysis

The values limb darkening coefficients $\boldsymbol{\alpha}$ **&** $\boldsymbol{\beta}$ are approximated as **0.4015 & 0.5985** (Method 1) and **0.4543 & 0.5457** (Method 2) These values are obtained by substitution of values in the equation and also by drawing a line of slope equal to average of all the points in the curve. The real values obtained from the source [3] are shown as below

$\boldsymbol{\alpha_{real} = 0.39\ and\ \beta_{real} = 0.61}$ **(They vary from range 0.6 to 0.7)**

From Method 1

The error in the values of $\boldsymbol{\alpha}$ is

$$\frac{\alpha - \alpha_{real}}{\alpha_{real}} = \frac{0.4015 - 0.39}{0.39} = \mathbf{0.0295\ or\ 2.95\%}$$

The error in the values of $\boldsymbol{\beta}$ is

$$\frac{\beta - \beta_{real}}{\beta_{real}} = \frac{0.5985 - 0.61}{0.61} = \mathbf{-0.0188\ or -1.88\%}$$

From Method 2

The error in the values of $\boldsymbol{\beta}$ is

$$\frac{\beta - \beta_{real}}{\beta_{real}} = \frac{0.5457 - 0.61}{0.61} = \mathbf{-0.0643\ or -6.43\%}$$

The error in the values of $\boldsymbol{\alpha}$ is

$$\frac{\alpha - \alpha_{real}}{\alpha_{real}} = \frac{0.4543 - 0.39}{0.39} = \mathbf{0.0165\ or\ 1.65\%}$$

From the above results it is clear that the approximation taken in case 2 is slightly unhealthy for further calculations but any method can be employed in finding the true values. I will be taking the values from method 1 as the final ones as their net error (1.07%) is smaller compared to 2 (-4.78%).

1.5 Results

The value of $\boldsymbol{\alpha}$ **= 0.4015**

The value of $\boldsymbol{\beta}$ **= 0.5985**

1.6 Conclusion

A number of approx. are taken for better understanding of the intensity curve of the sun. I have taken the intensity at a pixel as the sum of its R, G, B values divide by 3. Fig. 1 and Fig. 2 show that the intensity of light is maximum at the center and it falls as we move away from it.

The values of α & β are approximated from *Fig. 1.3.2* using summation of all the partial slopes between the any two points. This is not an accurate way of measuring as the resultant line of the same slope (orange line) deviates from the main curve. The measurement of the slopes is taken from the center to the right most corner of the sun and the values of their corresponding intensity start to change rapidly as we approach the center.

Though there are no dark spots are present in *Fig. 1.2.2*, their position can be detected by observing *the Fig. 1.3.1.* The dark spots are relatively cooler than most of the sun's surface and thus emit low intensity light which could be detected by a sudden decrease in the intensity at a point R. Its temperature could be easily be determined and this makes our study on the sun's surface more farfetched. One such observation is shown below

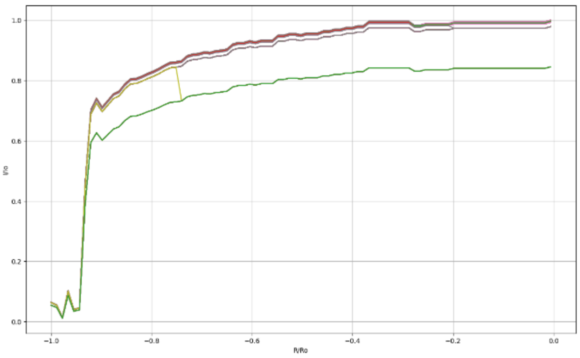


Fig. (yellow) showing the values of intensity dropping at a dark spot. (Another Image)

Cites

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2. https://astronomy.com/magazine/news/2022/01/solar-filters-for-observing-the-sun

3. https://www.bhu.ac.in/research_pub/jsr/Volumes/JSR_64_01_2020/32.pdf

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All the graphs are taken from Python