Advanced Communication Theory
Multipath Spatiotemporal SIMO Wireless Systems

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Aims

The main objective of this assignment-study is to simulate a QPSK-DS-CDMA communication system and design space-time array receivers to handle multipaths, suppress MAI (Multiple Access Interference) and improve the overall capacity of the system.

Definitions

Throughout Tasks 1-3 certain variables are kept constant. For Task 4 the variables are imported from the personal data file provided.

To set the modulation angle and the shift used for gold sequence generation and the relative delay of multipaths in Tasks 1-3 two variables are defined:

- $\mathcal{X}$ = alphabetical order of the 1^{st} letter of your surname = 22 (V)
- $\mathcal{Y}$ = alphabetical order of the 1^{st} letter of your formal firstname = 18 (R)

The bits associated with the information being transmitted is modulated and demodulated according to QPSK coding of bit pairs. Bit pairs are encoded according to Equation 1.

$$s(t) = \sqrt{2}\exp(2\pi F_c t + \frac{\pi}{2}(i-1) + \phi) \tag{1}$$

Where $i = 1, 2, 3$ and 4 and $\phi = \mathcal{X} + 2\mathcal{Y} = 58$. Additionally using the above variables the gold sequence's shift is defined as $d \geq 1 + (\mathcal{X} + \mathcal{Y})\ mod\ 12$, where the m-sequences used to form the gold sequence are determined by the polynomials of Table 1.

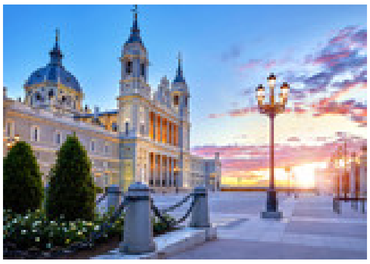
1^{st} Polynomial	2^{nd} Polynomial
$D^4 + D + 1$	$D^4 + D^3 + 1$

Table 1: Polynomials for m-sequence generation

Photos

The three photos used corresponding to each of the users are shown below.

(a) Photo user 1



(b) Photo user 2

(c) Photo user 3

For Tasks 1-3 the desired user is User 1 and so the first image is the desired image to be recovered.

Task 1

This task implies considering three different users each transmitting a digital photo, at the same time, on the same frequency band. The aim is, using the above definitions, to simulate the communication system and design a receiver to discard unwanted signals and receive only User 1's photo.

The channel is defined by Table 2.

signal-paths arriving at the receiver	relative delay	fading coefficient	(azimuth, elevation) (θ, ϕ)
one path at $s_1(t)$	$(\tau_1 \bmod 15) = 5$	$\beta_1 = 0.4$	$(30^o, 0^o)$
one path at $s_2(t)$	$(\tau_2 \bmod 15) = 7$	$\beta_2 = 0.7$	$(90^o, 0^o)$
one path at $s_3(t)$	$(\tau_3 \bmod 15) = 12$	$\beta_3 = 0.2$	$(150^o, 0^o)$

Table 2: Channel Parameters (no multipath effects)

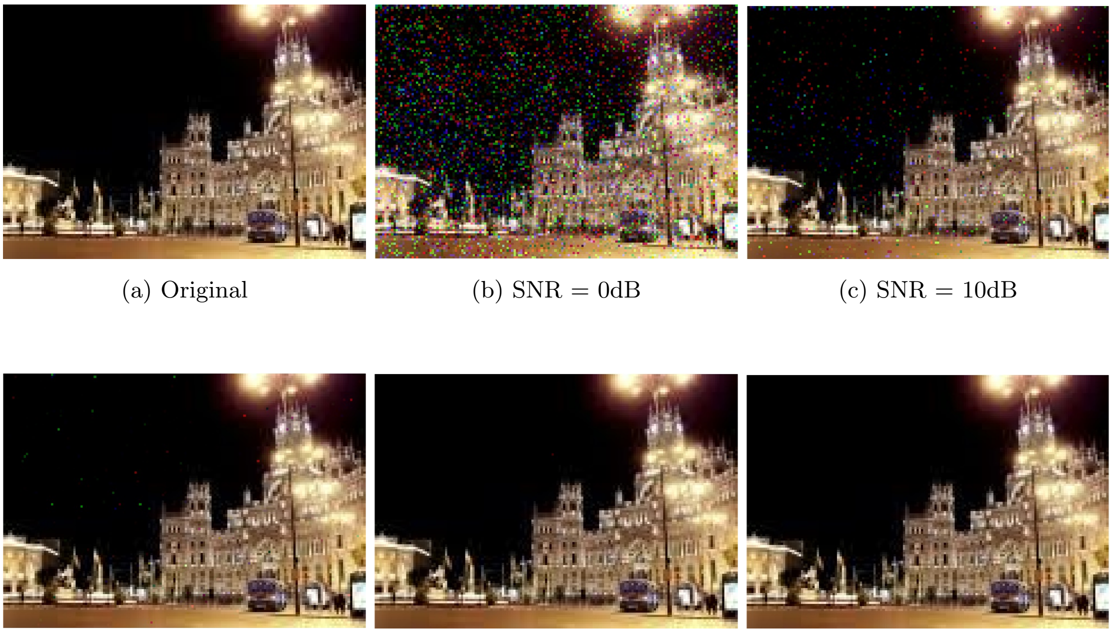
By simulating the communication channel and varying the Signal-to-Noise Ratio (SNR) the desired user's photo quality varies, as shown in Figure 2 and the corresponding Bit Error Rate (BER) and Number of Bits in Error (NBE) in Table 3.

SNR(dB)	NBE	BER
0	32902.2	0.06276
10	5584.4	0.01064
20	424	8.086×10^{-4}
30	2.2	4.196×10^{-6}
40	0	0

Table 3: Bit Error Rate and Number of Bits in Error for varying values of SNR (Average of 5 executions)

As can be seen the quality of the received image improves as SNR increases, as it implies a lower noise interference. For higher SNR, little interference is present and the demodulation can be performed correctly, with the recovered photo for $SNR = 40$dB being identical to the original photo and $SNR =$ 20 & 30dB showing little noise interference.

On the contrary, for low SNR, noise interferes greatly affecting the channel estimation and hence affecting the final result. With considerable noise the delay estimate isn't precise and the resulting demodulated photo has components of all photos, as can be seen for $SNR = 0$dB and in a lower degree for $SNR = 10$dB.



(a) Original (b) SNR = 0dB (c) SNR = 10dB

(d) SNR = 20dB (e) SNR = 30dB (f) SNR = 40dB

Figure 2: Received photo for varying values of SNR

Task 2

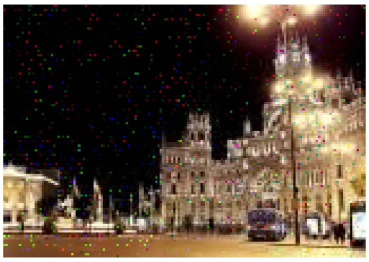
This task implies a similar problem to Task 1 except with the presence of multipaths for user 1, aiming to resolve them to achieve perfect reception. Table 4 shows the channel specifications.

signal-paths arriving at the receiver	relative delay	fading coefficient	(azimuth, elevation) (θ, ϕ)
1^{st} path of $s_1(t)$	$(\mathcal{X} + \mathcal{Y})$ mod 4	$\beta_{11} = 0.8$	$(30^o, 0^o)$
2^{nd} path of $s_1(t)$	$4 + (\mathcal{X} + \mathcal{Y})$ mod 5	$\beta_{12} = 0.4 \exp(-j40^o)$	$(45^o, 0^o)$
3^{rd} path of $s_1(t)$	$9 + (\mathcal{X} + \mathcal{Y})$ mod 6	$\beta_{13} = 0.8 \exp(+j80^o)$	$(20^o, 0^o)$
$s_2(t)$	8	$\beta_2 = 0.5$	$(80^o, 0^o)$
$s_3(t)$	13	$\beta_3 = 0.2$	$(150^o, 0^o)$

Table 4: Channel parameters (with multipath effects)

For this task the presence of multipaths provides an opportunity to achieve a better demodulation. After channel estimation, by resolving the different multipaths the demodulated image is a combination of the different multipaths using a Rake receiver, which allows for a better distinction between the desired user and the other interfering photos.

(a) Original



(b) SNR = 0dB

(c) SNR = 10dB

(d) SNR = 20dB

(e) SNR = 30dB

(f) SNR = 40dB

Figure 3: Received photo for varying values of SNR

As can be seen from Figure 3, the presence of multipaths is an opportunity for better demodulation. For SNR $\geq$ 20dB the recovered image is identical to the original image, whilst for lower SNR the presence of noise is not very significant. Task 2 clearly gives much better results than Task 1, showing that by resolving the multipaths a better reception is achievable. Table 5 shows the BER and NBE for Task2.

SNR(dB)	NBE	BER
0	4245.6	8.1×10^{-3}
10	62.4	1.19×10^{-4}
20	0	0
30	0	0
40	0	0

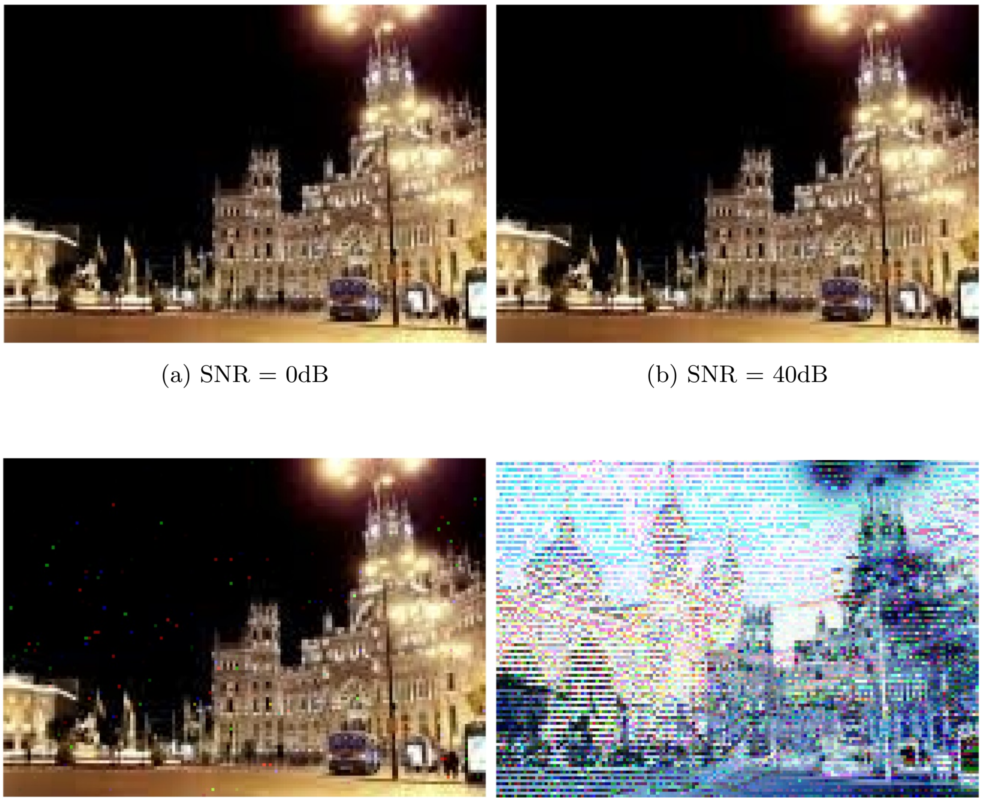
Table 5: Bit Error Rate and Number of Bits in Error for varying values of SNR (Average of 5 executions)

Task 3

Task 3 extends the problem and channel described in Task 1 except in a SIMO (Single In Multiple Out) architecture, with a uniform circular receiver array of 5 isotropic elements (antennas) with half-wavelength inter-antenna spacing. As such the antenna position are defined by Equation 2 in the xy-plane and 0 in the z-plane, where A denotes the amplitude, $i = 0, 1, 2, 3$ *and* 4, and $halfwavelength = \frac{360}{5} = 72^o$.

$$Array\ Position = A\exp(30 + (i \times halfwavelength)) \tag{2}$$

The channel parameters are the same as for Task 1 as described by Table 2. The communication system is simulated and a RAKE receiver is designed to demodulate accordingly the signals detected by each of the antennas in the array.



(a) SNR = 0dB (b) SNR = 40dB

(c) SNR = -15dB (d) SNR = -22dB

Figure 4: Received photo for varying values of SNR

From Figure 4 it is clear that the SIMO architecture gives improved results when comparing with tasks 1 and 2. For SNR = 40 and 0dB the reception gives perfect results as shown above. The array architecture is in fact so good that for SNR = -15dB residual noise interference is present, and it is only after SNR = -22dB that real incorrect results are obtained, due to incorrect channel estimation. Table 6 shows the BER and NBE for varying values of SNR.

SNR(dB)	NBE	BER
-22	407493.6	0,77724
-15	447	8.526×10^{-4}
0	0	0
40	0	0

Table 6: Bit Error Rate and Number of Bits in Error for varying values of SNR (Average of 5 executions)

Task 4

This task 4 implies a different problem where 3 users transmit a text message of sixty 8-bit characters. Again with the same SIMO receiver array structure the aim is to estimate the channel parameters and then demodulate the message from user 1.

The gold sequence is generated from the m-sequences generated from the polynomials of Table 7, and the "phase-shift" from the personal data file. The personal data file, additionally has angle ϕ for demodulation, the fading coefficients (β) of the multipaths and the received signal vector from the antenna array.

1^{st} Polynomial	2^{nd} Polynomial
$D^5 + D^2 + 1$	$D^5 + D^3 + D^2 + D + 1$

Table 7: Polynomials for m-sequence generation

The first step is to extend the received signal for spatiotemporal conditions from N to NN_{ext}, where $N_{ext} = 2N_c$. The data extension is done as shown in Figure 5.

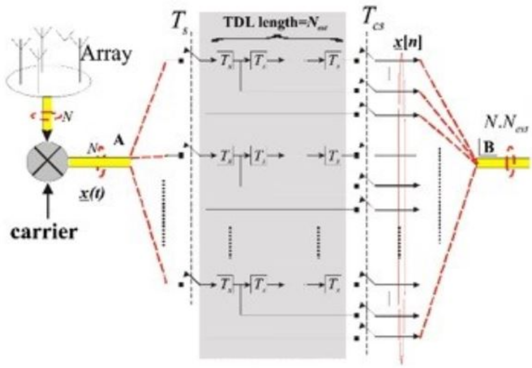


Figure 5: Data extension

In order to correctly decode the message sent the first thing to do is the channel parameters estimation. In order to do this the manifold is extended to apply spatiotemporal parameters and then the 'STAR' subspace cost function is calculated for every azimuth and delay, Equation 3.

$$\zeta(\theta, l) = \frac{1}{\underline{h}(\theta, l)^H \mathbb{P}_n \underline{h}(\theta, l)} \tag{3}$$

Where $\underline{h}(\theta, l)$ is defined by equation 4 and denotes the extended manifold as a function of azimuth (θ) and delay (l). $\mathbb{J}$ denotes the shifting matrix, and $\mathbb{P}_n$ the projection operator associated with the 'noise subspace' of $\mathbb{R}_{\text{xx}}$.

$$\underline{h}(\theta, l) = \underline{S}_{ij} \otimes \mathbb{J}^{l_{ij}} \underline{\mathfrak{c}}_i \tag{4}$$

When estimating the channel parameters of the received signals Figure 6 is obtained, showing clearly the 3 different multipaths of the received signal. By selecting the peaks of the cost function the appropriate delays and azimuths can be estimated for the beamforming in the further steps.

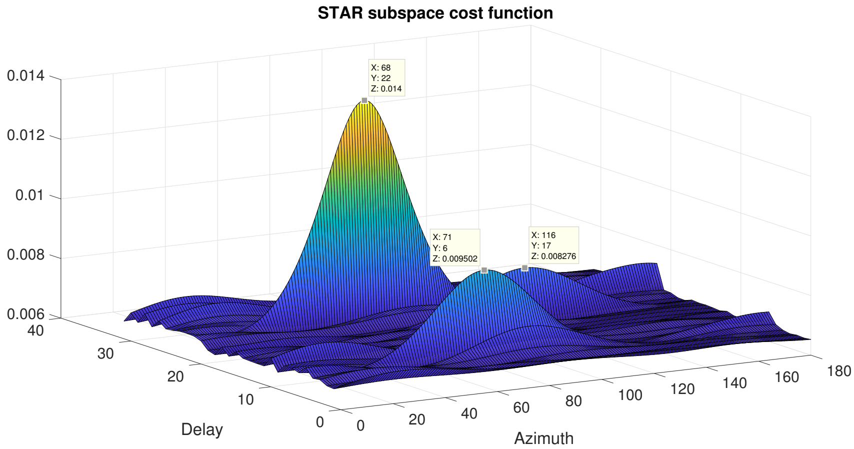


Figure 6: Cost function for varying azimuth and delay

From Figure 6 it can be seen the 3 different multipaths are $\theta = 68^o, 71^o, 116^o$ and $delay = 22, 6, 17$. From that the matrix $\mathbb{H} = [\underline{h}_1, \underline{h}_2, \underline{h}_3]$ is formed with each of the multipath's extended manifold, in order to do a beamformer in each direction. Using this the weight vectors to receive the multipaths are formed, Equation 5.

$$\underline{w} = \mathbb{H}\underline{\beta} \tag{5}$$

After applying the weights to the extended data the only step left is to demodulate the message according to the QPSK map using the given ϕ. After this the message can be converted to characters to get the final message:

Ricardo, you are awesome! You have completed the mission!!!!