

9.1 Angle Relationships



TEKS

Equations, expressions, and relationships—7.11.C
Write and solve equations using geometry concepts, including the sum of the angles in a triangle, and angle relationships.



ESSENTIAL QUESTION

How can you use angle relationships to solve problems?

EXPLORE ACTIVITY



TEKS 7.11.C

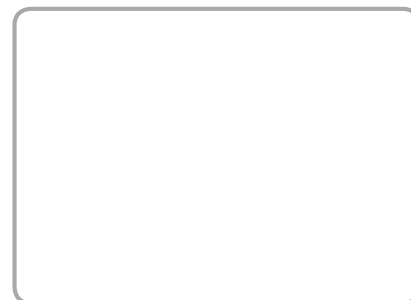
Measuring Angles

It is useful to work with pairs of angles and to understand how pairs of angles relate to each other. **Congruent angles** are angles that have the same measure.

STEP 1 Using a ruler, draw a pair of intersecting lines. Label each angle from 1 to 4.

STEP 2 Use a protractor to help you complete the chart.

Angle	Measure of Angle
$m\angle 1$	
$m\angle 2$	
$m\angle 3$	
$m\angle 4$	
$m\angle 1 + m\angle 2$	
$m\angle 2 + m\angle 3$	
$m\angle 3 + m\angle 4$	
$m\angle 4 + m\angle 1$	



Reflect

- Conjecture** Share your results with other students. Make a conjecture about pairs of angles that are opposite each other.

- Conjecture** When two lines intersect to form two angles, what conjecture can you make about the pairs of angles that are next to each other?



Math On the Spot

my.hrw.com

Angle Pairs and One-Step Equations

Vertical angles are the opposite angles formed by two intersecting lines. Vertical angles are congruent because the angles have the same measure.

Adjacent angles are pairs of angles that share a vertex and one side but do not overlap.

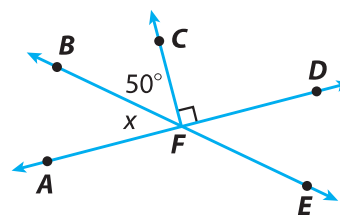
Complementary angles are two angles whose measures have a sum of 90° .

Supplementary angles are two angles whose measures have a sum of 180° . You discovered in the Explore Activity that adjacent angles formed by two intersecting lines are supplementary.

EXAMPLE 1



Use the diagram.



- A** Name a pair of vertical angles.

$\angle AFB$ and $\angle DFE$

- B** Name a pair of adjacent angles.

$\angle AFB$ and $\angle BFD$

- C** Name a pair of supplementary angles.

$\angle AFB$ and $\angle BFD$

- D** Find the measure of $\angle AFB$.

Use the fact that $\angle AFB$ and $\angle BFD$ in the diagram are supplementary angles to find $m\angle AFB$.

$$m\angle AFB + m\angle BFD = 180^\circ \quad \text{They are supplementary angles.}$$

$$x + 140^\circ = 180^\circ \quad m\angle BFD = 50^\circ + 90^\circ = 140^\circ$$

$$\underline{-140^\circ} \quad \underline{-140^\circ} \quad \text{Subtract } 140^\circ \text{ from both sides.}$$

$$x = 40^\circ$$

The measure of $\angle AFB$ is 40° .

Reflect

- 3. Analyze Relationships** What is the relationship between $\angle AFB$ and $\angle BFC$? Explain.

- 4. Draw Conclusions** Are $\angle AFC$ and $\angle BFC$ adjacent angles? Why or why not?

Math Talk

Mathematical Processes

Are $\angle BFD$ and $\angle AFE$ vertical angles? Why or why not?

My Notes

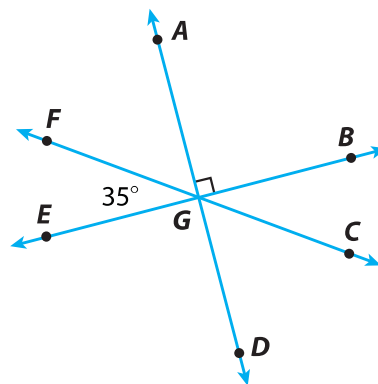
YOUR TURN

Use the diagram.

5. Name a pair of supplementary angles.

6. Name a pair of vertical angles.

7. Find the measure of $\angle CGD$. _____



Personal Math Trainer

Online Assessment and Intervention

my.hrw.com

Angle Pairs and Two-Step Equations

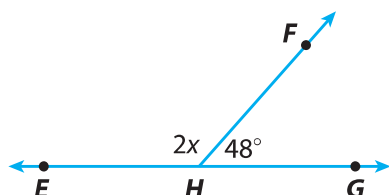
Sometimes solving an equation is only the first step in using an angle relationship to solve a problem.

EXAMPLE 2



TEKS 7.11.C

- A** Find the measure of $\angle EHF$.



$\angle EHF$ and $\angle FHG$ form a straight line.

STEP 1

Identify the relationship between $\angle EHF$ and $\angle FHG$.

$\angle EHF$ and $\angle FHG$ are supplementary angles.

Since angles $\angle EHF$ and $\angle FHG$ form a straight line, the sum of the measures of the angles is 180° .

STEP 2

Write and solve an equation to find x .

$$m\angle EHF + m\angle FHG = 180^\circ$$

$$2x + 48^\circ = 180^\circ$$

$$\underline{-48^\circ} \quad \underline{-48^\circ}$$

$$2x = 132^\circ$$

$$x = 66^\circ$$

The sum of the measures of supplementary angles is 180° .

Subtract 48° from both sides.

Divide both sides by 2.

- o Since $m\angle EHF = 2x$, then $m\angle EHF = 132^\circ$.



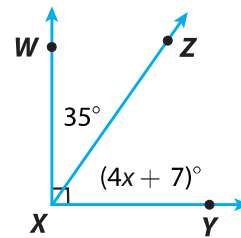
Math On the Spot

my.hrw.com

B Find the measure of $\angle ZXY$.

STEP 1 Identify the relationship between $\angle WXZ$ and $\angle ZXY$.

$\angle WXZ$ and $\angle ZXY$ are complementary angles.



STEP 2 Write and solve an equation to find x .

$$m\angle WXZ + m\angle ZXY = 90^\circ$$

$$35^\circ + (4x + 7)^\circ = 90^\circ$$

$$4x + 42^\circ = 90^\circ$$

$$\underline{-42^\circ} \quad \underline{-42^\circ}$$

$$4x = 48^\circ$$

$$x = 12^\circ$$

The sum of the measures of complementary angles is 90° .

Substitute the values.

Combine like terms.

Subtract 42 from both sides.

Divide both sides by 4.

STEP 3 Find the measure of $\angle ZXY$.

$$m\angle ZXY = (4x + 7)^\circ$$

$$= (4(12) + 7)^\circ$$

$$= 55^\circ$$

Substitute 12 for x .

Use the Order of Operations.

- The measure of $\angle ZXY$ is 55° .

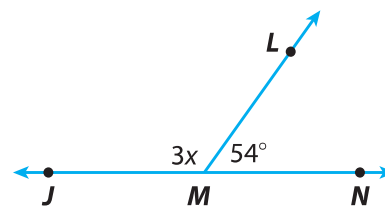
YOUR TURN

8. Find the measure of $\angle JML$.

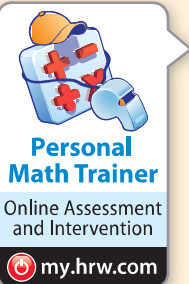
$$x =$$

$$m\angle JML = 3x$$

$$=$$



9. **Critique Reasoning** Cory says that to find $m\angle JML$ above you can stop when you get to the solution step $3x = 126^\circ$. Explain why this works.



Using Angle Measures in Triangles

You learned earlier that the sum of the measures of the angles in any triangle is 180° . You can use this property in many real-world situations.



Math On the Spot

my.hrw.com

EXAMPLE 3

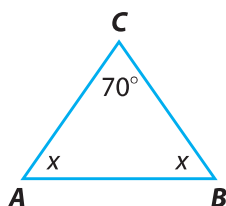


TEKS 7.11.C

The front of the top story of a house is shaped like an isosceles triangle. The measure of the angle at the top of the triangle is 70° . Find the measure of each of the base angles.



STEP 1 Make a sketch.



STEP 2 Write an equation.

$$m\angle A + m\angle B + m\angle C = 180^\circ$$

$$x + x + 70^\circ = 180^\circ$$

The sum of the angle measures in a triangle is 180° .

Substitute values.

STEP 3 Solve the equation to find x .

$$x + x + 70^\circ = 180^\circ$$

$$2x + 70^\circ = 180^\circ$$

$$\underline{-70^\circ} \quad \underline{-70^\circ}$$

$$2x = 110^\circ$$

$$\underline{\frac{2x}{2}} = \underline{\frac{110^\circ}{2}}$$

$$x = 55^\circ$$

Combine like terms.

Subtract 70° from both sides.

Divide both sides of the equation by 2.

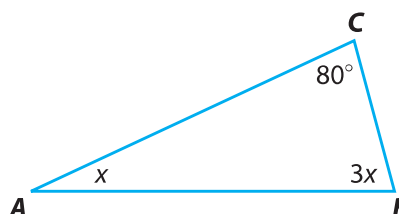
- Each of the base angles measures 55° .

YOUR TURN

Use the diagram.

10. Find the value of x . _____

11. Find the measures of $\angle A$ and $\angle B$. _____



Personal Math Trainer

Online Assessment and Intervention

my.hrw.com

Guided Practice

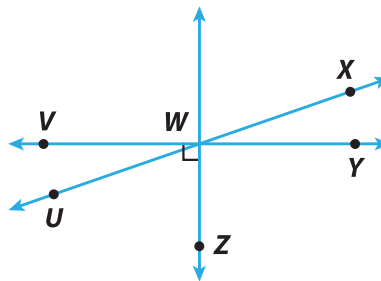
For Exercises 1–2, use the figure. (Example 1)

1. **Vocabulary** The sum of the measures of $\angle UWV$ and $\angle UWZ$ is 90° , so $\angle UWV$ and $\angle UWZ$ are

_____ angles.

2. **Vocabulary** $\angle UWV$ and $\angle VWX$ share a vertex and one side. They do not overlap, so $\angle UWV$ and $\angle VWX$ are

_____ angles.



For Exercises 3–4, use the figure.

3. $\angle AGB$ and $\angle DGE$ are _____ angles,

so $m\angle DGE =$ _____. (Example 1)

4. Find the measure of $\angle EGF$. (Example 2)

$$m\angle CGD + m\angle DGE + m\angle EGF = 180^\circ$$

$$______ + ______ + ______ = 180^\circ$$

$$______ + 2x = 180^\circ$$

$$2x = ______$$

$$m\angle EGF = 2x = ______$$

5. Find the measures of $\angle A$ and $\angle B$. (Example 3)

$$m\angle A + m\angle B + m\angle C = 180^\circ$$

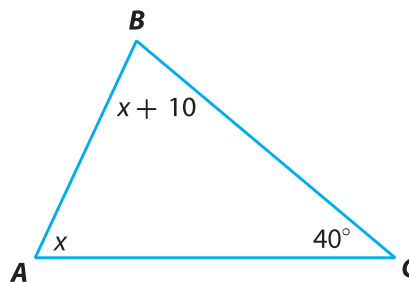
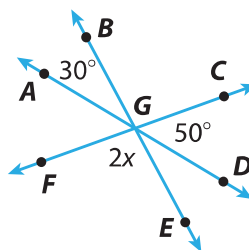
$$______ + ______ + ______ = 180$$

$$2x + ______ = 180$$

$$2x = ______$$

$$x = ______, \text{ so } m\angle A = ______.$$

$$x + 10 = ______, \text{ so } m\angle B = ______.$$



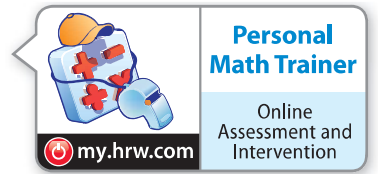
ESSENTIAL QUESTION CHECK-IN

6. Suppose that you know that $\angle T$ and $\angle S$ are supplementary, and that $m\angle T = 3 \cdot (m\angle S)$. How can you find $m\angle T$?

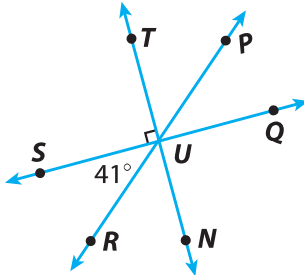
9.1 Independent Practice



7.11.C



For Exercises 7–11, use the figure.



7. Name a pair of adjacent angles. Explain why they are adjacent.

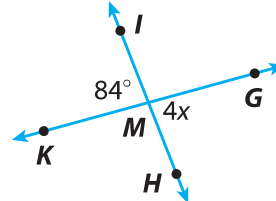
8. Name a pair of acute vertical angles.

9. Name a pair of supplementary angles.

10. **Justify Reasoning** Find $m\angle QUR$. Justify your answer.

11. **Draw Conclusions** Which is greater, $m\angle TUR$ or $m\angle RUQ$? Explain.

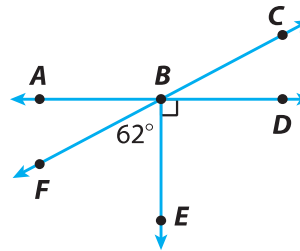
Solve for each indicated angle measure or variable in the figure.



12. x _____

13. $m\angle KMH$ _____

Solve for each indicated angle measure or variable in the figure.

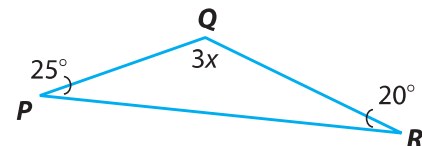


14. $m\angle CBE$ _____

15. $m\angle ABF$ _____

16. $m\angle CBA$ _____

Solve for each indicated angle measure or variable in the figure.



17. x _____

18. $m\angle Q$ _____

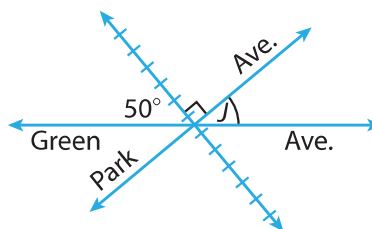
Let $\triangle ABC$ be a right triangle with $m\angle C = 90^\circ$.

- 19. Critical Thinking** An equilateral triangle has three congruent sides and three congruent angles. Can $\triangle ABC$ be an equilateral triangle? Explain your reasoning.

- 20. Counterexample** An isosceles triangle has two congruent sides, and the angles opposite those sides are congruent. River says that right triangle ABC cannot be an isosceles triangle. Give a counterexample to show that his statement is incorrect.

- 21. Make a Conjecture** In a scalene triangle, no two sides have the same length, and no two angles have the same measure. Do you think a right triangle can be a scalene triangle? Explain your reasoning.

- 22. Represent Real-World Problems** The railroad tracks meet the road as shown. The town will allow a parking lot at angle J if the measure of angle J is greater than 38° . Can a parking lot be built at angle J ? Why or why not?



- 23. Analyze Relationships** In triangle XYZ , $m\angle X = 30^\circ$, and all the angles have measures that are whole numbers. Angle Y is an obtuse angle. What is the greatest possible measure that angle Z can have? Explain your answer.
