LESSON

4.4 Ratios and Pi

TEKS **Proportionality—7.5.B** Describe π as the ratio of the circumference of a circle to its diameter.

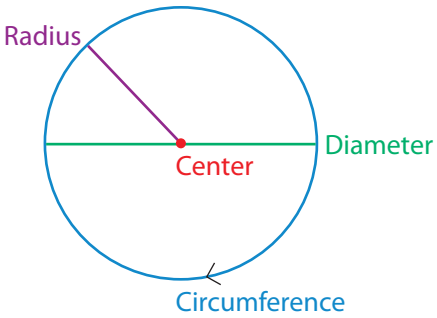
ESSENTIAL QUESTION

What is the relationship between the circumference of a circle and its diameter?

EXPLORE ACTIVITY Real World TEKS 7.5.B

Exploring Circumference

A circle is a set of points in a plane that are a fixed distance from the center.



A **radius** is a line segment with one endpoint at the center of the circle and the other endpoint on the circle. The length of a radius is called the radius of the circle.

A **diameter** of a circle is a line segment that passes through the center of the circle and whose endpoints lie on the circle. The length of the diameter is twice the length of the radius. The length of a diameter is called the diameter of the circle.

The **circumference** of a circle is the distance around the circle.

A Use a measuring tape to find the circumference of five circular objects. Then measure the distance across each item to find its diameter. Record the measurements of each object in the table below.

Object	Circumference C	Diameter d	$\frac{C}{d}$

B Divide the circumference of each object by its diameter. Round your answer to the nearest hundredth.

Reflect

1. **Make a Conjecture** Describe what you notice about the ratio $\frac{C}{d}$ in your table.

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Determining Proportionality in Circles

The ratio of the circumference to the diameter $\frac{C}{d}$ of any circle is the same for all circles. This ratio $\frac{C}{d}$ is represented by the Greek letter π, called **pi**. As you calculated in the Explore Activity, the value of π is close to 3. You can approximate π as 3.14 or $\frac{22}{7}$.

Later, you can use this ratio to find a formula for circumference. Circles do not have sides and angles, but every circle has a radius and diameter whose lengths you can measure and compare.

EXAMPLE 1

TEKS 7.5.B

Determine if the radius and diameter of the two circles are proportional.

$r = 5$ in.

$r = 3$ in.

STEP 1 Find the radius and diameter of each circle.

Larger circle		Smaller circle
$r = 5$ in.	The radius is given.	$r = 3$ in.
$d = 10$ in.	To find the diameter, multiply the radius by 2.	$d = 6$ in.

The diameter is 2 times the length of the radius.

STEP 2 Set up a proportion using the corresponding lengths of the radius and diameter.

$\frac{r}{r} = \frac{d}{d}$ Set up a proportion.

$\frac{5}{3} \stackrel{?}{=} \frac{10}{6}$ Substitute the values for r and d.

$\frac{5}{3} \stackrel{?}{=} \frac{10 \div 2}{6 \div 2}$ Simplify.

$\frac{5}{3} = \frac{5}{3}$ ✓ The two ratios are equal so the corresponding lengths are proportional.

The radius and diameter of the two circles are proportional.

Reflect

2. In every circle, what is the ratio of the radius to the diameter? What is the ratio of the circumference to the diameter?

_____

_____

3. **Draw Conclusions** Based on the ratios of the radius to diameter and circumference to diameter, are all circles proportional? Is the ratio of circumference to the diameter of any circle the same for all circles?

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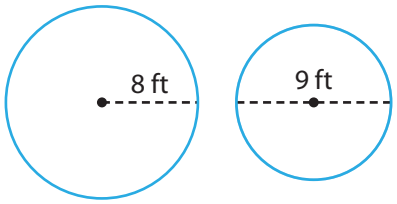
YOUR TURN

4. Determine if the radius and diameter of the two circles are proportional.



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Finding Missing Parts of a Circle

You know that $\frac{C}{d} = \pi$ for all circles. You can use this ratio to find unknown measures.

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EXAMPLE 2

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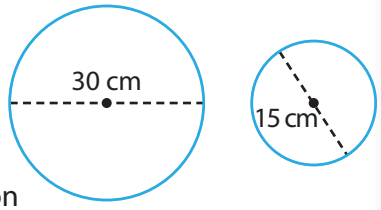
A The circumference of the larger circle is approximately 94 cm. Use a proportion to find the approximate circumference of the smaller circle.



Set up a proportion using the given information for the larger and smaller circles.

$\frac{C}{d} = \frac{C}{d}$ — *Set up a proportion.*

$\frac{94}{30} = \frac{C}{15}$ — *Substitute the given values.*

$\frac{94 \div 2}{30 \div 2} = \frac{C}{15}$ — *Since 30 divided by 2 is 15, divide 94 by 2 to find the value of C.*

$\frac{94}{30} = \frac{47}{15}$

$C = 47$

The circumference of the smaller circle is approximately 47 cm.

B A larger circle has a circumference of 50.24 cm and a diameter of 16 cm. The circumference of a smaller circle is 25.12 cm. Use a proportion to find the diameter of the smaller circle.

$\frac{C}{d} = \frac{C}{d}$ — *Set up a proportion.*

$\frac{50.24}{16} = \frac{25.12}{d}$ — *Substitute the given values.*

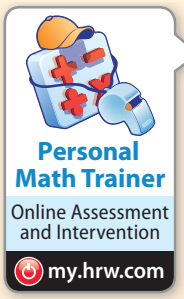
$\frac{50.24 \div 2}{16 \div 2} = \frac{25.12}{d}$ — *Since 50.24 divided by 2 is 25.12, divide 16 by 2 to find the value of d.*

$\frac{50.24}{16} = \frac{25.12}{8}$

$d = 8$

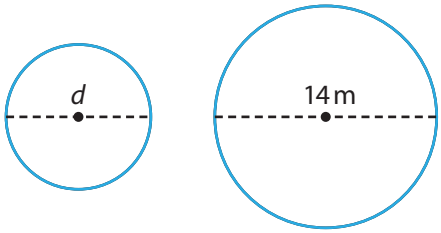
The diameter of the smaller circle is 8 cm.

My Notes



YOUR TURN

5. The circumference of the larger circle is approximately 44 meters and the circumference of the smaller circle is approximately 11 meters. Use a proportion to find the approximate diameter of the smaller circle. _____



Guided Practice

Fill in the blanks. (Explore Activity)

1. **Vocabulary** In any circle, the ratio of the _____ to the diameter is pi.

2. **Vocabulary** In any circle, the ratio of the _____ to the radius is 2.

3. You can use the decimal number _____ or the fraction _____ as an approximation for pi.

4. Determine if the radius and diameter of a circle with a diameter of 100 mm and a circle with a diameter of 10 mm are proportional. (Example 1)

_____ = _____

5. Is the circle represented by a penny similar to the one of a quarter? Explain. (Example 1)

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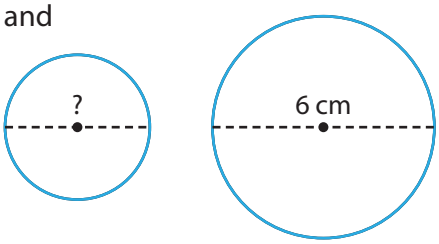
6. The circumference of the larger circle is about 18.8 centimeters and the circumference of the smaller circle is about 6.3 centimeters. Find the approximate diameter of the smaller circle.

(Example 2) _____



ESSENTIAL QUESTION CHECK-IN

7. What is the result of dividing the distance around a circle by the distance across the same circle? What number can you use as an approximate value for this ratio?

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Name _____ Class _____ Date _____

4.4 Independent Practice

TEKS 7.5.B

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8. **Measurement** Jillian measured the distance around a small fishpond as 27 yards. Which would be a good estimate for the distance across the pond, 14 yards, 9 yards, or 7 yards? Explain how you decided.

9. A rotating wind turbine has a diameter of about 185 feet and its circumference is about 580 feet. A smaller model of the turbine has a circumference of about 10 feet. What will the diameter of the model be?

10. **Multistep** Andrew has a flying disc with a radius of 10 centimeters. What is the circumference of the disc? (Remember $\frac{C}{d} = 3.14$.)

11. Mandie wants to put some lace trim around the outside of a round tablecloth she expanded. The original tablecloth had a radius of 2 feet and a circumference of 12.56 feet. If the tablecloth now has a radius of 3 feet, is 15 feet of lace enough? Explain.

12. Marta is making two different charms for a necklace. One charm has a 1 centimeter diameter and a 3.14 centimeter circumference. A similar charm has a diameter of 4 centimeters. What is the circumference?

13. Randy is putting bricks around the outside of his round flower bed to protect the plants.

a. If the diameter of his flower bed is 100 inches, what is the distance around the garden? (Remember $\frac{C}{d} = 3.14$.)

b. If the curved bricks he wants to buy are each half a foot long, how many will he need to put around the outside of the garden? Explain.

c. If each brick costs $0.68, and he can only buy whole bricks, how much will it cost him to get the material to put around the outside of his garden?

d. If his mother decides he can only have half of that diameter for his flower bed, how will the cost of the bricks be affected? Explain?

14. Your grandmother is teaching you how to make a homemade pie. The pie pan has a diameter of 9 inches.

a. If she asks you to cut a strip of pie crust long enough to go around the outside of the pan, how long does it need to be? (Remember $\frac{C}{d} = 3.14$.)

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b. If another pie pan is 8 inches across the diameter, how long does that piece of crust need to be?

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H.O.T. FOCUS ON HIGHER ORDER THINKING

Work Area

15. **Make a Conjecture** You know that all squares are similar and all circles are similar. An equilateral triangle has 3 equal sides and 3 angles of 60 degrees each. Are all equilateral triangles similar?

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16. **Multiple Representations** You know three different number representations for pi that you can use to approximate the answer to a problem. Describe a situation when you might choose to use $\frac{22}{7}$.

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17. **Represent Real-World Problems** Describe an example in your daily life where you might be able to measure around something but could not measure across it.

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18. **Critical Thinking** Every morning Jesse runs 3 laps on a circular track. One morning the track is closed, but the straight path from one side of the track to the other is open. How many times should Jesse run across the path if he wants to run his usual distance? Explain your answer.

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