QDCS : Digrammatic Calculus and Error Correction

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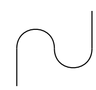
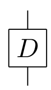
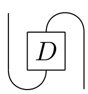
TD 1

1 Cups and Caps

Question 1. Write the following diagram as a composition of identities, cups and caps: .

Question 2. Compute its interpretation, and check it is the identity. What can we say about the mirrored version of the diagram?

Question 3. Let  be a diagram representing an arbitrary 2×2 matrix M. Compute the interpretation of . What operation is applied on M?

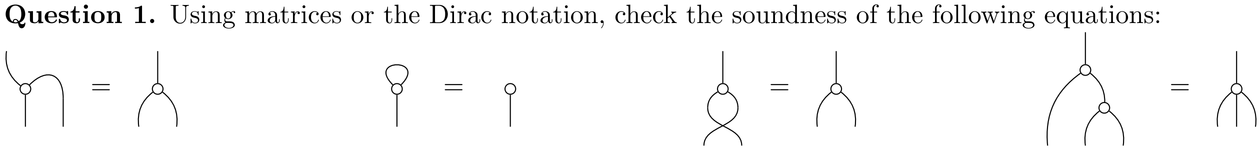
2 Tautology in Graphical Notation

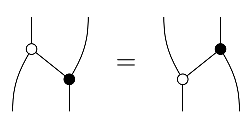
Question 1. Show by matrix computation that $|0\rangle \circ \langle 0| = |0\rangle \otimes \langle 0| = \langle 0| \otimes |0\rangle$. Generalise and show that $|\psi\rangle \circ \langle\varphi| = |\psi\rangle \otimes \langle\varphi| = \langle\varphi| \otimes |\psi\rangle$ for any states $|\psi\rangle$ and $|\varphi\rangle$ using properties of the tensor.

Question 2. Write the above terms using the graphical notation. What can we say about it?

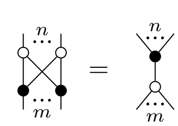
3 Only Connectivity Matters

Question 1. Using matrices or the Dirac notation, check the soundness of the following equations:



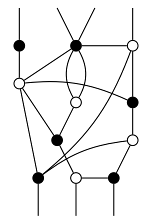
Question 2. Show diagrammatically that: . What is the interpretation of these diagrams?

4 Generalised Bialgebra

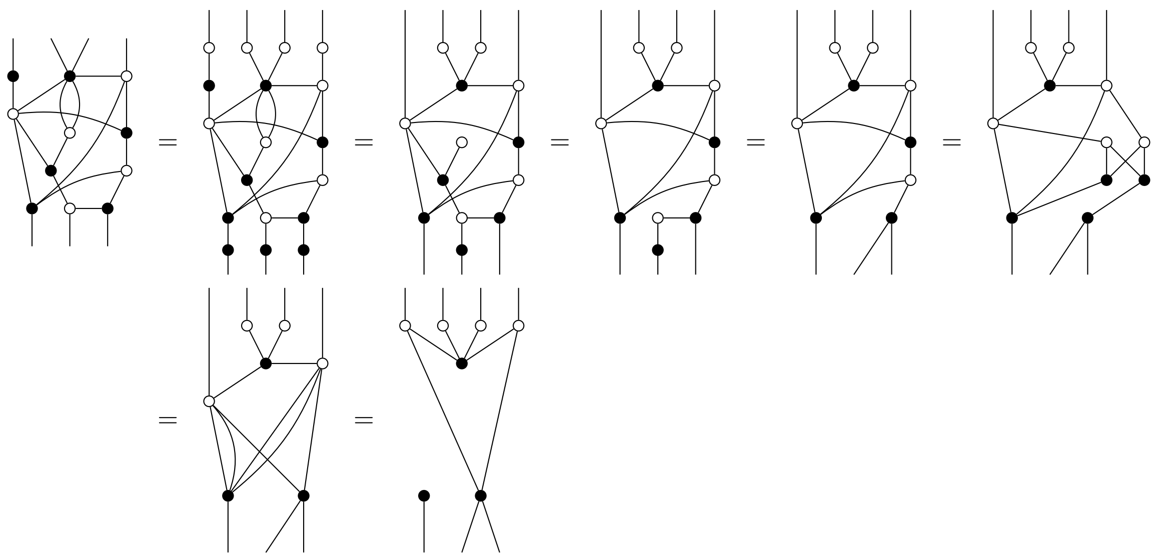
Question 1. By an induction, show that we can generalise the bialgebra rule to:  up to a global scalar. What is this scalar?

5 Normal Form

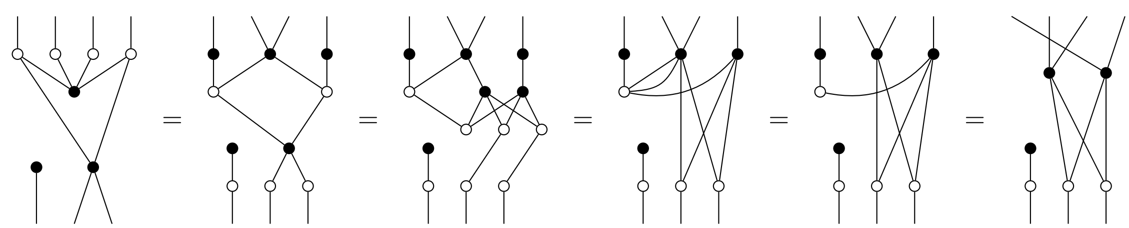
Question 1. Put the following diagram in Z-X normal form, then put it in X-Z normal form:



Answer: First, in the Z-X normal form:

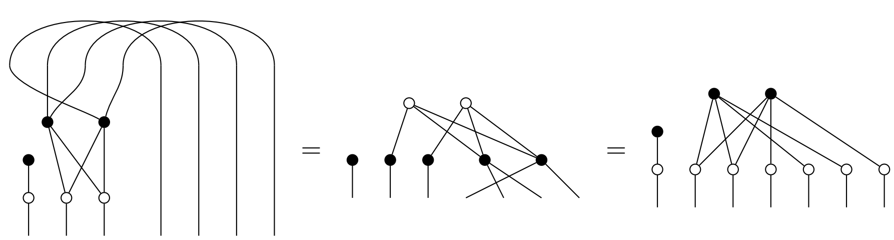


To put it in the X-Z normal form, we can either start from the first diagram, or from the one in Z-X normal form:

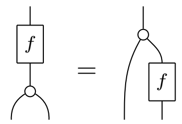


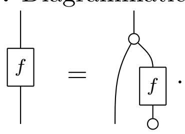
Question 2. Bend all input wires into outputs. Put the resulting diagram in Z-X and X-Z normal form.

Answer: Since all previous diagrams are equal up to deformation, we can once again choose which to start with. It is then easy to put them in either the Z-X or the X-Z normal form:

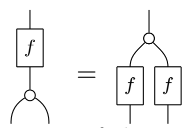


6 Phases

Let f be an arbitrary 1-qubit operator such that:  . f is called a Z-phase.

Question 1. Diagrammatically show the left-right mirrored version of the above equation. Diagrammatically show that:  . Diagrammatically show that f is self-transpose.

Question 2. By computing the interpretation of the diagrams in the first equation, show that f has to be diagonal.

Question 3. Suppose instead f distributes over the Z-spider:  . What are the only two possibilities for such f that are also invertible? Are they a phase to one of the spiders?