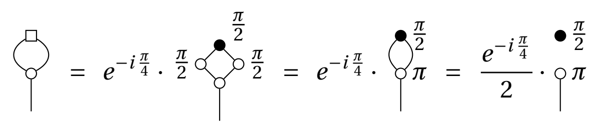
QMI/QDCS : ZX-Calculus

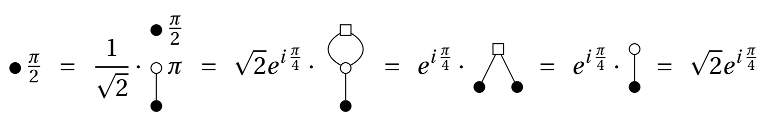
Renaud Vilmart

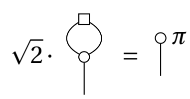
TD4: Clifford ZX-Calculus

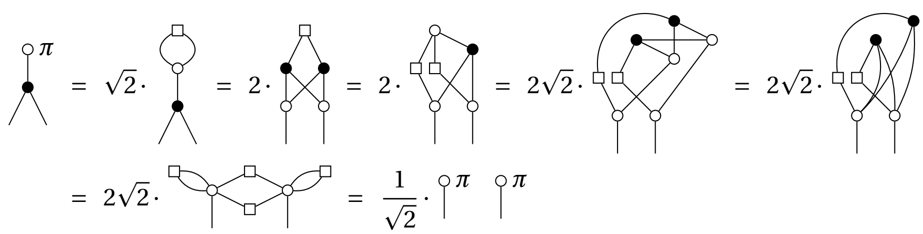
1 π-distribution

Consider the following derivations:



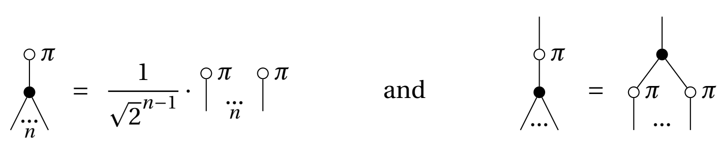






Question 1. Explain what rules are used at each step.

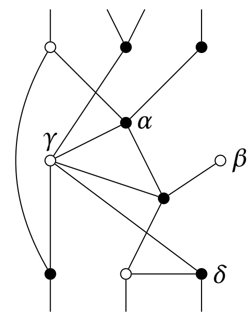
Question 2. Use the above to prove:



Hint: First equation should be a simple induction. Second should require the use of the first equation and the (generalised) bialgebra.

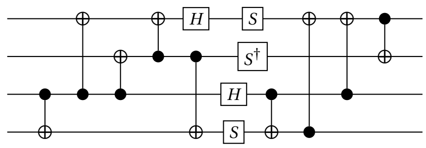
2 Graph-like structure

Question 1. Put the following diagram in graph-like form:

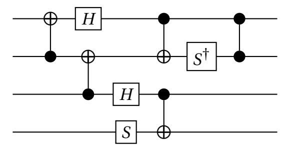


3 Verification

We are told that the first of the following circuits can be simplified into the second, which hence should implement the same operation:



and



We want to check that claim. To do so, we propose the following approach:

1. Build the circuit consisting of the first, followed by the dagger of the second.
2. Turn the obtained circuit into a ZX-diagram.
3. Put the diagram in normal form.
4. Finish by applying H-involutions and the Id rule wherever possible.
5. Check whether the diagram is reduced to the identity.

Question 1. Explain briefly why a reduction to the identity means the two starting circuits are equivalent.

Question 2. In the case of Clifford circuits, what about the converse? (I.e. what can we conclude when the diagram does not reduce to the identity?)

Question 3. Apply the above protocol, to check whether the two circuits are equivalent.

4 Stabilisers

Question 1. Show that the group generated by the stabilisers ZXX, XZX, XXZ does not contain $-III$. Build the ZX-state whose stabilisers are generated by the above three.