QDCS : Digrammatic Calculus and Error Correction

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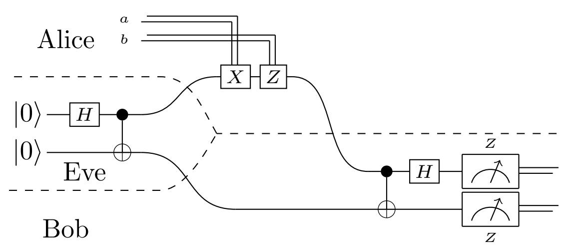
Exam 2

1 Super Dense Coding

Consider the following protocol, with three parties, Alice, Bob and Eve. The goal is for Alice to send two bits of (classical) data to Bob, by only sending him a qubit. For this she can use the fact that they share an entangled state (prepared in advanced by the third party Eve):

- Eve prepares an EPR pair, and sends each half to the other two parties
- Alice applies on her qubit an X gate if her first bit is 1; then a Z gate if her second bit is 1. She sends her qubit to Bob.
- Bob applies a CNot between Alice's qubit and his, then applies an H gate on Alice's qubit. He measures both qubits in the canonical basis.

The expectation is that at the end, the two classical bits from the measurements of Bob are exactly the bits Alice used on her half of the protocol. This protocol can be represented by the following circuit:



Question 1. Recall how to represent $|0\rangle$ and $|1\rangle$ (we don't care about the overall scalar here).

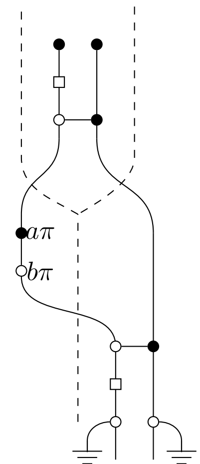
Answer: Up to global scalars:

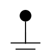
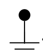
Question 2. Recall how to represent the X gate in ZX. Using variable a, suggest a way to represent $X^a = \begin{cases} I & \text{if } a = 0 \\ X & \text{if } a = 1 \end{cases}$. Similarly, explain how to represent Z^b.

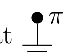
Answer: The X-gate is represented as π. X^a can then be represented by $a\pi$ since when $a = 0$, we get a binary spider with 0-rotation, which equals the identity. Similarly, we can represent Z^b by $b\pi$.

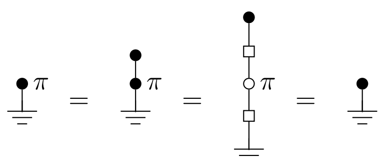
Question 3. Assuming a Z-measurement can be represented by , and using Question 2, turn the above circuit into a ZX-diagram.

Answer:

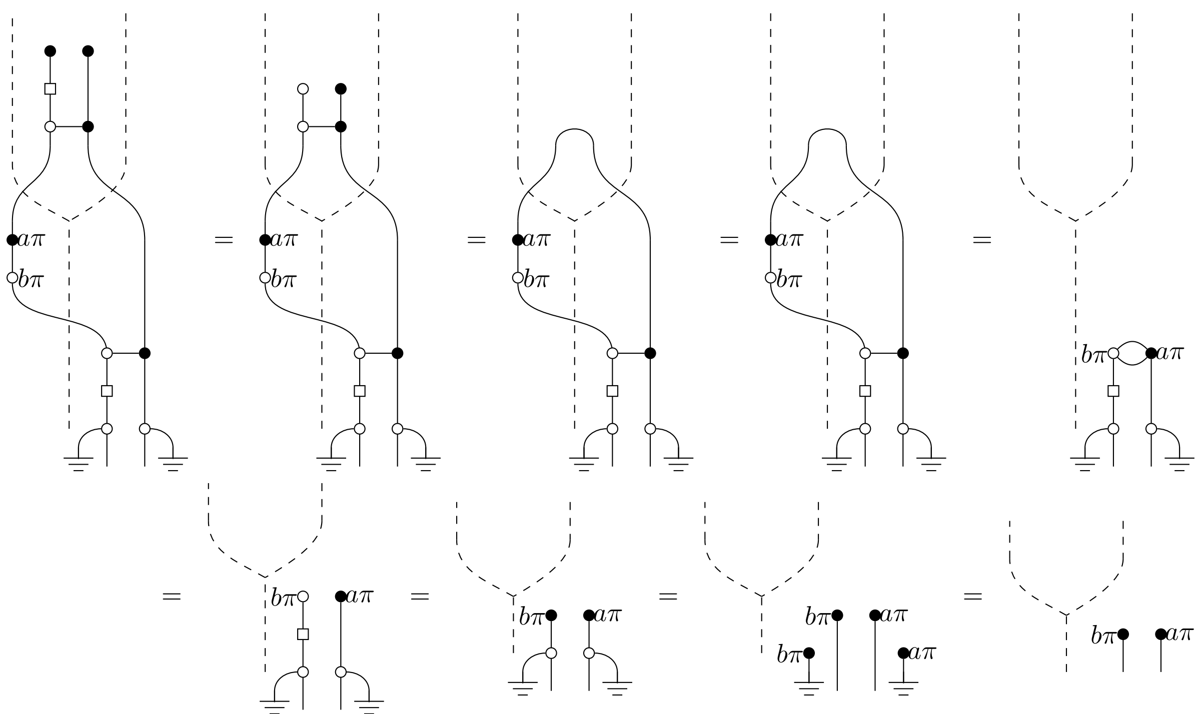


Recall that from the equational theory, we have  $= \sqrt{2}$. Since we ignore non-zero scalars here, this equation means can simply get rid of . Recall also that the Hadamard gate and the phase gates are "consumed" by the discard (aka partial trace).

Question 4. Using the above, together with the fact that $X = HZH$, show that  can also be removed.

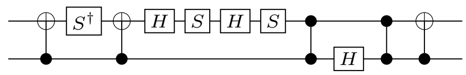
Answer: This translates into:  which can then be removed

Question 5. Simplify the diagram to show that Bob indeed gets Alice's bits a and b at the end.



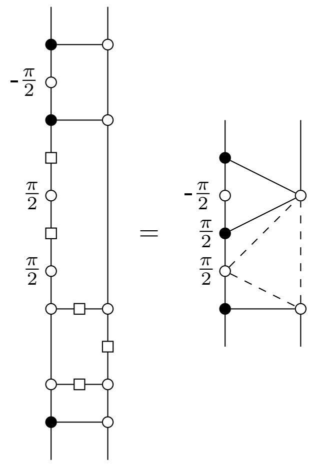
2 Clifford Simplification

Consider the following circuit:



Question 1. Turn the circuit into a ZX-diagram (you can ignore overall scalars). Do all the "obvious" simplifications.

Answer: Using the colour-change rule, the spider fusion, and replacing Hadamard gates by Hadamard edges, we get:



Question 2. Use the algorithm from the course to remove all inner spiders from this diagram.
Hint: Assuming all obvious simplifications are done, the best first step is to apply a local complementation on the most central $\frac{\pi}{2}$-spider.

Answer: Doing in order:

- a local complementation on the black $\frac{\pi}{2}$ spider, and subsequent spider fusions
- removal of parallel edges
- a pivoting (aka bialgebra) on the 2 left Z-spiders and subsequent spider fusions
- removal of parallel edges

we get: