QMI/QDCS : ZX-Calculus

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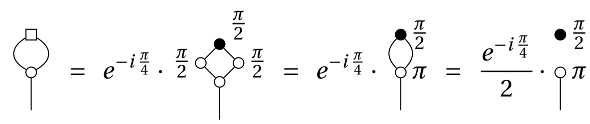
TD4: Clifford ZX-Calculus

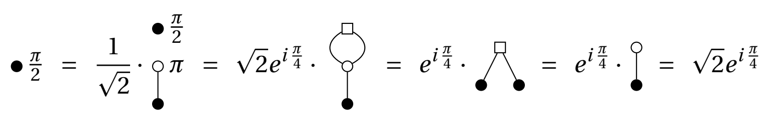
1 π-distribution

Consider the following derivations:



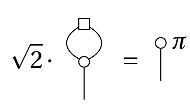
Answer:

1. H-decomp
2. fusion
3. Hopf-law

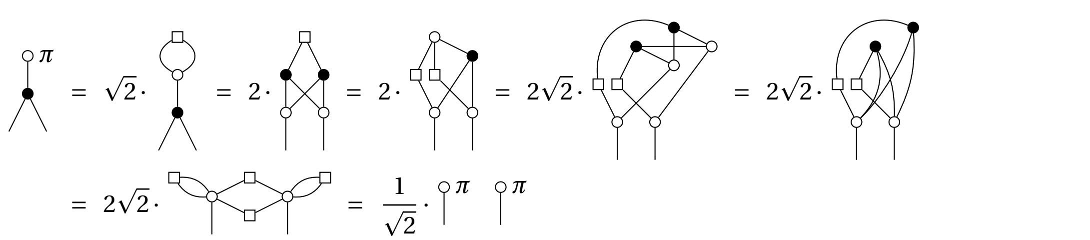


Answer:

1. a scalar equation
2. the previous equation
3. copy
4. H
5. a scalar equation



Answer: The first equation to turn the "H-loop" into a π Z-phase, followed by the second equation to get rid of the non-connected part of the diagram.



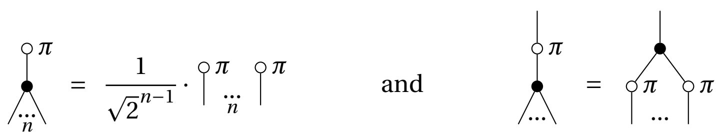
Answer:

1. previous equation
2. bialgebra
3. H
4. bialgebra
5. fusion
6. H and fusion
7. H-Hopf law and previous equation

Question 1. Explain what rules are used at each step.

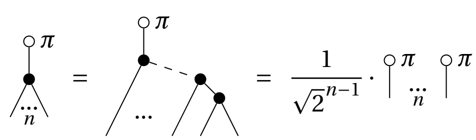
Answer: Written above in between the equations.

Question 2. Use the above to prove:

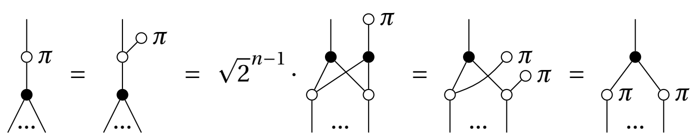


Hint: First equation should be a simple induction. Second should require the use of the first equation and the (generalised) bialgebra.

Answer: First:



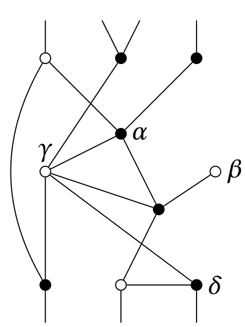
Then:



assuming the diagram has n outputs, using the spider fusion, $(2, n)$-bialgebra, above π-copy, and fusion again.

2 Graph-like structure

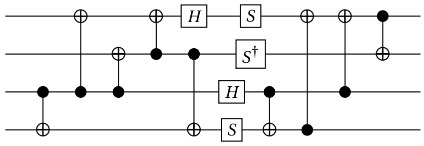
Question 1. Put the following diagram in graph-like form:



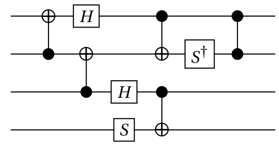
Answer: Done in class.

3 Verification

We are told that the first of the following circuits can be simplified into the second, which hence should implement the same operation:



and



We want to check that claim. To do so, we propose the following approach:

1. Build the circuit consisting of the first, followed by the dagger of the second.
2. Turn the obtained circuit into a ZX-diagram.
3. Put the diagram in normal form.
4. Finish by applying H-involutions and the Id rule wherever possible.
5. Check whether the diagram is reduced to the identity.

Question 1. Explain briefly why a reduction to the identity means the two starting circuits are equivalent.

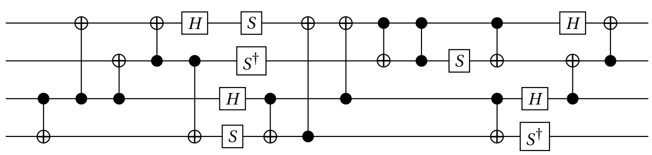
Answer: Since the circuits represent unitaries, their dagger are their inverse. Hence, if the first followed by the dagger of the second yields the identity, that means the two starting diagrams are equivalent.

Question 2. In the case of Clifford circuits, what about the converse? (I.e. what can we conclude when the diagram does not reduce to the identity?)

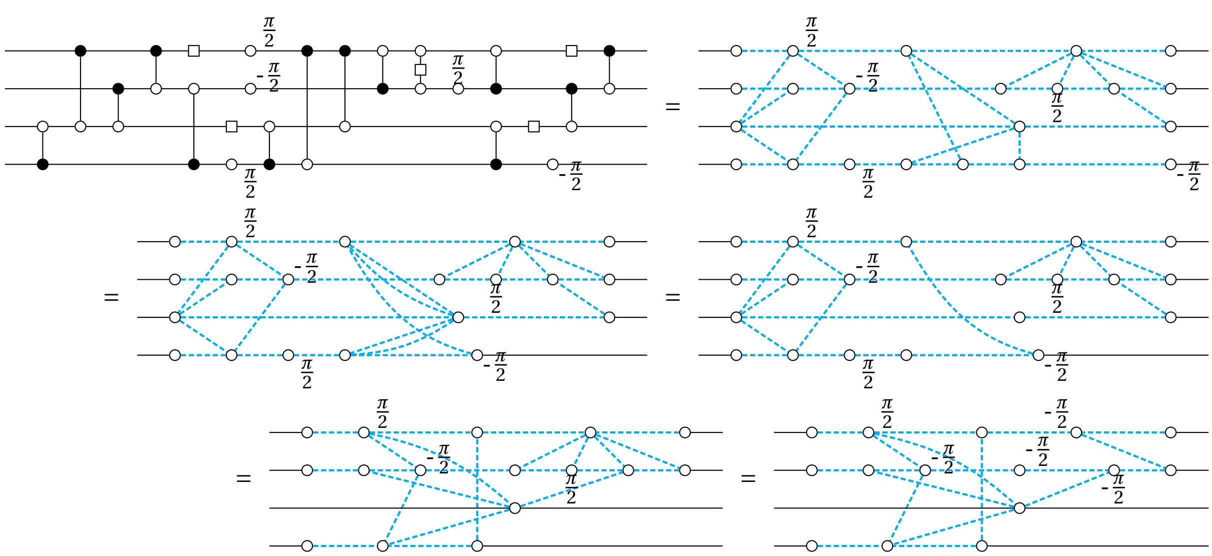
Answer: We haven't talked about the uniqueness of the normal forms during the class, so the answer is, we can't conclude. (It is however possible to enforce uniqueness by adding a few steps to the normal form algorithm.)

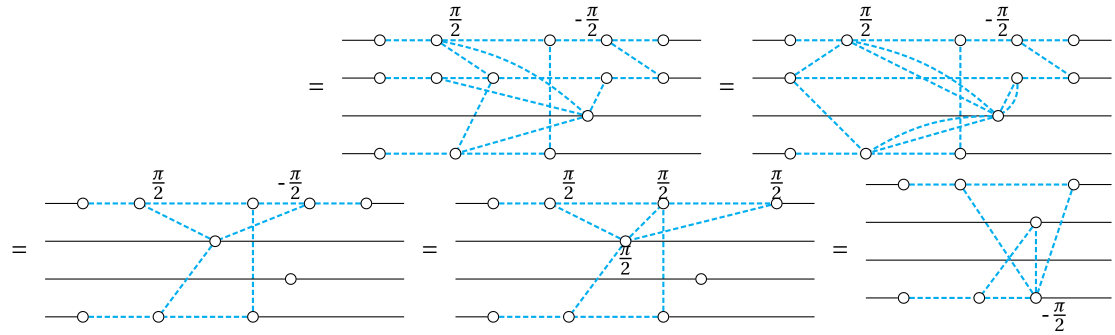
Question 3. Apply the above protocol, to check whether the two circuits are equivalent.

Answer: The circuit:



can be turned into a ZX-diagram, and then reduced, as follows:





The diagram does not seem to reduce to the identity. We can actually find a witness that it is not equivalent to the identity: if we apply an X gate on the first qubit, on the left, we should expect the X gate to come out on the right on the first qubit. However, we instead get $XIIZ$ on the right. We hence do not have the identity.

4 Stabilisers

Question 1. Show that the group generated by the stabilisers ZXX, XZX, XXZ does not contain $-III$. Build the ZX-state whose stabilisers are generated by the above three.