QDCS : Digrammatic Calculus and Error Correction

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Quick Exam 1

1 Computing an Interpretation

Question 1. Recall what the "only connectivity matters" meta-rule states about ZX-diagrams.

Answer: The only connectivity matters meta rule states that we can deform diagrams at will (while keeping the inputs and outputs fixed), without modifying the interpretation of the diagram.

Question 2. Compute the interpretation of the following diagram:

Answer: We can use the only connectivity matters meta-rule to compute the interpretation of the diagram as follows:

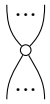
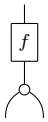
$$\left[\!\left[\;\right]\!\right] = \left(id \otimes \left[\!\left[\;\right]\!\right]\right) \circ (id \otimes H \otimes id) \circ \left(\left[\!\left[\;\right]\!\right] \otimes id\right)$$

$$= \begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix} \circ \frac{1}{\sqrt{2}} \begin{pmatrix} 1&0&1&0&0&0&0&0\\ 0&1&0&1&0&0&0&0\\ 1&0&-1&0&0&0&0&0\\ 0&1&0&-1&0&0&0&0\\ 0&0&0&0&1&0&1&0\\ 0&0&0&0&0&1&0&1\\ 0&0&0&0&1&0&-1&0\\ 0&0&0&0&0&1&0&-1 \end{pmatrix} \circ \left(\left[\!\left[\;\right]\!\right] \otimes id\right)$$

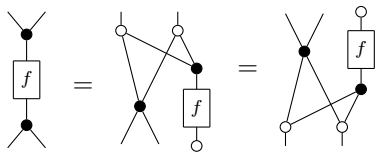
$$= \frac{1}{\sqrt{2}} \begin{pmatrix} 1&0&1&0&0&0&0&0\\ 0&1&0&-1&0&0&0&0\\ 0&0&0&0&1&0&1&0\\ 0&0&0&0&0&1&0&-1 \end{pmatrix} \begin{pmatrix} 1&0&0&0\\ 0&1&0&0\\ 0&0&0&0\\ 0&0&0&0\\ 0&0&0&0\\ 0&0&0&0\\ 0&0&1&0\\ 0&0&0&1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1&0&0&0\\ 0&1&0&0\\ 0&0&1&0\\ 0&0&0&-1 \end{pmatrix}$$

This is a controlled-Z gate (up to colinearity).

2 Back to phases

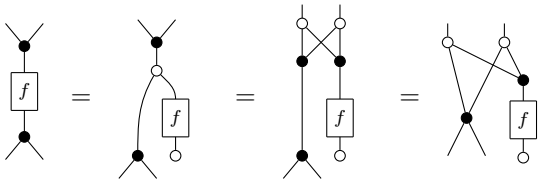
Let there be two spiders:  and  interacting through a bialgebra:  =  . Let f be a phase for the white spider:  =  .

Question 1. Show the following equalities:



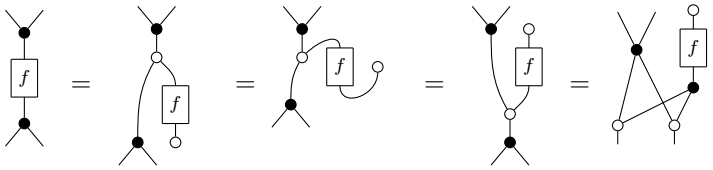
Hint: you can reuse properties of phases from previous TD sheets.

Answer:



The equations are obtained in order as follows:

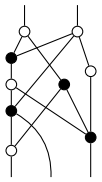
- using an equation from TD sheet 1/2
- using the bialgebra rule
- using the spider fusion rule



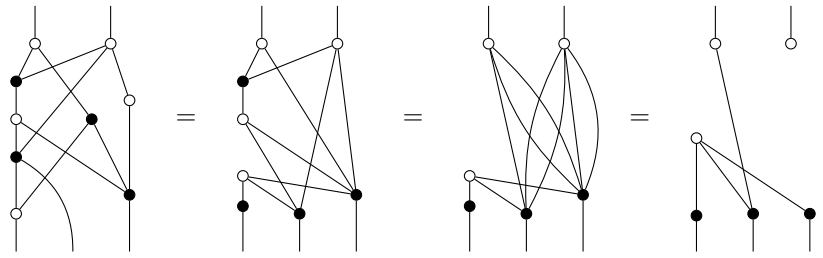
Here we have to use the "only connectivity matter" principle on the 2 Z-spiders, use the fact that f is self transpose, then again use the bialgebra and spider fusion rules.

3 Z-X Normal Form

Question 1. Put the following diagram in Z-X normal form:



Answer:



The qualities are obtained in order by:

- Adding a binary X-spider (equivalent to identity) on the first output, so that the input spiders are all Z spiders, and output spiders are all X spiders; merging together all adjacent spiders of the same type

- Applying a bialgebra on the topmost X-spider and the Z-spider right below; and then merging together adjacent spiders of the same colour
- Removing parallel edges between pairs of X and Z spiders

The only remaining inner spider is a Z-spider that is only connected to boundary X-spiders, so the last diagram is in Z-X normal form.