QDCS : Digrammatic Calculus and Error Correction

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TD 4

1 A Small Linear Code

Let $G = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ be the generating matrix of code C.

Question 1. Enumerate all codewords in C. What is the minimal distance of C?

Question 2. What are the dimension and the length of C?

Question 3. Give a parity-check matrix associated to C.

2 A Linear Code

Let C be the binary code with parity-check matrix

$$H = \begin{pmatrix} 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Note that any column of H has weight 3.

Question 1. Prove that the code has minimum distance > 3.

Question 2. Give a codeword of weight 4 of C.

Question 3. Prove that any word of C has an even weight.

3 Intuitions on Linear Codes

Let $C \subseteq \mathbb{F}_2^n$ be an $[n, k, d]$ code and G, H be respectively a generator and a parity check matrix of C. In what follow we list operations on G yielding a new matrix G'. For any one:

- does G' generate the same code?
- if not,
  - has the new code generated by G' the same length?
  - a larger dimension?
  - a smaller dimension?
  - might this code have a larger minimum distance?
  - a smaller minimum distance?

(1) Removing a row;

(2) swapping two rows;

(3) removing a column;

(4) swapping two columns;

(5) adding an additional row drawn at random;

(6) adding an additional row defined as the sum of all the other rows;

(7) adding an additional column defined as the sum of all the other columns.

Same questions when the operations are applied to H.

4 Y Errors

Consider the Pauli operator $Y = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$

Question 1. Compute the eigenvalues and eigenstates of Y.

Question 2. Give an orthonormal basis of $\mathcal{H}$ whose elements are swapped by Y.

Question 3. Deduce an encoding of one qubit into three which permits to correct an error in Y on one qubit.

5 Admissible Pauli Subgroup

Question 1. Show that any subgroup of $\mathcal{P}_n$ that does not contain $-I \otimes ... \otimes I$ is abelian.
Hint: First show that any element in the subgroup is involutive.