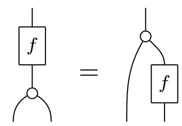
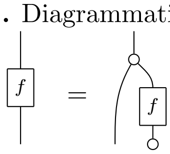
QDCS : Digrammatic Calculus and Error Correction

Renaud Vilmart

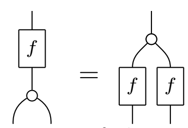
TD 2

1 Phases

Let f be an arbitrary 1-qubit operator such that:  . f is called a Z-phase.

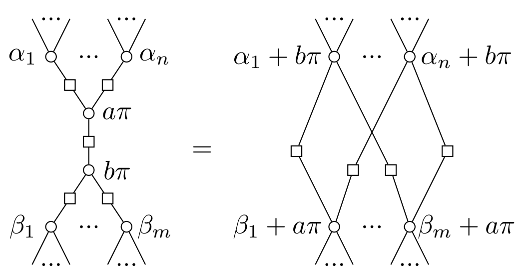
Question 1. Diagrammatically show the left-right mirrored version of the above equation. Diagrammatically show that:  . Diagrammatically show that f is self-transpose.

Question 2. By computing the interpretation of the diagrams in the first equation, show that f has to be diagonal.

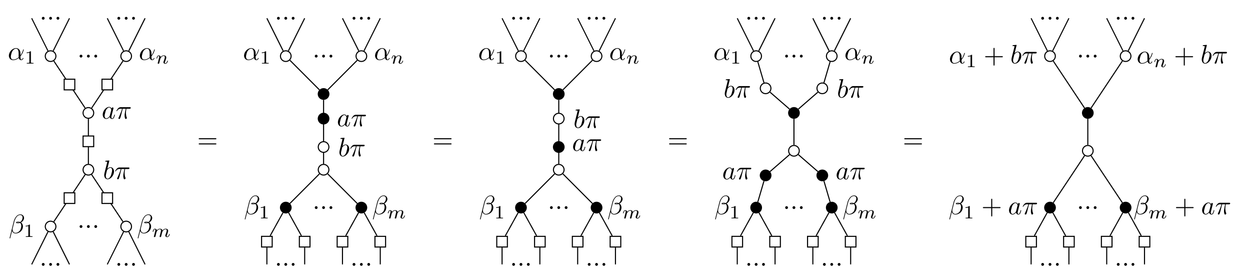
Question 3. Suppose instead f distributes over the Z-spider:  . What are the only two possibilities for such f that are also invertible? Are they a phase to one of the spiders?

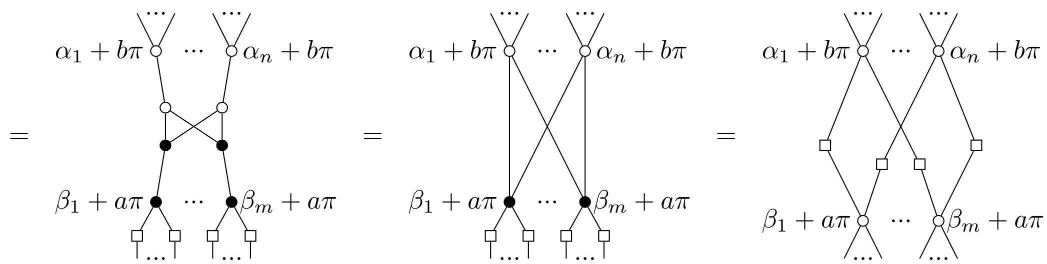
2 Pivoting

Question 1. With $a, b \in \{0, 1\}$, using the colour-change rule, the π-commutation rule and the generalised bialgebra rule, show that, up to a global scalar:



Answer: This can be proven diagrammatically, using the colour-change, the spider fusion, the π-commutation, the π-distribution, and the generalised bialgebra rules:

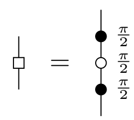




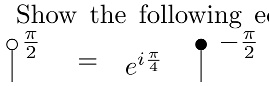
3 Local Complementation

In this exercise, we don't want to use the interpretation of the diagrams, only the equational theory of ZX.

Question 1. Using the Hadamard decomposition, et the Hadamard involution, show that:

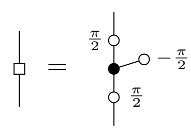


Question 2. Show the following equation, using the Hadamard decomposition, the spider rule and the π-copy rule:

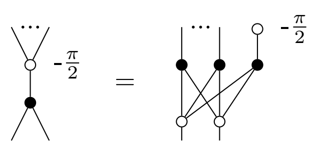


. What is its colour-swapped version?

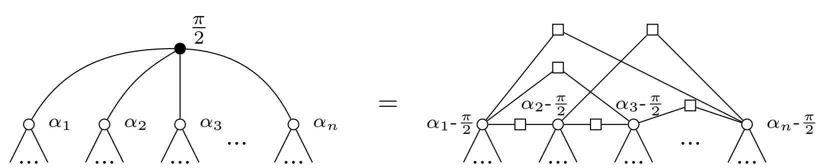
Question 3. Using the Hadamard decomposition and the previous question, show that:



Question 4. Using the spider rule and the generalised bialgebra rule, show that, up to a global scalar:

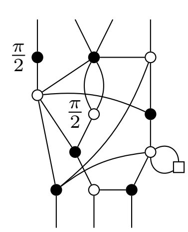


Question 5. By induction on n, using the equations of ZX, and the results from Questions 4 and 3, show the following equation where n is the number of Z-spiders (up to a global scalar):

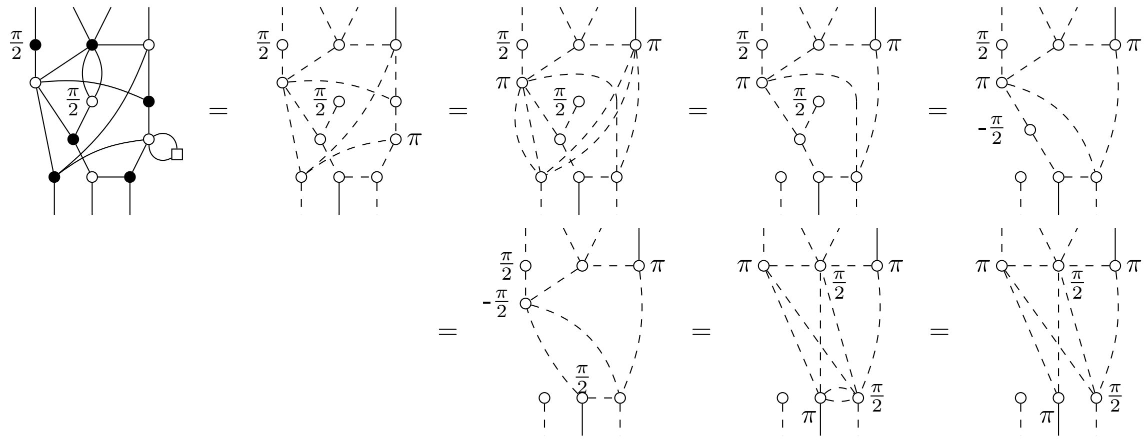


4 Reduction of Clifford Diagrams

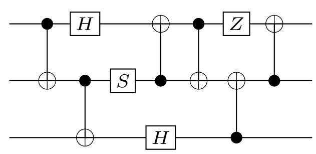
Question 1. Reduce the following diagram to remove all inner spiders:



Answer: Doing all "obvious" simplifications, removing all X-spiders by colour-change, and applying pivoting and local complementation eagerly on inner spiders, we get:

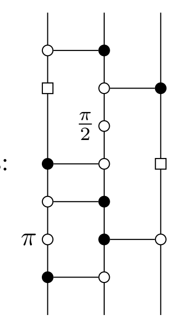


Question 2. Consider the following circuit:

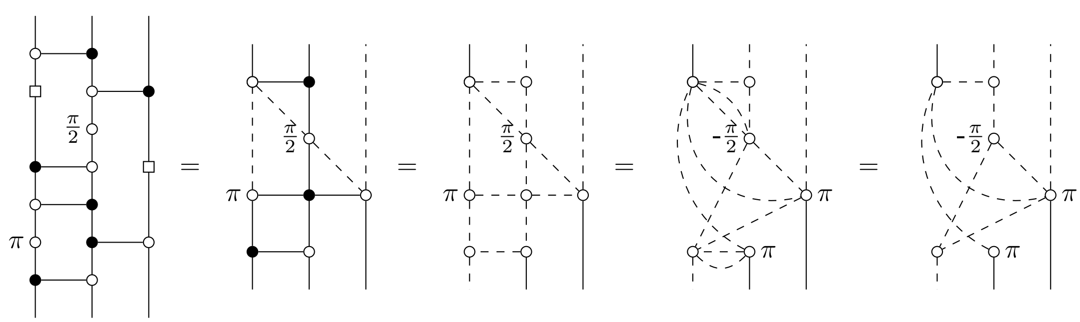


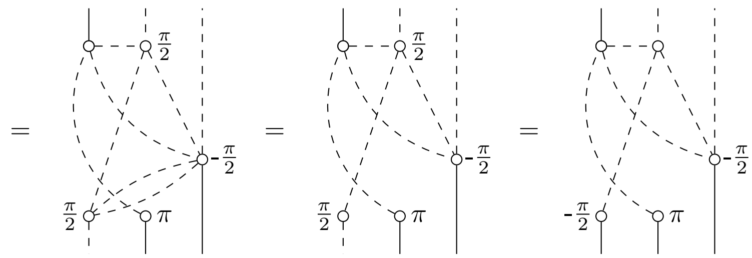
1. Turn it into a ZX-diagram
2. Use the algorithm to remove all inner spiders
3. Try to "extract" a circuit out of it

Answer:

1. First, using the translation from the course, we can turn the circuit into a ZX-diagram as: 

2. Then, we can get rid of the inner spiders using the algorithm from the course:





where the last step is technically not necessary, but simplifies the result of the next question.

3. We have to make the diagram circuit-like: