Actuarial Methods: Case Study

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The goal of the study is to calculate the annual net premium for 10-year endowment for a person at any age selected from any population. Thus, we are starting with listing assumptions for this study,

- our individual is a 20-year old US male, who bought insurance in 2017,
- 10-year term life insurance is assumed to have a benefit in amount of \$100 000, payable at the end of the year of death,
- 10-year pure endowment is assumed to have a benefit in amount of \$10 000, payable at the maturity,
- premiums are paid at the beginning of each year,
- the technical rate of interest must be equal to the minimum interest rate guaranteed is the same as the statutory rate of interest – 3.75% [2].

The following step is to derive an appropriate formula. First of all, let x denote the individual's age at the commencement of of the overall time period, n the duration of an insurance, and r the technical rate of interest. Second, we can calculate the annual net premium for 10-year endowment as a sum of the annual net premiums for 10-year term life insurance (P_1) and 10-year pure endowment (P_2). Thus, the formula for the net annual benefit premium is following,

$$P = P_1 + P_2 \tag{1}$$

Also, by the equivalence principle we have,

$$\ddot{a}_{x:\overline{n}|} \cdot P_1 + \ddot{a}_{x:\overline{n}|} \cdot P_2 = A^{1}_{x:\overline{n}|} \cdot B_1 + A_{x:\overset{1}{\overline{n}|}} \cdot B_2 \tag{2}$$

$$P_1 + P_2 = \frac{1}{\ddot{a}_{x:\overline{n}|}} \times \left[A^1_{x:\overline{n}|} \cdot B_1 + A_{x:\overset{1}{\overline{n}|}} \cdot B_2 \right] \tag{3}$$

Where $A^1_{x:\overline{n}|}$ is the actuarial present value of n-year term life insurance, $A_{x:\overset{1}{\overline{n}|}}$ is the actuarial present value of n-year pure endowment, $\ddot{a}_{x:\overline{n}|}$ is the n-year temporary life annuity-due, and B_1, B_2 are benefits.

We can express the n-year temporary life annuity-due via the n-year endowment applying the actuarial identity.

$$1 = d \cdot \ddot{a}_{x:\overline{n}|} + A_{x:\overline{n}|} \tag{4}$$

After some basic manipulations, we have,

$$\ddot{a}_{x:\overline{n}|} = \frac{1 - A_{x:\overline{n}|}}{d} \tag{5}$$

$$= \frac{1 - A^1_{x:\overline{n}|} - A_{x:\overset{1}{\overline{n}|}}}{d} \tag{6}$$

Where d is an effective annual discount rate, and $A_{x:\overline{n}|}$ is the actuarial present value of n-year endowment.

Now we should find the appropriate representation of the actuarial net present values. Let us begin with the actuarial present value of n-year term life insurance. In this case, as we know, benefits are paid only if the insured dies within a given period of time, but we cannot be sure about the exact time of death k, thus it is a random variable. However, we are interested in discounting factor v, therefore the random variable of our primary interest is v^k.

$$A^1_{x:\overline{n}|} = \mathbb{E}(v^k) \tag{7}$$

$$= \sum_{i=0}^{n-1} v^{i+1} \cdot f(v^{i+1}) \tag{8}$$

Note, benefits are equal to zero after the final year of the insurance, thus we are interested only the n-year horizon. Also, the benefit is paid at the end of the year of death.

Another important element is the mass function, which we define as

$$f(x) = P(\{\text{attains age } x + i\}) \cdot P(\{\text{ dies within the following year}\}) \tag{9}$$

$$= {}_ip_x \cdot q_{x+i} \tag{10}$$

Finally, by (8) and (10) we have,

$$A^{1}_{x:\overline{n}|} = \sum_{i=0}^{n-1} v^{i+1} \cdot {}_{i}p_{x} \cdot q_{x+i} \tag{11}$$

The actuarial present value of n-year pure endowment calculates the same way, but we are only interested in one case, when the individual attains the final year. Therefore,

$$A_{x:\overline{n}|}^{\;\;1} = v^{n} \cdot {}_{n}p_{x} \tag{12}$$

Since we are going to use the life table, the functions ${}_{n}p_{x}$, q_x can be expressed as

$${}_{n}p_{x} = \frac{\ell_{x+n}}{\ell_{x}} \tag{13}$$

$$q_{x} = 1 - \frac{\ell_{x+1}}{\ell_{x}} \tag{14}$$

Where ℓ_x represents the expected number of survivors to age x from the ℓ_0 newborns.

At last, we have everything to find the benefit premium. By (3),(5),(11),(12), (13) and (14) we get,

$$P = \frac{d}{1 - A_{x:\overline{n}|}} \times \left[\sum_{i=0}^{n-1} v^{i+1} \cdot \left(1 - \frac{\ell_{x+i+1}}{\ell_{x+i}}\right) \cdot \frac{\ell_{x+i}}{\ell_{x}} \cdot B_1 + v^{n} \cdot \frac{\ell_{x+n}}{\ell_{x}} \cdot B_2 \right] \tag{15}$$

After the implementation of the formula (15) in LibreOffice, the estimated premium amounts to \$822.99, for more information, please, see the .xls file.

Case study: extended

In this section we are supposed to calculate net premium reserves (${}_{k}V$) for $k = 3$ periods from now. It can be calculated as,

$${}_{k}V = {}_{k}V^{1} + {}_{k}V^{2} \tag{16}$$

where ${}_{k}V^{1}$ denotes the net premium reserve of a n-year term life insurance, and ${}_{k}V^{2}$ is the net premium reserve of a n-year pure endowment. The formulas of ${}_{k}V^{1}$ and ${}_{k}V^{2}$ are following:

$${}_{k}V^{1} = B_1 \cdot A^{\;\;1}_{x+k:\overline{n-k}|} - P_1 \cdot \ddot{a}_{x+k:\overline{n-k}|} \tag{17}$$

$$ {}_{k}V^2 = B_2 \cdot A_{x+k:\overset{1}{\overline{n-k}|}} - P_2 \cdot \ddot{a}_{x+k:\overline{n-k}|} \tag{18} $$

The calculations in LibreOffice yielded the net premium reserve in the amount of $4,487.27.

References

[1] Newton L. Bowers, Hans U. Gerber, James C. Hickman, Donald A. Jones, Cecil J. Nesbitt. *Actuarial Mathematics*. The Society of Actuaries, 1997.

[2] *Prescribed U.S. Statutory and Tax Interest Rates for the Valuation of Life Insurance and Annuity Products*. WillisTowersWatson, October 2019.