Association Analysis: Basic Concepts

Li, Jia

DSA Thrust, Information Hub
HKUST Guangzhou

Oct 27, 2025

Association Rule Mining

- **Given a set of transactions, find rules that will predict the occurrence of an item based on the occurrences of other items in the transaction**

Market-Basket transactions

TID	*Items*
1	**Bread, Milk**
2	**Bread, Diaper, Beer, Eggs**
3	**Milk, Diaper, Beer, Coke**
4	**Bread, Milk, Diaper, Beer**
5	**Bread, Milk, Diaper, Coke**

Example of Association Rules

$\{Diaper\} \rightarrow \{Beer\}$,
$\{Milk, Bread\} \rightarrow \{Eggs, Coke\}$,
$\{Beer, Bread\} \rightarrow \{Milk\}$,

Implication means co-occurrence, not causality!

Definition: Frequent Itemset

- **Itemset**
  - A collection of one or more items
    - Example: {Milk, Bread, Diaper}
  - k-itemset
    - An itemset that contains k items
- **Support count (σ)**
  - Frequency of occurrence of an itemset
  - E.g. σ({Milk, Bread,Diaper}) = 2
- **Support**
  - Fraction of transactions that contain an itemset
  - E.g. s({Milk, Bread, Diaper}) = 2/5
- **Frequent Itemset**
  - An itemset whose support is greater than or equal to a *minsup* threshold

TID	*Items*
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Definition: Association Rule

- **Association Rule**
  - An implication expression of the form X → Y, where X and Y are itemsets
  - Example:
    {Milk, Diaper} → {Beer}
- **Rule Evaluation Metrics**
  - Support (s)
    - Fraction of transactions that contain both X and Y
  - Confidence (c)
    - Measures how often items in Y appear in transactions that contain X

TID	*Items*
1	**Bread, Milk**
2	**Bread, Diaper, Beer, Eggs**
3	**Milk, Diaper, Beer, Coke**
4	**Bread, Milk, Diaper, Beer**
5	**Bread, Milk, Diaper, Coke**

Example:

$$\{\mathrm{Milk}, \mathrm{Diaper}\} \Rightarrow \{\mathrm{Beer}\}$$

$$s = \frac{\sigma(\mathrm{Milk}, \mathrm{Diaper}, \mathrm{Beer})}{|\mathrm{T}|} = \frac{2}{5} = 0.4$$

$$c = \frac{\sigma(\mathrm{Milk}, \mathrm{Diaper}, \mathrm{Beer})}{\sigma(\mathrm{Milk}, \mathrm{Diaper})} = \frac{2}{3} = 0.67$$

Association Rule Mining Task

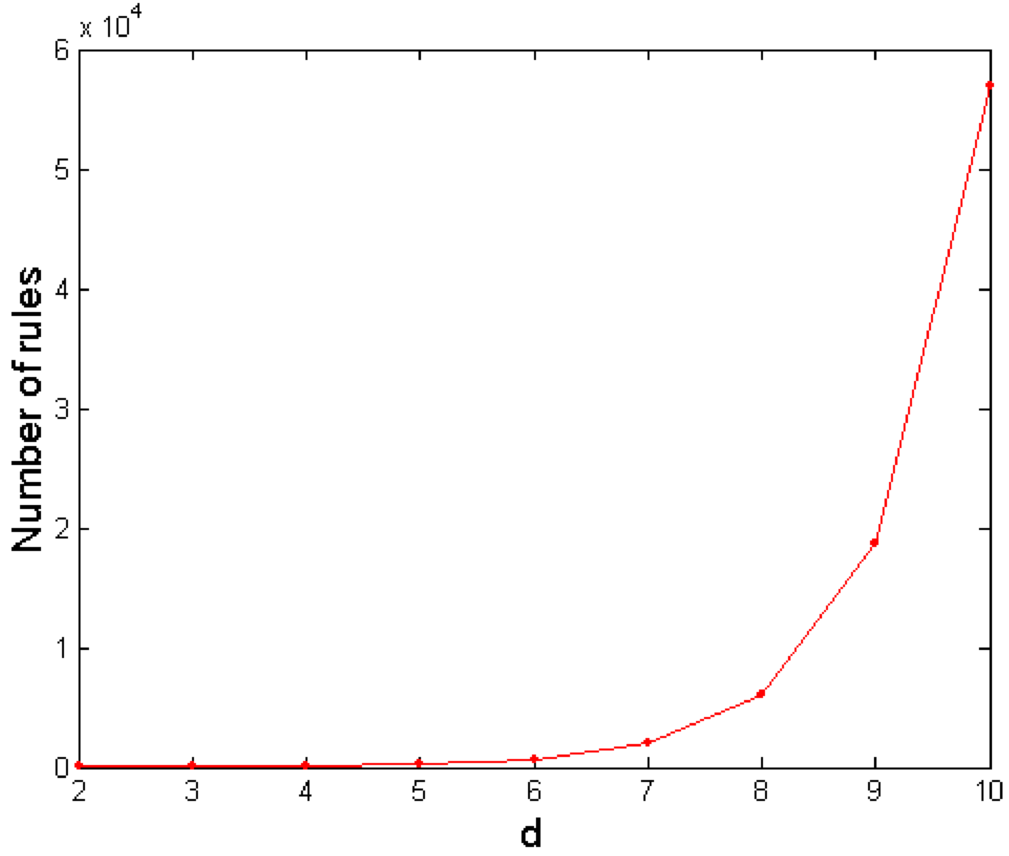
- **Given a set of transactions T, the goal of association rule mining is to find all rules having**
  - support ≥ *minsup* threshold
  - confidence ≥ *minconf* threshold

- **Brute-force approach:**
  - List all possible association rules
  - Compute the support and confidence for each rule
  - Prune rules that fail the *minsup* and *minconf* thresholds

  ⇒ Computationally prohibitive!

Computational Complexity

- **Given d unique items:**
  - Total number of itemsets = 2^d
  - Total number of possible association rules:



$$R = \sum_{k=1}^{d-1}\left[\binom{d}{k} \times \sum_{j=1}^{d-k}\binom{d-k}{j}\right]$$

$$= 3^d - 2^{d+1} + 1$$

If d=6, R = 602 rules

Mining Association Rules

TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Example of Rules:

{Milk,Diaper} $\rightarrow$ {Beer} (s=0.4, c=0.67)
{Milk,Beer} $\rightarrow$ {Diaper} (s=0.4, c=1.0)
{Diaper,Beer} $\rightarrow$ {Milk} (s=0.4, c=0.67)
{Beer} $\rightarrow$ {Milk,Diaper} (s=0.4, c=0.67)
{Diaper} $\rightarrow$ {Milk,Beer} (s=0.4, c=0.5)
{Milk} $\rightarrow$ {Diaper,Beer} (s=0.4, c=0.5)

Observations:

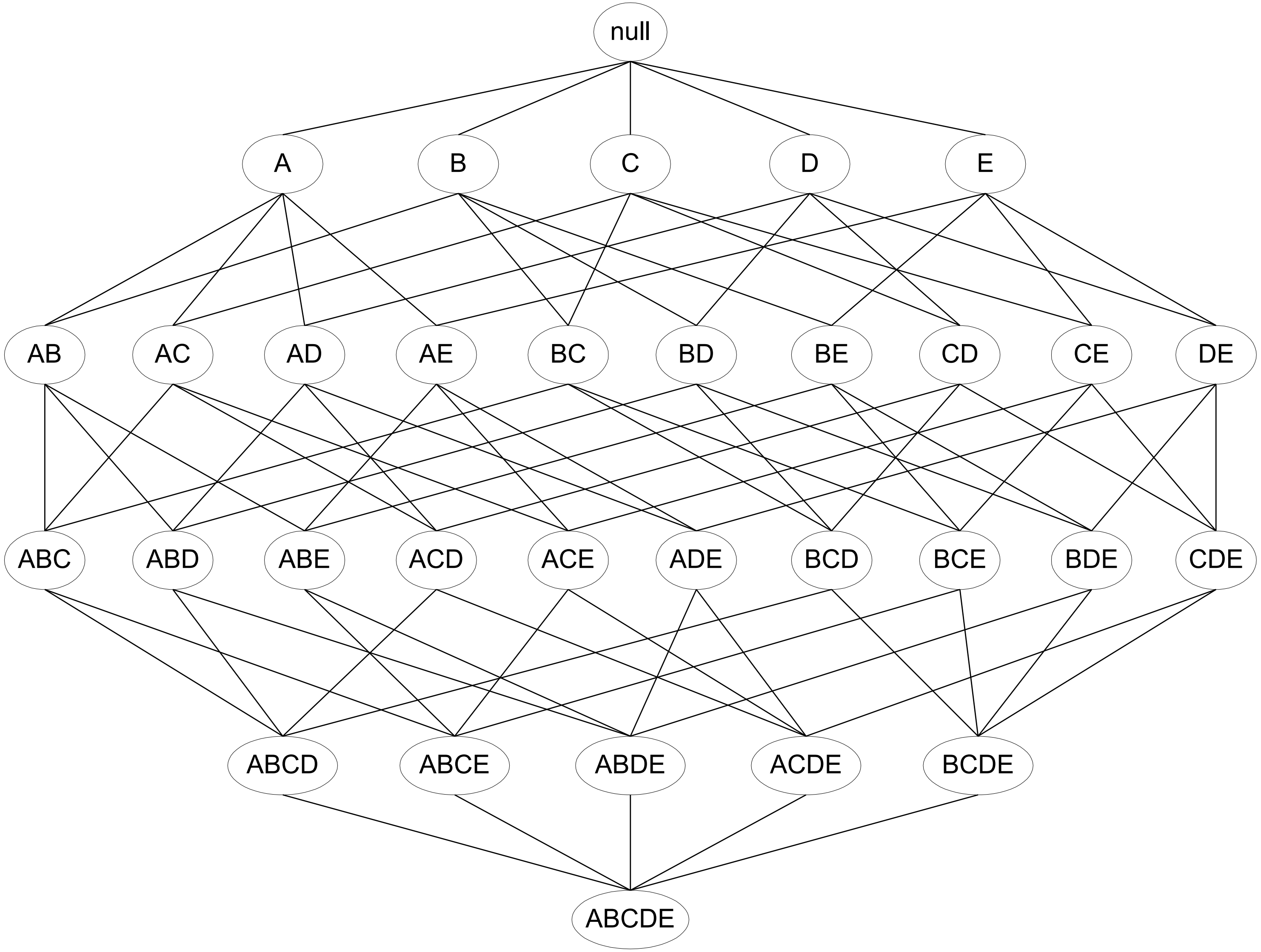
- All the above rules are binary partitions of the same itemset: {Milk, Diaper, Beer}
- Rules originating from the same itemset have identical support but can have different confidence
- Thus, we may decouple the support and confidence requirements

Mining Association Rules

- **Two-step approach:**
  1. Frequent Itemset Generation
     - Generate all itemsets whose support $\geq$ minsup
  2. Rule Generation
     - Generate high confidence rules from each frequent itemset, where each rule is a binary partitioning of a frequent itemset

- **Frequent itemset generation is still computationally expensive**

Frequent Itemset Generation



Given d items, there are 2^d possible candidate itemsets

Frequent Itemset Generation

- **Brute-force approach:**
  - Each itemset in the lattice is a candidate frequent itemset
  - Count the support of each candidate by scanning the database

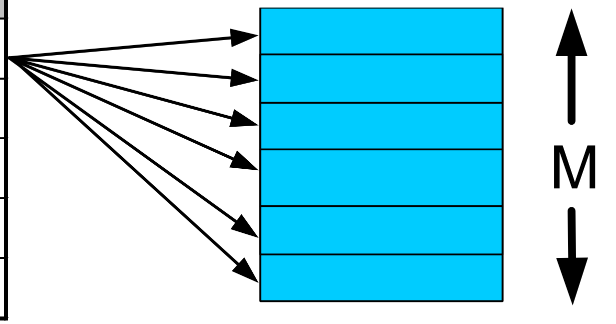
Transactions

TID	*Items*
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

N

w

List of Candidates



  - Match each transaction against every candidate
  - Complexity ~ O(NMw) => Expensive since $M = 2^d$!!!

Frequent Itemset Generation Strategies

- **Reduce the number of candidates (M)**
  - Complete search: $M=2^d$
  - Use pruning techniques to reduce M
- **Reduce the number of transactions (N)**
  - Reduce size of N as the size of itemset increases
  - Used by DHP and vertical-based mining algorithms
- **Reduce the number of comparisons (NM)**
  - Use efficient data structures to store the candidates or transactions
  - No need to match every candidate against every transaction

Reducing Number of Candidates

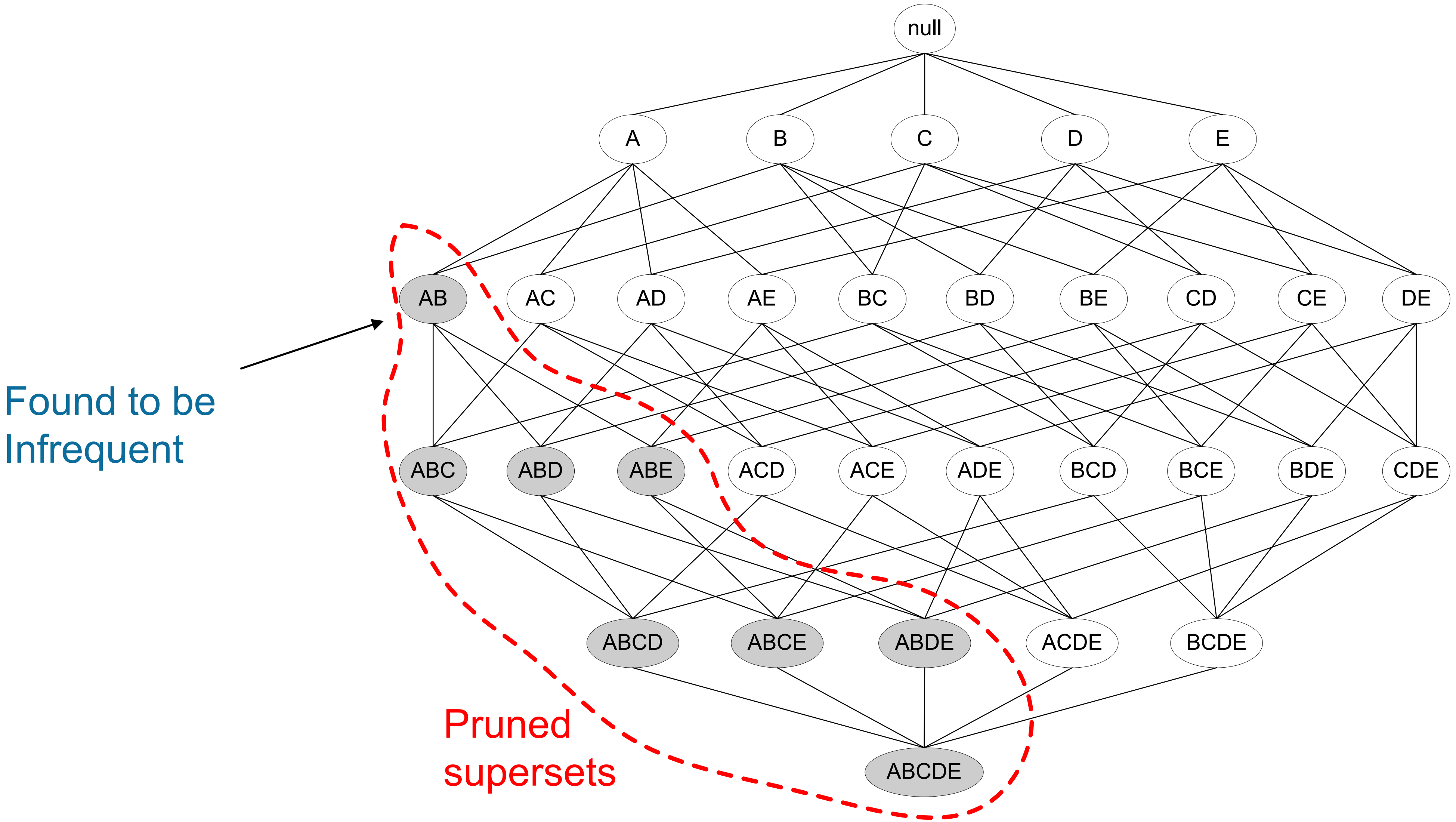
- **Apriori principle:**
  - If an itemset is frequent, then all of its subsets must also be frequent

- **Apriori principle holds due to the following property of the support measure:**

$$\forall X, Y : (X \subseteq Y) \Rightarrow s(X) \geq s(Y)$$

  - Support of an itemset never exceeds the support of its subsets
  - This is known as the anti-monotone property of support

Illustrating Apriori Principle



Illustrating Apriori Principle

TID	Items
1	Bread, Milk
2	Beer, Bread, Diaper, Eggs
3	Beer, Coke, Diaper, Milk
4	Beer, Bread, Diaper, Milk
5	Bread, Coke, Diaper, Milk

Minimum Support = 3

If every subset is considered,

$^6C_1 + {}^6C_2 + {}^6C_3$

$6 + 15 + 20 = 41$

With support-based pruning,

$6 + 6 + 4 = 16$

Items (1-itemsets)

Item	Count
Bread	4
Coke	2
Milk	4
Beer	3
Diaper	4
Eggs	1

Illustrating Apriori Principle

TID	*Items*
1	Bread, Milk
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Items (1-itemsets)

Item	Count
Bread	**4**
Coke	**2**
Milk	**4**
Beer	**3**
Diaper	**4**
Eggs	**1**

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Illustrating Apriori Principle

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3	Beer, Coke, Diaper, Milk
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5	Bread, Coke, Diaper, Milk

Items (1-itemsets)

Item	Count
Bread	4
Coke	2
Milk	4
Beer	3
Diaper	4
Eggs	1

Minimum Support = 3

Itemset
{Bread,Milk}
{Bread, Beer }
{Bread,Diaper}
{Beer, Milk}
{Diaper, Milk}
{Beer,Diaper}

Pairs (2-itemsets)

(No need to generate candidates involving Coke or Eggs)

If every subset is considered,

$^6C_1 + {}^6C_2 + {}^6C_3$

6 + 15 + 20 = 41

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Illustrating Apriori Principle

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Items (1-itemsets)

Item	Count
Bread	4
Coke	2
Milk	4
Beer	3
Diaper	4
Eggs	1

Minimum Support = 3

Pairs (2-itemsets)

(No need to generate candidates involving Coke or Eggs)

Itemset	Count
{Bread,Milk}	3
{Beer, Bread}	2
{Bread,Diaper}	3
{Beer,Milk}	2
{Diaper,Milk}	3
{Beer,Diaper}	3

If every subset is considered,

$^6C_1 + {}^6C_2 + {}^6C_3$

$6 + 15 + 20 = 41$

With support-based pruning,

$6 + 6 + 4 = 16$

Illustrating Apriori Principle

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Items (1-itemsets)

Item	Count
Bread	4
Coke	2
Milk	4
Beer	3
Diaper	4
Eggs	1

Minimum Support = 3

Pairs (2-itemsets)

(No need to generate candidates involving Coke or Eggs)

Itemset	Count
{Bread,Milk}	3
{Beer, Bread}	2
{Bread,Diaper}	3
{Beer,Milk}	2
{Diaper,Milk}	3
{Beer,Diaper}	3

Triplets (3-itemsets)

Itemset
{ Beer, Diaper, Milk}
{ Beer,Bread,Diaper}
{Bread,Diaper,Milk}
{ Beer, Bread, Milk}

If every subset is considered,

$^6C_1 + {}^6C_2 + {}^6C_3$

$6 + 15 + 20 = 41$

With support-based pruning,

$6 + 6 + 4 = 16$

Illustrating Apriori Principle

TID	Items
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Items (1-itemsets)

Item	Count
Bread	4
Coke	2
Milk	4
Beer	3
Diaper	4
Eggs	1

Minimum Support = 3

Pairs (2-itemsets)

(No need to generate candidates involving Coke or Eggs)

Itemset	Count
{Bread,Milk}	3
{Beer, Bread}	2
{Bread,Diaper}	3
{Beer,Milk}	2
{Diaper,Milk}	3
{Beer,Diaper}	3

Triplets (3-itemsets)

Itemset	Count
{ Beer, Diaper, Milk}	2
{ Beer,Bread, Diaper}	2
{Bread, Diaper, Milk}	2
{Beer, Bread, Milk}	1

If every subset is considered,

$^6C_1 + {}^6C_2 + {}^6C_3$

6 + 15 + 20 = 41

With support-based pruning,

6 + 6 + 4 = 16

Illustrating Apriori Principle

TID	Items
1	Bread, Milk
2	Beer, Bread, Diaper, Eggs
3	Beer, Coke, Diaper, Milk
4	Beer, Bread, Diaper, Milk
5	Bread, Coke, Diaper, Milk

Items (1-itemsets)

Item	Count
Bread	4
Coke	2
Milk	4
Beer	3
Diaper	4
Eggs	1

Minimum Support = 3

If every subset is considered,
$^6C_1 + {}^6C_2 + {}^6C_3$
6 + 15 + 20 = 41
With support-based pruning,
6 + 6 + 4 = 16
6 + 6 + 1 = 13

Pairs (2-itemsets)

(No need to generate candidates involving Coke or Eggs)

Itemset	Count
{Bread,Milk}	3
{Beer, Bread}	2
{Bread,Diaper}	3
{Beer,Milk}	2
{Diaper,Milk}	3
{Beer,Diaper}	3

Triplets (3-itemsets)

Itemset	Count
{ Beer, Diaper, Milk}	2
{ Beer,Bread, Diaper}	2
{Bread, Diaper, Milk}	2
{Beer, Bread, Milk}	1

Apriori Algorithm

- F_k: frequent k-itemsets
- L_k: candidate k-itemsets

- **Algorithm**
  - Let k=1
  - Generate F_1 = {frequent 1-itemsets}
  - Repeat until F_k is empty
    - **Candidate Generation**: Generate L_{k+1} from F_k
    - **Candidate Pruning**: Prune candidate itemsets in L_{k+1} containing subsets of length k that are infrequent
    - **Support Counting**: Count the support of each candidate in L_{k+1} by scanning the DB
    - **Candidate Elimination**: Eliminate candidates in L_{k+1} that are infrequent, leaving only those that are frequent => F_{k+1}

Candidate Generation: Brute-force method

TID	Items
1	Bread, Milk
2	Beer, Bread, Diaper, Eggs
3	Beer, Coke, Diaper, Milk
4	Beer, Bread, Diaper, Milk
5	Bread, Coke, Diaper, Milk

Items

Item
Beer
Bread
Cola
Diapers
Eggs
Milk

→

Candidate Generation

Itemset
{Beer, Bread, Cola}
{Beer, Bread, Diapers}
{Beer, Bread, Eggs}
{Beer, Bread, Milk}
{Beer, Cola, Diapers}
{Beer, Cola, Eggs}
{Beer, Cola, Milk}
{Beer, Diapers, Eggs}
{Beer, Diapers, Milk}
{Beer, Eggs, Milk}
{Bread, Cola, Diapers}
{Bread, Cola, Eggs}
{Bread, Cola, Milk}
{Bread, Diapers, Eggs}
{Bread, Diapers, Milk}
{Bread, Eggs, Milk}
{Cola, Diapers, Eggs}
{Cola, Diapers, Milk}
{Cola, Eggs, Milk}
{Diapers, Eggs, Milk}

→

Candidate Pruning

Itemset
{Bread, Diapers, Milk}

Frequent 2-itemset

Itemset
{Beer, Diapers}
{Bread, Diapers}
{Bread, Milk}
{Diapers, Milk}

Figure 5.6. A brute-force method for generating candidate 3-itemsets.

Candidate Generation: Merge Fk-1 and F1 itemsets

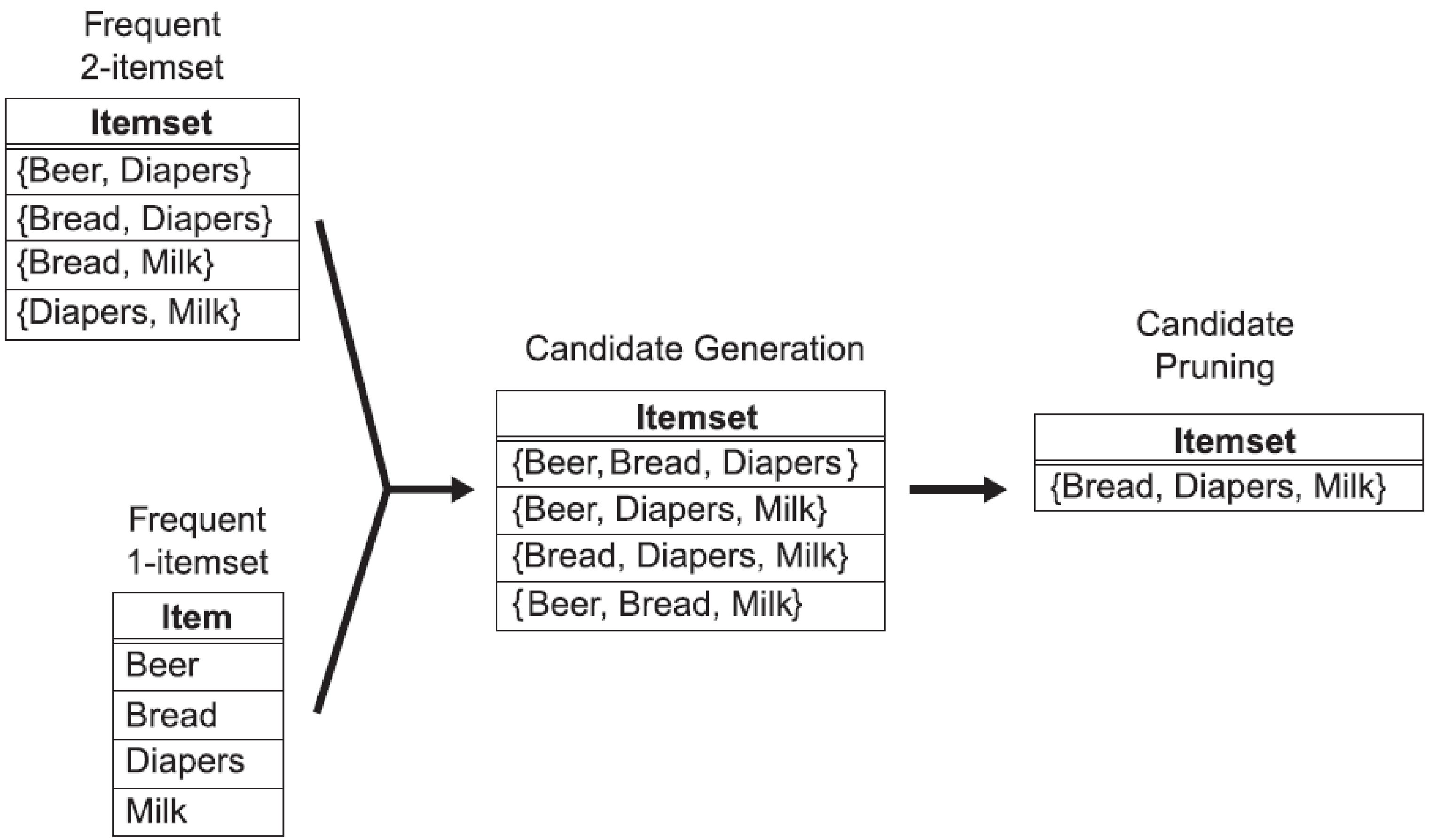


Figure 5.7. Generating and pruning candidate k-itemsets by merging a frequent $(k-1)$-itemset with a frequent item. Note that some of the candidates are unnecessary because their subsets are infrequent.

Candidate Generation: F_{k-1} x F_{k-1} Method

- **Merge two frequent (k-1)-itemsets if their first (k-2) items are identical**

- **F_3 = {ABC,ABD,ABE,ACD,BCD,BDE,CDE}**
  - Merge(**AB**C, **AB**D) = **AB**CD
  - Merge(**AB**C, **AB**E) = **AB**CE
  - Merge(**AB**D, **AB**E) = **AB**DE

  - Do not merge(**A**BD,**A**CD) because they share only prefix of length 1 instead of length 2

Candidate Pruning

- **Let F_3 = {ABC,ABD,ABE,ACD,BCD,BDE,CDE} be the set of frequent 3-itemsets**
- **L_4 = {ABCD,ABCE,ABDE} is the set of candidate 4-itemsets generated (from previous slide)**
- **Candidate pruning**
  - Prune ABCE because ACE and BCE are infrequent
  - Prune ABDE because ADE is infrequent
- **After candidate pruning: L_4 = {ABCD}**

Candidate Generation: F_{k-1} x F_{k-1} Method

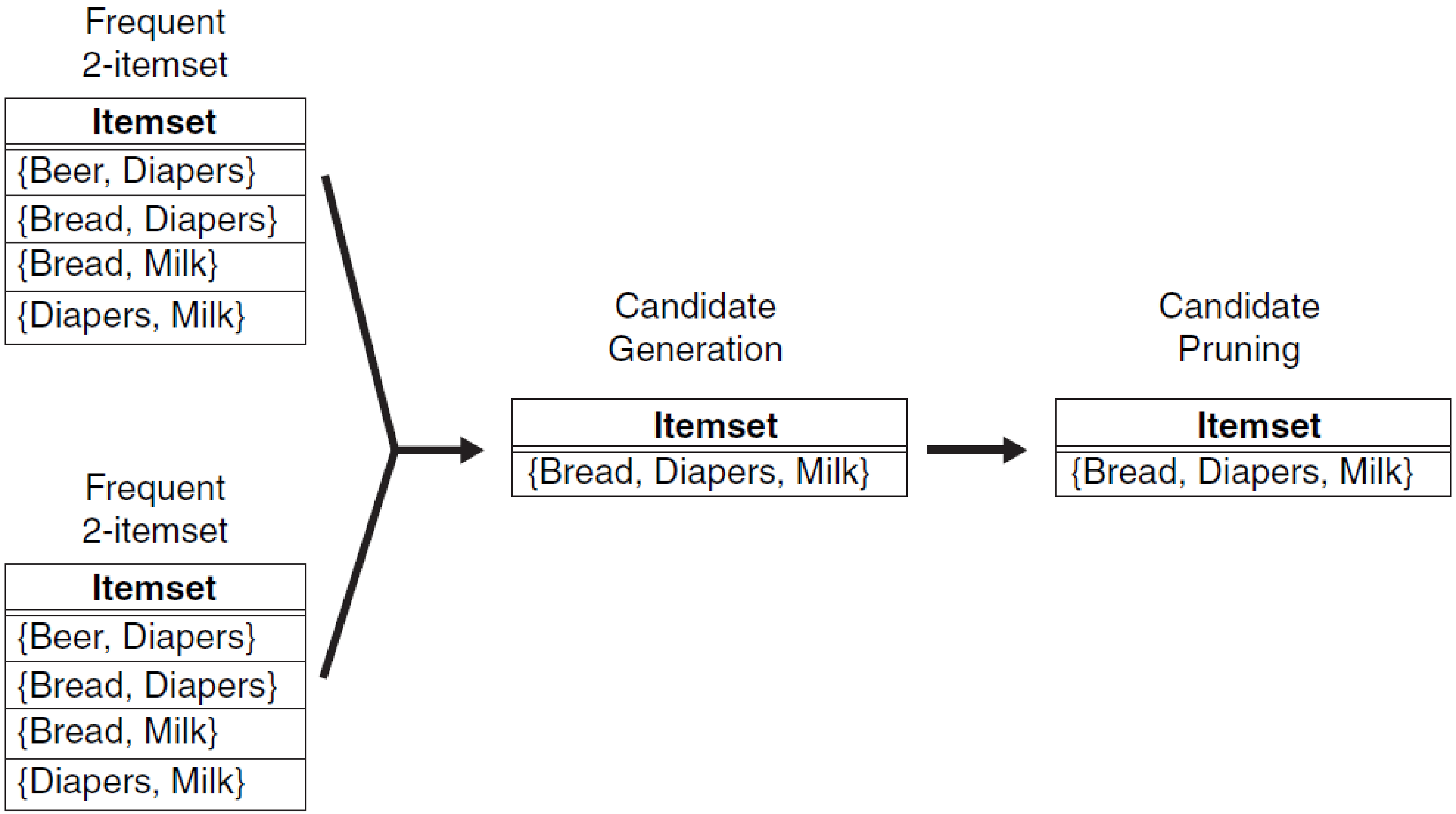


Figure 5.8. Generating and pruning candidate k-itemsets by merging pairs of frequent $(k-1)$-itemsets.

Illustrating Apriori Principle

TID	Items
1	Bread, Milk
2	Beer, Bread, Diaper, Eggs
3	Beer, Coke, Diaper, Milk
4	Beer, Bread, Diaper, Milk
5	Bread, Coke, Diaper, Milk

Items (1-itemsets)

Item	Count
Bread	**4**
Coke	**2**
Milk	**4**
Beer	**3**
Diaper	**4**
Eggs	**1**

Minimum Support = 3

Pairs (2-itemsets)

(No need to generate candidates involving Coke or Eggs)

Itemset	Count
{Bread,Milk}	**3**
{Beer, Bread}	**2**
{Bread,Diaper}	**3**
{Beer,Milk}	**2**
{Diaper,Milk}	**3**
{Beer,Diaper}	**3**

Triplets (3-itemsets)

Itemset	Count
{Bread, Diaper, Milk}	**2**

If every subset is considered,
$^6C_1 + {}^6C_2 + {}^6C_3$
6 + 15 + 20 = 41
With support-based pruning,
6 + 6 + 4 = 16

Use of $F_{k-1}xF_{k-1}$ method for candidate generation results in only one 3-itemset. This is eliminated after the support counting step.

Alternate F_{k-1} x F_{k-1} Method

- **Merge two frequent (k-1)-itemsets if the last (k-2) items of the first one is identical to the first (k-2) items of the second.**

- **F_3 = {ABC,ABD,ABE,ACD,BCD,BDE,CDE}**
  - Merge(A**BC**, **BC**D) = A**BC**D
  - Merge(A**BD**, **BD**E) = A**BD**E
  - Merge(A**CD**, **CD**E) = A**CD**E
  - Merge(B**CD**, **CD**E) = B**CD**E

Candidate Pruning for Alternate F_{k-1} x F_{k-1} Method

- **Let F_3 = {ABC,ABD,ABE,ACD,BCD,BDE,CDE} be the set of frequent 3-itemsets**

- **L_4 = {ABCD,ABDE,ACDE,BCDE} is the set of candidate 4-itemsets generated (from previous slide)**
- **Candidate pruning**
  - Prune ABDE because ADE is infrequent
  - Prune ACDE because ACE and ADE are infrequent
  - Prune BCDE because BCE
- **After candidate pruning: L_4 = {ABCD}**

Rule Generation

- **Given a frequent itemset L, find all non-empty subsets $f \subset L$ such that $f \rightarrow L - f$ satisfies the minimum confidence requirement**
  - If {A,B,C,D} is a frequent itemset, candidate rules:

ABC →D,	ABD →C,	ACD →B,	BCD →A,
A →BCD,	B →ACD,	C →ABD,	D →ABC
AB →CD,	AC → BD,	AD → BC,	BC →AD,
BD →AC,	CD →AB,		

- **If $|L| = k$, then there are $2^k - 2$ candidate association rules (ignoring $L \rightarrow \emptyset$ and $\emptyset \rightarrow L$)**

Rule Generation

- **In general, confidence does not have an anti-monotone property**

  $c(ABC \rightarrow D)$ can be larger or smaller than $c(AB \rightarrow D)$

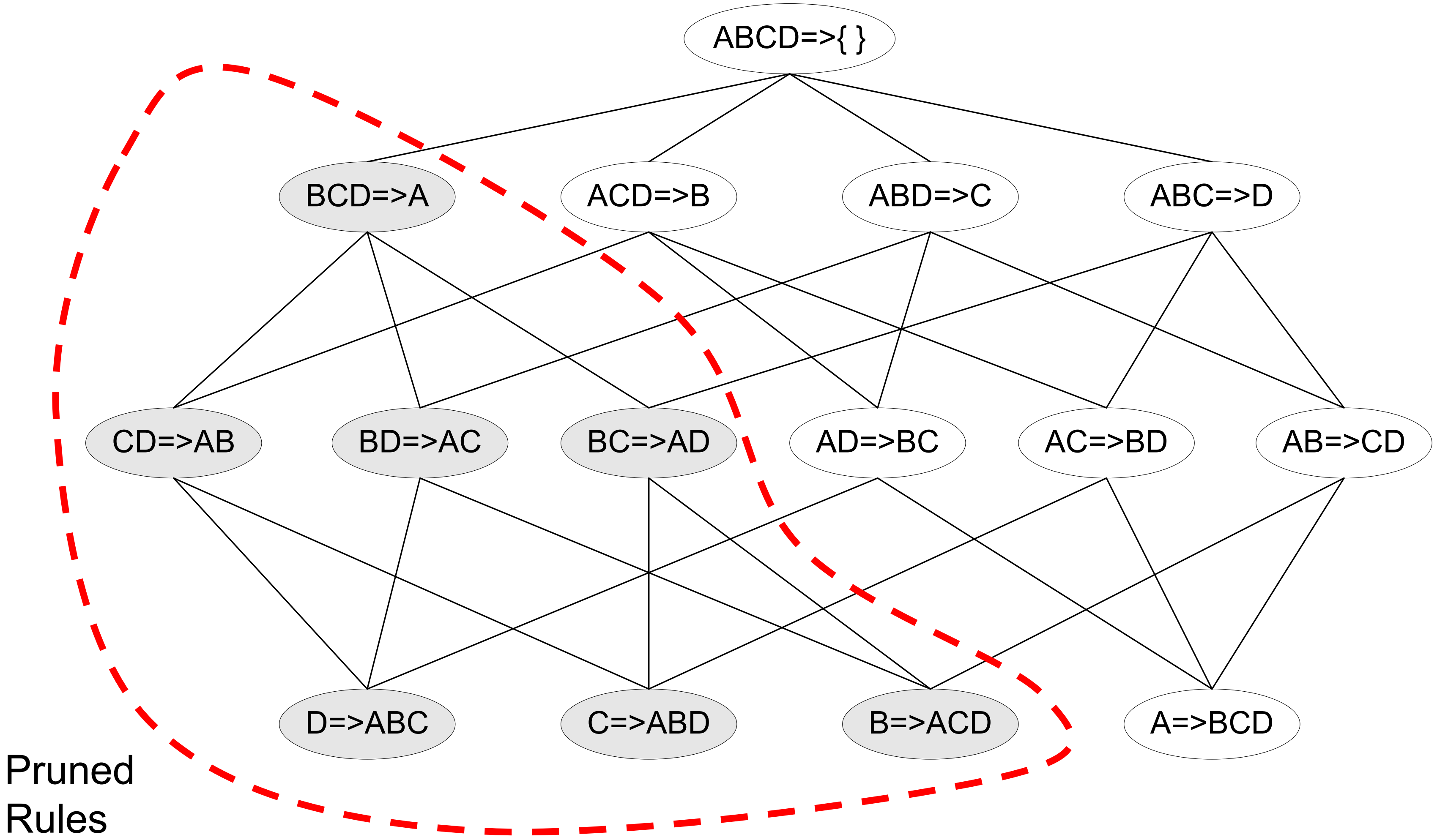
- **But confidence of rules generated from the same itemset has an anti-monotone property**
  - E.g., Suppose {A,B,C,D} is a frequent 4-itemset:

$$c(ABC \rightarrow D) \geq c(AB \rightarrow CD) \geq c(A \rightarrow BCD)$$

  - Confidence is anti-monotone w.r.t. number of items on the RHS of the rule

Rule Generation for Apriori Algorithm

Lattice of rules



Q&A