Derivation of maximum stable CFL for aeroacoustic and scalar advection test cases

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Repository:
https://gitlab.com/sloede/dg_calc_dt.git

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Contents

1 Introduction

In this report, the empirical derivation of the maximum stable CFL numbers for aeroacoustic test cases in the ZFS Discontinuous Galerkin block is described and the results are exposed. The CFL numbers were obtained using a routine based on the bisection method which receives the CFL interval, polynomial degree, velocity components, and maximum error threshold as inputs, and returns the maximum CFL supported for the test case.
The results were used to construct a table filled with the maximum stable CFL numbers for a range of polynomial degrees varying from 0 to 15. The correspondent CFL value is then introduced in the calculation of the dt_{stable} so the user can specity a CFL number equal to 1.0 in virtually all cases and obtain an optimazed stable solution.

2 Formulas

- $dt = \frac{1}{2p+1} * \frac{2}{invJac * \lambda_{max}}$: Time step, where p is the polynomial degree, $invJac$ is the inverse Jacobian, and λ_{max} was calculated using three different methods:

Method	Aeroacosutics	Scalar advection
1(2D)	$\lambda_{max} = u_0 + v_0 + 2$	$\lambda_{max} = u_0 + v_0$
1(3D)	$\lambda_{max} = u_0 + v_0 + w_0 + 3$	$\lambda_{max} = u_0 + v_0 + w_0$
2	$\lambda_{max} = max(u_0, v_0) + 1$	$\lambda_{max} = max(u_0, v_0)$
3	$\lambda_{max} = \sqrt{u_0^2 + v_0^2} + 1$	$\lambda_{max} = \sqrt{u_0^2 + v_0^2}$

- $CFL_{factor} = dt * (2p+1) * \frac{invJac * \lambda_{max}}{2}$: Maximum stable CFL number, obtained after running the bisection routine.

- $dt_{stable} = CFL_{factor} * \frac{1}{2p+1} * \frac{2}{invJac * \lambda_{max}}$: Maximum stable time step, obtained by introducing the maximum stable CFL number into the time step calculation.

3 Results

The maximum CFL number for a stable solution was obtained for each test case in order to determine the largest stable time step dt_{stable}. To acheive that, a wide range of velocity components and polynomial degrees was adopted. In addition, the two integration schemes in the DG block were used, the Gauss and Gauss-Lobatto schemes.

3.1 Acoustic perturbation

The evaluation of the different methods for the λ_{max} calculation showed that the method 1 generates the best results among the three for the aeroacoustic test cases. Therefore, this method was used to calculate dt.
Two significantly different setups for 2D and two more for 3D were selected to perform the evaluations, which were done using the branch revision r17157.
The results have shown that the grid refinement can have a significant dependency on the results for the first polynomial degrees whereas for the higher polynomial degrees this dependency is not so significant. For this reason, the maximum CFL numbers were obtained increasing the grid refinement of the test cases in one level for polynomial degrees up to 5. In addition, it was observed that for aeroacoustic test cases the polynomial degree and the mean velocity components have an impact on the CFL and that the polynomial degree is dominant over the velocity. Concerning the integration schemes for the DG block, it was observed that the maximum stable CFL numbers are higher for the Gauss-Lobatto integration scheme.

Acoustic perturbation test cases:

- 2D pressure pulse for wall reflection: initialCondition=201, t=30s, name=2D_square_acousticperturb_wall
- 2D convergence test with sourceTerm: initialCondition=5, t=1.5s, name=2D_square_acousticperturb_convergence_np1
- 3D pressure pulse for wall reflection: initialCondition=115, t=10s, name=3D_cube_acousticperturb_wall
- 3D convergence test with sourceTerm: initialCondition=5, t=0.4s, name=3D_cube_acousticperturb_convergence_np8

The plots depicted below in Fig. 1 and 2 represents the minimum CFL number for every test case and polynomial degree. The CFL curves are quite similar in all four cases for polynomial degrees above 2 using both integration schemes.

Table 1 was then implemented in revision r18571. It takes into account the test case spatial dimension and the integration scheme, where the CFL factor is the minimum stable CFL among all velocity components for each polynomial degree considered.

p	2D Gauss	3D Gauss	2D Gauss-Lobatto	3D Gauss-Lobatto
0	2.497	2.976	5.916	7.445
1	2.390	2.574	5.916	7.445
2	1.978	2.055	3.868	4.113
3	1.652	1.697	2.753	2.825
4	1.413	1.444	2.116	2.176
5	1.230	1.251	1.722	1.760
6	1.097	1.123	1.458	1.502
7	0.984	1.003	1.259	1.293
8	0.890	0.904	1.107	1.131
9	0.812	0.827	0.990	1.008
10	0.747	0.758	0.895	0.906
11	0.691	0.700	0.815	0.823
12	0.643	0.650	0.750	0.756
13	0.602	0.607	0.692	0.698
14	0.565	0.570	0.644	0.649
15	0.533	0.537	0.602	0.606

Table 1: CFL factor values for acoustic pertubation problems.

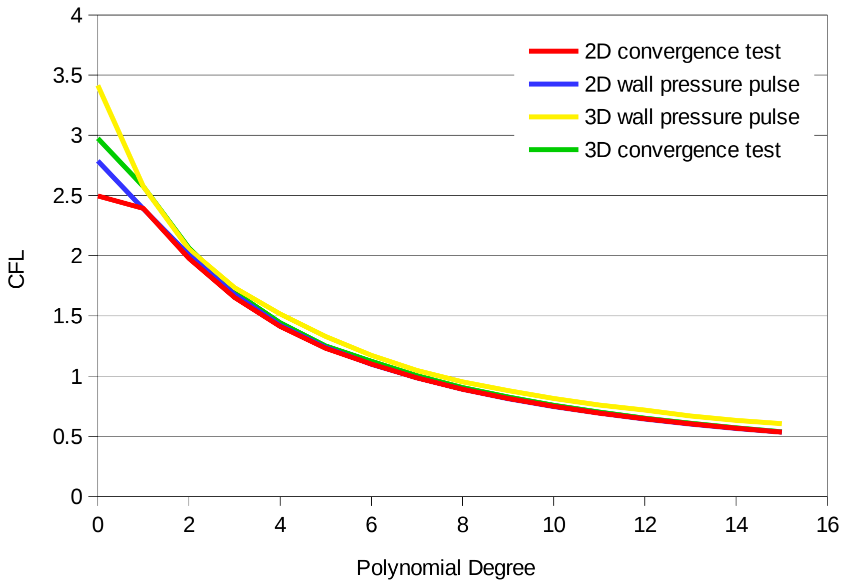

Figure 1: Maximum stable CFL number with Gauss scheme.

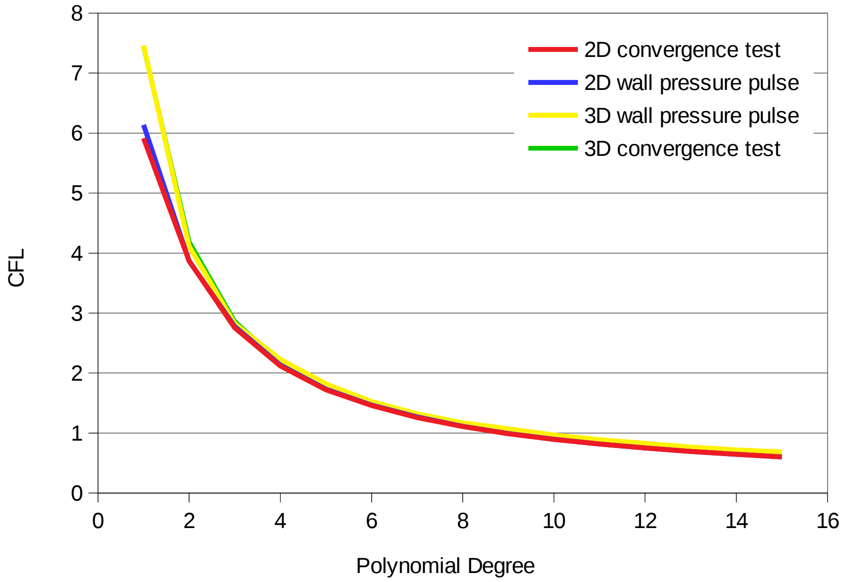

Figure 2: Maximum stable CFL number with Gauss-Lobatto scheme.

3.2 Scalar advection

For the scalar advection test cases method 1, used for the λ_{max} calculation, also showed the best results, and thus was used for the *dt* calculation.
Here, since there are less scalar advection test cases and those generally take less time to be evaluated, all scalar advection problems where used to perform the CFL evaluations, which were also done using the branch revision r17157. The test cases listed below represents only those that contribute to the final CFL table, i.e., those with the minimum CFL for each polynomial degree. The same procedure described in the previous section for the aeroacoustic test cases was used for the scalar advection. The difference on the results lies in the fact that the scalar advection cases have only a dependency on the polynomial degree used, i.e., the velocity has no impact on the maximum stable CFL number. In addition, it was again observed that the maximum stable CFL numbers are higher for the Gauss-Lobatto integration scheme.

Scalar advection test cases:

- 2D convergence test: initialCondition=1, t=3s, name=2D_square_linearscalaradv_convergence_np8
- 2D linear x-direction: initialCondition=4, t=3s, name=2D_square_linearscalaradv_linear_pref1_np1

- 2D convergence test: initialCondition=1, t=3s, name=2D_square_linearscalaradv_gausslobatto_np1
- 3D convergence test: initialCondition=1, t=4s, name=3D_cube_linearscalaradv_convergence_np8
- 3D convergence test: initialCondition=1, t=2s, name=3D_cube_linearscalaradv_convergence_np8_lshape
- 3D convergence test: initialCondition=1, t=4s, name=3D_cube_linearscalaradv_gausslobatto_np1

Table 2, also implemented in revision r18571, represents the CFL numbers obtained for the scalar advection problems using the same methodology as for the acoustic perturbation.

p	2D Gauss	3D Gauss	2D Gauss-Lobatto	3D Gauss-Lobatto
0	2.448	2.891	5.277	5.848
1	2.420	2.716	5.277	5.848
2	2.171	2.454	3.908	3.948
3	1.834	2.081	3.056	3.621
4	1.652	1.877	2.852	3.244
5	1.497	1.702	2.486	2.897
6	1.369	1.523	2.232	2.618
7	1.246	1.382	2.036	2.298
8	1.151	1.281	1.856	2.114
9	1.071	1.203	1.748	1.972
10	1.015	1.132	1.624	1.834
11	0.959	1.060	1.542	1.741
12	0.904	1.002	1.455	1.669
13	0.856	0.960	1.365	1.539
14	0.814	0.912	1.307	1.470
15	0.783	0.868	1.243	1.406

Table 2: CFL factor values for linear scalar advection problems.

3.3 Acoustic perturbation and scalar advection comparison

Finally, the CFL factors found for both acoustic peturbation and linear scalar advection problems are plotted in Fig. 3, in this plot one can see that the linear advection problems have higher CFL factors for polynomial degrees from 3 and above while maintaining the same tendency as the respective CFL values for the acoustic pertubation problems.

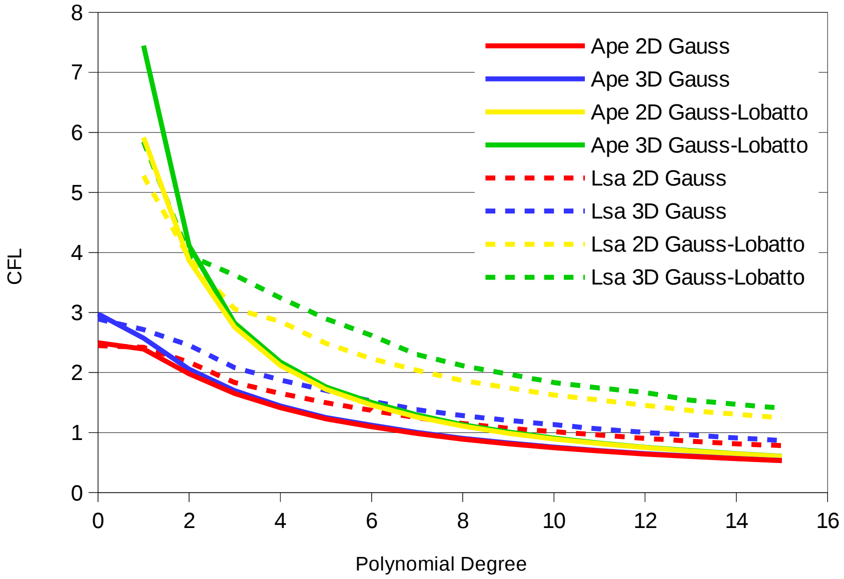

Figure 3: Maximum stable CFL numbers for acoustic perturbation (Ape) and scalar advection problems (Lsa).