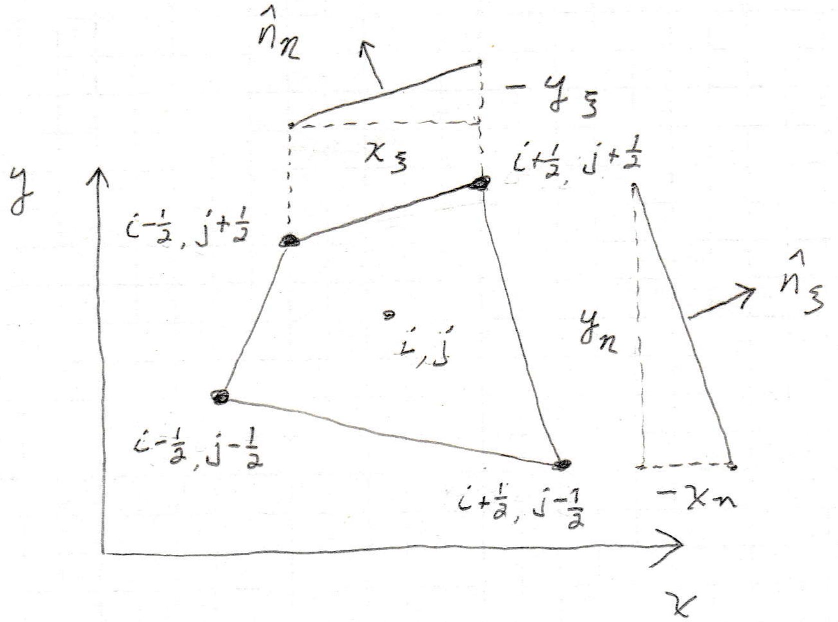
2d Finite-Volume Cell Volume and Projected Face Areas

1 Cell volumes and projected cell-face areas (2-D F-V)



$$\left.\begin{aligned}
y_{\eta\, i+\frac{1}{2}, j} &= y_{i+\frac{1}{2}\, j+\frac{1}{2}} - y_{i+\frac{1}{2}\, j-\frac{1}{2}} \\
x_{\eta\, i+\frac{1}{2}\, j} &= x_{i+\frac{1}{2}\, j+\frac{1}{2}} - x_{i+\frac{1}{2}\, j-\frac{1}{2}} \\
y_{\xi\, i\, j+\frac{1}{2}} &= y_{i+\frac{1}{2}\, j+\frac{1}{2}} - y_{i-\frac{1}{2}\, j+\frac{1}{2}} \\
x_{\xi\, i\, j+\frac{1}{2}} &= x_{i+\frac{1}{2}\, j+\frac{1}{2}} - x_{i-\frac{1}{2}\, j+\frac{1}{2}}
\end{aligned}\right\} \text{Inverse Metrics (2D)} \qquad (1)$$

→ These are the projected cell-face areas

Projected cell-face areas:

$$S_{\xi_x} = y_\eta \qquad S_{\eta_x} = -y_\xi$$
$$S_{\xi_y} = -x_\eta \qquad S_{\eta_y} = x_\xi \tag{2}$$

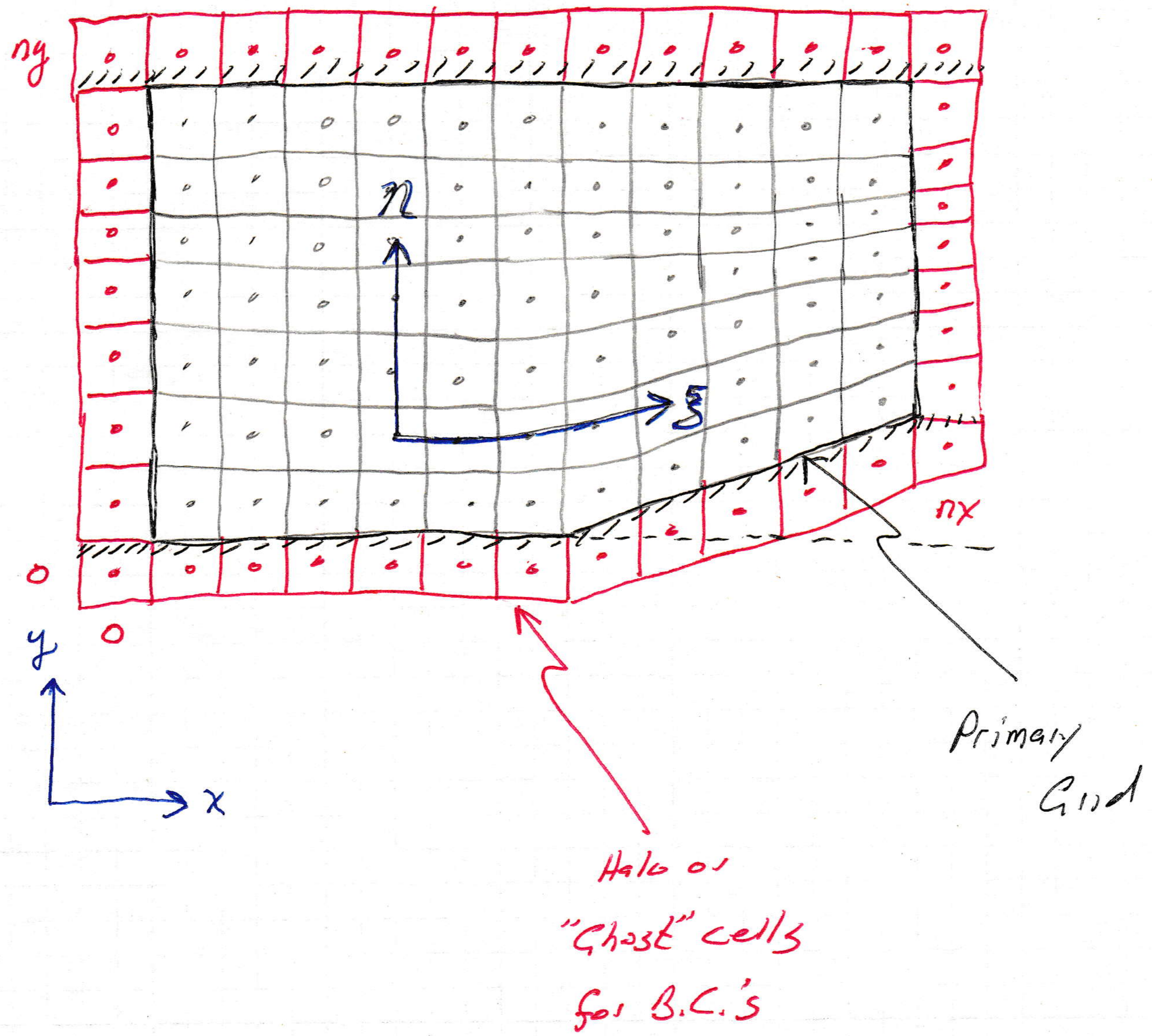
Cell-face areas:

$$S_\xi = \left(S_{\xi_x}^2 + S_{\xi_y}^2\right)^{\frac{1}{2}}$$
$$S_\eta = \left(S_{\eta_x}^2 + S_{\eta_y}^2\right)^{\frac{1}{2}} \tag{3}$$

2D cell-"volumes":

$$\Delta V = \frac{1}{2}\left|\begin{array}{l}(x_{i+\frac{1}{2},j+\frac{1}{2}} - x_{i-\frac{1}{2},j-\frac{1}{2}})(y_{i+\frac{1}{2}\,j+\frac{1}{2}} - y_{i-\frac{1}{2}\,j-\frac{1}{2}}) \\ (x_{i-\frac{1}{2},j+\frac{1}{2}} - x_{i+\frac{1}{2}\,j-\frac{1}{2}})(y_{i-\frac{1}{2}\,j+\frac{1}{2}} - y_{i+\frac{1}{2}\,j-\frac{1}{2}})\end{array}\right| \tag{4}$$

Assuming the increments $\Delta\xi = \Delta\eta = 1$ in generalized coordinates, these quantities describe the transformation of a quadrilateral cell with a volume ΔV in $x-y$ coordinates to a square cell with a volume of unity in $\xi-\eta$ coordinates


ny
nx
0
0
η
ξ
y
x
Primary Grid
Halo or "Ghost" cells for B.C.'s