AE 4803 – Advanced Aircraft Propulsion

Preliminary Component Design of a Turbojet Engine

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INTRODUCTION

This report details the requirements, methodology, and results of the preliminary component design and off-design evaluation of an aircraft turbojet engine with performance optimized for takeoff and ground roll. The performance requirements for the engine, as stipulated by the high-level cycle design, include a single spool, maximum thrust of 40 kN, air mass flow rate of 46 kg/s, a compressor stagnation pressure ratio of 11, and a maximum turbine inlet stagnation temperature of 1450 K, and takeoff and ground roll at sea level, standard atmospheric conditions. The off-design engine performance is to be predicted at cruise conditions, with an altitude of 35 kft, turbine inlet stagnation temperature of 1220 K, and flight Mach number of 0.80.

APPROACH

Constraints and Assumptions

Prior to beginning the design phase, the team analyzed the requirements and organized a set of assumptions and constraints using information derived from the problem specification and background knowledge to help guide the design of the engine. The following are additional requirements and assumptions for each engine component. Numerical constraints and constants are presented in Table 1.

The compressor maintains a constant axial velocity throughout all stages after the inlet guide vane (IGV). The design of the compressor shall use the constant pitchline blade radius approach, meaning a constant, mean blade radius is to be used for all associated calculations. Secondary stagnation pressure losses will not be explicitly predicted, since such predictions require higher fidelity computational methods, but will still be accounted for by increasing the total pressure loss parameter resulting from cascade analysis by a factor of 2.3. The maximum stage pressure ratio is limited to 1.7 to reflect reasonable values for modern compressors.

The combustor shall be a dump-type combustor containing an aerodynamic prediffuser. 60% of the air entering the combustor will enter the primary zone.

The turbine shall not possess any form of cooling, and thus the maximum stagnation temperature at the turbine inlet is limited to 1450 K. The turbine shall feature a constant blade tip radius. The combusted fuel-air gas mixture in the turbine is assumed to be a calorically perfect gas.

The engine nozzle shall be converging with a fixed exit area.

Table 1. Additional requirements and constraints by component.

Diffuser			
r_d	0.985		
Compressor			
$M_{inlet, rel}$	0.75	**$D_{f,max}$**	0.55
Exit Swirl	0˚	**Taper Ratio**	0.8
$(t/b)_{max}$	10%		
Combustor			
$\boldsymbol{\eta_b}$	0.99	$\boldsymbol{\Delta h_R}$	43.8 MJ/kg
$\boldsymbol{\tau_{residence}}$	3.5 ms	**PLF_{cold}**	5
Turbine			
$\boldsymbol{\eta_{shaft}}$	0.995	**MW_{gas}**	28.7 g/mol
Exit Swirl	0˚	$\boldsymbol{\gamma}$	1.28
$(t/b)_{max}$	20%	**Taper Ratio**	0.5
Nozzle			
$\boldsymbol{\eta_{n,adiab}}$	0.95	$\boldsymbol{\gamma}$	1.35

For off-design analysis, it is assumed that the compressor and turbine have the same polytropic efficiency as the design point, and that the combustor experiences the same stagnation pressure loss as the design point.

All dimensional units were in SI units.

Methodology

The design of all engine components involved the use of Python, and optimization was performed using the Python based open-source optimization library, OpenMDAO. The following components were considered for the design of the single spool turbojet engine: inlet diffuser, inlet guide vane, compressor, combustor, turbine, and nozzle. The goal of the design is to meet all the performance requirements while achieving the maximum overall efficiency and thrust specific fuel consumption.

The team approached the design process by beginning at the component level and gradually integrating each component into an overall engine optimization program. Initially, for each component, a script was written that calculated its performance, flow angles, flow conditions, stresses, and geometric parameters given a set of inlet conditions. The full set of equations used to calculate these values is supplied in the Appendices. Each script was validated by inputting values from a test case and ensuring the script outputs matched the test outputs. The design process then diverges for each component; the following explains the methodology for each component.

For the compressor, the initial script calculated the performance of a single stage. After validation, the script was adapted into an optimization code maximizing stage efficiency. The independent design variables chosen were the compressor inlet flow angle α_1, ψ, ϕ, r_m, σ for the rotor and stator, and axial velocity c_z. The single stage optimization code was then adapted into a multistage compressor optimizer. The first stage of the compressor searches for the optimal set design variable values and c_z, and the exit conditions α_3, T_{o3}, and P_{o3} comprise the inlet conditions into the subsequent stage. This is repeated for the desired number of stages. The same foundational single stage optimizer code is called for each stage of the multistage optimizer. The search for the optimal number of stages is not included in the optimizer to reduce code complexity— the optimal number of stages was found by running a loop through a range of possible number of stages and searching for the result with the highest efficiency. Equation 1 displays the equation for overall compressor efficiency.

$$\eta_{compressor} = \frac{Pr_c^{\,(\gamma-1)/\gamma} - 1}{\frac{T_{o3}}{T_{o2}} - 1} \tag{1}$$

The design of the turbine followed a similar structure. A single stage turbine optimizer, with independent design variables r_m, *Re* at the nozzle and rotor opening, and $°R$, was adapted into a multistage turbine optimizer. The target turbine power generation was set equal to the amount of work required by the compressor to achieve the compressor pressure ratio, accounting for the shaft efficiency. The turbine was optimized to maximize overall efficiency. The turbine overall efficiency is defined below in Equation 2.

$$\eta_{turbine} = \frac{1 - \frac{T_{o5}}{T_{o4}}}{1 - Pr_t^{\,(\gamma-1)/\gamma}} \tag{2}$$

The combustor design incorporated all the requirements indicated in the Constrains and Assumptions subsection into an optimization code that found an ideal combustor geometry with the minimum stagnation pressure loss. The independent design variables included an array of dimensions related to the prediffuser, prediffuser angle, the dump gap ratio, fuel-to-air ratio, and pre-combustion Mach number.

The compressor, combustor, and turbine codes are next integrated into an overall engine optimizer framework. The outputs of the compressor are linked to the combustor, and the outputs of the combustor are linked to the turbine, creating a system with cyclic dependencies. Each component is defined as a ‘cycle,’ which OpenMDAO optimizes independently with each iteration and searches for a solution among them through the Non-Linear Block Gauss-Seidel method. Through this system level optimization program with automatic feedback loops, a feasible design is identified with compatible parameters among all the components with minimal need for manual verification among various independently designed codes.

The inlet guide vane, diffuser, and nozzle were designed statically, entirely in context of the compressor-combustor-turbine system and did not have an independent optimizer. For instance, the system optimizer had specified the optimal inlet compressor conditions. Therefore, given the desired flight condition, the inlet and exit geometry of the diffuser can be determined based on how much the flow must be decelerated.

RESULTS

This section provides comprehensive data detailing the geometric and performance parameters that define each component and the overall engine.

General Engine Values

The general design values for the engine are given in Table 2. The engine exceeds the 40 kN takeoff thrust requirement by 1.2 kN and operates at approximately 6620 RPM with a mass flow rate of 46 kg/s.

Table 2. General Engine Results

RPM	$\dot{W}_{comp}$ (MW)	$\dot{W}_{turb}$ (MW)	Fuel-Air Ratio	ST (kN*s/kg)	TSFC (kg/kN*s)	Thrust (kN)
6620	14.8	-14.9	0.02	0.896	1.12	41.2

Diffuser

Table 3. Diffuser Results

Engine Inlet Area (m^2)	Diffuser Exit Area (m^2)	p_{o2} (kPa)	M_2	$c_{z,2}$ (m/s)
0.64	0.65	101.8	0.17	57.5

Compressor

The compressor achieves the required pressure ratio of 11 in 8 stages at a reasonable adiabatic and polytropic efficiency of 88% and 87%, respectively. An inlet guide vane was required to induce swirl before the first compressor stage—the IGV parameters are given in Table 4. The overall compressor performance is given in Table 5. Table 6 details the geometric, flow conditions, and loss factors associated with each stage of the compressor.

Table 4. Inlet Guide Vane Results

$c_{z,exit}$	r_h (m)	r_t (m)	r_m (m)	# Blades	χ_{out}	α_2	p_{o2}/p_{o1}
199	0.530	0.248	0.42	172	40.7	40.7	0.992

Table 5. Overall Compressor Performance

# Stages	c_z (m/s)	p_{o3} (MPa)	T_{o3} (K)	η	η_{poly}
8	199	1.11	610	0.880	0.873

Table 6. Compressor Parameters for each Stage

1st Stage	# Blades	χ_{out} (°)	D_f	σ	$\frac{h}{b}$	$p_{o,exit}$ (kPa)	AN^2 (m^{2}RPM^2)	$\frac{\sigma_c}{\rho_{blade}}$ (kPa/kg/m^3)	σ_{blade} (kPa)
Rotor	15	-4.21	0.440	0.631	1.01	101	1.08E7	16.9	56.5
Stator	14	28.6	0.550	0.569	1.01	130	~	~	~
r_h (m)	r_t (m)	r_m(m)	ϕ	ψ	**Pr**	η	**°R**	$M_{1,rel}$	α_3 (°)
0.329	0.431	0.383	0.751	0.331	1.28	0.919		0.75	28.6

2nd Stage	# Blades	χ_{out}	D_f	σ	$\frac{h}{b}$	$p_{o,exit}$	AN^2	$\frac{\sigma_c}{\rho_{blade}}$	σ_{blade}
Rotor	23	0.024	0.459	0.976	0.816	130	8.64E6	12.6	64.7
Stator	26	20.2	0.521	1.08	0.816	181	~	~	~
r_h	r_t	r_m	ϕ	ψ	**Pr**	η	**°R**	$M_{1,rel}$	α_3
0.340	0.422	0.383	0.751	0.476	1.39	0.916		0.75	23.1
3rd Stage	**# Blades**	χ_{out}	D_f	σ	$\frac{h}{b}$	$p_{o,exit}$	AN^2	$\frac{\sigma_c}{\rho_{blade}}$	σ_{blade}
Rotor	25	-0.047	0.481	1.05	0.623	181	6.60E6	10.4	84.3
Stator	30	15.9	0.512	1.25	0.623	250	~	~	~
r_h	r_t	r_m	ϕ	ψ	**Pr**	η	**°R**	$M_{1,rel}$	α_3
0.351	0.413	0.383	0.751	0.515	1.38	0.915		0.75	17.5
4th Stage	**# Blades**	χ_{out}	D_f	σ	$\frac{h}{b}$	$p_{o,exit}$	AN^2	$\frac{\sigma_c}{\rho_{blade}}$	σ_{blade}
Rotor	26	-0.138	0.493	1.10	0.484	250	5.13E6	8.05	107
Stator	31	11.4	0.503	1.31	0.484	339	~	~	~
r_h	r_t	r_m	ϕ	ψ	**Pr**	η	**°R**	$M_{1,rel}$	α_3
0.358	0.407	0.383	0.751	0.536	1.35	0.915		0.75	11.9
5th Stage	**# Blades**	χ_{out}	D_f	σ	$\frac{h}{b}$	$p_{o,exit}$	AN^2	$\frac{\sigma_c}{\rho_{blade}}$	σ_{blade}
Rotor	28	-0.151	0.524	1.18	0.384	339	4.07E6	6.39	140
Stator	33	9.22	0.498	1.40	0.384	463	~	~	~
r_h	r_t	r_m	ϕ	ψ	**Pr**	η	**°R**	$M_{1,rel}$	α_3
0.364	0.402	0.383	0.751	0.600	1.36	0.914		0.75	9.55
6th Stage	**# Blades**	χ_{out}	D_f	σ	$\frac{h}{b}$	$p_{o,exit}$	AN^2	$\frac{\sigma_c}{\rho_{blade}}$	σ_{blade}
Rotor	27	-0.183	0.537	1.15	0.306	463	3.24E6	5.09	183
Stator	31	9.00	0.497	1.31	0.306	615	~	~	~

r_h	r_t	r_m	ϕ	ψ	**Pr**	η	**˚R**	$M_{1,rel}$	α_3
0.368	0.398	0.383	0.751	0.607	1.33	0.915		0.72	9.16
7th Stage	**# Blades**	χ_{out}	D_f	σ	$\frac{h}{b}$	$p_{o,exit}$	AN^2	$\frac{\sigma_c}{\rho_{blade}}$	σ_{blade}
Rotor	25	-0.194	0.548	1.07	0.249	615	2.63E6	4.14	237
Stator	38	0	0.496	1.59	0.249	796	~	~	~
r_h	r_t	r_m	ϕ	ψ	**Pr**	η	**˚R**	$M_{1,rel}$	α_3
0.371	0.396	0.383	0.751	0.599	1.29	0.907		0.697	0
8th Stage	**# Blades**	χ_{out}	D_f	σ	$\frac{h}{b}$	$p_{o,exit}$	AN^2	$\frac{\sigma_c}{\rho_{blade}}$	σ_{blade}
Rotor	102	-0.027	0.478	4.25	0.206	796	2.18E6	3.43	108
Stator	48	0	0.550	2.00	0.206	1110	~	~	~
r_h	r_t	r_m	ϕ	ψ	**Pr**	η	**˚R**	$M_{1,rel}$	α_3
0.373	0.394	0.383	0.751	0.897	1.40	0.873		0.723	0

From Table 6, clear trends can be delineated. As expected, the stagnation pressure increases downstream the compressor. Moreover, stagnation pressure only increases across the rotor blades since only the rotor blades are imparting work onto the flow. Since c_z, r_m, and RPM are constant through the compressor, ϕ remains constant. ψ increases downstream the compressor, from 0.33 to 0.89, indicating that later stages are experiencing a higher pressure change as well as doing more work to the flow. This stage loading increase is caused by the incoming flow angle to each stage approaching 0˚. It follows that the stage efficiency also decreases from about 92% to 87% from the first to last stage. The typical ψ range for a compressor is 0.3 to 0.5, so by the 5th stage the stage loading is already significantly higher than normal. It also makes sense that the bending stress increases since the stage loading is increasing.

It can also be seen that blade height decreases down the compressor. This is because, assuming constant c_z and that density increases in a compressor, the flow area must decrease to maintain constant mass flow. Another trend is that the number of blades increase down the compressor, with the number of stator blades usually higher than the number of rotor blades.

Combustor

Table 7 displays the combustor design results. The maximum combustor exit temperature is 1436 K, which is lower than the maximum allowed 1450 K to avoid the need for bleed air cooling. The combustor flame tube is very long—a result which the team attributes to the optimization objective function being to minimize stagnation pressure losses.

Table 7. Combustor Parameters

L_{diff} (cm)	$r_{inner,3'}$(cm)	$r_{m,3'}$(cm)	$r_{outer,3'}$(m)	$M_{3'}$	$L_{fl,tube}$(m)
7.61	1.63	02.5	0.033	0.146	1.26
D_L(cm)	**$A_{h,eff}$ (cm^2)**	**T_{o4} (K)**	**p_{o4} (MPa)**	**p_{o4}/p_{o3}**	**Total Length (m)**
5.25	19.6	1436	1.09	0.983	1.34

Turbine

The turbine produced enough energy, 14.9 MW, to power the turbine in 2 stages with a relatively low, but reasonable adiabatic and polytropic efficiency of 83% and 80%, respectively.

Table 8. Overall Turbine Performance

# Stages	c_{z4} (m/s)	p_{o5} (kPa)	T_{o5} (K)	η	η_{poly}
2	106	416	1191	0.83	0.80

Table 9. Turbine Parameters for each Stage

1st Stage	r_h (m)	r_t (m)	r_m (m)	# Blades	χ_{out} (°)	α_{out} (°)	σ	$\frac{h}{b}$	$p_{o,exit}$ (kPa)	M_{exit}	AN^2	$\frac{\sigma_c}{\rho_{blade}}$ (kPa/kg/m^3)	σ_{bend} (MPa)
Nozzle	0.310	0.384	0.348	30	80.8	80.8	0.998	1.04	1070	1.00	~	~	~
Rotor	0.250	0.384	0.324	23	-73.9	-72.7	1.14	1.32	544	0.551	7.18E6	15.6	2.61

ϕ	ψ	η_{st}	°R	$c_{z,exit}$
0.441	-4.27	0.919	0.332	107

2nd Stage	r_h (m)	r_t (m)	r_m (m)	# Blades	χ_{out} (°)	α_{out} (°)	σ	$\frac{h}{b}$	$p_{o,exit}$ (kPa)	M_{exit}	AN^2	$\frac{\sigma_c}{\rho_{blade}}$ (kPa/kg/m^3)	σ_{bend} (MPa)
Nozzle	0.237	0.384	0.319	24	73.1	73.1	14.1	0.124	526	0.550	~	~	~
Rotor	0.196	0.384	0.305	22	-0.97	0.00	7.82	0.271	417	0.160	1.26E7	20.7	5.38 E-2

ϕ	ψ	η_{st}	°R	$c_{z,exit}$
0.497	-1.63	0.803	0.184	107

As expected, the stagnation pressure decreases across the turbine as work is extracted from the flow to power the turbine. The work done by the first and second stage was 80% and 20% of the total compressor work requirement, respectively. The reason for this ratio was because of the zero exit swirl requirement for the turbine. It would be very difficult for the flow to be "straightened" if the rotor must extract a significant amount of work. Some trends are opposite of the compressor. Here, the blade height is increasing, the number of blades is tending to decrease, and the bending stress is decreasing axially downstream.

There are a few uncertainties regarding the results, however. The loading coefficient for the first stage is very high, at -4.27, whereas typical turbine stage loading, though higher than compressors, is between -2.2 to -2.6. The solidities are also very high.

Nozzle

Table 10. Nozzle Parameters

A_e (m^2)	p_e (kPa)	T_e (K)	u_e (m/s)	M_e
0.0956	224	878	627	1.10

Off-Design Performance

The off-design performance for the design is reported below in Table 1l. The off-design thrust decreased from sea level by a factor of 4: 10.7 kN for a specific thrust of 239, specific fuel consumption of 6.86E-5, and a fuel-to-air ratio of 0.0164. To achieve the thrust characteristics, the mass flow rate decreased slightly to about 45 kg/s, the diffuser exit mach number accelerates to twice that at sea level, whereas the RPM, $Pr_{c,}$, and Pr_t have changed slightly.

Table 11. Off Design Performance Parameters

Air Mass Flow Rate (kg/s)	Diffuser Exit Mach No.	RPM	FAR	Pr_c	Pr_t	ST	SFC	Thrust (kN)
44.9	0.265	6691	0.0164	10.6	0.455	239	6.86E-5	10.7

CONCLUSIONS

As shown in our results, the ideal turbojet for the given conditions and requirements has eight compressor stages, two turbine stages, and a radius of roughly half a meter. Our approach to determining the ideal engine focused on improving the efficiency of each component starting with the compressor, which may not always be the best approach. For example, since the compressor was optimized first, the rpm chosen is ideal for the compressor and not necessarily the turbine, and a designer seeking a more efficient turbine may arrive at a different engine with a different rpm. Additionally, maximizing each component separately may not provide the best overall engine, and maximizing an overall performance parameter such as TSFC may be more appropriate. Our approach was chosen because it produces reasonably efficient results and is simpler than optimizing for an overall performance parameter.

The Pratt and Whitney JT3C-7 engine is a civil turbojet which is somewhat comparable to the engine designed here, as it produces 53 kN of thrust at takeoff and has an overall pressure ratio of 12.5. It has a radius of 0.49 meters including the casing, which means is slightly smaller than our designed engine. It has nine low pressure and seven high pressure stages, which differs significantly from our single spool eight stage design. A two-spool design would likely perform significantly better than our single spool design as the later compressor stages can be run at a higher RPM to improve efficiency. The turbine on the JT3C-7 has one high pressure stage and two low pressure stages, which is like ours except for the dual spool. The extra stage may be due to power requirements from other parts of the plane which we did not account for, or for higher power requirement from the higher stage count compressor.

APPENDIX

The following equations, organized by engine component, were implemented into the code to calculate component performance, geometry, flow angles, and flow conditions. The "General" category contains many equations that are common throughout many components. For instance, many of the flow conditions at different locations within each component were calculated using the isentropic relations. If an equation for static or stagnation pressure or temperature is not explicitly defined under the component, it is calculated using the isentropic equations. The table below defines the number used to reference the exit of each station.

Ambient/Engine Inlet – 1, Diffuser Exit – 2, IGV Exit – 2.1, Compressor Exit – 3, Combustor Exit – 4, Turbine Exit – 5, Nozzle Exit – 6

<u>General</u>

$$\mathrm{M}_i = \sqrt{\gamma R T_i} \qquad \mathrm{T}_{oi} = \mathrm{T}_i(1+\tfrac{\gamma-1}{2}M_i{}^2) \qquad \mathrm{P}_{oi} = \mathrm{P}_i(1+\tfrac{\gamma-1}{2}M_i{}^2)^{\gamma/\gamma-1}$$

$$\mathrm{c}_{zi} = \mathrm{M}_i\sqrt{\gamma R T_i}$$

$$\mathrm{r}_t = \sqrt{\mathrm{r}_m{}^2 + \frac{\dot{m}}{2\pi\rho\mathrm{c}_z}} \qquad \mathrm{r}_h = \sqrt{\mathrm{r}_m{}^2 - \frac{\dot{m}}{2\pi\rho\mathrm{c}_z}} \qquad \mathrm{h} = \mathrm{r}_t - \mathrm{r}_h$$

$$\tau = \dot{m}_a\left[(1+f)u_e - u_{flight}\right] + (p_e - p_a)A_e \qquad \mathrm{ST} = \tau/\dot{m}_a$$

<u>Diffuser</u>

$$\mathrm{P}_{o2} = \mathrm{P}_{oa}\mathrm{r}_d(1+\tfrac{\gamma-1}{2}M_a{}^2)^{\gamma/\gamma-1} \qquad \mathrm{T}_{o2} = \mathrm{T}_{oa}\mathrm{r}_d{}^{\gamma/\gamma-1}$$

<u>Inlet Guide Vane</u>

$$\mu = \frac{0.000001458 * T_3^{\frac{3}{2}}}{T_3+110.4} \qquad r_{tip} = \sqrt{r_m^2 + \frac{\dot{m}}{2\pi\rho_3 c_{z3}}} \qquad r_{hub} = \sqrt{r_m^2 - \frac{\dot{m}}{2\pi\rho_3 c_{z3}}} \qquad h = r_t - r_h$$

$$\sigma = 2\psi_z\cos(\alpha_1)\sin(\alpha_1) \qquad s = \frac{b}{\sigma} \qquad C_{nozzle} = 0.993 + 0.021 * \frac{b}{h}$$

$$D_h = \frac{2sh*\cos(\alpha_2)}{s*\cos(\alpha_2)+h} \qquad Re_e = \frac{\rho_3 c_3 D_h}{\mu} \qquad \zeta^* = 1.04 + 0.06\left(\frac{\alpha_1+\alpha_2}{100}\right)^2$$

$$\zeta = (\zeta^* C_j - 1)\left(\frac{10^5}{Re_e}\right)^{1/4}$$

<u>Compressor</u>

$$\alpha_2 = \tan^{-1}(\tan\beta_2 + 1/\phi) \qquad \beta_1 = \tan^{-1}(\tan\alpha_1 - 1/\phi) \qquad \beta_2 = \tan^{-1}(\psi/\phi + \tan\beta_1)$$

$$\chi_1 = \beta_1 + i_r \qquad \chi_{2r} = \beta_2 + \delta_r \qquad \chi_{2s} = \alpha_2 - i_s$$

$$\chi_3 = \alpha_3 - \delta_s \qquad \varphi_r = \chi_1 - \chi_{2r} \qquad \varphi_s = \chi_{2s} - \chi_3$$

$$\delta_r = (\beta_2 - \chi_1)/\left(\sqrt{\sigma_r}/0.25 - 1\right) \qquad \delta_s = (\chi_2 - \alpha_3)/\left(\sqrt{\sigma_r}/0.25 - 1\right)$$

$$N = \frac{c_z}{\phi}\frac{30}{\pi r_m} \qquad U = c_z/\phi$$

$$D_{f,r} = 1 - \frac{\cos\beta_1}{\cos\beta_2} + \frac{1}{\sigma_r}\frac{\cos\beta_1}{2}(\tan\beta_2 - \tan\beta_1) \qquad D_{f,s} = 1 - \frac{\cos\alpha_2}{\cos\alpha_3} + \frac{1}{\sigma_s}\frac{\cos\alpha_2}{2}(\tan\alpha_2 - \tan\alpha_3)$$

$$\sigma_r = \left(\frac{\cos\beta_1}{2}(\tan\beta_2 - \tan\beta_1)\right)\Big/\left(D_{f,r} - 1 + \frac{\cos\beta_1}{\cos\beta_2}\right)$$

$$\varpi_r = 2.3\times\left\{\left(\frac{\cos\beta_1}{\cos\beta_2}\right)^2 \frac{\sigma_r}{\cos\beta_2}\left(0.012 + 0.0004e^{7.5D_{f,r}}\right)\right\} \qquad \varpi_s = 2.3\times\left\{\left(\frac{\cos\alpha_2}{\cos\alpha_3}\right)^2 \frac{\sigma_s}{\cos\alpha_3}\left(0.012 + 0.0004e^{7.5D_{f,s}}\right)\right\}$$

$$T_1 = T_{o1} - (c_z/\cos\alpha_1)^2/(2c_p) \qquad T_2 = T_{o2} - (c_z/\cos\alpha_2)^2/(2c_p) \qquad T_{o2} = T_{o1} + (c_z/\phi)^2(\psi/c_p)$$

$$M_{1,rel} = (c_z/\cos\beta_1)/\sqrt{\gamma R T_1} \qquad M_2 = (c_z/\cos\alpha_2)/\sqrt{\gamma R T_2} \qquad M_{2,rel} = (c_z/\cos\beta_2)/\sqrt{\gamma R T_2}$$

$$p_{o3}/p_{o2} = 1 - \varpi_s(1 - p_2/p_{o2})$$

$$\frac{p_{o2}}{p_{o1}} = \left(\frac{T_{o2}/T_2}{1 + \frac{\gamma-1}{2}M_{2,rel}^2}\right)^{\frac{\gamma}{\gamma-1}}\left[1 - \varpi_r\left\{1 - \left(1 + \frac{\gamma-1}{2}M_{1,rel}^2\right)^{\frac{\gamma}{1-\gamma}}\right\}\right]\left(\frac{1 + \frac{\gamma-1}{2}M_{1,rel}^2}{T_{o1}/T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$C_{p,r} = \frac{p_2 - p_1}{p_{o1,rel} - p_1} \qquad C_{p,s} = \frac{p_3 - p_2}{p_{o2} - p_2}$$

$$\dot{W}_{st} = \dot{m}c_p\left(T_{o2} - T_{o1}\right)$$

Combustor

$$K_t = \left(1 - \frac{A_3}{A_{3'}}\right)^2 + \left(1 - \frac{A_3}{A_{3'}}\right)^6 \qquad \frac{P_{03'}}{P_{03}} = e^{\frac{-\gamma_3 M_3^2}{2} * K_t} \qquad \frac{P_{03'} - P_{03}}{P_{03}}$$

$$PLF = \frac{\Delta P_{03,4}}{q_{ref}} \qquad q_{ref} = \frac{\dot{m}_3}{2\rho_3 A_{ref}}$$

Turbine

$$\alpha_2 = \cos^{-1}\left(\frac{M_1 T_1}{M_2 T_2}\right) \qquad \alpha_3 = \tan^{-1}(\tan\beta_3 + 1/\phi) \qquad \beta_2 = \tan^{-1}(\tan\alpha_2 - 1/\phi)$$

$$\beta_3 = \tan^{-1}((\psi\text{-}2R)/(2\phi)) \qquad \psi = \Delta h_o/U^2 = 2\,(1 - R - \phi\tan\alpha_2)$$

$$s_n = o_n / \cos\alpha_2 \qquad o_n = Re_n\,(\mu/\rho_2 c_2) \qquad n_{blades,n} = round(2\pi r_m/s_n)$$

$$\sigma_{z,n} = \frac{b_{z,n}}{s_n} = \frac{1}{\psi_{z,n}}\left(\frac{\tan\alpha_1}{\tan\alpha_2} - 1\right)\sin(2\alpha_2)\left\{\frac{\gamma/2M_2^2}{\left(1 + \frac{\gamma - 1}{2}M_2^2\right)^{\gamma/(\gamma-1)} - 1}\right\}$$

$$\sigma_n = \sigma_{z,n} / \cos(\tan^{-1}((\tan\alpha_1 + \tan\alpha_2)/2))$$

$$b_{z,n}/h_1 = (s_n\sigma_{z,n})/\,h_1 \qquad (h/b)_n = h_1\,/\,(s_n\sigma_n)$$

$$\mathrm{Re}_{e,n} = \frac{\rho_2 c_2 D_{h,n}}{\mu} \cong \mathrm{Re}_{o,n}\frac{D_{h,n}}{o_n} \qquad D_{h,n} = \frac{2s_n h_1 \cos\alpha_2}{s_n \cos\alpha_2 + h_1}$$

$$\zeta_n = \left(\left[1.04+0.06\left(\frac{\alpha_1+\alpha_2}{100}\right)^2\right]\left[0.993+0.021\left(\frac{b_{z,n}}{h_1}\right)\right]-1\right)\left(\frac{10^5}{\mathrm{Re}_{e,n}}\right)^{1/4}$$

$$p_{o2} = p_{o1}\left[\frac{1-\dfrac{c_2^2}{2c_pT_{o1}}\dfrac{1}{1-\zeta_n}}{1-\dfrac{c_2^2}{2c_pT_{o1}}}\right]^{\gamma/\gamma-1}$$

$w_2 = c_z / \cos\beta_2$ $\qquad w_3 = c_z / \cos\beta_3$ $\qquad c_3 = c_z / \cos\alpha_3$

$M_{2,\mathrm{rel}} = M_2\,(w_2 / c_2)$ $\qquad M_{3,\mathrm{rel}} = w_3 / \sqrt{\gamma R T_3}$

$s_r = o_r / \cos\beta_3$ $\qquad o_r = Re_r\,(\mu / \rho_3 w_3)$ $\qquad n_{blades,r} = round(2\pi r_m / s_r)$

$$\sigma_{z,r} = \frac{b_{z,r}}{h_2} = \frac{1}{\psi_{z,r}}\left(\frac{\tan\beta_2}{\tan\beta_3}-1\right)\sin(2\beta_3)\left\{\frac{\gamma/2\,M_{3,rel}^2}{\left(1+\frac{\gamma-1}{2}M_{3,rel}^2\right)^{\gamma/(\gamma-1)}-1}\right\}$$

$\sigma_r = \sigma_{z,r} / \cos(\tan^{-1}((\tan\beta_2+\tan\beta_3)/2))$

$b_{z,r}/h_2 = (s_r\sigma_{z,r}) / h_2$ $\qquad (h/b)_r = h_2 / (s_r\sigma_r)$

$T_{o3} = T_{o1} + \Delta h_o / c_p$ $\qquad T_3 = T_{o3} - c_3^2 / 2c_p$ $\qquad T_{o2,rel} = T_{o1} - w_2^2 / 2c_p$

$$\mathrm{Re}_{e,r} = \frac{\rho_3 w_3 D_{h,r}}{\mu} \cong \mathrm{Re}_{o,r}\frac{D_{h,r}}{o_r} \qquad D_{h,r} = \frac{2s_r h_2\cos\beta_3}{s_r\cos\beta_3 + h_2}$$

$$\zeta_r = \left(\left[1.04+0.06\left(\frac{\beta_2+\beta_3}{100}\right)^2\right]\left[0.975+0.075\left(\frac{b_{z,r}}{h_2}\right)\right]-1\right)\left(\frac{10^5}{\mathrm{Re}_{e,r}}\right)^{1/4}$$

$$\frac{p_{o3,rel}}{p_{o2,rel}} = \left[\frac{1 - \frac{w_3^2}{2c_p T_{o2,rel}} \frac{1}{1-\zeta_r}}{1 - \frac{w_3^2}{2c_p T_{o2,rel}}} \right]^{\gamma/\gamma-1}$$

$$p_{o3} / p_{o3,rel} = (T_{o3}/T_{o2,rel})^{\gamma/(\gamma-1)}$$

$$p_{o2,rel} / p_{o2} = (T_{o2,rel} / T_{o1})^{\gamma/(\gamma-1)}$$

$$p_{o3} = \frac{p_{o3}}{p_{o3,rel}} \frac{p_{o3,rel}}{p_{o2,rel}} \frac{p_{o2,rel}}{p_{o2}} p_{o2}$$

$$\eta_p = \frac{\ln(T_{o1}/T_{03})}{\ln\left(1 + \frac{1}{\eta_{st}}(T_{o1}/T_{03} - 1)\right)}$$

$$AN^2 = \pi/2 \, (r_{tip,2}^2 - r_{hub,2}^2) \, N^2$$

<u>Nozzle</u>

$$u_6 = \sqrt{2c_p T_{o5}(1 - (\frac{1}{1 + \frac{(\gamma-1)}{2} {M_6}^2})}$$

$$\frac{A_e}{A_5} = \frac{P_{o5}}{P_{oe}} \frac{M_5}{M_e} \sqrt{2c_p T_{o5}(1 - (\frac{1}{1 + \frac{(\gamma-1)}{2} {M_6}^2})}$$