(1)The problem can be expressed as

$$\text{minimize } ||Ax-b||_2^2$$
$$\text{subject to } Gx \leq h$$

Where

$$A = \begin{bmatrix} g(t_1) \\ g(t_2) \\ . \\ . \\ g(t_N) \end{bmatrix}, x = \begin{bmatrix} x_1 \\ x_2 \\ . \\ . \\ x_{M+3} \end{bmatrix}, b = \begin{bmatrix} t_1 \\ t_2 \\ . \\ . \\ t_N \end{bmatrix}$$
$$G = \begin{bmatrix} -g^{''}(t_1) \\ -g^{''}(t_2) \\ . \\ . \\ -g^{''}(t_N) \end{bmatrix}, h = \vec{0}$$

(2)

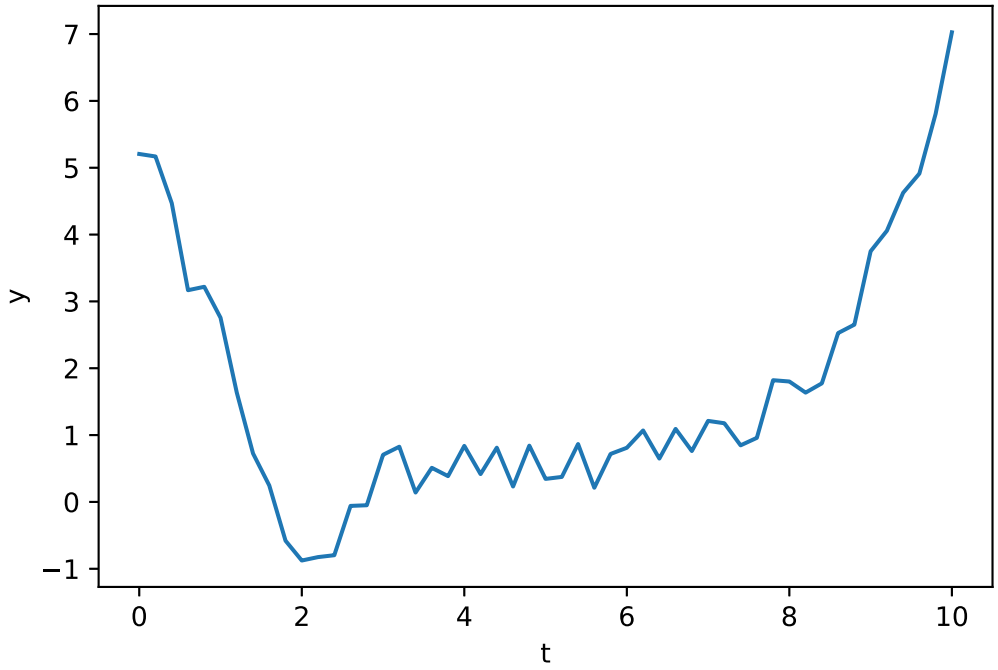
In [128…

```
import numpy
import matplotlib.pyplot as plt
from spline_data import t, y
from bsplines import *
import cvxpy as cp
%matplotlib inline
```

```
/Users/deyuyang/Desktop/convex_optimization/hw2_code/.venv/lib/python3.8/site-pa
ckages/ipykernel/ipkernel.py:283: DeprecationWarning: `should_run_async` will no
t call `transform_cell` automatically in the future. Please pass the result to `
transformed_cell` argument and any exception that happen during thetransform in
`preprocessing_exc_tuple` in IPython 7.17 and above.
  and should_run_async(code)
```

In [129…

```
plt.plot(t, y)
plt.xlabel("t")
plt.ylabel("y")
plt.show()
```



In [130…

```
m = 13
n = t.shape[0]
A = np.zeros((n,m))
G = np.zeros((n,m))
for i in np.arange(A.shape[0]):
    g,_,gpp = bsplines(t[i])
    A[i]= g
    G[i] = -gpp
```

```
/Users/deyuyang/Desktop/convex_optimization/hw2_code/.venv/lib/python3.8/site-pa
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transformed_cell` argument and any exception that happen during thetransform in
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  and should_run_async(code)
```

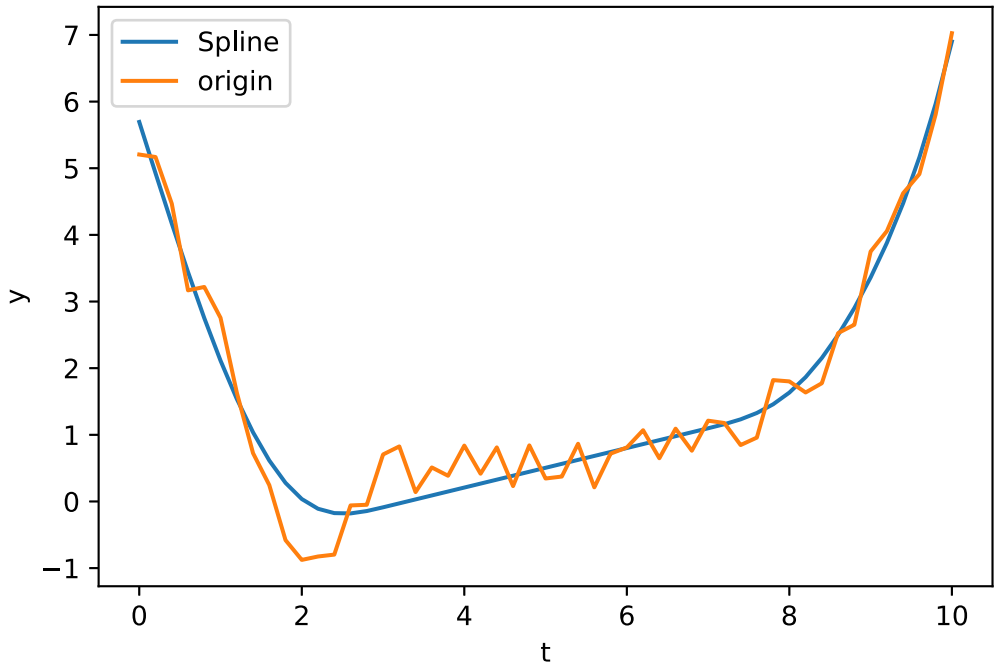
In [131…

```
x = cp.Variable(m)
objective = cp.Minimize(cp.sum_squares(A@x - y))
constraints = [G@x<=0]
prob = cp.Problem(objective, constraints)
result = prob.solve()
print(x.value)
```

```
[ 5.69438888  4.41060672  1.8430424  -0.38409468 -0.08762342  0.20884784
  0.5053191   0.80179036  1.09826161  1.39473287  3.12555339  5.22124366
  6.90143774]
```

In [132…

```
opt_x = x.value
opt_y = A@opt_x
spline_combine,  = plt.plot(t, opt_y,label = 'Spline')
origin = plt.plot(t,y, label = 'origin')
plt.xlabel("t")
plt.ylabel("y")
plt.legend()
plt.show()
```



In [133…

```
opt_x = x.value
opt_y = A*opt_x
plt.plot(t, opt_y)
plt.xlabel("t")
plt.ylabel("gk(t)")
plt.show()
```

```
/Users/deyuyang/Desktop/convex_optimization/hw2_code/.venv/lib/python3.8/site-pa
ckages/ipykernel/ipkernel.py:283: DeprecationWarning: `should_run_async` will no
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`preprocessing_exc_tuple` in IPython 7.17 and above.
  and should_run_async(code)
```

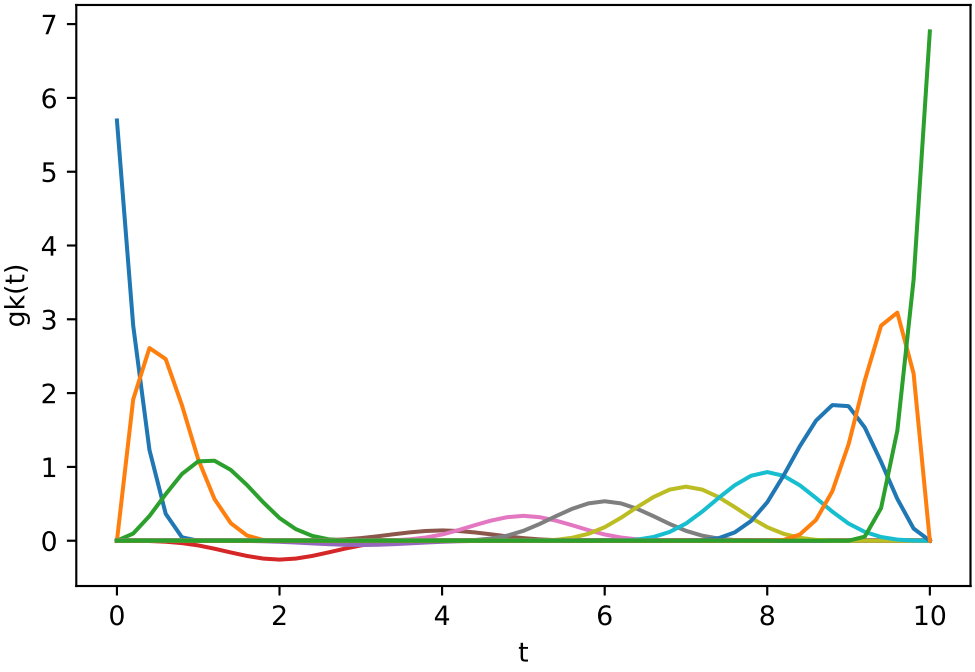

In [ ]: