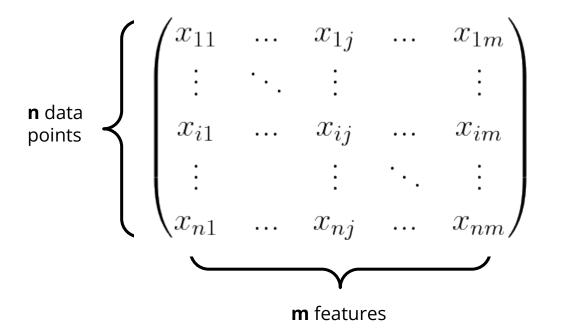
Distance & Similarity

Boston University CS 506 - Lance Galletti

Data



Feature Space

From our data we can generate a **feature space** of all possible values for the set of features in our data.

name	age	balance
Jane	25	150
John	30	100

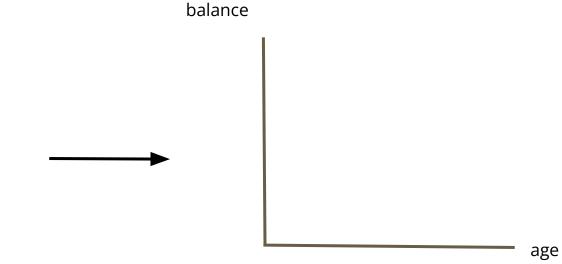
Feature Space

From our data we can generate a **feature space** of all possible values for the set of features in our data.

name age balance

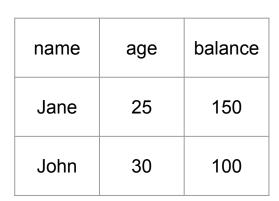
Jane 25 150

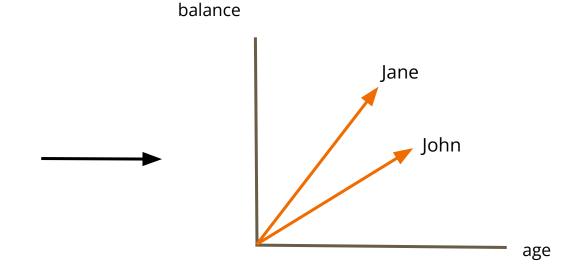
John 30 100



Feature Space

From our data we can generate a **feature space** of all possible values for the set of features in our data.





Our feature space is the Euclidean plane

Distance

In order to uncover interesting structure from our data, we need a way to **compare** data points.

A **dissimilarity function** is a function that takes two objects (data points) and returns a **large value** if these objects are **dissimilar**.

A special type of dissimilarity function is a **distance** function

Distance

d is a distance function if and only if:

- d(i, j) = 0 if and only if i = j
- $\bullet \quad d(i,j) = d(j,i)$
- $d(i, j) \le d(i, k) + d(k, j)$

We don't **need** a distance function to compare data points, but why would we prefer using a distance function?

For **x**, **y** points in **d**-dimensional real space

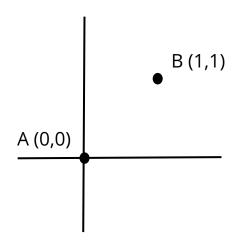
I.e.
$$x = [x_1, ..., x_d]$$
 and $y = [y_1, ..., y_d]$

$$L_p(x,y) = \left(\sum_{i=1}^{d} |x_i - y_i|^p\right)^{\frac{1}{p}}$$

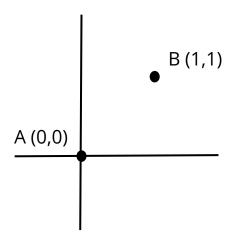
When $\mathbf{p} = 2$ -> Euclidean Distance

When $\mathbf{p} = 1$ -> Manhattan Distance

d = 2



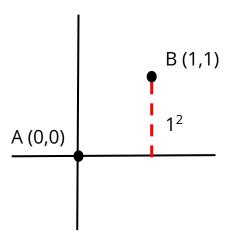
$$d = 2$$



$$p = 2$$

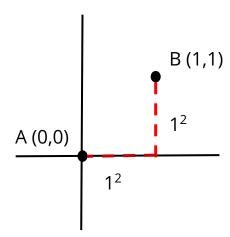
$$L_p(x,y) = \left(\sum_{i=1}^{d} |x_i - y_i|^p\right)^{\frac{1}{p}}$$

$$d = 2$$



$$L_p(x,y) = \left(\sum_{i=1}^{d} |x_i - y_i|^p\right)^{\frac{1}{p}}$$

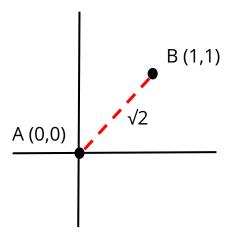
$$d = 2$$



$$p = 2$$

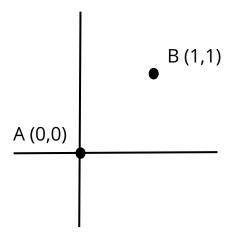
$$L_p(x,y) = \left(\sum_{i=1}^{d} |x_i - y_i|^p\right)^{\frac{1}{p}}$$

$$d = 2$$



$$L_p(x,y) = \left(\sum_{i=1}^{d} |x_i - y_i|^p\right)^{\frac{1}{p}}$$

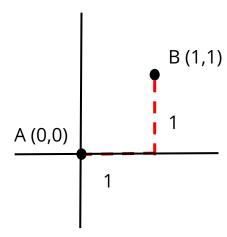
$$d = 2$$



$$p = 1$$

$$L_p(x, y) = \left(\sum_{i=1}^{d} |x_i - y_i|^p\right)^{\frac{1}{p}}$$

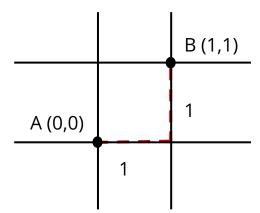
$$d = 2$$



$$p = 1$$

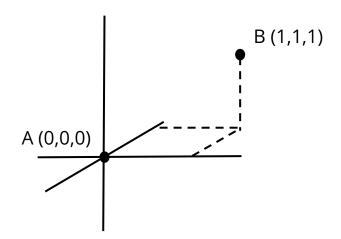
$$L_p(x,y) = \left(\sum_{i=1}^{d} |x_i - y_i|^p\right)^{\frac{1}{p}}$$

$$d = 2$$



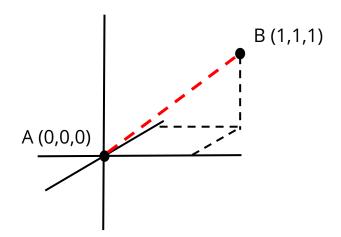
$$L_p(x, y) = \left(\sum_{i=1}^{d} |x_i - y_i|^p\right)^{\frac{1}{p}}$$

$$d = 3$$



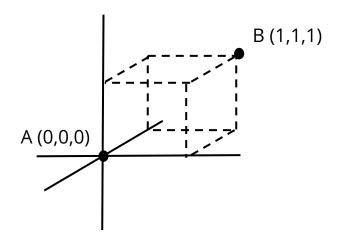
$$L_p(x,y) = \left(\sum_{i=1}^{d} |x_i - y_i|^p\right)^{\frac{1}{p}}$$

$$d = 3$$



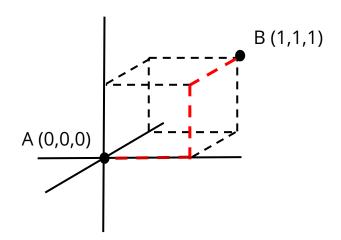
$$L_p(x, y) = \left(\sum_{i=1}^{d} |x_i - y_i|^p\right)^{\frac{1}{p}}$$

$$d = 3$$



$$L_p(x, y) = \left(\sum_{i=1}^{d} |x_i - y_i|^p\right)^{\frac{1}{p}}$$

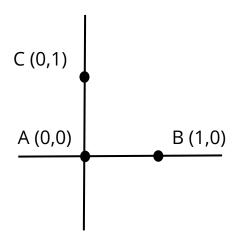
$$d = 3$$



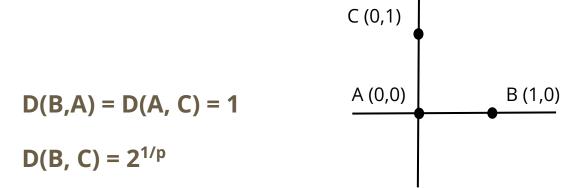
$$L_p(x, y) = \left(\sum_{i=1}^{d} |x_i - y_i|^p\right)^{\frac{1}{p}}$$

Is L_p a distance function when 0 ?

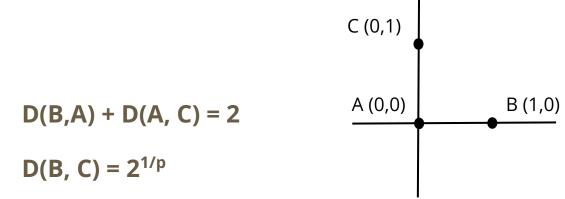
Is L_p a distance function when 0 ?



Is L_p a distance function when 0 ?

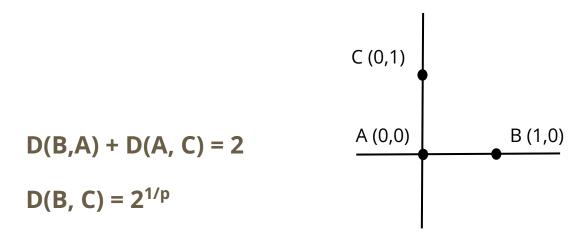


Is L_p a distance function when 0 ?



But... if **p < 1** then **1/p > 1**

Is L_p a distance function when 0 ?



So D(B, C) > D(B, A) + D(A, C) which violates the triangle inequality

A **similarity** function is a function that takes two objects (data points) and returns a **large value** if these objects are **similar**.

$$s(x, y) = cos(\theta)$$

where θ is the angle between \mathbf{x} and \mathbf{y}

To get a corresponding **dissimilarity** function, we can usually try

$$d(x, y) = 1 / s(x, y)$$

or

$$d(x, y) = k - s(x, y)$$
 for some k

Here, we can use

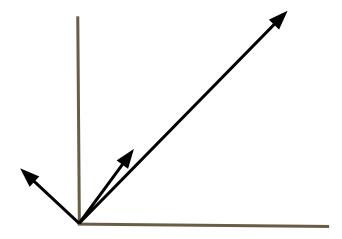
$$d(x, y) = 1 - s(x, y)$$

When should you use **cosine (dis)similarity** over **euclidean distance**?

When direction matters more than magnitude

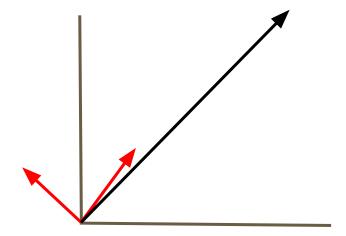
When should you use **cosine (dis)similarity** over **euclidean distance**?

When direction matters more than magnitude



When should you use **cosine (dis)similarity** over **euclidean distance**?

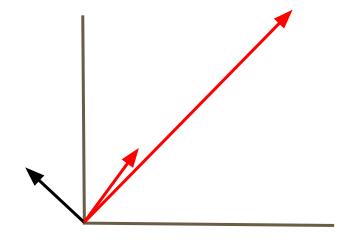
When direction matters more than magnitude



Close under Euclidean distance

When should you use **cosine (dis)similarity** over **euclidean distance**?

When direction matters more than magnitude



Close under Cosine Similarity

How similar are the following documents?

	w ₁	W ₂	•••	w _d
X	1	0	•••	1
у	1	1		0

One way is to use the Manhattan distance which will return the size of the set difference

	w ₁	W_2	•••	w _d
X	1	0	•••	1
у	1	1	•••	0

$$L_1(x,y) = \sum_{i=1}^d |x_i - y_i|$$

One way is to use the Manhattan distance which will return the size of the set difference

	W ₁	W ₂		w _d
X	1	0	•••	1
у	1	1		0

$$L_1(x,y) = \sum_{i=1}^d (x_i - y_i)$$
 Will only be 1 when $\mathbf{x_i} \neq \mathbf{y_i}$

But how can we distinguish between these two cases?

	W ₁	W_2		W _{d-1}	w _d
х	1	1	1	0	1
у	1	1	1	1	0

	W ₁	W_2
x	0	1
у	1	0

Only differ on the last two words

Completely different

But how can we distinguish between these two cases?

	w ₁	W_2		W _{d-1}	w _d
х	1	1	1	0	1
у	1	1	1	1	0

	W ₁	W_2
x	0	1
у	1	0

Only differ on the last two words

Completely different

Both have Manhattan distance of 2

We need to account for the size of the intersection!

$$JSim(x,y) = \frac{|x \cap y|}{|x \cup y|}$$

$$JDist(x,y) = 1 - \frac{|x \cap y|}{|x \cup y|}$$

A quick note on Norms

- Distance from the origin
 - Minkowski Distance <=> Lp Norm
 - Not all distances can create a norm
- Notion of size
- Has the following properties:

 - o p(x) = 0 iff x = 0
 - o p(x) ≥ 0 for all x