Question 2 **(6 marks)**

The Cartesian equations for three planes are $x - y - z = 2$, $2x - y + z = 7$ and $3x + y + z = 2$.

(a) Show that none of these planes is parallel to another. (2 marks)

(b) Solve the three equations simultaneously. (3 marks)

(c) State the geometric interpretation of the solution obtained in part (b). (1 mark)

Question 2 **(6 marks)**

The Cartesian equations for three planes are $x - y - z = 2$, $2x - y + z = 7$ and $3x + y + z = 2$.

(a) Show that none of these planes is parallel to another. (2 marks)

Solution
The planes have normal vectors $\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$ and since none of these are scalar multiples of each other, then none of the three planes is parallel to one of the others.
Specific behaviours
✓ correctly states all normal vectors ✓ correct explanation

(b) Solve the three equations simultaneously. (3 marks)

Solution
$x - y - z = 2$ $3x + y + z = 2$ $4x = 4, \quad x = 1$ $1 - y - z = 2$ $2 - y + z = 7$ $3 - 2y = 9, \quad y = -3$ $3 - 3 + z = 2, \quad z = 2$ $x = 1, \quad y = -3, \quad z = 2$
Specific behaviours
✓ uses elimination to obtain value of x ✓ uses elimination to obtain a second value ✓ states correct solution set

(c) State the geometric interpretation of the solution obtained in part (b). (1 mark)

Solution
Three non-parallel planes intersecting at the point $(1, -3, 2)$.
Specific behaviours
✓ correctly interprets solution

Question 4 **(7 marks)**

The coordinates of three points in space are $L(0, 3, 3)$, $M(-2, 1, -1)$ and $N(-1, 1, 2)$.

(a) Determine the vector equation of the sphere with diameter LM. (3 marks)

(b) Determine the Cartesian equation of the plane that contains all three points. (4 marks)

Question 4 **(7 marks)**

The coordinates of three points in space are $L(0,3,3), M(-2,1,-1)$ and $N(-1,1,2)$.

(a) Determine the vector equation of the sphere with diameter LM. (3 marks)

Solution
Centre: $$\frac{1}{2}\left(\begin{pmatrix}0\\3\\3\end{pmatrix}+\begin{pmatrix}-2\\1\\-1\end{pmatrix}\right)=\begin{pmatrix}-1\\2\\1\end{pmatrix}$$ Radius: $$\left\|\begin{pmatrix}0\\3\\3\end{pmatrix}-\begin{pmatrix}-1\\2\\1\end{pmatrix}\right\| = \sqrt{1^2+1^2+2^2}=\sqrt{6}$$ Equation: $$\left\|\underset{\sim}{r}-\begin{pmatrix}-1\\2\\1\end{pmatrix}\right\|=\sqrt{6}$$
Specific behaviours
✓ calculates centre ✓ calculates radius or diameter ✓ correct vector equation

(b) Determine the Cartesian equation of the plane that contains all three points. (4 marks)

Solution
$$\overrightarrow{ML}=\begin{pmatrix}0\\3\\3\end{pmatrix}-\begin{pmatrix}-2\\1\\-1\end{pmatrix}=\begin{pmatrix}2\\2\\4\end{pmatrix}, \quad \overrightarrow{NL}=\begin{pmatrix}0\\3\\3\end{pmatrix}-\begin{pmatrix}-1\\1\\2\end{pmatrix}=\begin{pmatrix}1\\2\\1\end{pmatrix}$$ Normal to plane: $$\underset{\sim}{n}=\begin{pmatrix}1\\1\\2\end{pmatrix}\times\begin{pmatrix}1\\2\\1\end{pmatrix}=\begin{pmatrix}-3\\1\\1\end{pmatrix}$$ Constant: $$\begin{pmatrix}-3\\1\\1\end{pmatrix}\cdot\begin{pmatrix}0\\3\\3\end{pmatrix}=6$$ Cartesian equation: $$-3x+y+z=6$$
Specific behaviours
✓ derives two vectors in the plane ✓ calculates normal to plane ✓ calculates constant ✓ correct cartesian equation

Question 4 **(9 marks)**

(a) Solve the following system of equations and interpret the solution geometrically.

(4 marks)

$$\begin{aligned} x + y + z &= 3 \\ 2x + y + 3z &= 4 \\ x - y + z &= -5 \end{aligned}$$

(b) The position vectors of points A, B and C are $\overrightarrow{OA} = \begin{pmatrix} 0 \\ -5 \\ 6 \end{pmatrix}, \overrightarrow{OB} = \begin{pmatrix} 3 \\ -3 \\ 2 \end{pmatrix}$ and $\overrightarrow{OC} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$.

Determine the Cartesian equation of the plane through the line AB and perpendicular to the plane OBC. (5 marks)

Question 4 **(9 marks)**

(a) Solve the following system of equations and interpret the solution geometrically. (4 marks)

$$x + y + z = 3$$
$$2x + y + 3z = 4$$
$$x - y + z = -5$$

Solution
$R_1 - R_3: 2y = 8 \rightarrow y = 4$ $R_2 - 2R_3: 3y + z = 14 \rightarrow z = 2$ $R_1: x + 4 + 2 = 3 \rightarrow x = -3$ Solution $x = -3, y = 4, z = 2$ represents the unique point at which the three planes meet.
Specific behaviours
✓ correctly eliminates at least one variable ✓ solves correctly for one variable ✓ solves correctly for all variables ✓ correctly interprets solution

(b) The position vectors of points A, B and C are $\overrightarrow{OA} = \begin{pmatrix} 0 \\ -5 \\ 6 \end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix} 3 \\ -3 \\ 2 \end{pmatrix}$ and $\overrightarrow{OC} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$.

Determine the Cartesian equation of the plane through the line AB and perpendicular to the plane OBC. (5 marks)

Solution
Vector perpendicular to plane OBC is $$\overrightarrow{OB} \times \overrightarrow{OC} = \begin{pmatrix} 3 \\ -3 \\ 2 \end{pmatrix} \times \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ 8 \\ 9 \end{pmatrix}$$ $$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix} 3 \\ -3 \\ 2 \end{pmatrix} - \begin{pmatrix} 0 \\ -5 \\ 6 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix}$$ Hence normal to required plane: $\begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix} \times \begin{pmatrix} 2 \\ 8 \\ 9 \end{pmatrix} = \begin{pmatrix} 50 \\ -35 \\ 20 \end{pmatrix} = 5\begin{pmatrix} 10 \\ -7 \\ 4 \end{pmatrix}$ Using point A: $\begin{pmatrix} 10 \\ -7 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ -5 \\ 6 \end{pmatrix} = 59$ Hence Cartesian equation is $10x - 7y + 4z = 59$.
Specific behaviours
✓ normal vector to plane OBC ✓ vector $\overrightarrow{AB}$ ✓ normal to required plane   ✓ evaluates constant   ✓ writes Cartesian equation

Question 1 **(6 marks)**

A system of equations, where b is a real constant, is as follows:

$$\begin{aligned} x - y + 3z &= 11 \\ x + 2y + 2z &= 3 \\ 2x + by + 4z &= 8 \end{aligned}$$

(a) Solve the system when $b = 3$. (4 marks)

(b) Interpret the system of equations geometrically when $b = 4$. (2 marks)

Question 1 **(6 marks)**

A system of equations, where b is a real constant, is as follows:

$$x - y + 3z = 11$$
$$x + 2y + 2z = 3$$
$$2x + by + 4z = 8$$

(a) Solve the system when $b = 3$. (4 marks)

Solution
$2(2) - (3)$: $y = -2$ Sub into $(1), (2)$: $x + 3z = 9$ $x + 2z = 7$ $z = 2$ $x + 4 = 7$ $x = 3$ Solution: $x = 3, \quad y = -2, \quad z = 2$
Specific behaviours
✓ indicates correct use of elimination techniques ✓ solves for y ✓ solves for z ✓ solves for x

(b) Interpret the system of equations geometrically when $b = 4$. (2 marks)

Solution
Consider last equation: $2x + 4y + 4z = 8 \Rightarrow x + 2y + 2z = 4$ Compare to second equation: $x + 2y + 2z = 3$ The system consists of two distinct parallel planes cut by a third plane.
Specific behaviours
✓ simplifies equation to enable comparison ✓ identifies parallel planes

Question 5 **(8 marks)**

Points A, B, C and D have position vectors $\overrightarrow{OA} = \begin{pmatrix} -1 \\ 2 \\ 4 \end{pmatrix}, \overrightarrow{OB} = \begin{pmatrix} -3 \\ 1 \\ 7 \end{pmatrix}, \overrightarrow{OC} = \begin{pmatrix} 3 \\ 4 \\ -2 \end{pmatrix}$ and $\overrightarrow{OD} = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix}$.

Note that $|\mathbf{u} \times \mathbf{v}| = |\mathbf{u}||\mathbf{v}| \sin\theta$, where θ is the angle between $\mathbf{u}$ and $\mathbf{v}$ given by $\theta = \cos^{-1}(\hat{\mathbf{u}} \cdot \hat{\mathbf{v}})$.

(a) Determine $|\overrightarrow{AB} \times \overrightarrow{AC}|$ and use the result to explain why A, B and C are collinear. (5 marks)

(b) Determine the Cartesian equation of the plane containing all four points. (3 marks)

Question 5 **(8 marks)**

Points A, B, C and D have position vectors $\overrightarrow{OA} = \begin{pmatrix} -1 \\ 2 \\ 4 \end{pmatrix}, \overrightarrow{OB} = \begin{pmatrix} -3 \\ 1 \\ 7 \end{pmatrix}, \overrightarrow{OC} = \begin{pmatrix} 3 \\ 4 \\ -2 \end{pmatrix}$ and $\overrightarrow{OD} = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix}$.

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(a) Determine $|\overrightarrow{AB} \times \overrightarrow{AC}|$ and use the result to explain why A, B and C are collinear. (5 marks)

Solution
$\overrightarrow{AB} = \begin{pmatrix} -3 \\ 1 \\ 7 \end{pmatrix} - \begin{pmatrix} -1 \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} -2 \\ -1 \\ 3 \end{pmatrix}, \quad \overrightarrow{AC} = \begin{pmatrix} 3 \\ 4 \\ -2 \end{pmatrix} - \begin{pmatrix} -1 \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ -6 \end{pmatrix}$ $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{pmatrix} -2 \\ -1 \\ 3 \end{pmatrix} \times \begin{pmatrix} 4 \\ 2 \\ -6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow |\overrightarrow{AB} \times \overrightarrow{AC}| = 0$ But $|\overrightarrow{AB} \times \overrightarrow{AC}| = |\overrightarrow{AB}| \cdot |\overrightarrow{AC}| \cdot \sin \theta$ and since $|\overrightarrow{AB}| \neq 0$ and $|\overrightarrow{AC}| \neq 0$ then $\sin \theta = 0 \Rightarrow \theta = 0$.   Hence vectors are parallel as the angle between them is zero, and as A is a point in common, then A, B and C are collinear.
Specific behaviours
✓ vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ ✓ cross product   ✓ magnitude of cross product   ✓ reasons that angle between vectors is zero   ✓ explains collinearity

(b) Determine the Cartesian equation of the plane containing all four points. (3 marks)

Solution
$\overrightarrow{AD} = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} - \begin{pmatrix} -1 \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}$ $\overrightarrow{AB} \times \overrightarrow{AD} = \begin{pmatrix} -2 \\ -1 \\ 3 \end{pmatrix} \times \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} = \begin{pmatrix} 6 \\ 3 \\ 5 \end{pmatrix}$ $\begin{pmatrix} 6 \\ 3 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} = 20$ $6x + 3y + 5z = 20$
Specific behaviours
✓ direction vector using D ✓ normal using cross product   ✓ Cartesian equation

Question 7 **(6 marks)**

The equation of line L is

$$\frac{x-2}{2}=\frac{y+1}{-3}=\frac{z-1}{6}.$$

(a) Determine the vector equation of the line in the form $\mathbf{r}=\mathbf{a}+\lambda\mathbf{b}$. (2 marks)

(b) The diameter of sphere S is the segment of line L between $x=2$ and $x=6$. Determine the equation of the sphere. (4 marks)

Question 7 **(6 marks)**

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$$\frac{x-2}{2}=\frac{y+1}{-3}=\frac{z-1}{6}.$$

(a) Determine the vector equation of the line in the form $\mathbf{r}=\mathbf{a}+\lambda\mathbf{b}$. (2 marks)

Solution
$\mathbf{r}=\begin{pmatrix}2\\-1\\1\end{pmatrix}+\lambda\begin{pmatrix}2\\-3\\6\end{pmatrix}$
Specific behaviours
✓ vector $\mathbf{a}$ ✓ vector $\mathbf{b}$

(b) The diameter of sphere S is the segment of line L between $x=2$ and $x=6$. Determine the equation of the sphere. (4 marks)

Solution
Using i-coefficient of line, $x=2\Rightarrow\lambda=0$ and $x=6\Rightarrow\lambda=2$. Hence centre of sphere when $\lambda=1$ at $(4,-4,7)$ Radius $=\left\lvert\begin{pmatrix}2\\-3\\6\end{pmatrix}\right\rvert=\sqrt{4+9+36}=7.$ Equation: $\left\lvert\mathbf{r}-\begin{pmatrix}4\\-4\\7\end{pmatrix}\right\rvert=7$ or $(x-4)^2+(y+4)^2+(z-7)^2=49$
Specific behaviours
✓ identifies location of centre ✓ identifies vector representing radius ✓ correct radius ✓ correct equation in either form

Question 4 **(6 marks)**

(a) Solve this system of equations. (3 marks)

$$
\begin{aligned}
x + y + 3z &= 10 \\
2x - y + z &= 8 \\
4x + y - z &= 4
\end{aligned}
$$

(b) Determine the value of constant a so that the following system of equations does not have a unique solution and give a brief geometric interpretation of the system of equations with this value. (3 marks)

$$
\begin{aligned}
x + y + 3z &= 10 \\
2x - y + z &= 8 \\
ax + y - z &= 4
\end{aligned}
$$

Question 4 **(6 marks)**

(a) Solve this system of equations. (3 marks)

$$x + y + 3z = 10$$
$$2x - y + z = 8$$
$$4x + y - z = 4$$

Solution
$Eq(2) + Eq(3) \Rightarrow\ 6x = 12 \Rightarrow x = 2$ $Eq(1) + Eq(2) \Rightarrow 3(2) + 4z = 18 \Rightarrow z = 3$ $2 + y + 3(3) = 10 \Rightarrow y = -1$ $x = 2, \qquad y = -1, \qquad z = 3$
Specific behaviours
✓ eliminates one variable using appropriate technique ✓ value of one variable ✓ values of second and third variables

(b) Determine the value of constant a so that the following system of equations does not have a unique solution and give a brief geometric interpretation of the system of equations with this value. (3 marks)

$$x + y + 3z = 10$$
$$2x - y + z = 8$$
$$ax + y - z = 4$$

Solution
$Eq(2) + Eq(3) \Rightarrow\ (2 + a)x = 12$ $a = -2\ \ (\text{Since } 0x = 12 \text{ impossible})$ Two parallel planes cut by the third non-parallel plane.
Specific behaviours
✓ adds second and third equations ✓ states value of a ✓ indicates parallel planes

Question 5 **(10 marks)**

The points A, B and C have position vectors $(1, 0, -2)$, $(b, -2, 1)$ and $(2, -1, 0)$ respectively.

(a) Determine the vector equation for the line through A and C. (2 marks)

(b) Determine, in terms of b, the Cartesian equation of the plane containing A, B and C. (5 marks)

(c) The line with equation $\mathbf{r} = (3, -5, 6) + \mu(2, q, -12)$ is perpendicular to the plane containing A, B and C. Determine the values of the constants b and q. (3 marks)

Question 5 **(10 marks)**

The points A, B and C have position vectors $(1, 0, -2)$, $(b, -2, 1)$ and $(2, -1, 0)$ respectively.

(a) Determine the vector equation for the line through A and C. (2 marks)

Solution
$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = (1, -1, 2)$ $\mathbf{r} = (1, 0, -2) + \lambda(1, -1, 2)$
Specific behaviours
✓ direction of line ✓ vector equation

(b) Determine, in terms of b, the Cartesian equation of the plane containing A, B and C. (5 marks)

Solution
$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = (b - 1, -2, 3)$ $\mathbf{n} = \overrightarrow{AB} \times \overrightarrow{AC} = \begin{pmatrix} b-1 \\ -2 \\ 3 \end{pmatrix} \times \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ 5-2b \\ 3-b \end{pmatrix}$ $\mathbf{n} \cdot \overrightarrow{OA} = -1 + 2b - 6 = 2b - 7$ $-x + (5 - 2b)y + (3 - b)z = 2b - 7$
Specific behaviours
✓ second vector in plane ✓ uses cross product ✓ normal vector ✓ constant ✓ Cartesian equation

(c) The line with equation $\mathbf{r} = (3, -5, 6) + \mu(2, q, -12)$ is perpendicular to the plane containing A, B and C. Determine the values of the constants b and q. (3 marks)

Solution
$(2, q, -12) = k(-1, 5 - 2b, 3 - b) \Rightarrow k = -2$ $-12 = -2(3 - b) \Rightarrow b = -3$ $q = -2\big(5 - 2(-3)\big) = -22$
Specific behaviours
✓ indicates normal and line direction parallel ✓ value of b ✓ value of q

Question 2 **(7 marks)**

A sphere has equation $2x^2 + 2y^2 + 2z^2 - 4x + 8y + 6z + 2 = 0$.

(a) Determine the coordinates of the centre and the radius of the sphere. (4 marks)

(b) Determine the vector equation of the straight line that passes through the points on the sphere where $y = -2$ and $z = 0$. (3 marks)

Question 2 **(7 marks)**

A sphere has equation $2x^2+2y^2+2z^2-4x+8y+6z+2=0$.

(a) Determine the coordinates of the centre and the radius of the sphere. (4 marks)

Solution
$x^2+y^2+z^2-2x+4y+3z+1=0$ $(x-1)^2+(y+2)^2+(z+1.5)^2=-1+1+4+1.5^2$ $=\dfrac{16+9}{4}=\left(\dfrac{5}{2}\right)^2$ Radius is 2.5 units Centre at (1, -2, -1.5)
Specific behaviours
✓ divides both sides by 2 ✓ completes the squares ✓ states the radius ✓ states centre

(b) Determine the vector equation of the straight line that passes through the points on the sphere where $y=-2$ and $z=0$. (3 marks)

Solution
$x^2+4-2x-8+1=0$ $x^2-2x-3=0$ $(x+1)(x-3)=0 \Rightarrow x=-1,\ 3$ Point on line is (3, -2, 0) Direction of line is $\langle 1,0,0\rangle$ $\mathbf{r}=3\mathbf{i}-2\mathbf{j}+\lambda\mathbf{i}=(3+\lambda)\mathbf{i}-2\mathbf{j}$
Specific behaviours
✓ determines x-coordinates of points on sphere ✓ states direction of line ✓ states vector equation of line

Question 4 **(7 marks)**

Consider the following system of equations, where k is a real constant.

$$\begin{aligned} x + 2y + z &= 3 \\ 2x - y - 3z &= k \\ x + 3y + kz &= 6 \end{aligned}$$

(a) Solve the system of equations when $k = 1$. (3 marks)

(b) Show that no value of k exists for the system of equations to represent three planes intersecting in a single straight line. (4 marks)

Question 4 **(7 marks)**

Consider the following system of equations, where k is a real constant.

$$\begin{aligned} x+2y+z&=3 \\ 2x-y-3z&=k \\ x+3y+kz&=6 \end{aligned}$$

(a) Solve the system of equations when $k=1$. (3 marks)

Solution
$x+2y+z=3 \quad (1)$ $2x-y-3z=1 \quad (2)$ $x+3y+z=6 \quad (3)$ $y=3 \quad (3)-(1)$ $x+z=-3$ $2x-3z=4$ $5x=-5 \Rightarrow x=-1,\ z=-2$ $x=-1,\ y=3,\ z=-2$
Specific behaviours
✓ eliminates x and z to find y ✓ eliminates and solves for another variable ✓ states values of all three variables

(b) Show that no value of k exists for the system of equations to represent three planes intersecting in a single straight line. (4 marks)

Solution
$\begin{matrix} 2(1)-(2)\to(2) \\ (3)-(1)\to(3) \end{matrix} : \begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & 5 & 5 & 6-k \\ 0 & 1 & k-1 & 3 \end{bmatrix}$ $5(3)-(2)\to(3) : \begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & 5 & 5 & 6-k \\ 0 & 0 & 5k-10 & k+9 \end{bmatrix}$ For infinite solns require $5k-10=0 \Rightarrow k=2$ and $k+9=0 \Rightarrow k=-9$. Hence no value of k exists.
Specific behaviours
✓ reduces second and third rows in initial matrix ✓ reduces third row in second matrix ✓ indicates condition for planes to intersect in single straight line ✓ shows that no value of k exists

Question 5 **(8 marks)**

(a) Determine the vector equation of the plane that contains the points A(1, -1, 2), B(2, 1, 0) and C(3, -1, 1). (4 marks)

(b) Plane Π has equation $x + 2y - z = 3$. Line L is perpendicular to Π and passes through the point (1, -6, 4). Determine where line L intersects plane Π. (4 marks)

Question 5 **(8 marks)**

(a) Determine the vector equation of the plane that contains the points A(1, -1, 2), B(2, 1, 0) and C(3, -1, 1). (4 marks)

Solution
$\mathbf{AB} = \langle 1, 2, -2 \rangle$
$\mathbf{AC} = \langle 2, 0, -1 \rangle$
$\mathbf{AC} \times \mathbf{AB} = \langle 2, 3, 4 \rangle$
$\mathbf{r} \cdot \langle 2, 3, 4 \rangle = \langle 2, 1, 0 \rangle \cdot \langle 2, 3, 4 \rangle$
$\mathbf{r} \cdot \langle 2, 3, 4 \rangle = 7$
Specific behaviours
✓ finds two vectors in plane ✓ calculates cross product of two vectors ✓ substitutes into vector equation of plane ✓ simplifies vector equation

(b) Plane Π has equation $x + 2y - z = 3$. Line L is perpendicular to Π and passes through the point (1, -6, 4). Determine where line L intersects plane Π. (4 marks)

Solution
$\mathbf{r}_P \cdot \langle 1, 2, -1 \rangle = 3$
$\mathbf{r}_L = \langle 1, -6, 4 \rangle + t \langle 1, 2, -1 \rangle$
$\langle 1 + t, 2t - 6, 4 - t \rangle \cdot \langle 1, 2, -1 \rangle = 3$ $1 + t + 4t - 12 - 4 + t = 3$ $6t = 18 \Rightarrow t = 3$
$\mathbf{r} = \langle 1, -6, 4 \rangle + 3 \langle 1, 2, -1 \rangle$ $= \langle 4, 0, 1 \rangle \Rightarrow$ At $(4, 0, 1)$
Specific behaviours
✓ writes vector equation of plane ✓ writes vector equation of line through point ✓ substitutes line into plane and solves for t ✓ determines coordinates of point

Question 7 **(10 marks)**

Particle A has position vector given by $\mathbf{r} = 3\cos(t)\mathbf{i} + 3\sin(t)\mathbf{j}$, where t is the time in seconds.

(a) Show that the path of the particle is circular. (2 marks)

Particle B is stationary, with position vector $3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$.

(b) Determine an expression for the distance between particles A and B in terms of t.
(2 marks)

(c) Determine the position vector of the A when it is (i) nearest and (ii) furthest from B.
(6 marks)

Question 7 **(10 marks)**

Particle A has position vector given by $\mathbf{r} = 3\cos(t)\mathbf{i} + 3\sin(t)\mathbf{j}$, where t is the time in seconds.

(a) Show that the path of the particle is circular. (2 marks)

Solution
$x = 3\cos t,\ y = 3\sin t \ \Rightarrow\ \dfrac{x}{3} = \cos t,\ \dfrac{y}{3} = \sin t$ $\left(\dfrac{x}{3}\right)^2 + \left(\dfrac{y}{3}\right)^2 = 1 \ \Rightarrow\ x^2 + y^2 = 3^2$, circle centre (0,0), radius 3.
Specific behaviours
✓ converts to Cartesian form   ✓ states centre and radius

Particle B is stationary, with position vector $3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$.

(b) Determine an expression for the distance between particles A and B in terms of t. (2 marks)

Solution
$\lvert\mathbf{BA}\rvert = \lvert\mathbf{OA} - \mathbf{OB}\rvert = \sqrt{(3\cos t - 3)^2 + (3\sin t - 4)^2 + (-5)^2}$
Specific behaviours
✓ determines vector **BA** (or **AB**)   ✓ states magnitude of vector

(c) Determine the position vector of the A when it is (i) nearest and (ii) furthest from B. (6 marks)

Solution
Let S be square of distance between particles: $\dfrac{dS}{dt} = 2(-3\sin t)(3\cos t - 3) + 2(3\cos t)(3\sin t - 4)$ $\dfrac{dS}{dt} = 0 \ \Rightarrow\ -\sin t(3\cos t - 3) + \cos t(3\sin t - 4) = 0$ $3\sin t - 4\cos t = 0$ $\tan t = \dfrac{4}{3} \ \Rightarrow\ \sin t = \pm\dfrac{4}{5},\ \cos t = \pm\dfrac{3}{5}$ Nearest: $\mathbf{OA} = 3\left(\dfrac{3}{5}\right)\mathbf{i} + 3\left(\dfrac{4}{5}\right)\mathbf{j} = \dfrac{9}{5}\mathbf{i} + \dfrac{12}{5}\mathbf{j}$ Furthest: $\mathbf{OA} = -\dfrac{9}{5}\mathbf{i} - \dfrac{12}{5}\mathbf{j}$
Specific behaviours
✓ differentiates S   ✓ simplifies and equates derivative to 0   ✓ determines solution for tan t   ✓ derives possible values for sin t and cos t   ✓ determines nearest position   ✓ determines furthest position