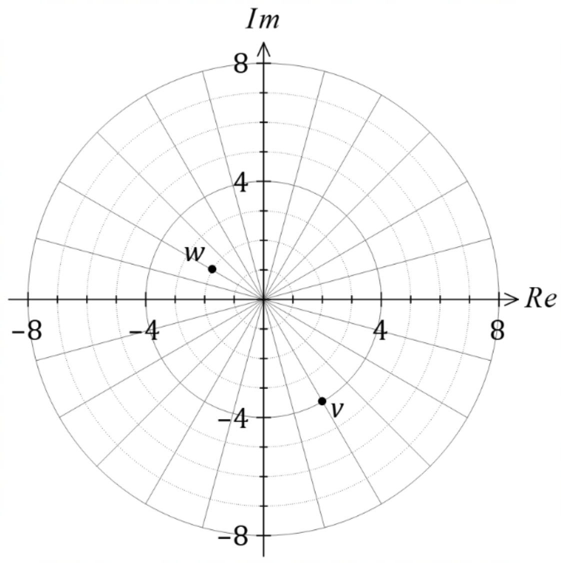
Question 1 **(7 marks)**

The diagram shows the complex numbers v and w in the Argand plane.



(a) Express v in

(i) polar form. (1 mark)

(ii) Cartesian form. (1 mark)

(b) Plot and label the following complex numbers on the diagram above:

(i) $z_1 = iv$. (1 mark)

(ii) $z_2 = vw$. (2 marks)

(iii) $z_3 = v - 2w$. (2 marks)

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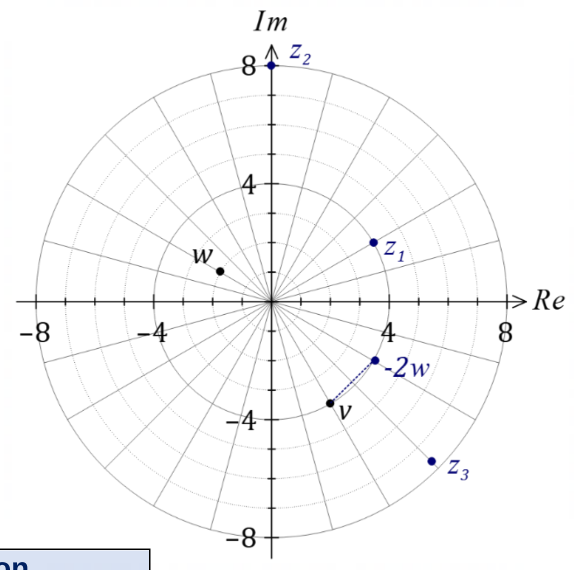


(a) Express v in

(i) polar form. (1 mark)

Solution
$v = 4\,\text{cis}\left(-\frac{\pi}{3}\right)$
Specific behaviours
✓ correct expression

(ii) Cartesian form. (1 mark)

Solution
$v = 2 - 2\sqrt{3}i$
Specific behaviours
✓ correct expression

(b) Plot and label the following complex numbers on the diagram above:

(i) $z_1 = iv$. (1 mark)

Solution
Rotates v by 90° about O.
Specific behaviours
✓ plots correctly

(ii) $z_2 = vw$. (2 marks)

Solution
$\lvert vw\rvert = 2 \times 4 = 8, \qquad \arg(vw) = -\frac{\pi}{3} + \frac{5\pi}{6} = \frac{\pi}{2}$
Specific behaviours
✓ correct argument ✓ correct modulus

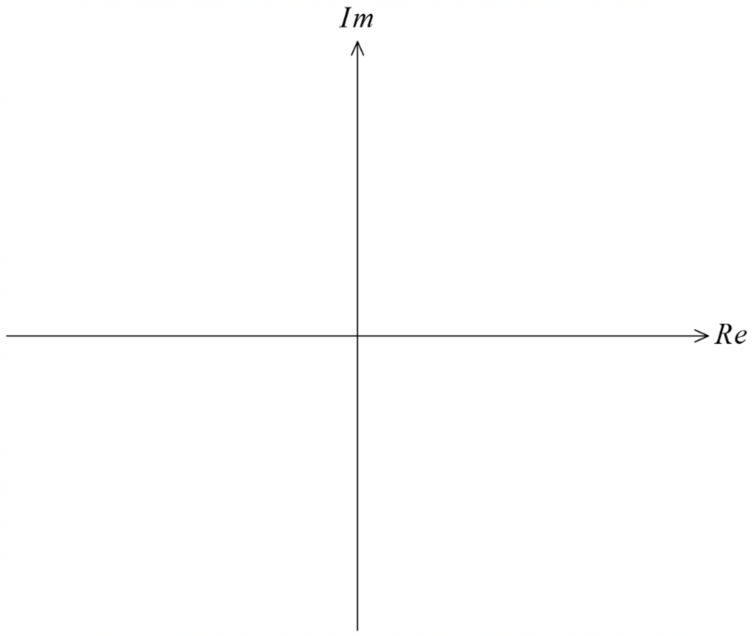
(iii) $z_3 = v - 2w$. (2 marks)

Solution
Use vector addition of v and $-2w$, so that $\arg(v - 2w) = -\frac{\pi}{4}, \qquad \lvert v - 2w\rvert \approx 7.7$
Specific behaviours
✓ correct argument ✓ correct modulus

Question 6 **(7 marks)**

(a) Given that $w = \dfrac{\sqrt{3} - i}{1 + i}$, determine the modulus and argument of w. (3 marks)

(b) Sketch the subset of the complex plane determined by $-2|z| = z + \overline{z} - 4$. (4 marks)

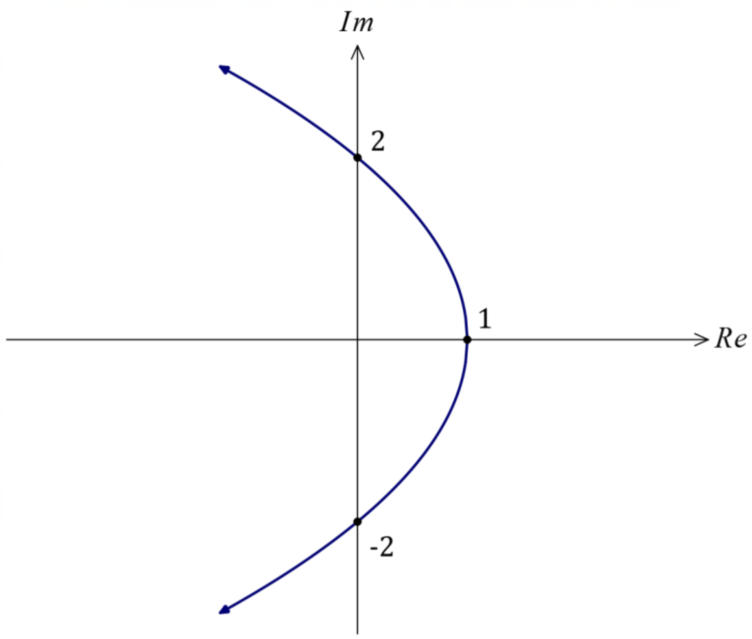


Question 6 **(7 marks)**

(a) Given that $w = \dfrac{\sqrt{3} - i}{1 + i}$, determine the modulus and argument of w. (3 marks)

Solution
$u = \sqrt{3} - i = 2\,\text{cis}\left(-\dfrac{\pi}{6}\right), \qquad v = 1 + i = \sqrt{2}\,\text{cis}\left(\dfrac{\pi}{4}\right)$ $\lvert w \rvert = \dfrac{2}{\sqrt{2}} = \sqrt{2}, \qquad \arg w = \arg u - \arg v = -\dfrac{\pi}{6} - \dfrac{\pi}{4} = -\dfrac{5\pi}{12}$
Specific behaviours
✓ expresses numerator and denominator in polar form   ✓ modulus   ✓ argument

(b) Sketch the subset of the complex plane determined by $-2|z| = z + \bar{z} - 4$. (4 marks)



Solution
Let $z = x + iy$ so that $-2\lvert x + iy \rvert = x + iy + x - iy - 4$ $\lvert x + iy \rvert = -x + 2$ $x^2 + y^2 = x^2 - 4x + 4$ $y^2 = 4 - 4x$ $x = 1 - \dfrac{y^2}{4}$
Specific behaviours
✓ uses Cartesian form to eliminate i ✓ obtains relationship   ✓ sketches parabolic curve   ✓ correct vertex and other intercepts

Question 1 **(6 marks)**

The polynomial $f(z) = g(z) \times h(z)$, where $h(z) = z^2 - 6z + 10$.

(a) Show that $z - 3 - i$ is a factor of $h(z)$. (2 marks)

(b) Given that $f(z) = z^4 - 8z^3 + 28z^2 - 56z + 60$, solve $f(z) = 0$, giving all solutions in Cartesian form. (4 marks)

Question 1 **(6 marks)**

The polynomial $f(z) = g(z) \times h(z)$, where $h(z) = z^2 - 6z + 10$.

(a) Show that $z - 3 - i$ is a factor of $h(z)$. (2 marks)

Solution
$h(3+i) = (3+i)^2 - 6(3+i) + 10$ $= 9 + 6i - 1 - 18 - 6i + 10$ $= 0$
Specific behaviours
✓ substitutes $h(3+i)$ ✓ fully expands all terms and then simplifies

(b) Given that $f(z) = z^4 - 8z^3 + 28z^2 - 56z + 60$, solve $f(z) = 0$, giving all solutions in Cartesian form. (4 marks)

Solution
Since $h(z)$ is a factor of $f(z)$ then $z = 3 + i$ and its conjugate $z = 3 - i$ will both be solutions. $f(z) = g(z)h(z)$ $z^4 - 8z^3 + 28z^2 - 56z + 60 = (z^2 + az + 6)(z^2 - 6z + 10)$ Comparing z coefficients, $-56 = 10a - 36 \Rightarrow a = -2 \Rightarrow g(z) = z^2 - 2z + 6$. $z^2 - 2z + 6 = 0$ $(z-1)^2 - 1 = -6$ $(z-1)^2 = 5i^2$ $z = 1 \pm \sqrt{5}i$ Hence $f(z) = 0$ when $z = 3 \pm i, z = 1 \pm \sqrt{5}i$.
Specific behaviours
✓ uses result from part (a) to state two solutions ✓ determines $g(z)$ ✓ one correct solution to $g(z) = 0$ ✓ states all correct solutions

Question 2 **(5 marks)**

(a) Express the complex number $\dfrac{8}{1-\sqrt{3}i}$ in the form $r\,\text{cis}\,\theta\,, -\pi < \theta \le \pi$. (3 marks)

(b) When $u = 4\,\text{cis}\left(\dfrac{\pi}{12}\right)$ and $v = 5\,\text{cis}\left(-\dfrac{\pi}{10}\right)$ determine

(i) $|uv^3|$. (1 mark)

(ii) $\arg(v \div u)$. (1 mark)

Question 2 **(5 marks)**

(a) Express the complex number $\dfrac{8}{1-\sqrt{3}i}$ in the form $r\operatorname{cis}\theta$, $-\pi < \theta \le \pi$. (3 marks)

Solution
$\dfrac{8}{1-\sqrt{3}i} \times \dfrac{1+\sqrt{3}i}{1+\sqrt{3}i} = \dfrac{8}{4}(1+\sqrt{3}i)$ $= 4\operatorname{cis}\left(\dfrac{\pi}{3}\right)$
Specific behaviours
✓ exposes real and imaginary parts   ✓ correct modulus   ✓ correct answer in polar form

(b) When $u = 4\operatorname{cis}\left(\dfrac{\pi}{12}\right)$ and $v = 5\operatorname{cis}\left(-\dfrac{\pi}{10}\right)$ determine

(i) $|uv^3|$. (1 mark)

Solution
$4 \times 5^3 = 500$
Specific behaviours
✓ correct value

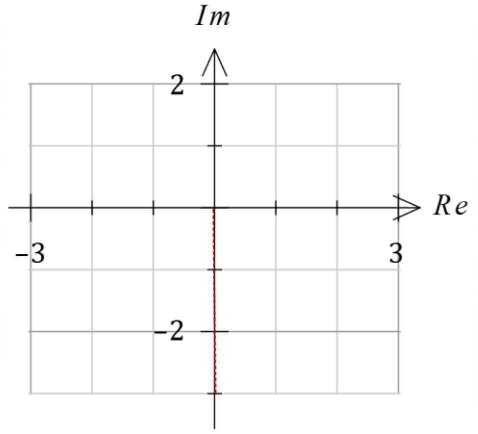
(ii) $\arg(v \div u)$. (1 mark)

Solution
$-\dfrac{\pi}{10} - \left(\dfrac{\pi}{12}\right) = \dfrac{(-6-5)\pi}{60} = -\dfrac{11\pi}{60}$
Specific behaviours
✓ correct value

Question 5 **(7 marks)**

Consider the complex number $z = -2 + 2i$.

(a) On the Argand diagram below, draw a line segment from the origin to z and from the origin to $z - 2\sqrt{2}i$. (2 marks)



(b) Determine the principal value of the argument of $z - 2\sqrt{2}i$. (3 marks)

(c) Determine the value of the modulus of $z - 2\sqrt{2}i$. (2 marks)

Question 5 **(7 marks)**

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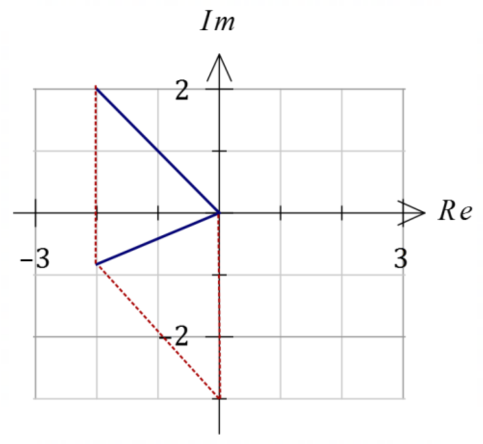
(a) On the Argand diagram below, draw a line segment from the origin to z and from the origin to $z - 2\sqrt{2}i$. (2 marks)



Solution
See diagram
Specific behaviours
✓ line O to z
✓ line O to $z - \sqrt{2}i$

(b) Determine the principal value of the argument of $z - 2\sqrt{2}i$. (3 marks)

Solution
$z = 2\sqrt{2}\,\mathrm{cis}\left({}^{3\pi}/_{4}\right)$. Using properties of rhombus, the line from O to $z - 2\sqrt{2}i$ bisects angle 2θ between Im axis and line from O to z: $$2\theta = {}^{3\pi}/_{4}, \qquad \theta = {}^{3\pi}/_{8}$$ Hence $\arg(z - \sqrt{2}i) = -{}^{\pi}/_{2} - {}^{3\pi}/_{8} = -{}^{7\pi}/_{8}$.
Specific behaviours
✓ indicates polar form of z ✓ uses properties of rhombus ✓ correct value

(c) Determine the value of the modulus of $z - 2\sqrt{2}i$. (2 marks)

Solution
$$\left\lvert z - 2\sqrt{2}i \right\rvert = \sqrt{2^2 + \left(2\sqrt{2} - 2\right)^2}$$ $$= \sqrt{16 - 8\sqrt{2}} = 2\sqrt{4 - 2\sqrt{2}}$$
Specific behaviours
✓ indicates lengths of suitable right triangle ✓ correct value, with some simplification

Question 6 **(7 marks)**

Let the complex number $v = \sqrt{2}\,\text{cis}\left(\frac{3\pi}{4}\right)$. Describe geometrically the locus of the complex number $z = x + iy$ in the Argand plane that is determined by the relation $|z - v^2| = \sqrt{2}|z - v|$.

Question 6 **(7 marks)**

Let the complex number $v = \sqrt{2}\,\text{cis}\left(\frac{3\pi}{4}\right)$. Describe geometrically the locus of the complex number $z = x + iy$ in the Argand plane that is determined by the relation $|z - v^2| = \sqrt{2}|z - v|$.

Solution
$v = \sqrt{2}\,\text{cis}\left(\frac{3\pi}{4}\right) = -1 + i$ $v^2 = 2\,\text{cis}\left(\frac{3\pi}{2}\right) = -2i$ $\lvert x + iy - (-2i)\rvert = \sqrt{2}\lvert x + iy - (-1 + i)\rvert$ $x^2 + (y+2)^2 = 2((x+1)^2 + (y-1)^2)$ $x^2 + y^2 + 4y + 4 = 2(x^2 + 2x + 1 + y^2 - 2y + 1)$ $x^2 + y^2 + 4y + 4 = 2x^2 + 4x + 2y^2 - 4y + 4$ $x^2 + 4x + y^2 - 8y = 0$ $(x+2)^2 - 4 + (y-4)^2 - 16 = 0$ $(x+2)^2 + (y-4)^2 = 20 = 2\sqrt{5}$ Hence the locus of z is a circle of radius $2\sqrt{5}$ units with centre at $-2 + 4i$.
Specific behaviours
✓ v in Cartesian form ✓ v^2 in Cartesian form ✓ uses $z = x + iy$ and modulus to eliminate i ✓ expands and simplifies equation ✓ factors squared terms ✓ describes locus as a circle ✓ states correct centre and radius of circle

Question 2 **(6 marks)**

Polynomial P is defined as $P(z) = z^4 - 4z^3 + 14z^2 - 36z + 45$.

(a) Show that $z - 3i$ is a factor of $P(z)$. (2 marks)

(b) Solve $P(z) = 0$, writing solutions in Cartesian form. (4 marks)

Question 2 **(6 marks)**

Polynomial P is defined as $P(z) = z^4 - 4z^3 + 14z^2 - 36z + 45$.

(a) Show that $z - 3i$ is a factor of $P(z)$. (2 marks)

Solution
$P(3i) = (3i)^4 - 4(3i)^3 + 14(3i)^2 - 36(3i) + 45$ $= 81 + 108i - 126 - 108i + 45 \quad ...(1)$ $= 0$
Specific behaviours
✓ correctly substitutes $z = 3i$ into $P(z)$ ✓ expands and simplifies terms to obtain (1) and deduces $P(3i) = 0$

(b) Solve $P(z) = 0$, writing solutions in Cartesian form. (4 marks)

Solution
Conjugate of above is another factor: $z + 3i$ Hence $(z + 3i)(z - 3i) = z^2 + 9$ is a factor. $z^4 - 4z^3 + 14z^2 - 36z + 45 = (z^2 + 9)(z^2 - 4z + 5)$ $z^2 - 4z + 5 = 0$ $(z - 2)^2 = -1$ $z = 2 \pm i$ Solutions: $z = \pm 3i, z = 2 \pm i$.
Specific behaviours
✓ indicates conjugate as second factor and obtains quadratic factor   ✓ obtains second quadratic factor   ✓ solves second quadratic   ✓ indicates all solutions

Question 6 **(7 marks)**

(a) Determine the complex cube roots of -1 in the form $a + bi$, where $a, b \in \mathbb{R}$. (3 marks)

(b) Let ω be a complex cube root of unity, $\operatorname{Im}\omega \neq 0$, so that $\omega^3 - 1 = 0$.

(i) Show that $(\omega - 1)(\omega^2 + \omega + 1) = \omega^3 - 1$ and hence explain why $\omega^2 + \omega + 1 = 0$. (2 marks)

(ii) Simplify $(2 + 5\omega)(2 + 5\omega^2)$. (2 marks)

Question 6 **(7 marks)**

(a) Determine the complex cube roots of -1 in the form $a + bi$, where $a, b \in \mathbb{R}$. (3 marks)

Solution
$z^3 = -1 = \text{cis}(\pi + 2n\pi), n \in \mathbb{Z}$ $z = \text{cis}\left(\dfrac{\pi + 2n\pi}{3}\right)$ $z_0 = \text{cis}\left(\dfrac{\pi}{3}\right) = \dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i$ $z_1 = \text{cis}(\pi) = -1$ $z_2 = \text{cis}\left(-\dfrac{\pi}{3}\right) = \dfrac{1}{2} - \dfrac{\sqrt{3}}{2}i$
Specific behaviours
✓ expresses -1 in polar form (or sketch) ✓ z_0 or z_2 ✓ all roots in required form

(b) Let ω be a complex cube root of unity, $\text{Im}\,\omega \neq 0$, so that $\omega^3 - 1 = 0$.

(i) Show that $(\omega - 1)(\omega^2 + \omega + 1) = \omega^3 - 1$ and hence explain why $\omega^2 + \omega + 1 = 0$. (2 marks)

Solution
$(\omega - 1)(\omega^2 + \omega + 1) = \omega^3 + \omega^2 + \omega - \omega^2 - \omega - 1$ $= \omega^3 - 1$ Since $\omega^3 - 1 = 0$, but $\text{Im}\,\omega \neq 0$ and so $\omega - 1 \neq 0$, then $\omega^2 + \omega + 1 = 0$.
Specific behaviours
✓ fully expands factors ✓ explanation using factors

(ii) Simplify $(2 + 5\omega)(2 + 5\omega^2)$. (2 marks)

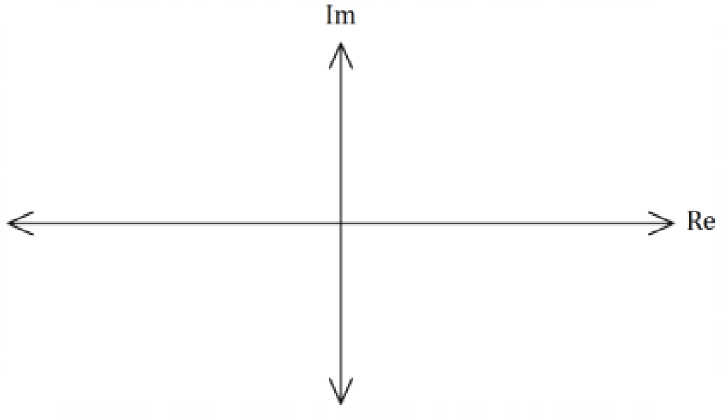
Solution
$(2 + 5\omega)(2 + 5\omega^2) = 4 + 10\omega + 10\omega^2 + 25\omega^3$ $= -6 + 10(1 + \omega + \omega^2) + 25$ $= 19$
Specific behaviours
✓ expands and uses $\omega^3 = 1$ and $\omega^2 + \omega + 1 = 0$ ✓ correct value

Question 8 **(7 marks)**

The locus L_1 of the complex number $z = x + iy$ has equation $|z - 6| = 2|z + 6|$.

(a) Show that L_1 is a circle with equation $x^2 + y^2 + 20x + 36 = 0$. (2 marks)

(b) Sketch L_1 on an Argand diagram. (2 marks)



Another locus L_2 has equation $w \cdot \bar{z} + \overline{w} \cdot z = 0$, where $w = 4 + 3i$.

(c) Show that L_2 is a tangent to L_1. (3 marks)

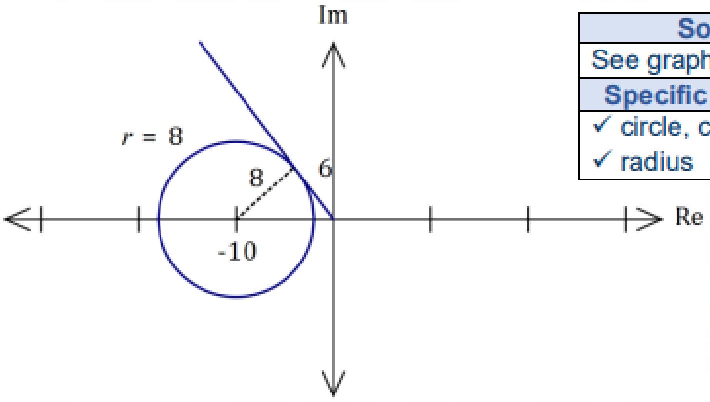
Question 8 **(7 marks)**

The locus L_1 of the complex number $z = x + iy$ has equation $|z - 6| = 2|z + 6|$.

(a) Show that L_1 is a circle with equation $x^2 + y^2 + 20x + 36 = 0$. (2 marks)

Solution
$(x-6)^2 + y^2 = 4((x+6)^2 + y^2)$ $x^2 - 12y + 36 + y^2 = 4x^2 + 48y + 144 + 4y^2$ $3x^2 + 3y^2 + 60y + 108 = 0$ $x^2 + y^2 + 20y + 36 = 0$
Specific behaviours
✓ equates square of magnitudes ✓ fully expands before simplification

(b) Sketch L_1 on an Argand diagram. (2 marks)



Solution
See graph
Specific behaviours
✓ circle, centre ✓ radius

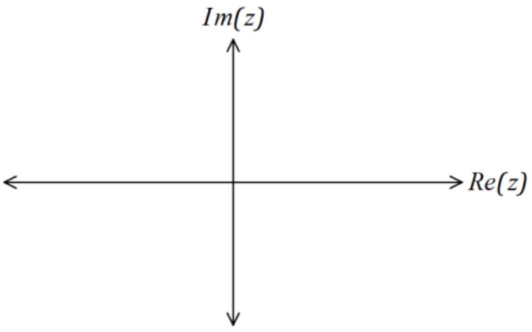
Another locus L_2 has equation $w \cdot \bar{z} + \bar{w} \cdot z = 0$, where $w = 4 + 3i$.

(c) Show that L_2 is a tangent to L_1. (3 marks)

Solution
$(4+3i)(x-iy) + (4-3i)(x+iy) = 0$ $4x - 4iy + 3ix + 3y + 4x + 4iy - 3ix + 3y = 0$ $6y = -8x$ $y = -\dfrac{4x}{3}$ From the Argand diagram, the gradient of the tangent to L_1 through the origin has slope $-\frac{8}{6} = -\frac{4}{3}$ and hence L_2 is a tangent to L_1.
Specific behaviours
✓ Cartesian equation of L_2 ✓ indicates tangent to circle through origin on Argand diagram ✓ derives gradient from geometry

Question 1 **(7 marks)**

(a) Locate the roots of the complex equation $z^5 - 1 = 0$ in the Argand plane below. (3 marks)



(b) State the sum of all the roots of the complex equation $z^5 - 1 = 0$. (1 mark)

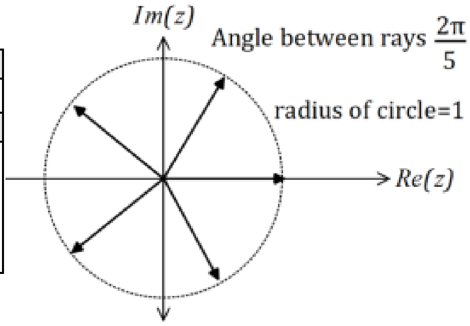
(c) Let u be any 5^{th} root of unity, where $\operatorname{Im} u \neq 0$.

Show that $(1+u)^2(1+u^3) = 1 + u + u^4$. (3 marks)

Question 1 **(7 marks)**

(a) Locate the roots of the complex equation $z^5 - 1 = 0$ in the Argand plane below. (3 marks)

Solution
See graph
Specific behaviours
✓ 5 evenly spaced rays ✓ indicates ray spacing ✓ indicates ray length *(May use rays or points)*



(b) State the sum of all the roots of the complex equation $z^5 - 1 = 0$. (1 mark)

Solution
$\text{Sum} = 0$
Specific behaviours
✓ states sum

(c) Let u be any 5th root of unity, where $\operatorname{Im} u \neq 0$.

Show that $(1+u)^2(1+u^3) = 1 + u + u^4$. (3 marks)

Solution
$\begin{aligned}(1+u)^2(1+u^3) &= (1+2u+u^2)(1+u^3) \\ &= 1+2u+u^2+u^3+2u^4+u^5 \\ &= (1+u+u^2+u^3+u^4)+u^5+u+u^4 \\ &= 0+1+u+u^4 \\ &= 1+u+u^4\end{aligned}$
Specific behaviours
✓ expands ✓ uses sum of roots ✓ uses $u^5 = 1$ and simplifies

Question 3 **(7 marks)**

Consider $f(z) = 3z^3 + 2z^2 + 15z + 10$, where z is a complex number.

(a) Determine, with reasons, which of the following are factors of $f(z)$.

(i) $z + 2$. (2 marks)

(ii) $z - \sqrt{5}i$. (2 marks)

(b) Solve the equation $f(z) = 0$. (3 marks)

Question 3 **(7 marks)**

Consider $f(z) = 3z^3 + 2z^2 + 15z + 10$, where z is a complex number.

(a) Determine, with reasons, which of the following are factors of $f(z)$.

(i) $z + 2$. (2 marks)

Solution
$f(-2) = -24 + 8 - 30 + 10 = -36$ $z + 2$ is NOT a factor since $f(-2) \neq 0$
Specific behaviours
✓ evaluates $f(-2)$ with evidence ✓ reason

(ii) $z - \sqrt{5}i$. (2 marks)

Solution
$f(\sqrt{5}i) = 3 \times 5\sqrt{5}i^3 + 2 \times 5i^2 + 15\sqrt{5}i + 10$ $= -15\sqrt{5}i - 10 + 15\sqrt{5}i + 10$ $= 0$ $z - \sqrt{5}i$ is a factor since $f(\sqrt{5}i) = 0$
Specific behaviours
✓ evaluates $f(\sqrt{5}i)$ with evidence ✓ reason

(b) Solve the equation $f(z) = 0$. (3 marks)

Solution
Second factor is $z + \sqrt{5}i$ $f(z) = (z - \sqrt{5}i)(z + \sqrt{5}i)(az + b)$ $= (z^2 + 5)(az + b)$ $= (z^2 + 5)(3z + 2)$ $f(z) = 0 \Rightarrow z = \pm\sqrt{5}i, \quad z = -\frac{2}{3}$
Specific behaviours
✓ indicates conjugate factor ✓ factorises $f(z)$ ✓ all three solutions

Question 7 **(5 marks)**

The complex numbers u and v satisfy the equations $u - v = 2i$ and $uv = 10$.

Solve the equations for u and v, giving your solution(s) in the form $x + yi$, where x and y are real.

Question 7 **(5 marks)**

The complex numbers u and v satisfy the equations $u - v = 2i$ and $uv = 10$.

Solve the equations for u and v, giving your solution(s) in the form $x + yi$, where x and y are real.

Solution
$u = v + 2i \Rightarrow v(v + 2i) = 10$ $v^2 + 2iv = 10$ $(v + i)^2 - i^2 = 10$ $(v + i)^2 = 9$ $v = \pm 3 - i$ $v = 3 - i$, $u = 3 + i$ or $v = -3 - i$, $u = -3 + i$
Specific behaviours
✓ eliminates u or v to form quadratic ✓ completes square ✓ solves quadratic correctly ✓ states one pair of solutions ✓ states second pairs of solutions

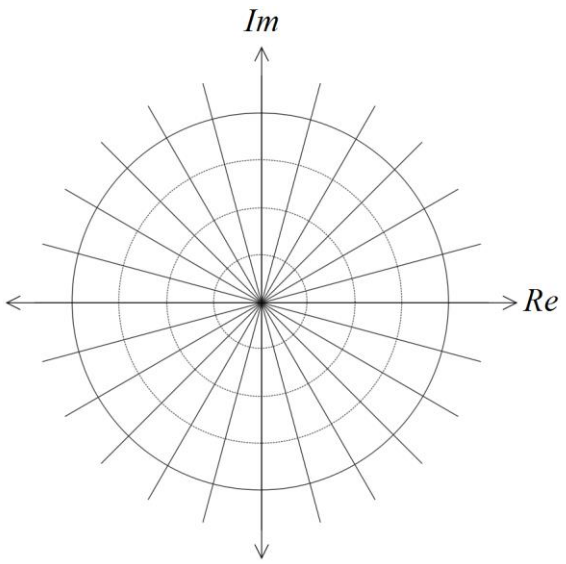
Question 3 **(8 marks)**

(a) Let $z = 2\cos\left(\frac{2\pi}{3}\right) + 2i\sin\left(\frac{2\pi}{3}\right)$.

(i) Express z in Cartesian form. (2 marks)

(ii) Determine z^5 in Cartesian form. (3 marks)

(b) If $w^3 + 1 = 0$, sketch the location of all roots of this equation on the axes below. (3 marks)



Question 3 **(8 marks)**

(a) Let $z = 2\cos\left(\frac{2\pi}{3}\right) + 2i\sin\left(\frac{2\pi}{3}\right)$.

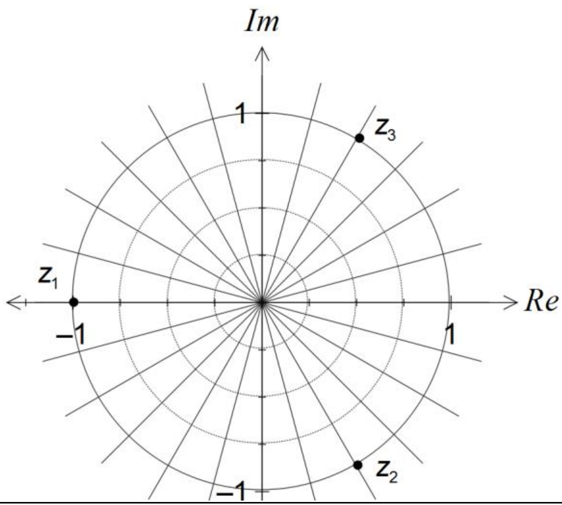
(i) Express z in Cartesian form. (2 marks)

Solution
$z = -1 + \sqrt{3}i$
Specific behaviours
✓ real part ✓ imaginary part

(ii) Determine z^5 in Cartesian form. (3 marks)

Solution
$z^5 = 2^5 cis\left(\frac{2\pi}{3} \times 5\right)$ $= 32cis\left(\frac{10\pi}{3}\right)$ $= 16 \times 2cis\left(-\frac{2\pi}{3}\right)$ $= 16 \times \overline{z}$ $= -16 - 16\sqrt{3}i$
Specific behaviours
✓ uses polar form to determine modulus ✓ uses polar form to determine argument $-\pi < \theta \le \pi$ ✓ converts to Cartesian form

(b) If $w^3 + 1 = 0$, sketch the location of all roots of this equation on the axes below. (3 marks)



Solution
See diagram - evenly spaced points on circle
Specific behaviours
✓ Adds scale to show real root at -1 ✓ Shows second root third way around circle ✓ Shows third root as conjugate of second

Question 5 **(7 marks)**

Consider the function $f(z) = z^4 + 4z^3 + 10z^2 + 20z + 25$.

(a) Determine the remainder when $f(z)$ is divided by $z - i$. (1 mark)

(b) Show that $z - \sqrt{5}i$ is a factor of f. (2 marks)

(c) Solve $f(z) = 0$. (4 marks)

Question 5 **(7 marks)**

Consider the function $f(z) = z^4 + 4z^3 + 10z^2 + 20z + 25$.

(a) Determine the remainder when $f(z)$ is divided by $z - i$. (1 mark)

Solution
$f(i) = i^4 + 4i^3 + 10i^2 + 20i + 25$ $= 1 - 4i - 10 + 20i + 25$ $= 16 + 16i$
Specific behaviours
✓ correct remainder

(b) Show that $z - \sqrt{5}i$ is a factor of f. (2 marks)

Solution
$f(\sqrt{5}i) = (\sqrt{5}i)^4 + 4(\sqrt{5}i)^3 + 10(\sqrt{5}i)^2 + 20\sqrt{5}i + 25$ $= 25 + 4(-5\sqrt{5}) + 10(-5) + 20\sqrt{5}i + 25$ $= 25 - 20\sqrt{5}i - 50 + 20\sqrt{5}i + 25$ $= 0$
Specific behaviours
✓ correctly evaluates powers of $\sqrt{5}i$ ✓ simplifies to show line that clearly sums to zero

(c) Solve $f(z) = 0$. (4 marks)

Solution
Since $z - \sqrt{5}i$ is a factor then $z + \sqrt{5}i$ must also be a factor. $z^4 + 4z^3 + 10z^2 + 20z + 25 = (z + \sqrt{5}i)(z - \sqrt{5}i)q(z)$ $= (z^2 + 5)q(z)$ $= (z^2 + 5)(z^2 + 4z + 5)$ $z^2 + 4z + 5 = 0$ $(z + 2)^2 - 4 = -5$ $(z + 2)^2 = -1 = i^2$ $z + 2 = \pm i$ $z = -2 \pm i$ Hence $f(z) = 0$ when $z = \pm\sqrt{5}i,\ z = -2 \pm i$.
Specific behaviours
✓ uses complex conjugate to obtain one quadratic factor of $f(z)$ ✓ determines second quadratic factor $q(z)$ ✓ shows use of appropriate method to solve $q(z) = 0$ ✓ states all solutions