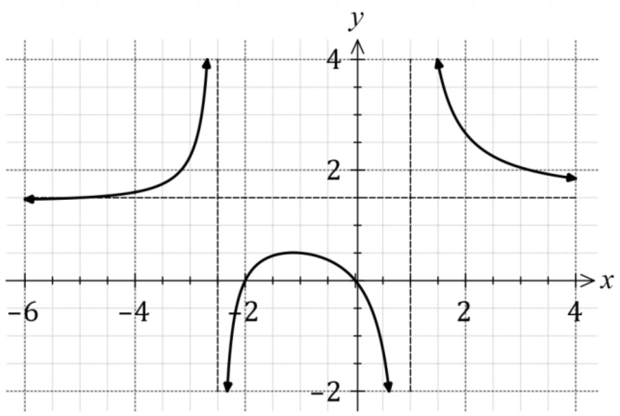
Question 11 **(8 marks)**

(a) Determine the equations of all asymptotes of the graph of $y = f(x)$ when

(i) $f(x) = \dfrac{1 + 2x^2}{x(1 - 3x)}$. (2 marks)

(ii) $f(x) = \dfrac{x^2 + 4}{x - 5}$. (2 marks)

(b) The graph of $y = g(x)$ is shown in the diagram, together with its three asymptotes.



The defining rule is given by

$$g(x) = \frac{ax(x + b)}{(2x + c)(x - d)}$$

where a, b, c and d are positive integer constants.

Determine, with brief reasons, the value of a, b, c and d. (4 marks)

Question 11 **(8 marks)**

(a) Determine the equations of all asymptotes of the graph of $y = f(x)$ when

(i) $f(x) = \dfrac{1 + 2x^2}{x(1 - 3x)}$. (2 marks)

Solution
$f(x) = \dfrac{2x^2 + 1}{-3x^2 + x}, \qquad \lim_{x \to \pm\infty} f(x) = -\dfrac{2}{3}$ Asymptotes: $x = 0,\ x = 1/3,\ y = -2/3.$
Specific behaviours
✓ horizontal asymptote   ✓ all asymptotes

(ii) $f(x) = \dfrac{x^2 + 4}{x - 5}$. (2 marks)

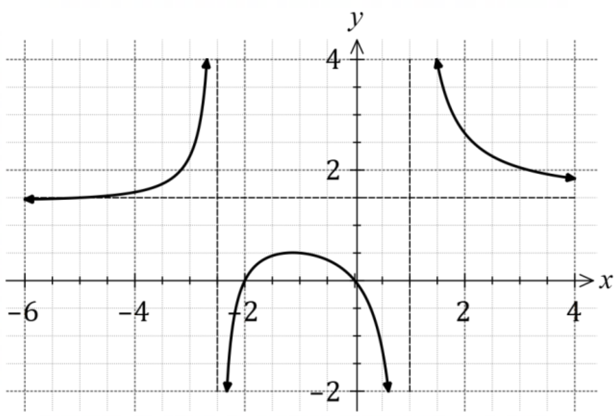
Solution
$f(x) = \dfrac{x^2 + 4}{x - 5} = x + 5 + \dfrac{29}{x - 5}$ Asymptotes: $x = 5,\ y = x + 5.$
Specific behaviours
✓ oblique asymptote   ✓ all asymptotes

(b) The graph of $y = g(x)$ is shown in the diagram, together with its three asymptotes.

The defining rule is given by

$$g(x) = \frac{ax(x + b)}{(2x + c)(x - d)}$$

where a, b, c and d are positive integer constants.

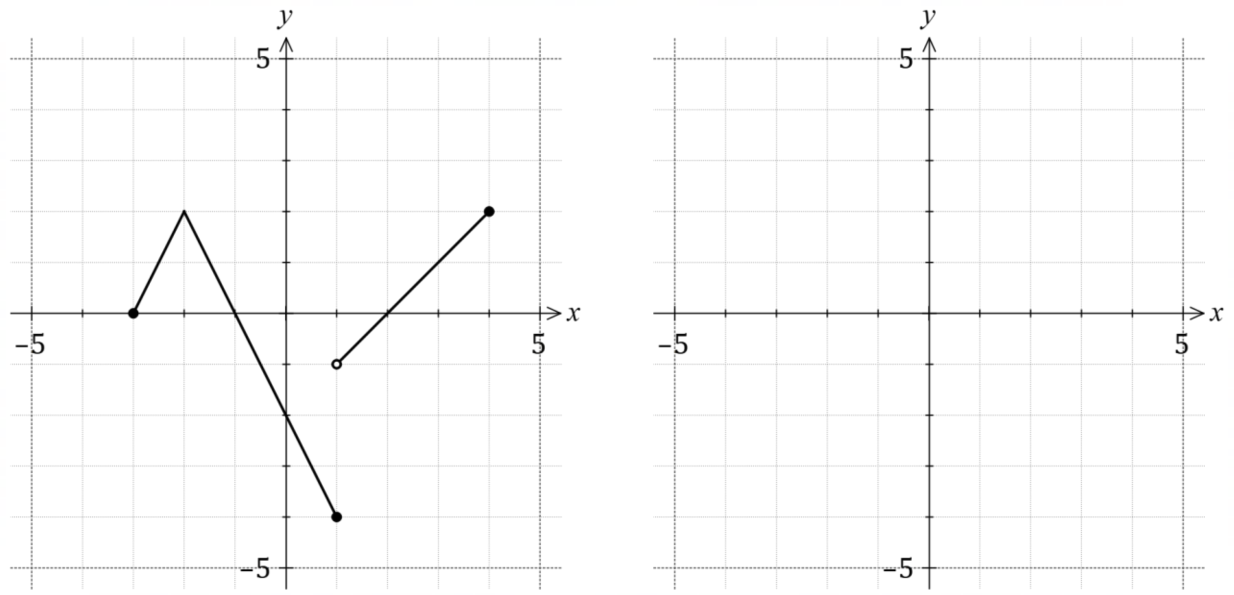


Determine, with brief reasons, the value of a, b, c and d. (4 marks)

Solution
Asymptote $y = 1.5 \to a/2 = 1.5 \to a = 3.$ Root at $(-2, 0) \to b = 2.$ Asymptote $x = -2.5 \to c = 5.$ Asymptote $x = 1 \to d = 1.$
Specific behaviours
✓✓✓✓ each value with appropriate reason

Question 14 **(9 marks)**

The graph of $y = f(x)$ is shown on the left-hand axes in the diagram below.



(a) Sketch the graph of $y = \dfrac{1}{f(x)}$ on the right-hand axes in the diagram. (5 marks)

(b) Solve the following equations.

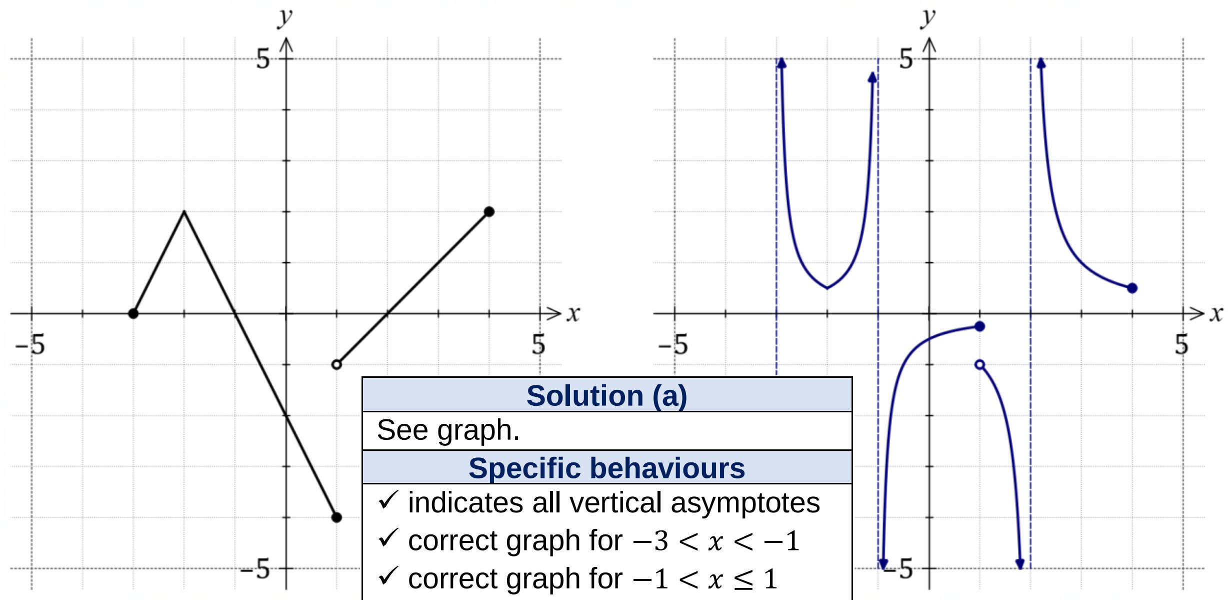
(i) $f(|x|) = 1.$ (1 mark)

(ii) $\left|\dfrac{1}{f(x)}\right| = 1.$ (1 mark)

(iii) $|f(x)| + f(x) = 0.$ (2 marks)

Question 14 **(9 marks)**

The graph of $y = f(x)$ is shown on the left-hand axes in the diagram below.



Solution (a)
See graph.
Specific behaviours
✓ indicates all vertical asymptotes
✓ correct graph for $-3 < x < -1$
✓ correct graph for $-1 < x \le 1$
✓ correct graph for $1 < x < 2$
✓ correct graph for $2 < x \le 4$

(a) Sketch the graph of $y = \dfrac{1}{f(x)}$ on the right-hand axes in the diagram. (5 marks)

(b) Solve the following equations.

(i) $f(|x|) = 1.$ (1 mark)

Solution
For $x \ge 0, f(x) = 1 \Rightarrow x = 3, \therefore x = \pm 3.$
Specific behaviours
✓ correct solution set

(ii) $\left|\dfrac{1}{f(x)}\right| = 1.$ (1 mark)

Solution
$\left\lvert\dfrac{1}{f(x)}\right\rvert = 1 \Rightarrow f(x) = \pm 1 \Rightarrow x = -2.5, -1.5, -0.5, 3$
Specific behaviours
✓ correct solution set

(iii) $|f(x)| + f(x) = 0.$ (2 marks)

Solution
Roots and intervals where $f(x) \le 0$: $(x = -3) \cup (-1 \le x \le 2).$
Specific behaviours
✓ includes 3 roots
✓ correct solution set

Question 16 **(8 marks)**

(a) Determine all solutions to the equation $z^3 - 8i = 0$ in exact polar form. (3 marks)

(b) Consider the ninth roots of unity expressed in polar form $r \text{ cis } \theta$.

(i) Determine the roots for which $0 < \theta < \frac{\pi}{2}$. (2 marks)

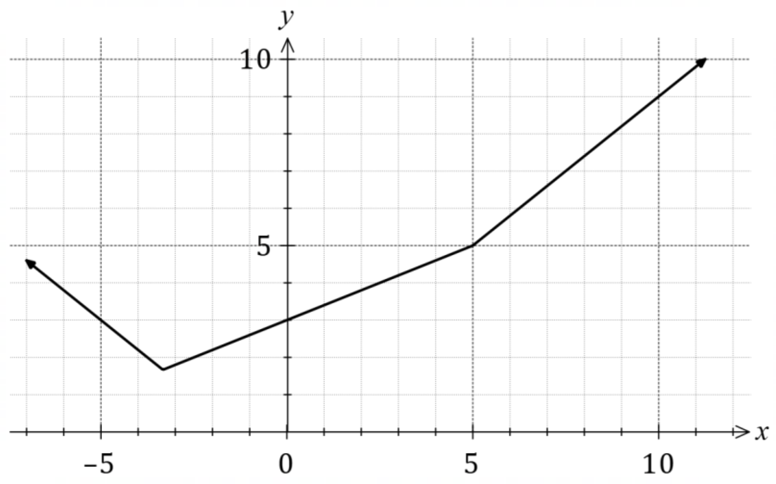
(ii) Use all nine roots to show that $\cos\left(\frac{2\pi}{9}\right) + \cos\left(\frac{4\pi}{9}\right) + \cos\left(\frac{8\pi}{9}\right) = 0$.

(3 marks)

Question 19 **(8 marks)**

Let $f(x) = |ax + b| + |cx + d|$ where a, b, c and d are constants such that $a \geq c \geq 0$.

The graph of $y = f(x)$ is shown below and passes through the points $(0, 3), (5, 5)$ and $(10,9)$.



(a) The equation $f(x) = kx + 1$ has an infinite number of solutions. State the value of the constant k. (1 mark)

(b) Determine the value of a, b, c and d. (5 marks)

(c) Determine the minimum value of $f(x)$. (2 marks)

Question 19 **(8 marks)**

Let $f(x) = |ax + b| + |cx + d|$ where a, b, c and d are constants such that $a \geq c \geq 0$.

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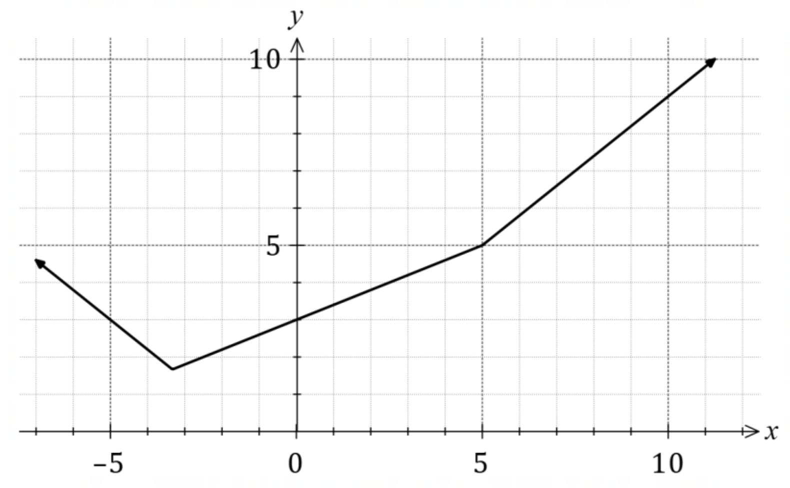


(a) The equation $f(x) = kx + 1$ has an infinite number of solutions. State the value of the constant k. (1 mark)

Solution
k is slope of RH part of f. $k = \frac{4}{5} = 0.8$.
Specific behaviours
✓ correct value

(b) Determine the value of a, b, c and d. (5 marks)

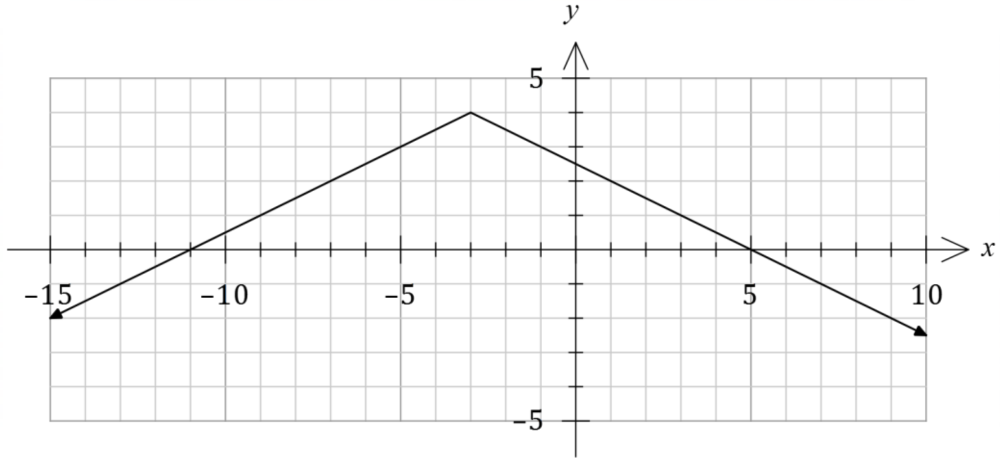
Solution
Equation of RH part of f is $y = 0.8x + 1$ and since $a \geq c \geq 0$ then $$(a + c)x + (b + d) = 0.8x + 1 \Rightarrow a + c = 0.8, \qquad b + d = 1$$ Equation of central part of f is $y = 0.4x + 3$ and so either $$(a - c)x + (b - d) = 0.4x + 3 \text{ or } (c - a)x + (d - b) = 0.4x + 3.$$ If $c - a = 0.4$ then $c = a + 0.4$ but this is impossible given that $a \geq c$. Solving $a + c = 0.8$ and $a - c = 0.4$ gives $a = 0.6, c = 0.2$. Solving $b + d = 1$ and $b - d = 3$ gives $b = 2$ and $d = -1$. Values: $a = 0.6,\ b = 2,\ c = 0.2,\ d = -1$.
Specific behaviours
✓ uses RH part to form equations for $a + c$ and $b + d$ ✓ uses central part to form equations for $a - c$ and $b - d$ ✓ repeats for $c - a$ and $d - b$ and eliminates impossible pair of equations ✓ correct values for a and c ✓ correct values for b and d

(c) Determine the minimum value of $f(x)$. (2 marks)

Solution
$$-0.8x - 1 = 0.4x + 3 \Rightarrow x = -\frac{10}{3} \Rightarrow f\left(-\frac{10}{3}\right) = \frac{5}{3}$$
Specific behaviours
✓ indicates correct method to obtain x-coordinate ✓ correct minimum

Question 9 **(8 marks)**

The graph of $y = f(x)$ is shown below, where $f(x) = a - |bx + c|$ and a, b and c are all positive constants.



(a) Determine the value of each of the constants a, b and c. (3 marks)

(b) Using the graph, or otherwise, solve

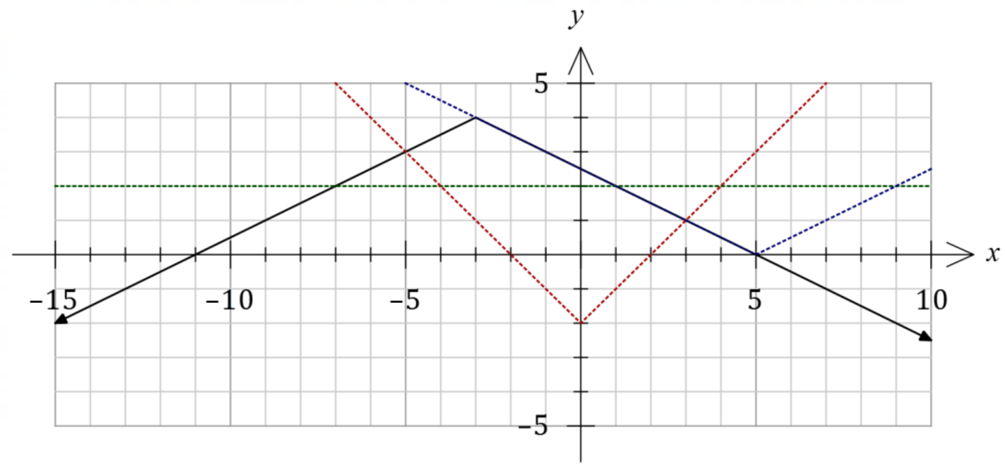
(i) $f(x) = 2$. (1 mark)

(ii) $f(x) = |x| - 2$. (2 marks)

(iii) $2f(x) = |x - 5|$. (2 marks)

Question 9 **(8 marks)**

The graph of $y = f(x)$ is shown below, where $f(x) = a - |bx + c|$ and a, b and c are all positive constants.



(a) Determine the value of each of the constants a, b and c. (3 marks)

Solution
$a = 4, \quad b = \frac{1}{2}, \quad c = \frac{3}{2}$
Specific behaviours
✓ value of a, ✓ value of b, ✓ value of c

(b) Using the graph, or otherwise, solve

(i) $f(x) = 2$. (1 mark)

Solution
$y = 2$ intersects $f(x)$ when $x = -7, x = 1$.
Specific behaviours
✓ correct solution

(ii) $f(x) = |x| - 2$. (2 marks)

Solution
$y = |x| - 2$ intersects $f(x)$ when $x = -5, x = 3$.
Specific behaviours
✓ indicates $y = |x| - 3$ on graph
✓ correct solution

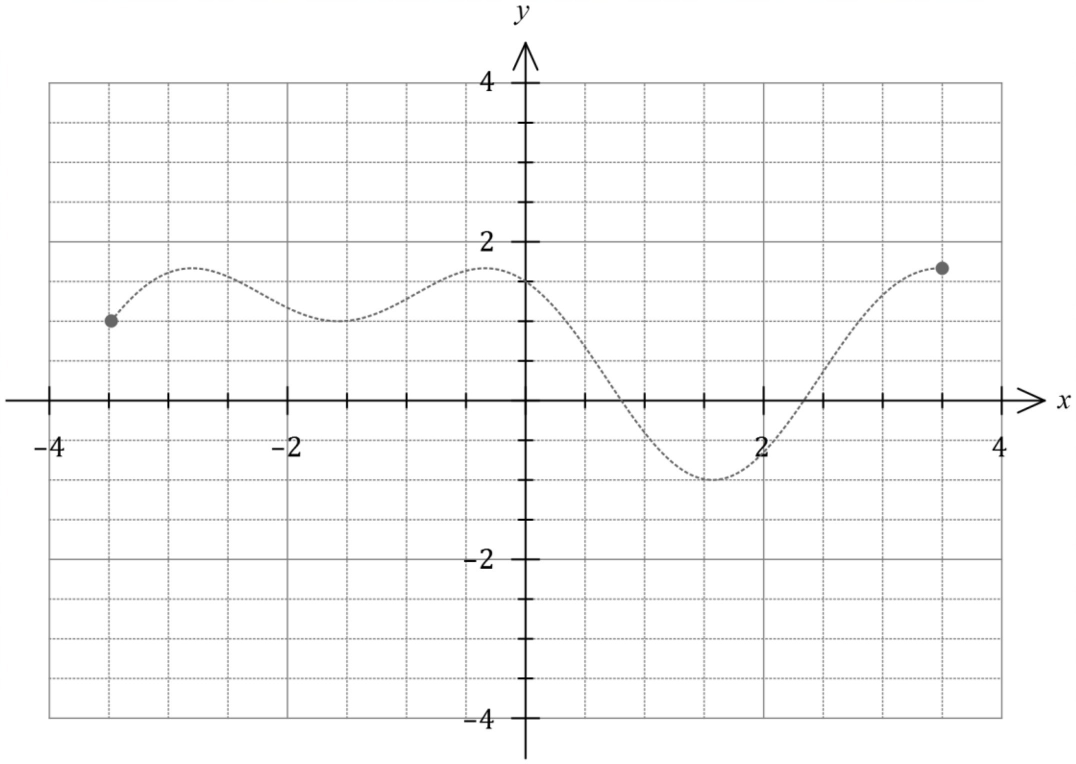
(iii) $2f(x) = |x - 5|$. (2 marks)

Solution
$y = \frac{1}{2}|x - 5|$ intersects $f(x)$ when $-3 \le x \le 5$.
Specific behaviours
✓ indicates $y = \frac{1}{3}|x - 3|$ on graph
✓ correct range of solutions

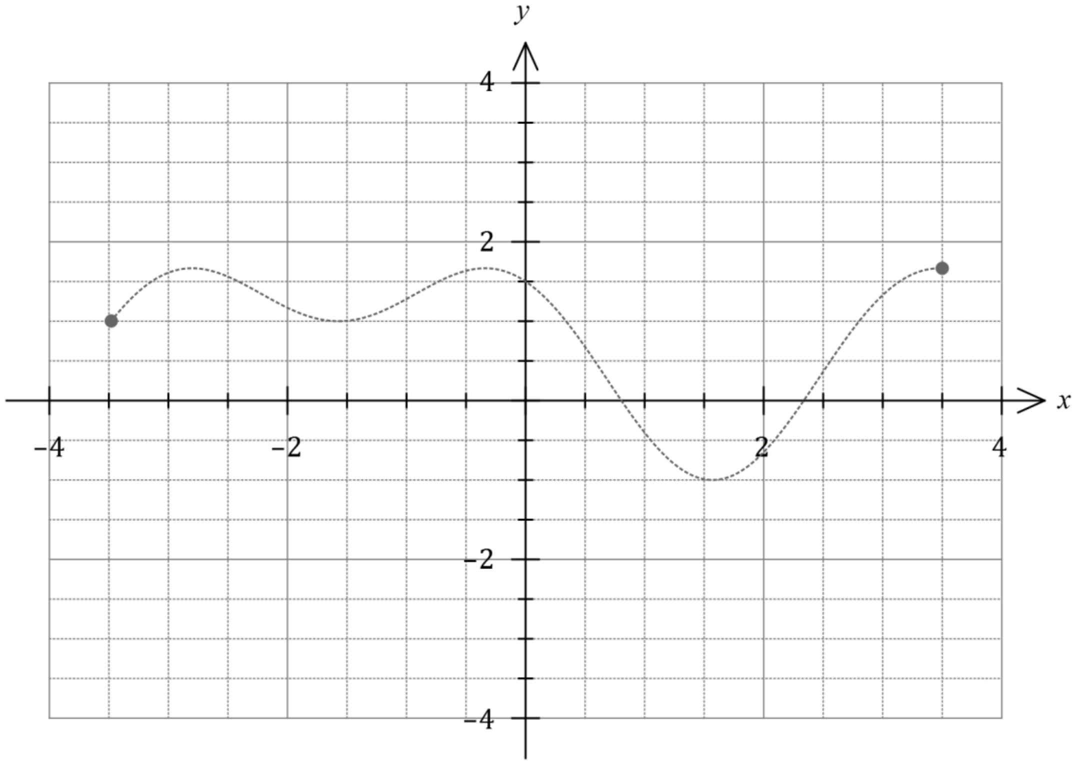
Question 12 **(8 marks)**

In each part of this question, the dotted curve shown is the graph of $y = f(x)$.

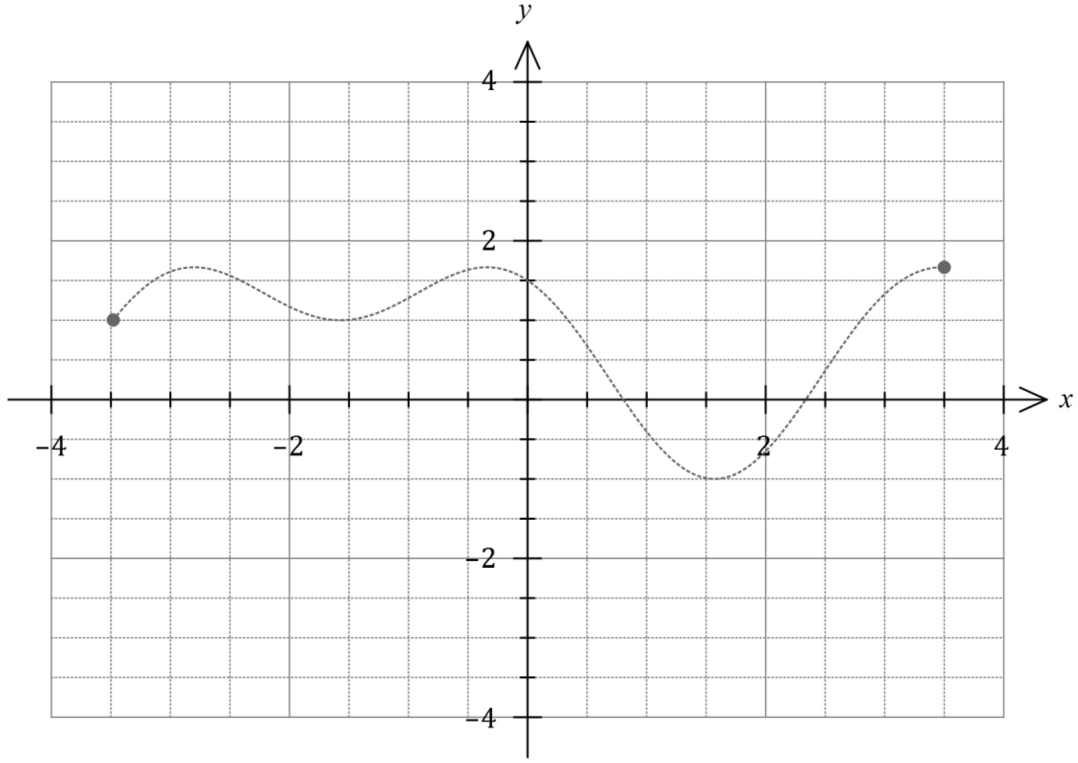
(a) Sketch the graph of $y = |f(x)|$. (2 marks)



(b) Sketch the graph of $y = \dfrac{1}{f(x)}$. (4 marks)



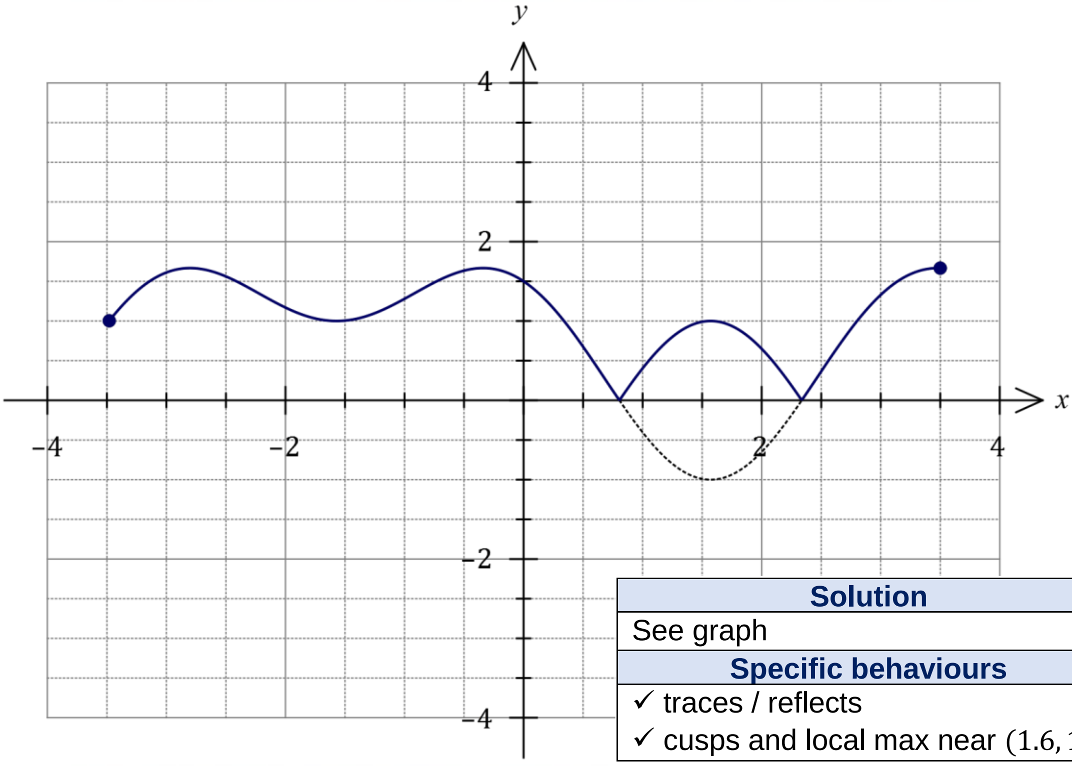
(c) Sketch the graph of $y = f(-|x|)$. (2 marks)



Question 12 **(8 marks)**

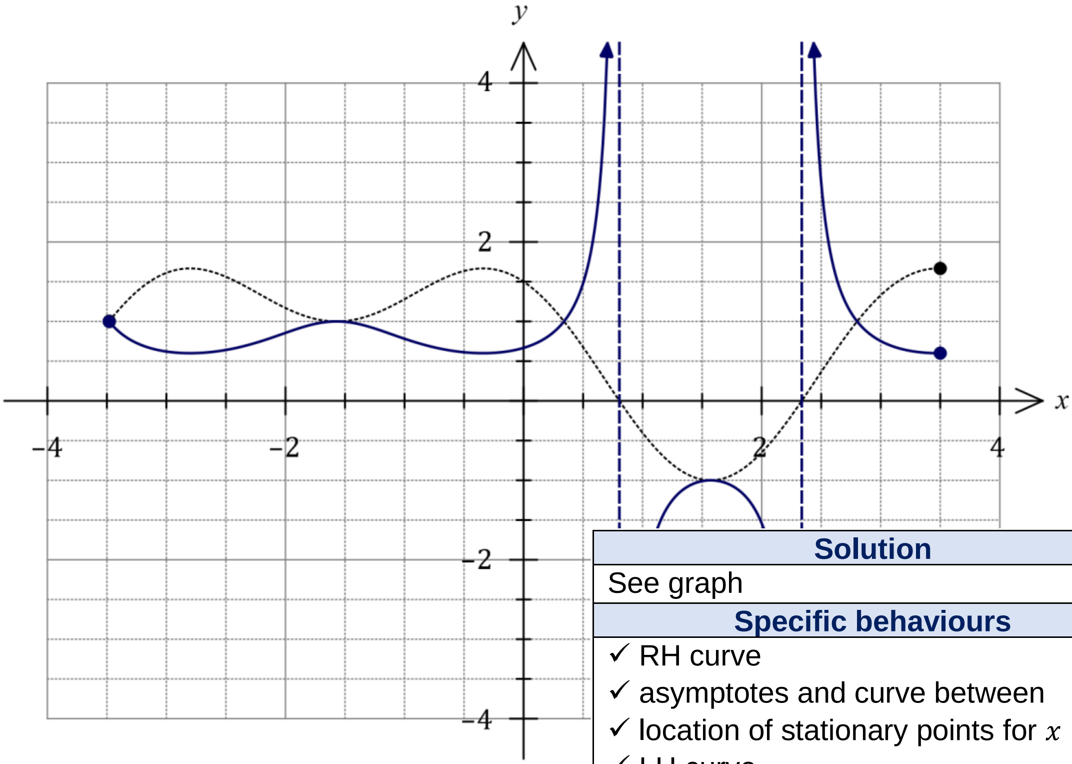
In each part of this question, the dotted curve shown is the graph of $y = f(x)$.

(a) Sketch the graph of $y = |f(x)|$. (2 marks)



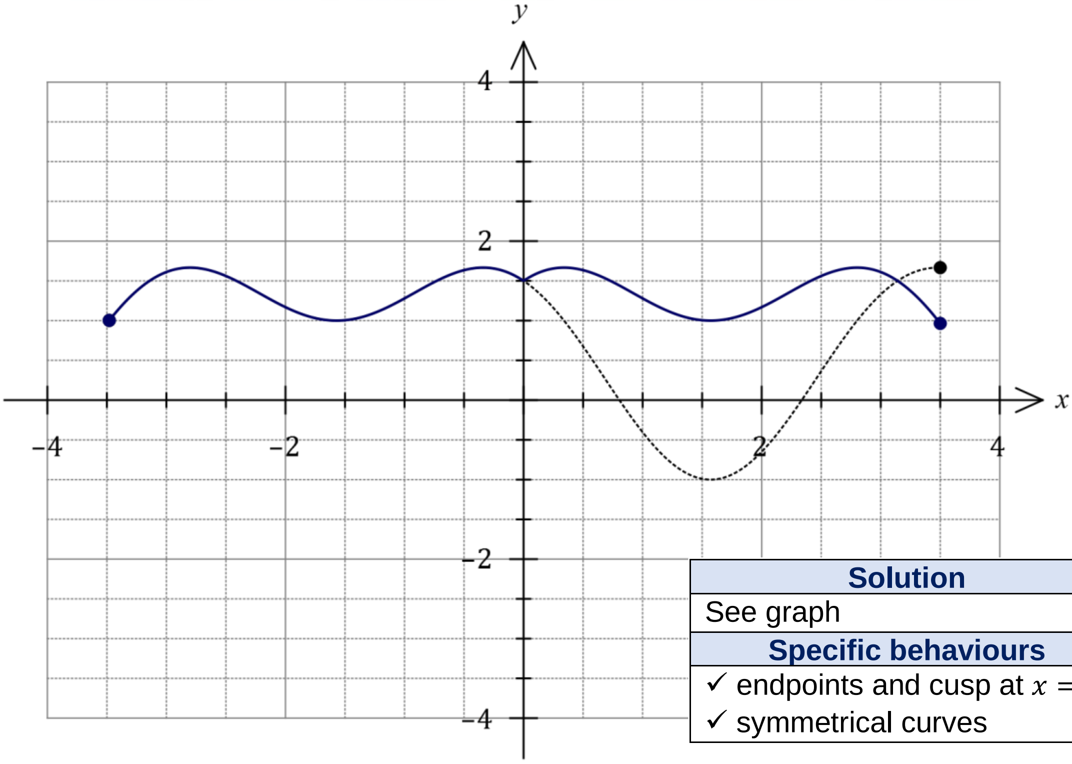
Solution
See graph
Specific behaviours
✓ traces / reflects
✓ cusps and local max near $(1.6, 1)$

(b) Sketch the graph of $y = \dfrac{1}{f(x)}$. (4 marks)



Solution
See graph
Specific behaviours
✓ RH curve
✓ asymptotes and curve between
✓ location of stationary points for $x < 0$
✓ LH curve

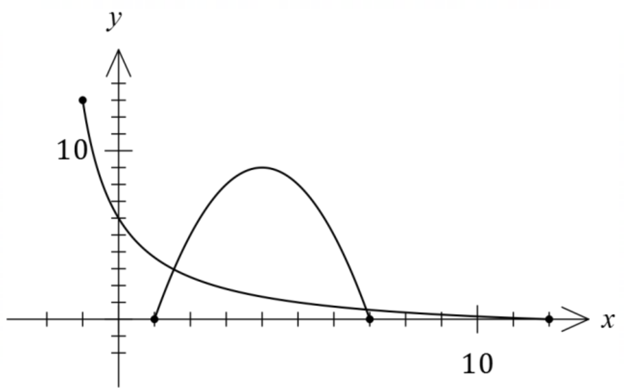
(c) Sketch the graph of $y = f(-|x|)$. (2 marks)



Solution
See graph
Specific behaviours
✓ endpoints and cusp at $x = 0$ ✓ symmetrical curves

Question 16 **(8 marks)**

The graphs of $y = f(x)$ and $y = g(x)$ are shown at right.



The functions are defined by

$$f(x) = \frac{12 - x}{x + 2}, \qquad -1 \le x \le 12$$

and

$$g(x) = -x^2 + 8x - 7, \qquad 1 \le x \le 7.$$

(a) Explain why the inverse of g is not a function. (1 mark)

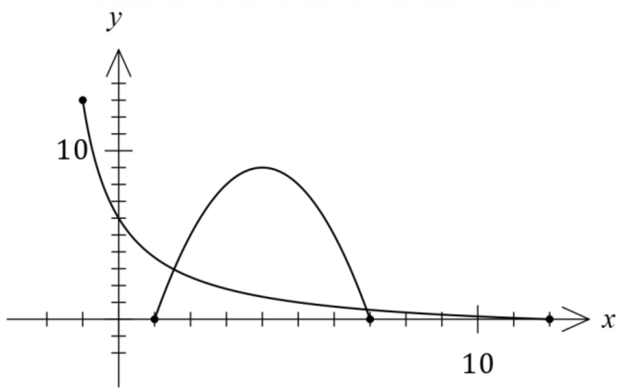
(b) Determine the definition for the inverse of f. (3 marks)

(c) Determine $g \circ f(0)$. (1 mark)

(d) Determine the domain for the function $g \circ f(x)$. (3 marks)

Question 16 **(8 marks)**

The graphs of $y = f(x)$ and $y = g(x)$ are shown at right.



The functions are defined by

$$f(x) = \frac{12 - x}{x + 2}, \qquad -1 \le x \le 12$$

and

$$g(x) = -x^2 + 8x - 7, \qquad 1 \le x \le 7.$$

(a) Explain why the inverse of g is not a function. (1 mark)

Solution
g is not a one-to-one function / g fails horizontal line test / etc.
Specific behaviours
✓ states valid reason

(b) Determine the definition for the inverse of f. (3 marks)

Solution
$x = \dfrac{12 - y}{y + 2}$ $xy + 2x + y = 12$ $y(x + 1) = 12 - 2x$ $y = \dfrac{12 - 2x}{x + 1}, \qquad 0 \le x \le 13.$
Specific behaviours
✓ interchanges x and y, cross multiplies and expands ✓ factors and obtains correct inverse ✓ limits domain to range of f

(c) Determine $g \circ f(0)$. (1 mark)

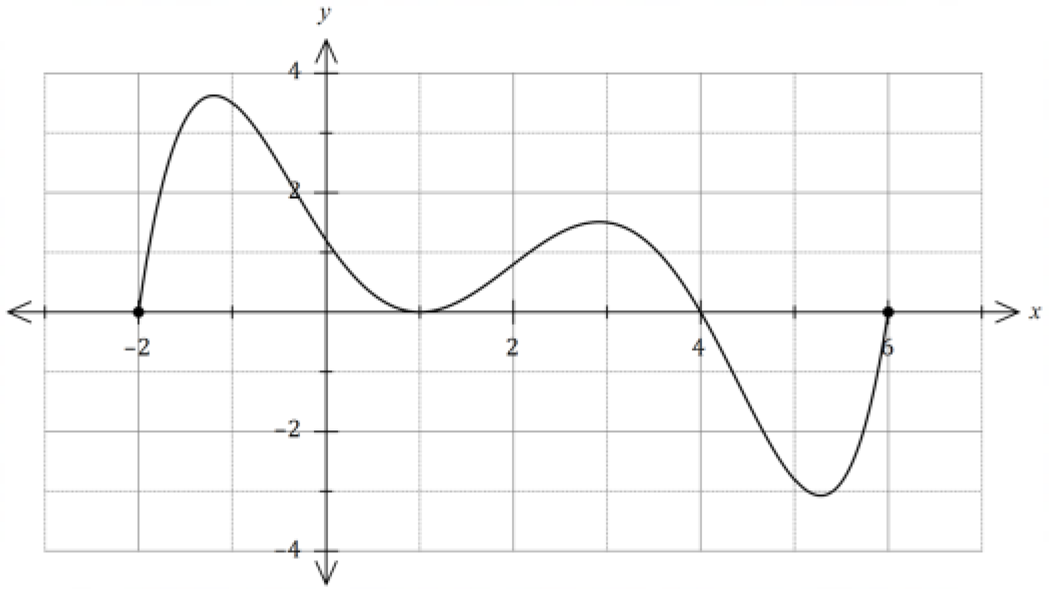
Solution
$g \circ f(0) = g(6) = 5$
Specific behaviours
✓ correct value

(d) Determine the domain for the function $g \circ f(x)$. (3 marks)

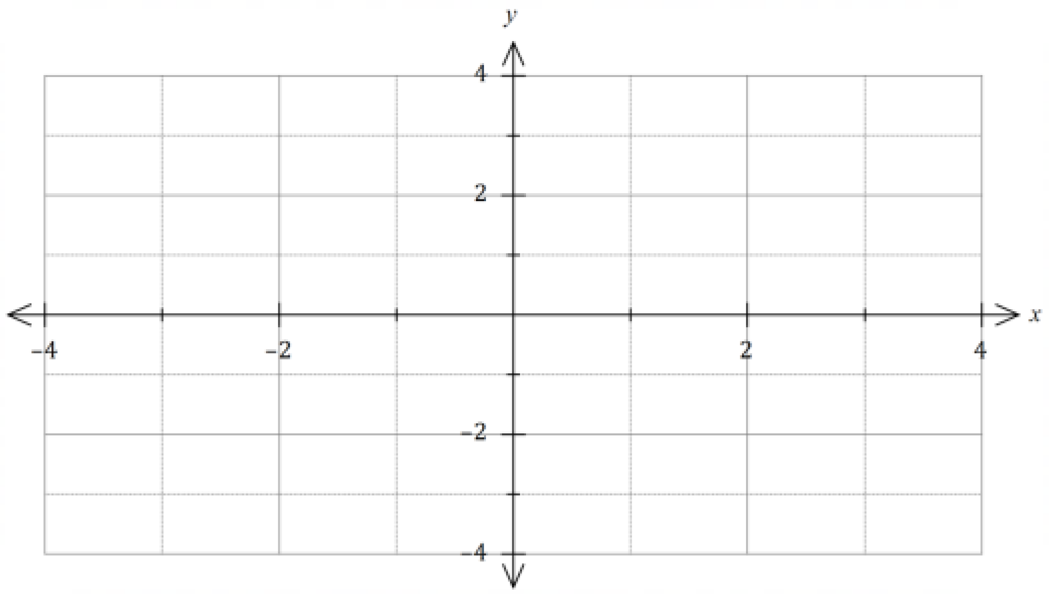
Solution
$1 \le R_f \le 7$ $\dfrac{12 - x}{x + 2} \ge 1 \Rightarrow x \le 5, \qquad \dfrac{12 - x}{x + 2} \le 7 \Rightarrow x \ge -\dfrac{1}{4}$ $D_{g \circ f} = \left\{x \in \mathbb{R}, -\dfrac{1}{4} \le x \le 5\right\}$
Specific behaviours
✓ indicates restriction on range of f ✓ indicates one correct bound of range ✓ correct range

Question 10 **(8 marks)**

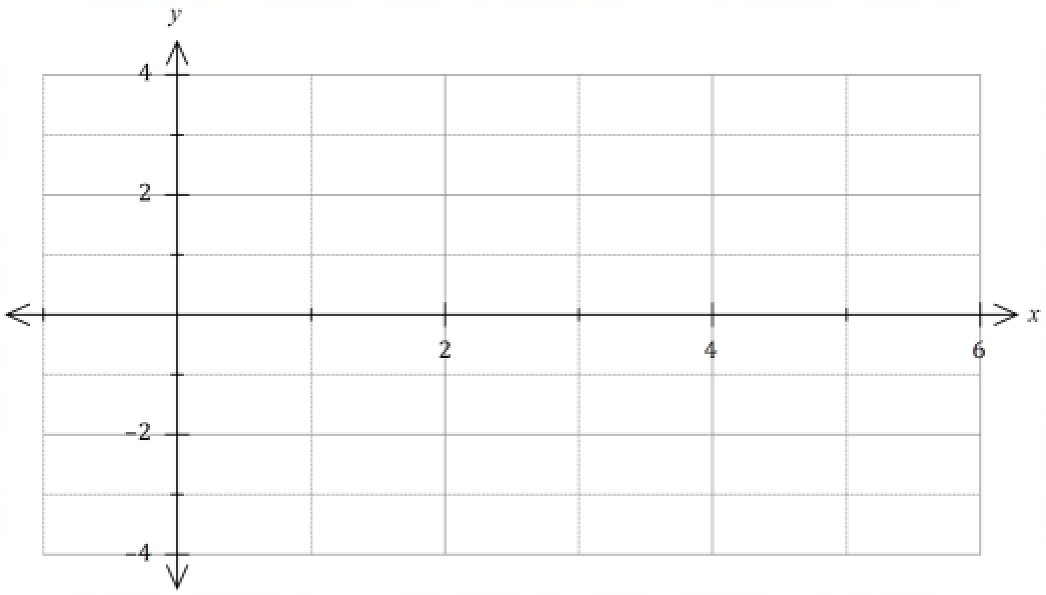
The graph of $y = f(x)$ is shown below over the domain $-2 \leq x \leq 6$.



(a) Sketch the graph of $y = f(|x|)$ over the domain $-3 \leq x \leq 3$ on the axes below. (2 marks)



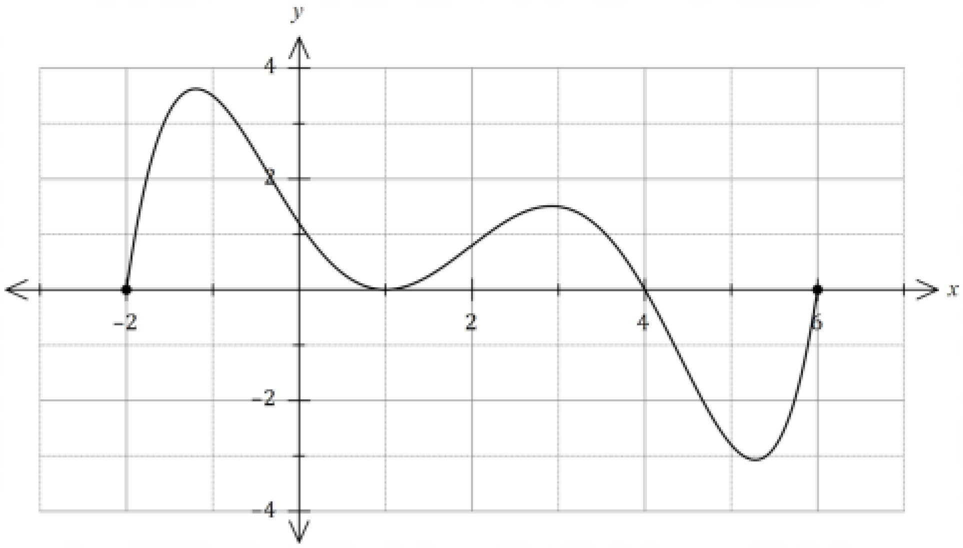
(b) Sketch the graph of $y = \dfrac{1}{f(x)}$ on the axes below over the domain $0 \le x \le 5$. (4 marks)



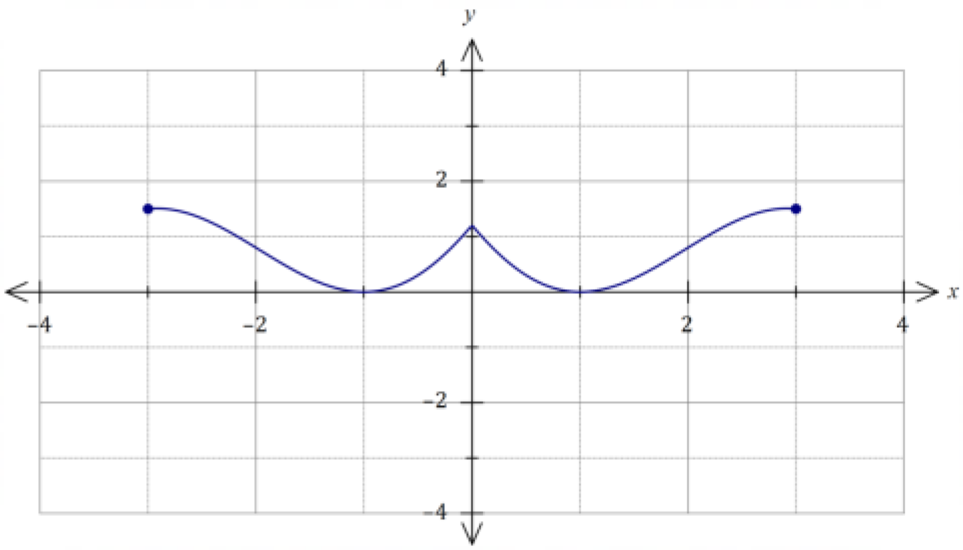
(c) List the equations of all asymptotes of the graph of $y = \dfrac{1}{f(|x|)}$ when drawn over the domain $-6 \le x \le 6$. (2 marks)

Question 10 **(8 marks)**

The graph of $y = f(x)$ is shown below over the domain $-2 \le x \le 6$.

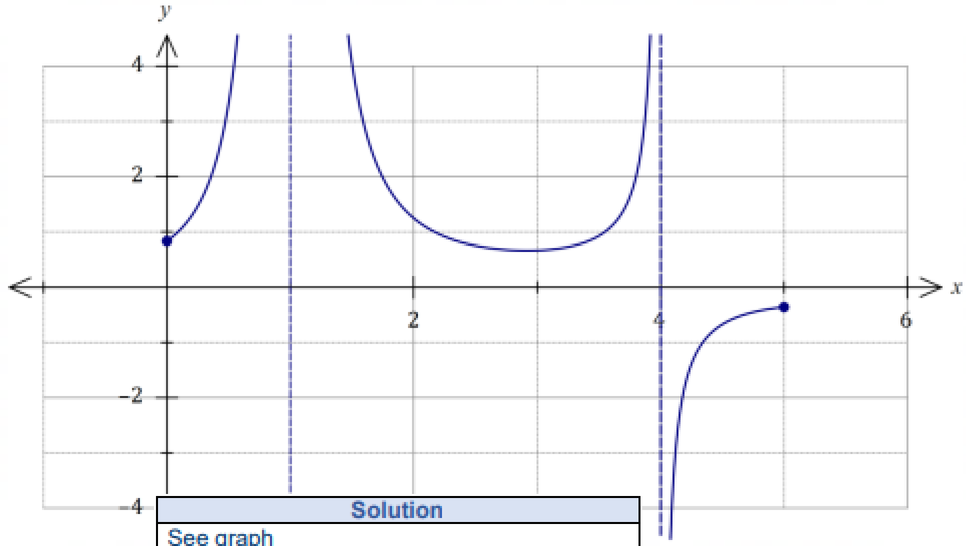


(a) Sketch the graph of $y = f(|x|)$ over the domain $-3 \le x \le 3$ on the axes below. (2 marks)



Solution
See graph
Specific behaviours
✓ cusp and curvature $-1 < x < 1$
✓ endpoints and symmetrical curve

(b) Sketch the graph of $y = \dfrac{1}{f(x)}$ on the axes below over the domain $0 \le x \le 5$. (4 marks)



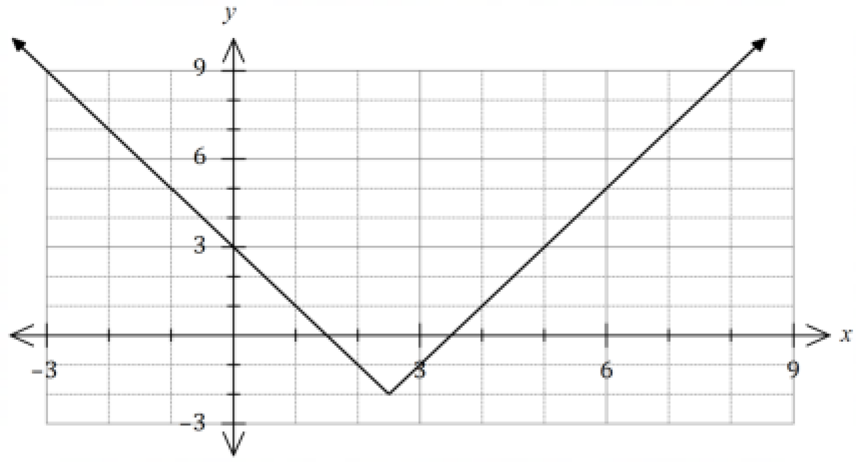
Solution
See graph
Specific behaviours
✓ endpoint and curvature $0 \le x < 1$
✓ endpoint and curvature $4 < x \le 5$
✓ indicates vertical asymptotes $x = 1, x = 4$
✓ minimum and curvature $1 < x < 4$

(c) List the equations of all asymptotes of the graph of $y = \dfrac{1}{f(|x|)}$ when drawn over the domain $-6 \le x \le 6$. (2 marks)

Solution
Zeroes of $f(x)$ for $0 \le x \le 6$ at $x = 1, 4, 6$ Hence six asymptotes: $x = \pm 1, \quad x = \pm 4, \quad x = \pm 6$
Specific behaviours
✓ four or more correct asymptotes
✓ lists exactly six asymptotes, all correct

Question 12 **(7 marks)**

The graph of $f(x) = |ax + b| + c$ is shown below.



(a) Determine all possible values of the constants a, b and c. (3 marks)

(b) Using the graph, or otherwise, solve

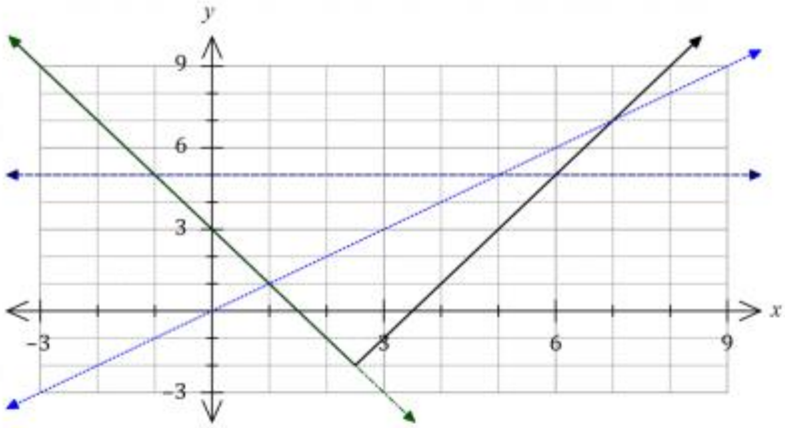
(i) $f(x) = 5$. (1 mark)

(ii) $f(x) = x$. (1 mark)

(iii) $f(x) + 2x = 3$. (2 marks)

Question 12 **(7 marks)**

The graph of $f(x) = |ax + b| + c$ is shown below.



(a) Determine all possible values of the constants a, b and c. (3 marks)

Solution
$c = -2$ Either $\{a = 2, b = -5\}$ or $\{a = -2, b = 5\}$
Specific behaviours
✓ value of c ✓ one correct set for a, b ✓ both correct sets for a, b

(b) Using the graph, or otherwise, solve

(i) $f(x) = 5$. (1 mark)

Solution
$x = -1, \quad x = 6$
Specific behaviours
✓ correct values

(ii) $f(x) = x$. (1 mark)

Solution
$x = 1, \quad x = 7$
Specific behaviours
✓ correct values

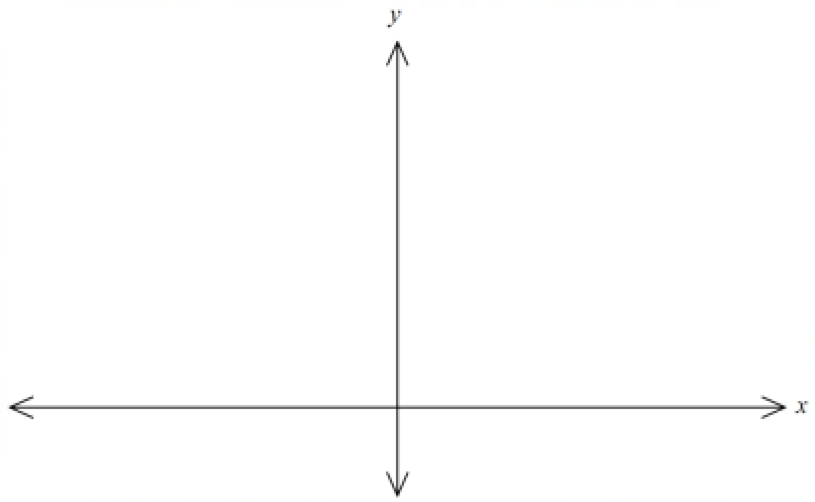
(iii) $f(x) + 2x = 3$. (2 marks)

Solution
$f(x) = 3 - 2x$ $x \leq 2.5$
Specific behaviours
✓ indicates sketch of line ✓ correct inequality

Question 14 **(7 marks)**

Let $f(x) = \left|\dfrac{x+2}{x-1}\right|$.

(a) Sketch the graph of $y = f(x)$ on the axes below. (3 marks)



(b) State the range of $f(x)$. (1 mark)

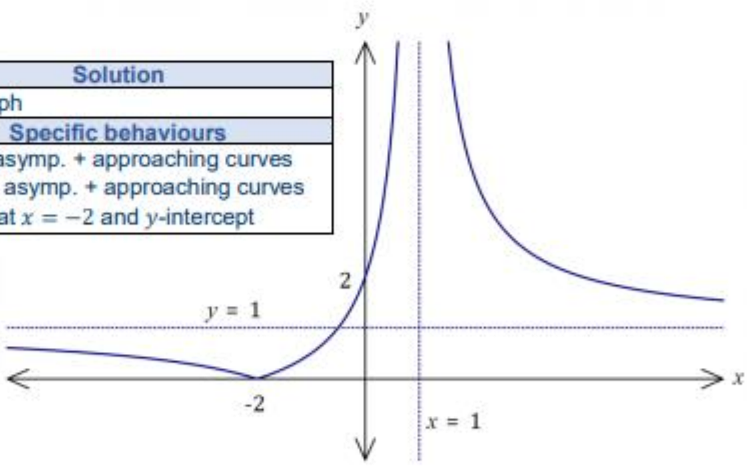
(c) The domain of f is restricted to $-2 \le x < b$ so that f^{-1} is a function. State the value of the constant b so that the domain of f is as large as possible and determine the domain and range for f^{-1}. (3 marks)

Question 14 **(7 marks)**

Let $f(x) = \left|\frac{x+2}{x-1}\right|$.

(a) Sketch the graph of $y = f(x)$ on the axes below. (3 marks)

Solution
See graph
Specific behaviours
✓ vert. asymp. + approaching curves
✓ horiz. asymp. + approaching curves
✓ cusp at $x = -2$ and y-intercept



(b) State the range of $f(x)$. (1 mark)

Solution
$R_f = \{y \in \mathbb{R}, y \geq 0\}$
Specific behaviours
✓ states $y \geq 0$

(c) The domain of f is restricted to $-2 \leq x < b$ so that f^{-1} is a function. State the value of the constant b so that the domain of f is as large as possible and determine the domain and range for f^{-1}. (3 marks)

Solution
$b = 1$ $D_{f^{-1}} = R_f = \{x \in \mathbb{R}, x \geq 0\}$ $R_{f^{-1}} = D_f = \{y \in \mathbb{R}, -2 \leq y < 1\}$
Specific behaviours
✓ value of b ✓ domain ✓ range

Question 16 **(9 marks)**

(a) Let $f(x) = \dfrac{x^2 - 4x - 2}{x - 1}$.

(i) Briefly describe the feature of the rule for $f(x)$ that indicates the graph of $y = f(x)$ will have an oblique (slanted) asymptote. (1 mark)

(ii) Determine the equations of all asymptotes of the graph of $y = f(x)$. (3 marks)

Question 16 **(9 marks)**

(a) Let $f(x) = \dfrac{x^2 - 4x - 2}{x - 1}$.

(i) Briefly describe the feature of the rule for $f(x)$ that indicates the graph of $y = f(x)$ will have an oblique (slanted) asymptote. (1 mark)

Solution
The degree of the polynomial in the numerator is one higher than that of the polynomial in the denominator.
Specific behaviours
✓ reasonable explanation

(ii) Determine the equations of all asymptotes of the graph of $y = f(x)$. (3 marks)

Solution
Vertical: $x = 1$ Oblique: $f(x) = \dfrac{x^2 - x}{x - 1} + \dfrac{-3x + 3}{x - 1} + \dfrac{-5}{x - 1}$ $= x - 3 - \dfrac{5}{x - 1}$ Hence asymptotes are $x = 1$ and $y = x - 3$.
Specific behaviours
✓ vertical asymptote ✓ expresses $f(x)$ to expose oblique asymptote ✓ oblique asymptote

(b) Let $g(x) = \dfrac{(x-2)(x+3)}{x^2+1}$.

(i) State the equation of the horizontal asymptote of the graph of $y = g(x)$. (1 mark)

(ii) State the values of $g(6), g(7)$ and $g(8)$. (1 mark)

(iii) Use your previous two answers to explain why the graph of $y = g(x)$ must have a local maximum to the right of $x = 7$. (3 marks)

(b) Let $g(x) = \dfrac{(x-2)(x+3)}{x^2+1}$.

(i) State the equation of the horizontal asymptote of the graph of $y = g(x)$. (1 mark)

Solution
$y = 1$
Specific behaviours
✓ asymptote

(ii) State the values of $g(6), g(7)$ and $g(8)$. (1 mark)

Solution
$g(6) = \dfrac{36}{37} \approx 0.97, \quad g(7) = 1, \quad g(8) = \dfrac{66}{65} \approx 1.02$
Specific behaviours
✓ correct values

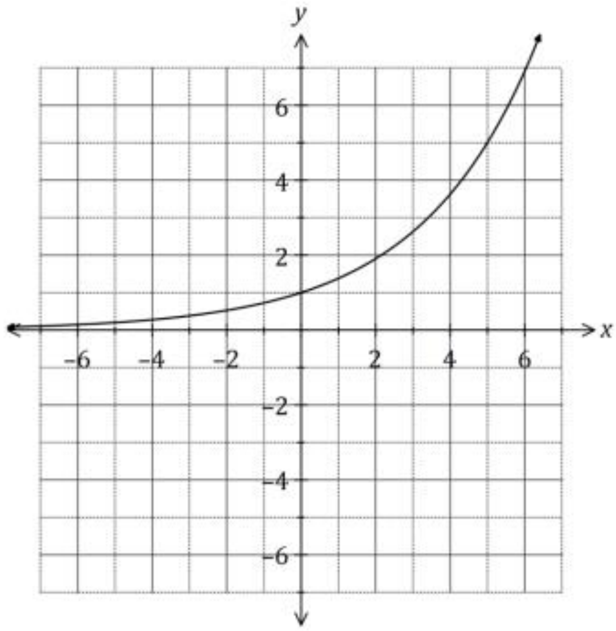
(iii) Use your previous two answers to explain why the graph of $y = g(x)$ must have a local maximum to the right of $x = 7$. (3 marks)

Solution
As $g(x)$ increases through $x = 7$, y is increasing and the curve cuts the horizontal asymptote $y = 1$. However, as $x \to \infty$, $y \to 1$ and since g is continuous for all x (has no vertical asymptotes) then at some point where $x > 7$ the curve must start to decrease to return to the asymptote and so a local maximum must exist. *NB Students may also use a sketch as part of their response, so long as it specifically uses the results from (i) and (ii).*
Specific behaviours
✓ indicates g increases through asymptote ✓ states g is continuous throughout ✓ explains why g must then decrease

Question 11 **(6 marks)**

(a) Explain why the function $f(x) = \sin x$, where $x \in \mathbb{R}$, is not one-to-one. (1 mark)

(b) The graph of $y = g(x)$ is shown below. Sketch the graph of $y = g^{-1}(x)$ on the same axes. (2 marks)



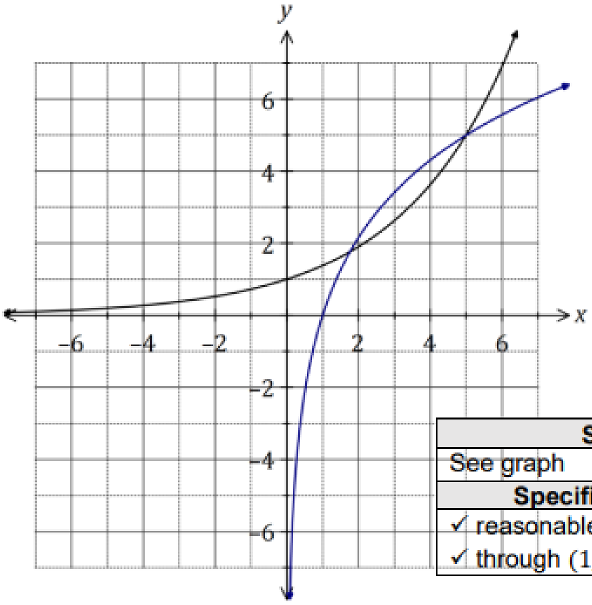
(c) The inverse function of h is defined as $h^{-1}(x) = x^2 + 10x + 22$ for $x \leq -5$. Determine the defining rule for $h(x)$ and state its domain. (3 marks)

Question 11 **(6 marks)**

(a) Explain why the function $f(x) = \sin x$, where $x \in \mathbb{R}$, is not one-to-one. (1 mark)

Solution
Graph of $f(x)$ fails horizontal line test, etc
Specific behaviours
✓ valid explanation

(b) The graph of $y = g(x)$ is shown below. Sketch the graph of $y = g^{-1}(x)$ on the same axes. (2 marks)



Solution
See graph
Specific behaviours
✓ reasonable reflection in $y = x$
✓ through $(1, 0)$ and $(5, 5)$

(c) The inverse function of h is defined as $h^{-1}(x) = x^2 + 10x + 22$ for $x \leq -5$. Determine the defining rule for $h(x)$ and state its domain. (3 marks)

Solution
$x = (y+5)^2 - 3 \Rightarrow y = \pm\sqrt{x+3} - 5$ (CAS) $D_{h^{-1}} = R_h \Rightarrow y \leq -5 \Rightarrow h(x) = -\sqrt{x+3} - 5$ $D_h = \{x : x \in \mathbb{R}, x \geq -3\}$
Specific behaviours
✓ using CAS or otherwise obtains two possible functions   ✓ uses range of h to determine $h(x)$ ✓ states that $x \geq -3$

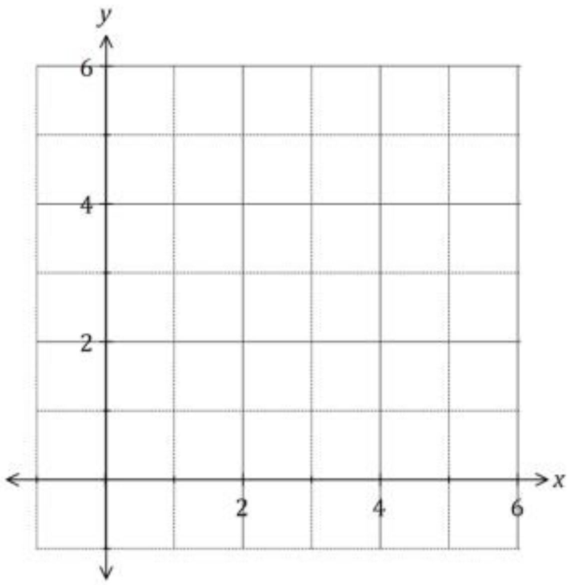
Question 16 **(8 marks)**

Let $f(x) = \sqrt{x-2}$, $g(x) = \dfrac{6}{x}$ and $h(x) = f \circ g(x)$.

(a) Determine an expression for $h(x)$ and show that the domain of $h(x)$ is $0 < x \leq 3$. (3 marks)

(b) Determine an expression for $h^{-1}(x)$, the inverse of $h(x)$. (1 mark)

(c) Sketch the graphs of $y = h(x)$ and $y = h^{-1}(x)$ on the axes below. (3 marks)



(d) Solve $h(x) = h^{-1}(x)$, correct to 0.01 where necessary. (1 mark)

Question 16 **(8 marks)**

Let $f(x) = \sqrt{x-2}$, $g(x) = \dfrac{6}{x}$ and $h(x) = f \circ g(x)$.

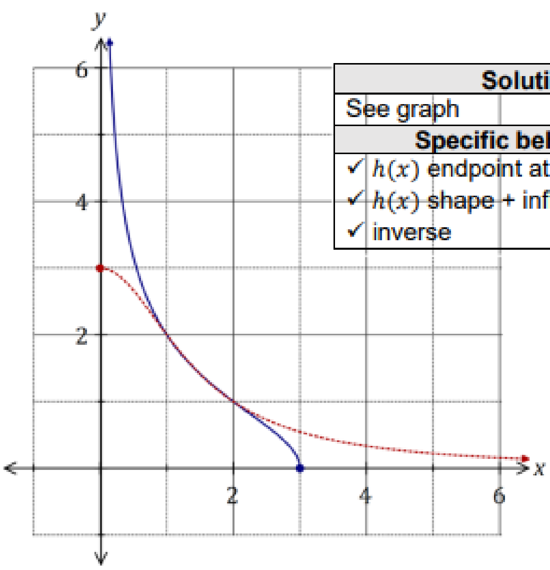
(a) Determine an expression for $h(x)$ and show that the domain of $h(x)$ is $0 < x \leq 3$. (3 marks)

Solution
$$h(x) = \sqrt{\frac{6}{x} - 2}$$ D_h: (i) require $x > 0$ so that $\frac{6}{x} - 2 > 0$ and (ii) $\frac{6}{x} \geq 2 \Rightarrow x \leq 3$ Hence $D_h: \{x \in \mathbb{R}: 0 < x \leq 3\}$
Specific behaviours
✓ $h(x)$ ✓ explains why $x > 0$ ✓ explains why $x \leq 3$

(b) Determine an expression for $h^{-1}(x)$, the inverse of $h(x)$. (1 mark)

Solution
$$h^{-1}(x) = \frac{6}{x^2 + 2} \quad \text{(CAS)}$$
Specific behaviours
✓ correct expression

(c) Sketch the graphs of $y = h(x)$ and $y = h^{-1}(x)$ on the axes below. (3 marks)



Solution
See graph
Specific behaviours
✓ $h(x)$ endpoint at $(3,0)$ ✓ $h(x)$ shape + inflection $x \approx 2$ ✓ inverse

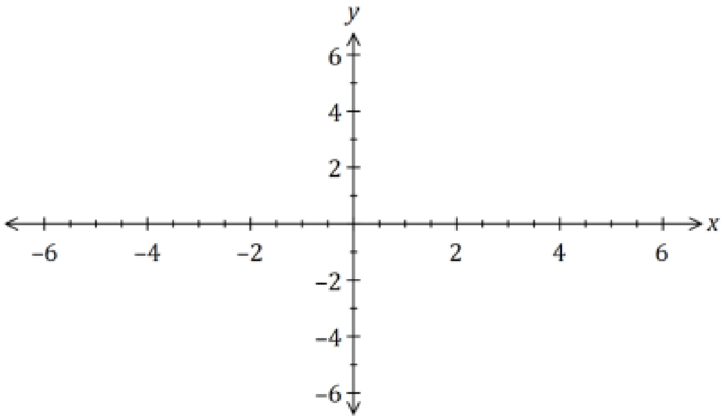
(d) Solve $h(x) = h^{-1}(x)$, correct to 0.01 where necessary. (1 mark)

Solution
$x = 1, \quad x = 2, \quad x \approx 1.46 \quad \text{(CAS)}$
Specific behaviours
✓ correct solutions

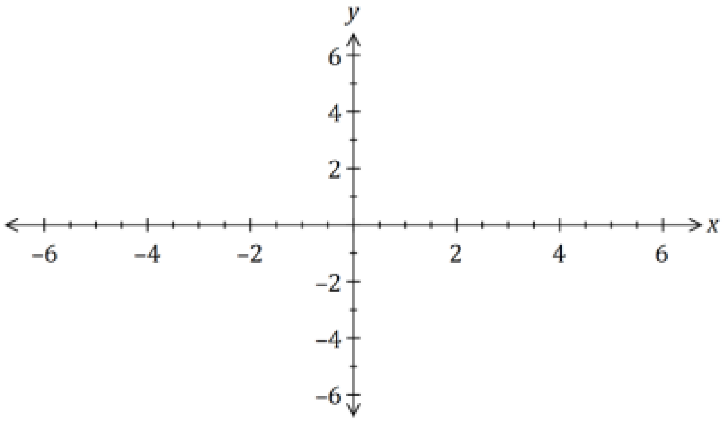
Question 19 **(8 marks)**

Let $f(x) = 3 - |2x - 6|$.

(a) Sketch the graph of $y = f(x)$ on the axes below. (2 marks)



(b) Sketch the graph of $y = f(|x|)$ and hence solve $f(|x|) - 3 = 0$. (3 marks)



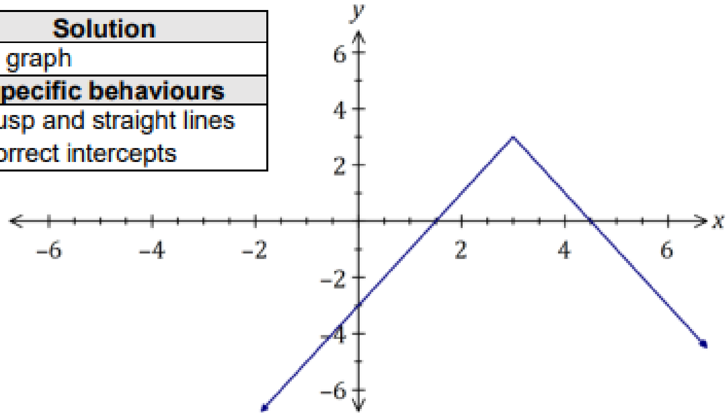
(c) The equation $f(x) = a|x + b| + c$ is true only for $0 \le x \le 3$. Determine the value of each of the constants a, b and c. (3 marks)

Question 19 **(8 marks)**

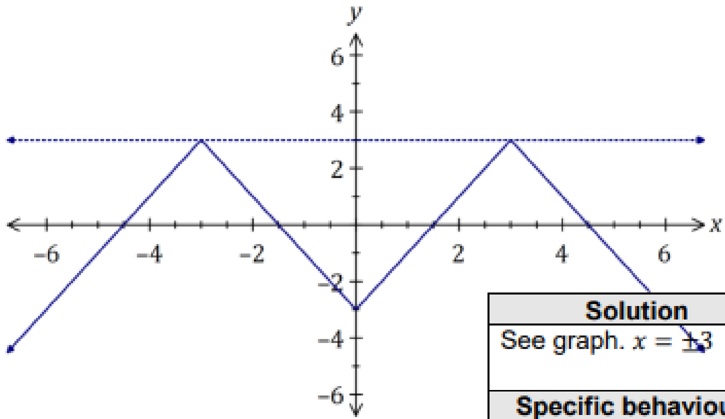
Let $f(x) = 3 - |2x - 6|$.

(a) Sketch the graph of $y = f(x)$ on the axes below. (2 marks)

Solution
See graph
Specific behaviours
✓ cusp and straight lines
✓ correct intercepts

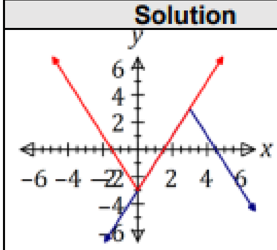


(b) Sketch the graph of $y = f(|x|)$ and hence solve $f(|x|) - 3 = 0$. (3 marks)



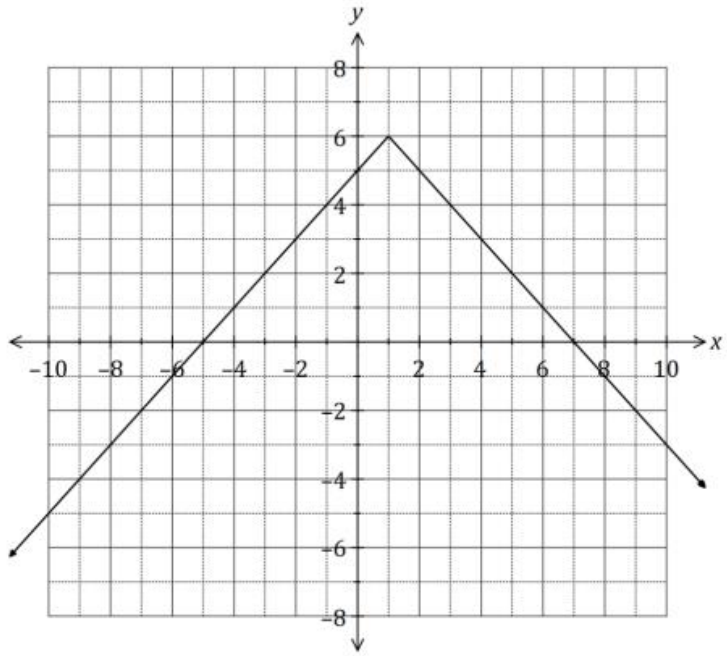
Solution
See graph. $x = \pm 3$
Specific behaviours
✓ required sketch
✓ adds $y = 3$
✓ solutions

(c) The equation $f(x) = a|x + b| + c$ is true only for $0 \le x \le 3$. Determine the value of each of the constants a, b and c. (3 marks)

Solution

$a = 2,\ b = 0,\ c = -3$
Specific behaviours
✓ sketch indicating situation
✓ two correct values
✓ all correct values

Question 11 **(7 marks)**

The graph of $y = f(x)$ is shown below, where $f(x) = a|x+b| + c$, where a, b and c are constants.



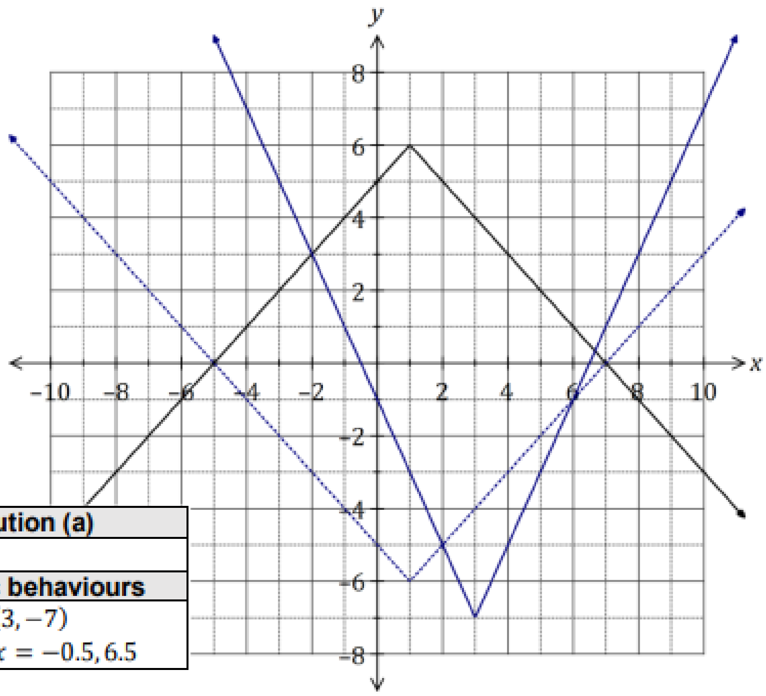
(a) Add the graph of $y = g(x)$ to the axes above, where $g(x) = 2|x-3| - 7$. (2 marks)

(b) Determine the values of a, b and c. (3 marks)

(c) Using your graph, or otherwise, solve $f(x) + g(x) = 0$. (2 marks)

Question 11 **(7 marks)**

The graph of $y = f(x)$ is shown below, where $f(x) = a|x + b| + c$, where a, b and c are constants.



Solution (a)
See graph
Specific behaviours
✓ cusp at $(3, -7)$
✓ roots at $x = -0.5, 6.5$

(a) Add the graph of $y = g(x)$ to the axes above, where $g(x) = 2|x - 3| - 7$. (2 marks)

(b) Determine the values of a, b and c. (3 marks)

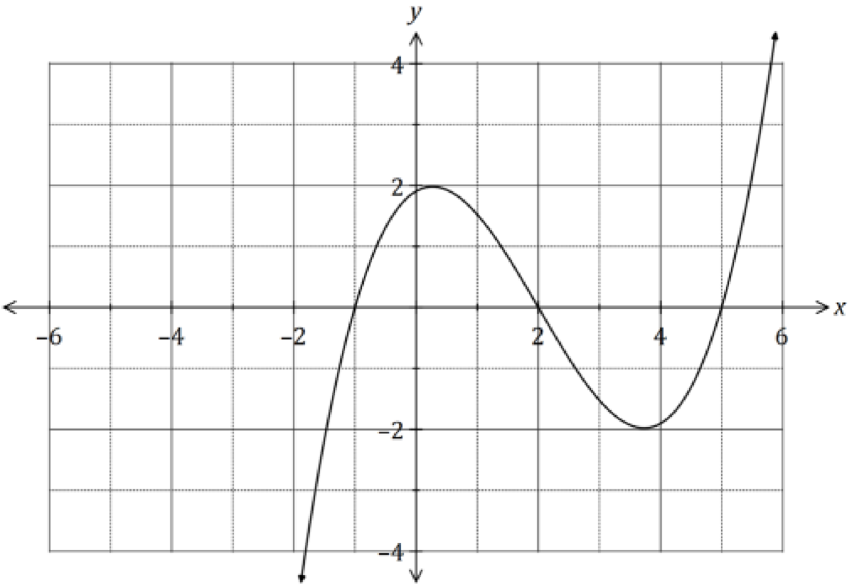
Solution
Slopes: $m \pm 1 \Rightarrow a = -1$ From cusp: $b = -1$ and $c = 6$
Specific behaviours
✓ correct value of a ✓ correct value of b ✓ correct value of c

(c) Using your graph, or otherwise, solve $f(x) + g(x) = 0$. (2 marks)

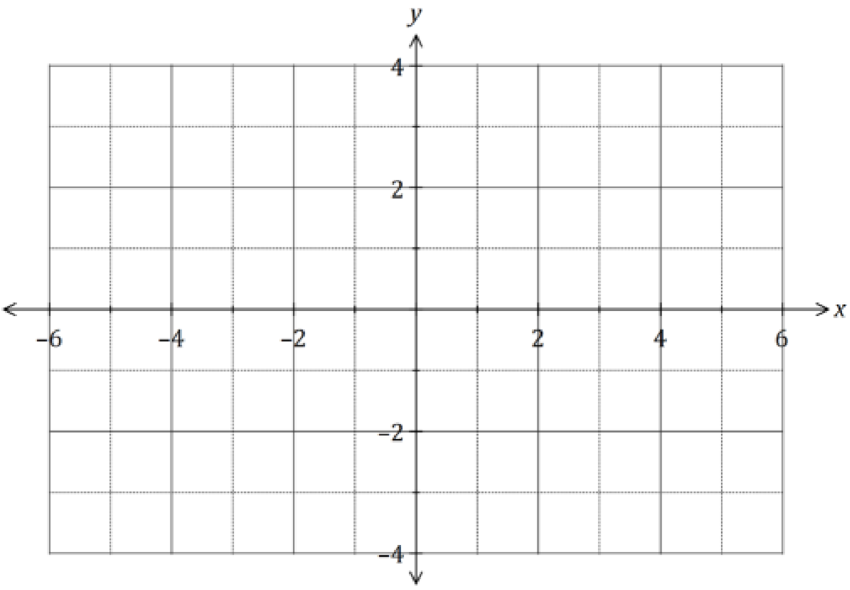
Solution
Using reflection of $y = f(x)$ in $y = 0$, graphs intersect when $x = 2, x = 6$.
Specific behaviours
✓ reflects $f(x)$ ✓ both solutions

Question 14 **(8 marks)**

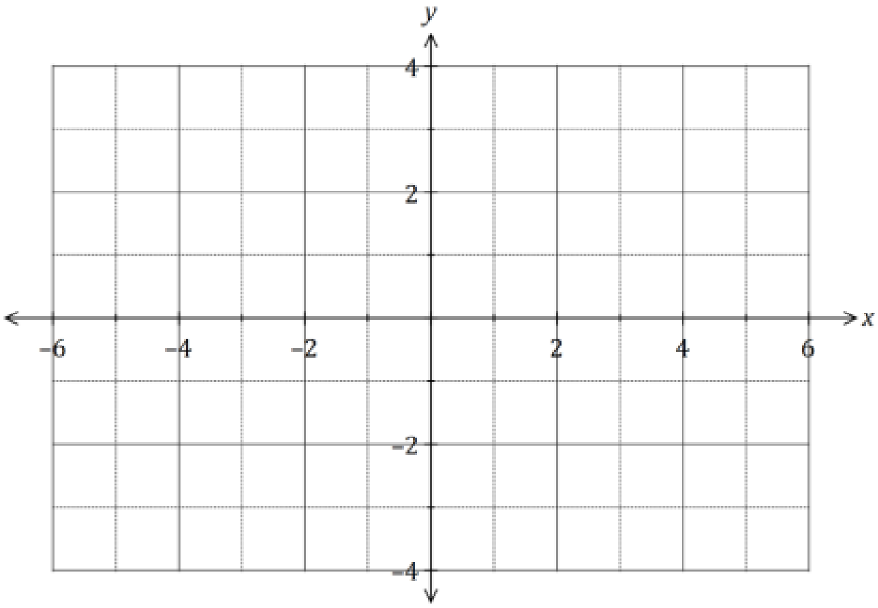
The graph of $y = f(x)$ is shown below.



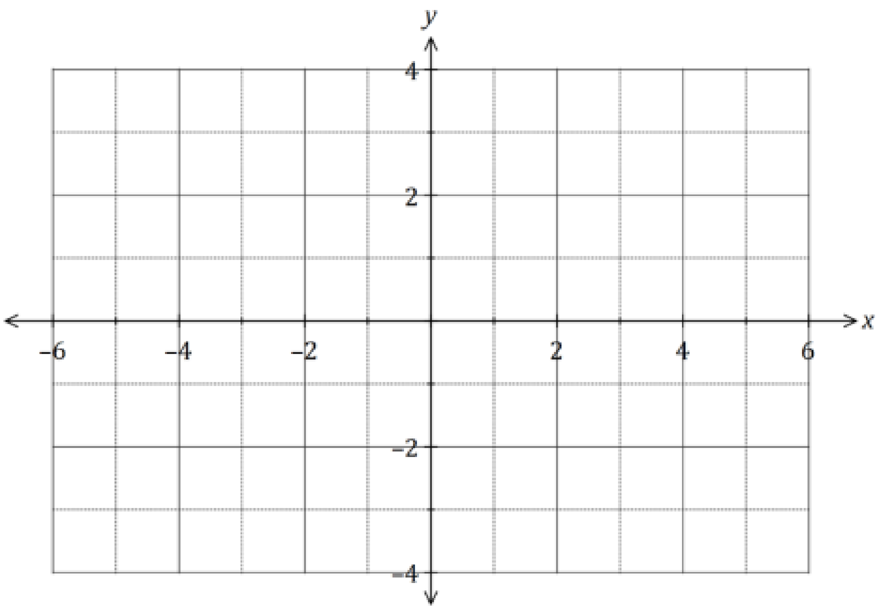
(a) Sketch the graph of $y = f(|x|)$ on the axes below. (2 marks)



(b) Sketch the graph of $y = \dfrac{1}{f(x)}$ on the axes below. (4 marks)

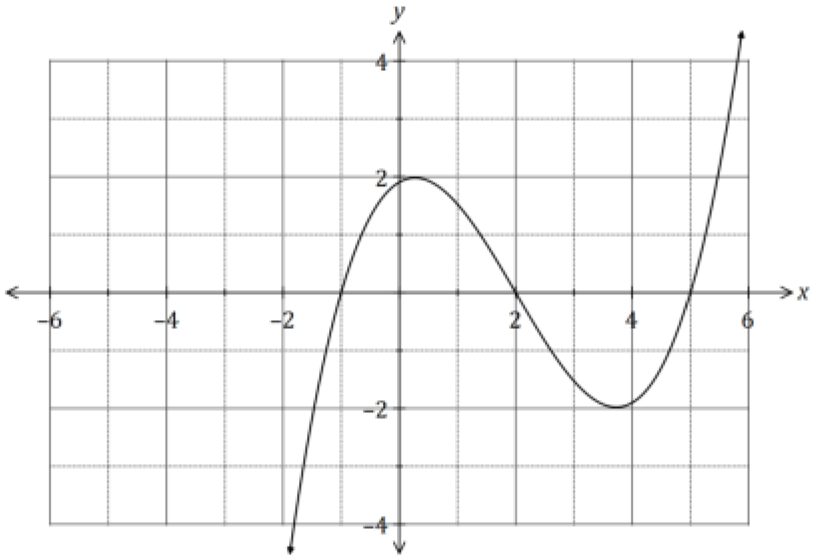


(c) Sketch the graph of $y = |f(|x|)|$ on the axes below. (2 marks)

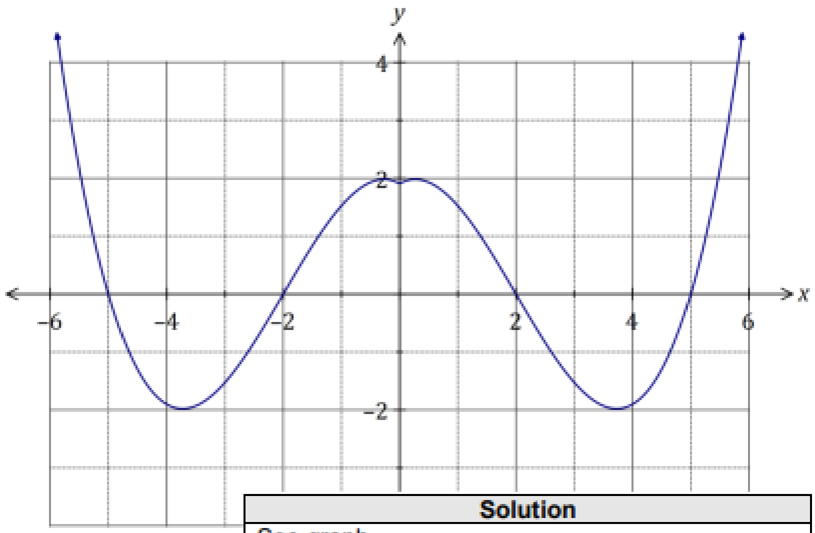


Question 14 **(8 marks)**

The graph of $y = f(x)$ is shown below.

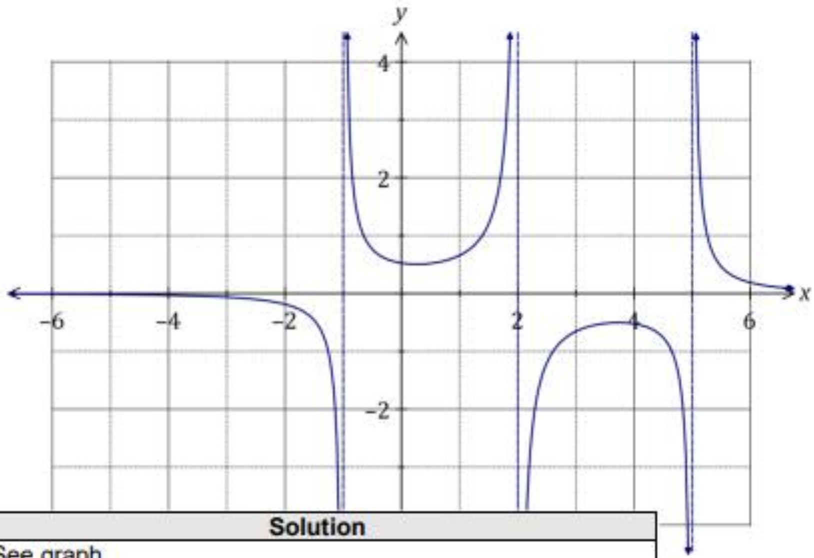


(a) Sketch the graph of $y = f(|x|)$ on the axes below. (2 marks)



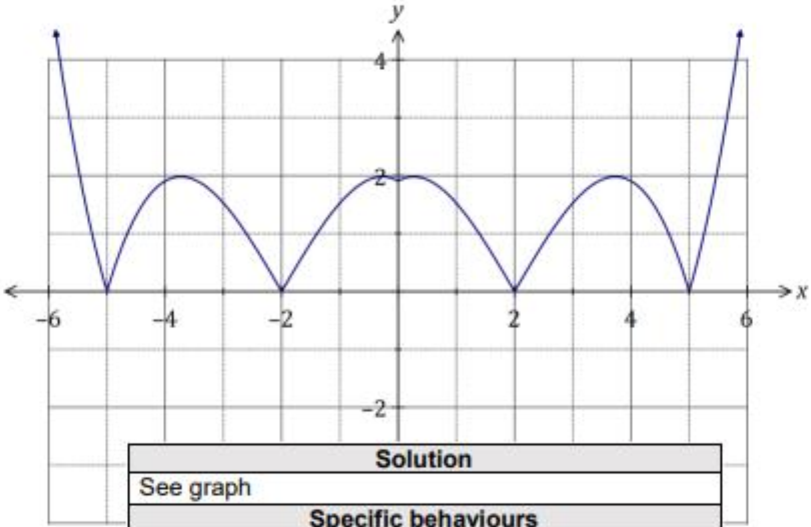
Solution
See graph
Specific behaviours
✓ indicates roots and slight cusp close to $(0, 1.9)$
✓ indicates symmetry about $x = 0$

(b) Sketch the graph of $y = \dfrac{1}{f(x)}$ on the axes below. (4 marks)



Solution
See graph
Specific behaviours
✓ indicates vertical asymptotes at $x = -1, 2$ and 5 ✓ indicates $y \to 0$ for $\lvert x \rvert \to \infty$ ✓ indicates turning points close to $(3.7, -0.5)$ and $(0.3, 0.5)$ ✓ indicates correct curvature between asymptotes

(c) Sketch the graph of $y = |f(|x|)|$ on the axes below. (2 marks)



Solution
See graph
Specific behaviours
✓ indicates symmetry about $x = 0$ ✓ reflects parts of graph (a) below $y = 0$ above axis

Question 18 **(8 marks)**

Functions f and g are defined as $f(x) = x^2 + ax - 10a$ and $g(x) = \dfrac{x}{x+b}$, where a and b are constants.

(a) Let $a = 2$ and $b = 5$.

(i) State, with reasons, whether the composition $f(g(x))$ is a one-to-one function over its natural domain.

(2 marks)

(ii) Determine any domain restrictions required so that the composition $g(f(x))$ is defined.

(3 marks)

(b) Determine the relationship between a and b so that the composition $g(f(x))$ is always defined for $x \in \mathbb{R}$.

(3 marks)

Question 18 **(8 marks)**

Functions f and g are defined as $f(x) = x^2 + ax - 10a$ and $g(x) = \dfrac{x}{x+b}$, where a and b are constants.

(a) Let $a = 2$ and $b = 5$.

(i) State, with reasons, whether the composition $f(g(x))$ is a one-to-one function over its natural domain.

(2 marks)

Solution
No because
- composite function has two roots at $x \approx -6.9, -4.2$
- horizontal line cuts graph twice (with sketch from CAS)
- etc
Specific behaviours
✓ reason
✓ support for reason

(ii) Determine any domain restrictions required so that the composition $g(f(x))$ is defined. (3 marks)

Solution
$g(f(x)) = \dfrac{x^2+2x-20}{x^2+2x-15}$ $x^2+2x-15 = (x+5)(x-3) \neq 0$ $x \neq -5, \quad x \neq 3$
Specific behaviours
✓ indicates composite function
✓ indicates denominator non-zero
✓ domain restrictions

(b) Determine the relationship between a and b so that the composition $g(f(x))$ is always defined for $x \in \mathbb{R}$. (3 marks)

Solution
$g(f(x)) = \dfrac{x^2+ax-10a}{x^2+ax-10a+b}$ $x^2+ax-10a+b \neq 0$ $a^2-4(-10a+b) < 0$ $b > \dfrac{1}{4}a^2 + 10a$
Specific behaviours
✓ indicates composite function denominator non-zero
✓ uses quadratic formula to create inequality
✓ states relationship