assignment_05

October 3, 2021

1 Logistic regression for multi-class classification

1.1 import libraries

[ ]:
```
import numpy as np
import matplotlib.pyplot as plt
import os
from tqdm import tqdm
```

1.2 load data

[ ]:
```
directory_data  = '/Users/lyuwan-u/Desktop/2021-2/
 ↪assignment-machine-learning-project/assignment05'
filename_data   = 'assignment_05_data.npz'
data            = np.load(os.path.join(directory_data, filename_data))

x_train = data['x_train']
y_train = data['y_train']

x_test  = data['x_test']
y_test  = data['y_test']

num_data_train  = x_train.shape[0]
num_data_test   = x_test.shape[0]

print('*************************************************')
print('size of x_train :', x_train.shape)
print('size of y_train :', y_train.shape)
print('*************************************************')
print('size of x_test :', x_test.shape)
print('size of y_test :', y_test.shape)
print('*************************************************')
print('number of training image :', x_train.shape[0])
print('height of training image :', x_train.shape[1])
print('width of training image :', x_train.shape[2])
print('*************************************************')
print('number of testing image :', x_test.shape[0])
```

```
print('height of testing image :', x_test.shape[1])
print('width of testing image :', x_test.shape[2])
print('*************************************************')
```

```
*************************************************
size of x_train : (20000, 28, 28)
size of y_train : (20000, 10)
*************************************************
size of x_test : (8000, 28, 28)
size of y_test : (8000, 10)
*************************************************
number of training image : 20000
height of training image : 28
width of training image : 28
*************************************************
number of testing image : 8000
height of testing image : 28
width of testing image : 28
*************************************************
```

1.3 number of classes

[ ]:
```
nClass = y_train.shape[1]

print('*************************************************')
print('number of classes :', nClass)
print('*************************************************')
```

```
*************************************************
number of classes : 10
*************************************************
```

1.4 vectorize image data

[ ]:
```
vector_x_train  = x_train.reshape(x_train.shape[0], x_train.shape[1] * x_train.
 ↪shape[2])
vector_x_test   = x_test.reshape(x_test.shape[0], x_test.shape[1] * x_test.
 ↪shape[2])

print('*************************************************')
print('dimension of the training data :', vector_x_train.shape)
print('dimension of the testing data :', vector_x_test.shape)
print('*************************************************')
print('dimension of the training label :', y_train.shape)
print('dimension of the testing label :', y_test.shape)
print('*************************************************')
```

```
*************************************************
```

```
dimension of the training data : (20000, 784)
dimension of the testing data : (8000, 784)
*************************************************
dimension of the training label : (20000, 10)
dimension of the testing label : (8000, 10)
*************************************************
```

1.5 index for each class

```
[ ]: index_train = {}
     index_test  = {}

     number_index_train  = np.zeros(nClass)
     number_index_test   = np.zeros(nClass)

     print('*************************************************')

     for i in range(nClass):

         index_train[i]  = np.where(y_train[:, i] == 1)
         index_test[i]   = np.where(y_test[:, i] == 1)

         number_index_train[i]   = np.shape(index_train[i])[1]
         number_index_test[i]    = np.shape(index_test[i])[1]

         print('number of the training data for class %2d : %5d' % (i,␣
      ↪number_index_train[i]))
         print('number of the testing data for class %2d : %5d' % (i,␣
      ↪number_index_test[i]))

     print('*************************************************')
```

```
*************************************************
number of the training data for class  0 :  2000
number of the testing data for class  0 :   800
number of the training data for class  1 :  2000
number of the testing data for class  1 :   800
number of the training data for class  2 :  2000
number of the testing data for class  2 :   800
number of the training data for class  3 :  2000
number of the testing data for class  3 :   800
number of the training data for class  4 :  2000
number of the testing data for class  4 :   800
number of the training data for class  5 :  2000
number of the testing data for class  5 :   800
number of the training data for class  6 :  2000
number of the testing data for class  6 :   800
number of the training data for class  7 :  2000
```

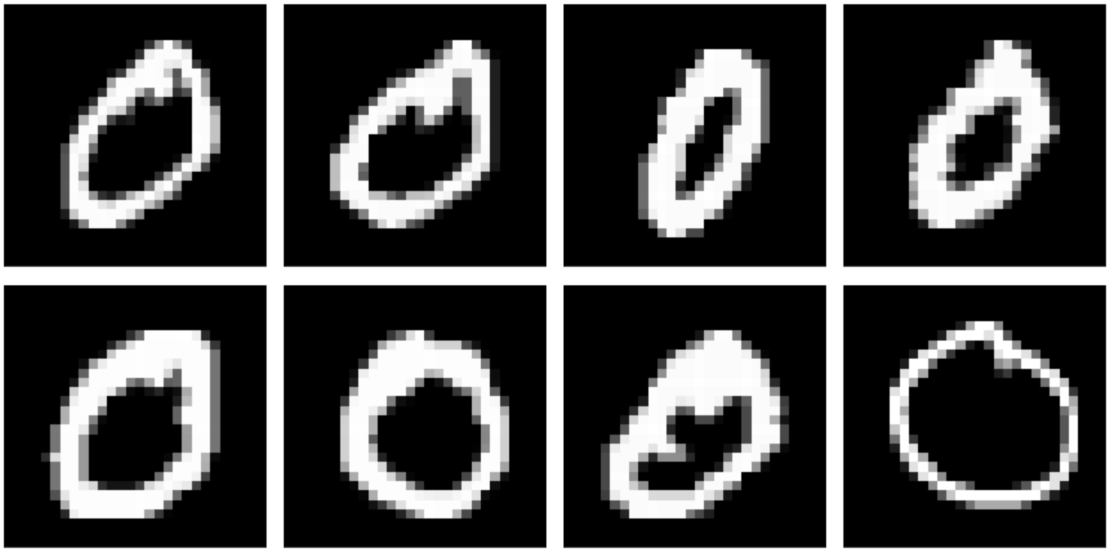
```
number of the testing data for class  7 :   800
number of the training data for class  8 :  2000
number of the testing data for class  8 :   800
number of the training data for class  9 :  2000
number of the testing data for class  9 :   800
*************************************************
```

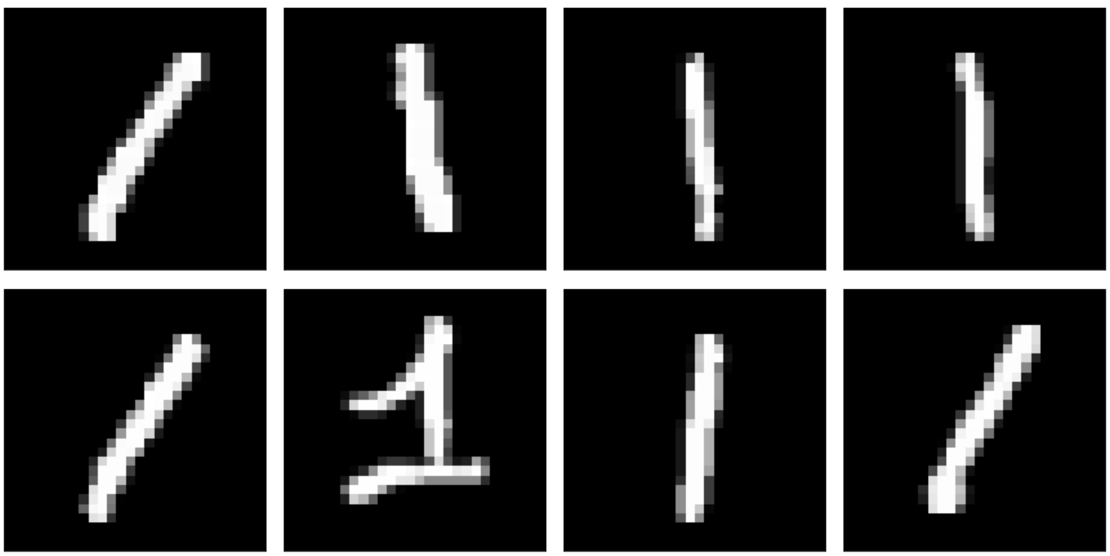
1.6 plot data

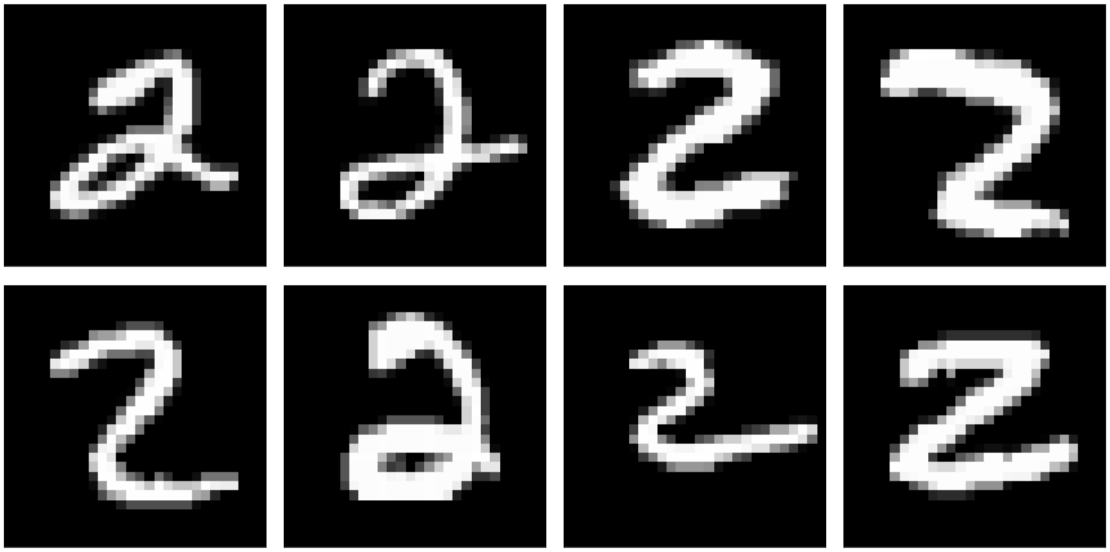
[ ]:
```
def plot_data_grid(data, index_data, nRow, nCol):

    fig, axes = plt.subplots(nRow, nCol, constrained_layout=True, figsize=(nCol␣
 ↪* 3, nRow * 3))

    for i in range(nRow):
        for j in range(nCol):

            k       = i * nCol + j
            index   = index_data[k]

            axes[i, j].imshow(data[index], cmap='gray', vmin=0, vmax=1)
            axes[i, j].xaxis.set_visible(False)
            axes[i, j].yaxis.set_visible(False)

    plt.show()
```

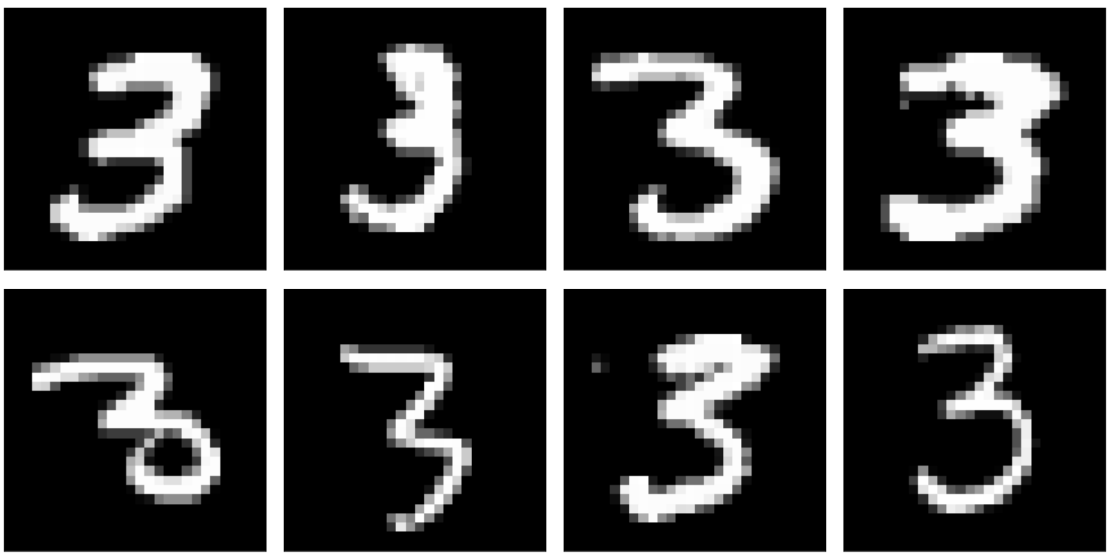
[ ]:
```
nRow    = 2
nCol    = 4
nPlot   = nRow * nCol
```

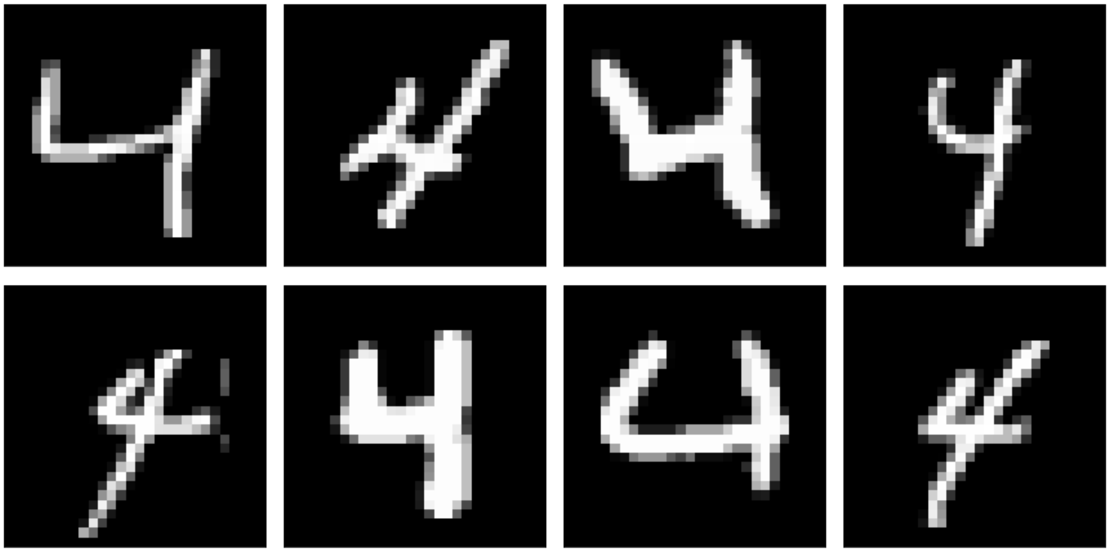
[ ]:
```
for i in range(nClass):

    index_class_plot = index_train[i][0][0:nPlot]
    plot_data_grid(x_train, index_class_plot, nRow, nCol)
```

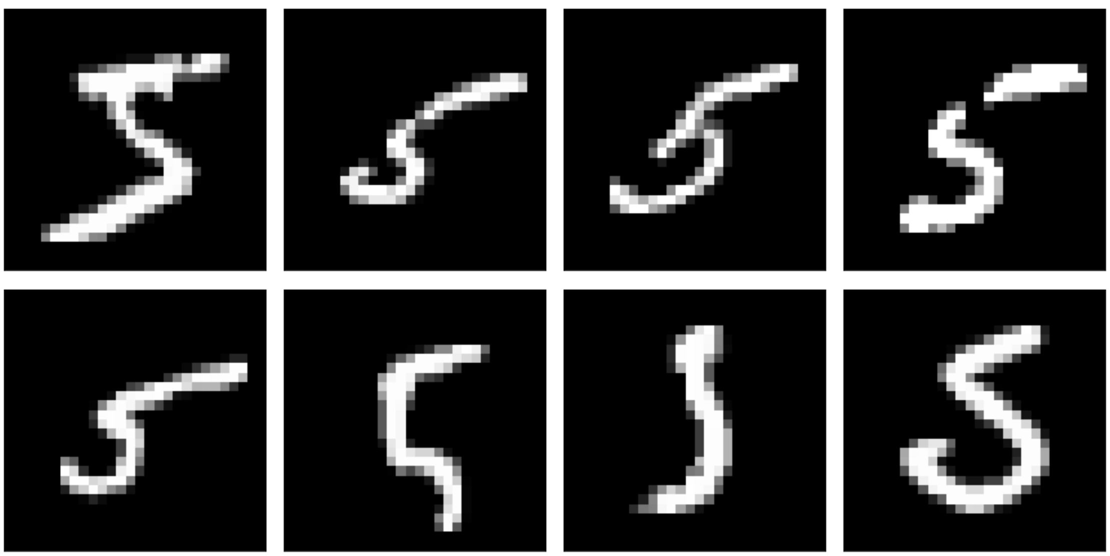


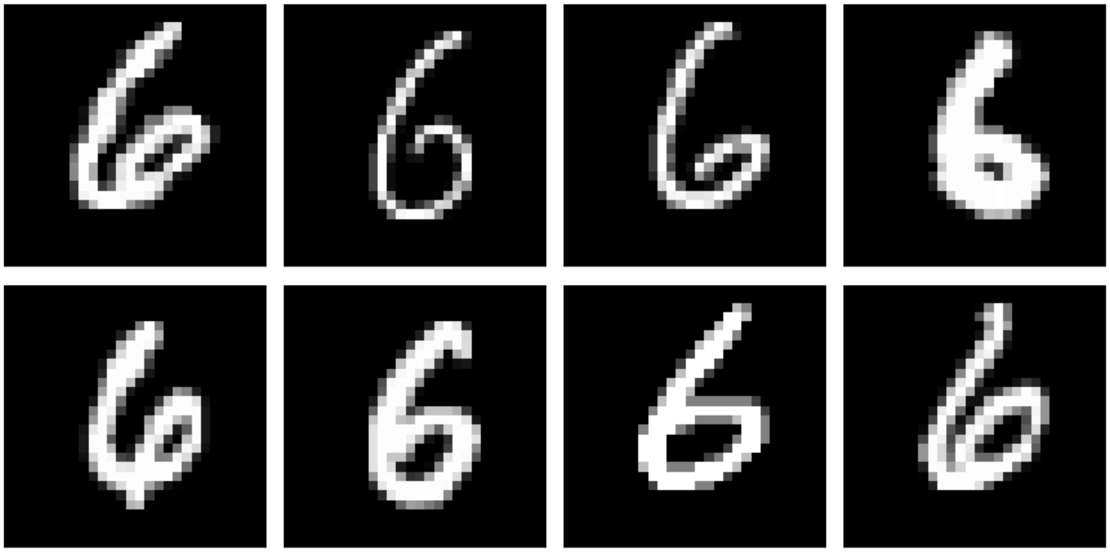


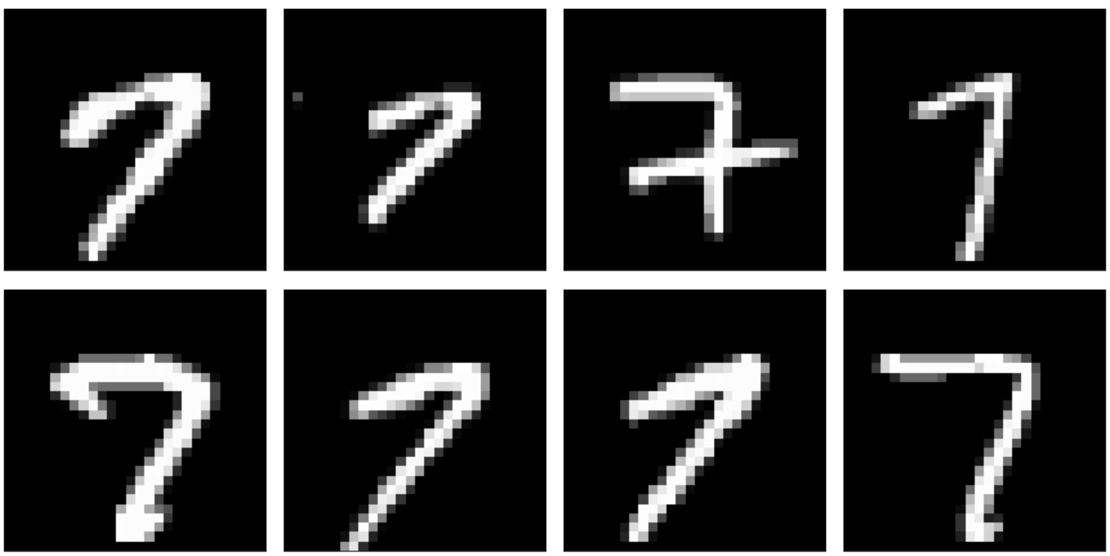


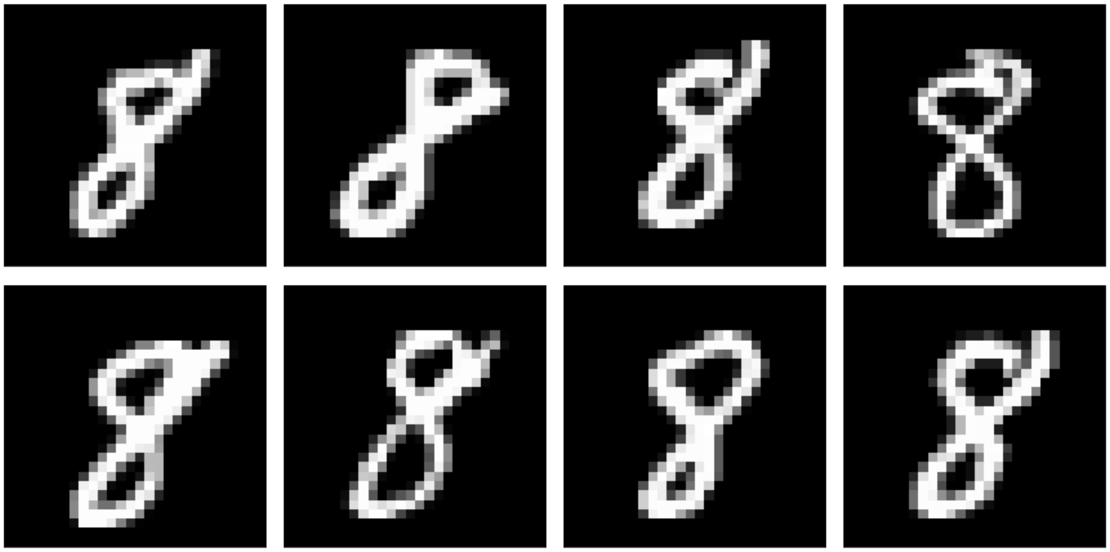


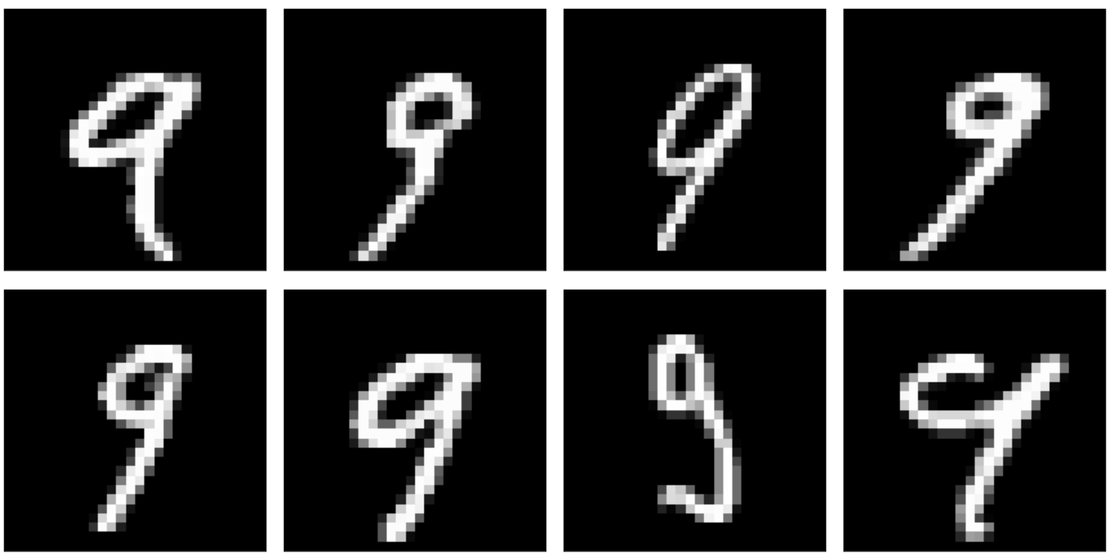












1.7 linear layer

[ ]:
```
one_train    = np.ones(shape=(len(vector_x_train),1))
one_test     = np.ones(shape=(len(vector_x_test),1))

vector_x_train_bias = np.hstack([vector_x_train,one_train])
vector_x_test_bias  = np.hstack([vector_x_test,one_test])

print('dimension of the training data with bias :', vector_x_train_bias.shape)
```

```
print('dimension of the testing data with bias :', vector_x_test_bias.shape)
```

```
dimension of the training data with bias : (20000, 785)
dimension of the testing data with bias : (8000, 785)
```

[ ]:
```
def get_weight(dim_input, dim_output):

    weight = np.ones((dim_output,dim_input+1))
    # initialize the model parameters (linear = 0.001, bias = 1)
    weight = weight * 0.001
    weight[:,-1] = 1

    return weight

dim_input   = vector_x_train.shape[1]
dim_output  = nClass

weight = get_weight(dim_input, dim_output)

print('dimension of the model parameters: ', weight.shape)
print('first row of the weight matrix: ', weight[0, :])
print(weight[:,-1])
```

```
dimension of the model parameters:  (10, 785)
first row of the weight matrix:  [0.001 0.001 0.001 0.001 0.001 0.001 0.001
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[1. 1. 1. 1. 1. 1. 1. 1. 1. 1.]
```

[ ]:
```python
def layer_linear(input, weight):
```

```
    output = np.dot(input,weight.T)

    return output
```

[ ]:
```
layer_linear(vector_x_train_bias,weight)
```

[ ]:
```
array([[1.12194118, 1.12194118, 1.12194118, …, 1.12194118, 1.12194118,
         1.12194118],
        [1.13895294, 1.13895294, 1.13895294, …, 1.13895294, 1.13895294,
         1.13895294],
        [1.14316471, 1.14316471, 1.14316471, …, 1.14316471, 1.14316471,
         1.14316471],
        …,
        [1.1306    , 1.1306    , 1.1306    , …, 1.1306    , 1.1306    ,
         1.1306    ],
        [1.16640784, 1.16640784, 1.16640784, …, 1.16640784, 1.16640784,
         1.16640784],
        [1.07769804, 1.07769804, 1.07769804, …, 1.07769804, 1.07769804,
         1.07769804]])
```

1.8 Softmax function

[ ]:
```
def activation_softmax(input):

    input_exp = np.exp(input)
    partition = input_exp.sum(axis=1,keepdims=True)
    output    = input_exp / partition

    return output
```

1.9 compute prediction by the forward propagation of the neural network

[ ]:
```
def compute_prediction(input, weight):

    output      = layer_linear(input,weight)
    prediction  = activation_softmax(output)

    return prediction
```

[ ]:
```
compute_prediction(vector_x_train_bias,weight)
```

[ ]:
```
array([[0.1, 0.1, 0.1, …, 0.1, 0.1, 0.1],
        [0.1, 0.1, 0.1, …, 0.1, 0.1, 0.1],
        [0.1, 0.1, 0.1, …, 0.1, 0.1, 0.1],
        …,
        [0.1, 0.1, 0.1, …, 0.1, 0.1, 0.1],
```

```
[0.1, 0.1, 0.1, …, 0.1, 0.1, 0.1],
[0.1, 0.1, 0.1, …, 0.1, 0.1, 0.1]])
```

1.10 compute loss for the cross-entropy term

```
[ ]: def compute_loss_cross_entropy(prediction, label):

         pred = np.log(prediction)
         loss = -(np.sum(np.sum(label*pred,axis=1)))/label.shape[0]
         return loss
```

1.11 compute loss for the regularization term

```
[ ]: def compute_loss_regularization(alpha, weight):

         L2_norm_square = np.linalg.norm(weight,ord=2) ** 2
         loss = L2_norm_square * alpha / 2

         return loss
```

```
[ ]: compute_loss_regularization(0.1,weight)
```

```
[ ]: 0.5003920000000002
```

1.12 compute loss function

```
[ ]: def compute_loss(prediction, label, alpha, weight):

         loss = compute_loss_cross_entropy(prediction,label) +␣
       ↪compute_loss_regularization(alpha,weight)

         return loss
```

1.13 compute gradient for the cross-entropy term

```
[ ]: def compute_gradient_cross_entropy(input, prediction, label):

         gradient = np.dot((prediction-label).T,input)
         gradient = gradient / len(label)

         return gradient
```

1.14 compute gradient for the regularization term

[ ]:
```
def compute_gradient_regularization(alpha, weight):

    gradient =  alpha * weight

    return gradient
```

[ ]:
```
compute_gradient_regularization(0.1,weight).shape
```

[ ]:
```
(10, 785)
```

1.15 compute gradient

[ ]:
```
def compute_gradient(input, prediction, label, alpha, weight):

    gradient = compute_gradient_cross_entropy(input,prediction,label) +␣
 ↪compute_gradient_regularization(alpha,weight)

    return gradient
```

1.16 compute accuracy

[ ]:
```
def compute_accuracy(prediction, label):

    bCorrect    = [True if list(prediction[i]).
 ↪index(max(prediction[i]))==list(label[i]).index(1) else False for i in␣
 ↪range(len(label))]
    accuracy    = np.sum(bCorrect) / len(label)
    return accuracy
```

1.17 consider bias in the data

[ ]:
```
one_train   = np.ones(shape=(len(vector_x_train),1))
one_test    = np.ones(shape=(len(vector_x_test),1))

vector_x_train_bias = np.hstack([vector_x_train,one_train])
vector_x_test_bias  = np.hstack([vector_x_test,one_test])

print('dimension of the training data with bias :', vector_x_train_bias.shape)
print('dimension of the testing data with bias :', vector_x_test_bias.shape)
```

```
dimension of the training data with bias : (20000, 785)
dimension of the testing data with bias : (8000, 785)
```

1.18 construct model parameters and initialize them

[ ]:
```python
def get_weight(dim_input, dim_output):

    weight = np.ones((dim_output,dim_input))
    # initialize the model parameters (linear = 0.001, bias = 1)
    weight = weight * 0.001
    weight[:,-1] = 1

    return weight
```

[ ]:
```python
dim_input   = vector_x_train_bias.shape[1]
dim_output  = nClass

weight = get_weight(dim_input, dim_output)

print('dimension of the model parameters: ', weight.shape)
print('first row of the weight matrix: ', weight[0, :])
```

```
dimension of the model parameters:  (10, 785)
first row of the weight matrix:  [0.001 0.001 0.001 0.001 0.001 0.001 0.001
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```

1.19 hyper-parameters

[ ]:
```
number_iteration    = 1000
learning_rate       = 0.001

list_size_minibatch = [50, 100, 200]
list_weight_decay   = [0.001, 0.01, 0.1]
```

```
num_size_minibatch  = len(list_size_minibatch)
num_weight_decay    = len(list_weight_decay)
```

1.20 variables for optimization information for different minibatch

[ ]:
```
train_loss_mean_minibatch     = np.zeros((num_size_minibatch, number_iteration))
train_loss_std_minibatch      = np.zeros((num_size_minibatch, number_iteration))

train_accuracy_mean_minibatch = np.zeros((num_size_minibatch, number_iteration))
train_accuracy_std_minibatch  = np.zeros((num_size_minibatch, number_iteration))

test_loss_minibatch           = np.zeros((num_size_minibatch, number_iteration))
test_accuracy_minibatch       = np.zeros((num_size_minibatch, number_iteration))
```

1.21 variables for optimization information for different weight decay

[ ]:
```
train_loss_mean_weight_decay        = np.zeros((num_weight_decay,␣
↪number_iteration))
train_loss_std_weight_decay         = np.zeros((num_weight_decay,␣
↪number_iteration))

train_accuracy_mean_weight_decay    = np.zeros((num_weight_decay,␣
↪number_iteration))
train_accuracy_std_weight_decay     = np.zeros((num_weight_decay,␣
↪number_iteration))

test_loss_weight_decay              = np.zeros((num_weight_decay,␣
↪number_iteration))
test_accuracy_weight_decay          = np.zeros((num_weight_decay,␣
↪number_iteration))
```

1.22 stochastic gradient descent iterations with different mini-batch (with alpha = 0)

[ ]:
```
print(num_data_train)
t = np.random.permutation(num_data_train)[0:0+list_size_minibatch[1]]
print(vector_x_train_bias[t].shape)
```

```
20000
(100, 785)
```

[ ]:
```
# iteration for mini-batch
for k in range(num_size_minibatch):

    size_minibatch  = list_size_minibatch[k]
```

```
    num_minibatch   = len(vector_x_train_bias) // size_minibatch
    alpha           = 0

    print('mini-batch size = %3d, alpha = %4.3f' % (size_minibatch, alpha))

    weight = get_weight(dim_input,nClass)
    # initialze seed for generating random number
    np.random.seed(0)
    # iteration for epoch
    for i in tqdm(range(number_iteration)):

        index_shuffle   = np.random.permutation(num_data_train)
        loss_epoch      = []
        accuracy_epoch  = []

        for j in range(num_minibatch):

            index_minibatch = index_shuffle[j*size_minibatch:
↪(j+1)*size_minibatch]
            data    = vector_x_train_bias[index_minibatch]
            label   = y_train[index_minibatch]

            prediction  = compute_prediction(data,weight)
            gradient    = compute_gradient(data,prediction,label,alpha,weight)
            weight      = weight  - learning_rate * gradient

            prediction  = compute_prediction(data,weight)
            loss        = compute_loss(prediction,label,alpha,weight)
            accuracy    = compute_accuracy(prediction,label)

            loss_epoch.append(loss)
            accuracy_epoch.append(accuracy)

        train_loss_mean_minibatch[k, i] = np.mean(loss_epoch)
        train_loss_std_minibatch[k, i]  = np.std(loss_epoch)

        train_accuracy_mean_minibatch[k, i] = np.mean(accuracy_epoch)
        train_accuracy_std_minibatch[k, i]  = np.std(accuracy_epoch)

        # testing
        data    = vector_x_test_bias
        label   = y_test

        prediction      = compute_prediction(data,weight)
        loss_test       = compute_loss(prediction,label,alpha,weight)
        accuracy_test   = compute_accuracy(prediction,label)
```

```
        test_loss_minibatch[k, i]       = loss_test
        test_accuracy_minibatch[k, i]   = accuracy_test
```

```
  0%|          | 0/1000 [00:00<?, ?it/s]

mini-batch size =  50, alpha = 0.000

100%|     | 1000/1000 [05:36<00:00,  2.97it/s]
  0%|          | 0/1000 [00:00<?, ?it/s]

mini-batch size = 100, alpha = 0.000

100%|     | 1000/1000 [04:43<00:00,  3.52it/s]
  0%|          | 0/1000 [00:00<?, ?it/s]

mini-batch size = 200, alpha = 0.000

100%|     | 1000/1000 [04:16<00:00,  3.89it/s]
```

1.23 stochastic gradient descent iterations with different regularization parameter (weight decay) (with mini-batch size = 100)

[ ]:
```
# iteration for mini-batch
for k in range(num_weight_decay):

    size_minibatch  = 100
    num_minibatch   = len(vector_x_train_bias) // size_minibatch
    alpha           = list_weight_decay[k]

    print('mini-batch size = %3d, alpha = %4.3f' % (size_minibatch, alpha))

    weight = get_weight(dim_input,nClass)
    # initialze seed for generating random number
    np.random.seed(0)
    # iteration for epoch
    for i in tqdm(range(number_iteration)):

        index_shuffle   = np.random.permutation(num_data_train)
        loss_epoch      = []
        accuracy_epoch  = []

        for j in range(num_minibatch):

            index_minibatch = index_shuffle[j*size_minibatch:
 ↪(j+1)*size_minibatch]

            data    = vector_x_train_bias[index_minibatch]
            label   = y_train[index_minibatch]

            prediction  = compute_prediction(data,weight)
```

```
            gradient    = compute_gradient(data,prediction,label,alpha,weight)
            weight      = weight - learning_rate * gradient

            prediction  = compute_prediction(data,weight)
            loss        = compute_loss(prediction,label,alpha,weight)
            accuracy    = compute_accuracy(prediction,label)

            loss_epoch.append(loss)
            accuracy_epoch.append(accuracy)

        train_loss_mean_weight_decay[k, i] = np.mean(loss_epoch)
        train_loss_std_weight_decay[k, i]  = np.std(loss_epoch)

        train_accuracy_mean_weight_decay[k, i] = np.mean(accuracy_epoch)
        train_accuracy_std_weight_decay[k, i]  = np.std(accuracy_epoch)

        # testing
        data    = vector_x_test_bias
        label   = y_test

        prediction      = compute_prediction(data,weight)
        loss_test       = compute_loss(prediction,label,alpha,weight)
        accuracy_test   = compute_accuracy(prediction,label)

        test_loss_weight_decay[k, i]       = loss_test
        test_accuracy_weight_decay[k, i]   = accuracy_test
```

```
  0%|          | 0/1000 [00:00<?, ?it/s]

mini-batch size = 100, alpha = 0.001

100%|     | 1000/1000 [05:07<00:00,  3.25it/s]
  0%|          | 0/1000 [00:00<?, ?it/s]

mini-batch size = 100, alpha = 0.010

100%|     | 1000/1000 [04:38<00:00,  3.59it/s]
  0%|          | 0/1000 [00:00<?, ?it/s]

mini-batch size = 100, alpha = 0.100

100%|     | 1000/1000 [04:36<00:00,  3.62it/s]
```

1.24 plot curve

[ ]:
```
def plot_curve(data, x_label, y_label, title):

    plt.figure(figsize=(8, 6))
    plt.title(title)
```

```
    plt.plot(range(len(data)), data, '-', color='red')

    plt.xlabel(x_label)
    plt.ylabel(y_label)

    plt.tight_layout()
    plt.show()
```

[ ]:
```
def plot_curve2(data1, label_data1, data2, label_data2, x_label, y_label,␣
↪title):

    plt.figure(figsize=(8, 6))
    plt.title(title)

    plt.plot(range(len(data1)), data1, '-', color = 'blue', label = label_data1)
    plt.plot(range(len(data2)), data2, '-', color = 'red', label = label_data2)

    plt.xlabel(x_label)
    plt.ylabel(y_label)

    plt.legend()
    plt.tight_layout()
    plt.show()
```

[ ]:
```
def plot_curve3(data1, label_data1, data2, label_data2, data3, label_data3,␣
↪x_label, y_label, title):

    plt.figure(figsize=(8, 6))
    plt.title(title)

    plt.plot(range(len(data1)), data1, '-', color = 'blue', label = label_data1)
    plt.plot(range(len(data2)), data2, '-', color = 'red', label = label_data2)
    plt.plot(range(len(data3)), data3, '-', color = 'green', label =␣
↪label_data3)

    plt.xlabel(x_label)
    plt.ylabel(y_label)

    plt.legend()
    plt.tight_layout()
    plt.show()
```

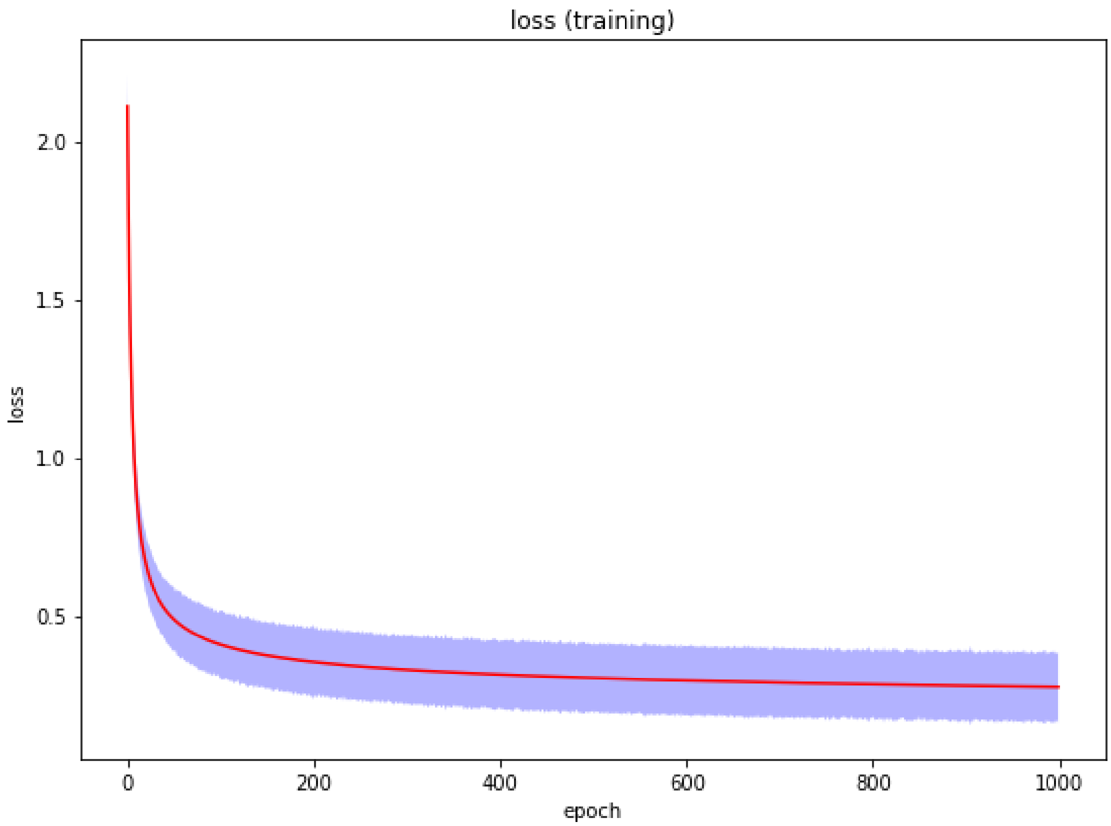
[ ]:
```
def plot_curve_error(data_mean, data_std, x_label, y_label, title):

    plt.figure(figsize=(8, 6))
    plt.title(title)
```

```
    alpha = 0.3

    plt.plot(range(len(data_mean)), data_mean, '-', color = 'red')
    plt.fill_between(range(len(data_mean)), data_mean - data_std, data_mean +␣
↪data_std, facecolor = 'blue', alpha = alpha)

    plt.xlabel(x_label)
    plt.ylabel(y_label)

    plt.tight_layout()
    plt.show()
```

[ ]:
```
def plot_curve_error2(data1_mean, data1_std, data1_label, data2_mean,␣
↪data2_std, data2_label, x_label, y_label, title):

    plt.figure(figsize=(8, 6))
    plt.title(title)

    alpha = 0.3

    plt.plot(range(len(data1_mean)), data1_mean, '-', color = 'blue', label =␣
↪data1_label)
    plt.fill_between(range(len(data1_mean)), data1_mean - data1_std, data1_mean␣
↪+ data1_std, facecolor = 'blue', alpha = alpha)

    plt.plot(range(len(data2_mean)), data2_mean, '-', color = 'red', label =␣
↪data2_label)
    plt.fill_between(range(len(data2_mean)), data2_mean - data2_std, data2_mean␣
↪+ data2_std, facecolor = 'red', alpha = alpha)

    plt.xlabel(x_label)
    plt.ylabel(y_label)

    plt.tight_layout()
    plt.show()
```

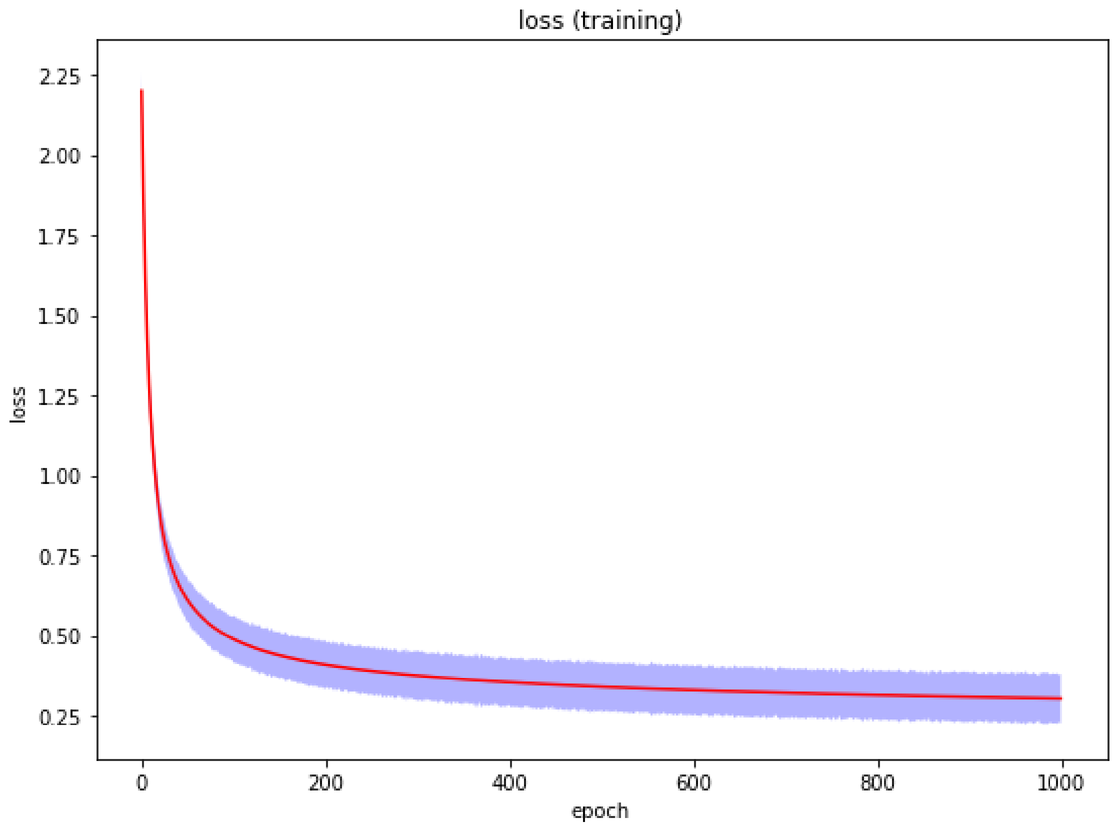
1.25 loss (training) curve : mini-batch size = 50, weight decay = 0

[ ]:
```
plot_curve_error(train_loss_mean_minibatch[0], train_loss_std_minibatch[0],␣
↪'epoch', 'loss', 'loss (training)')
```



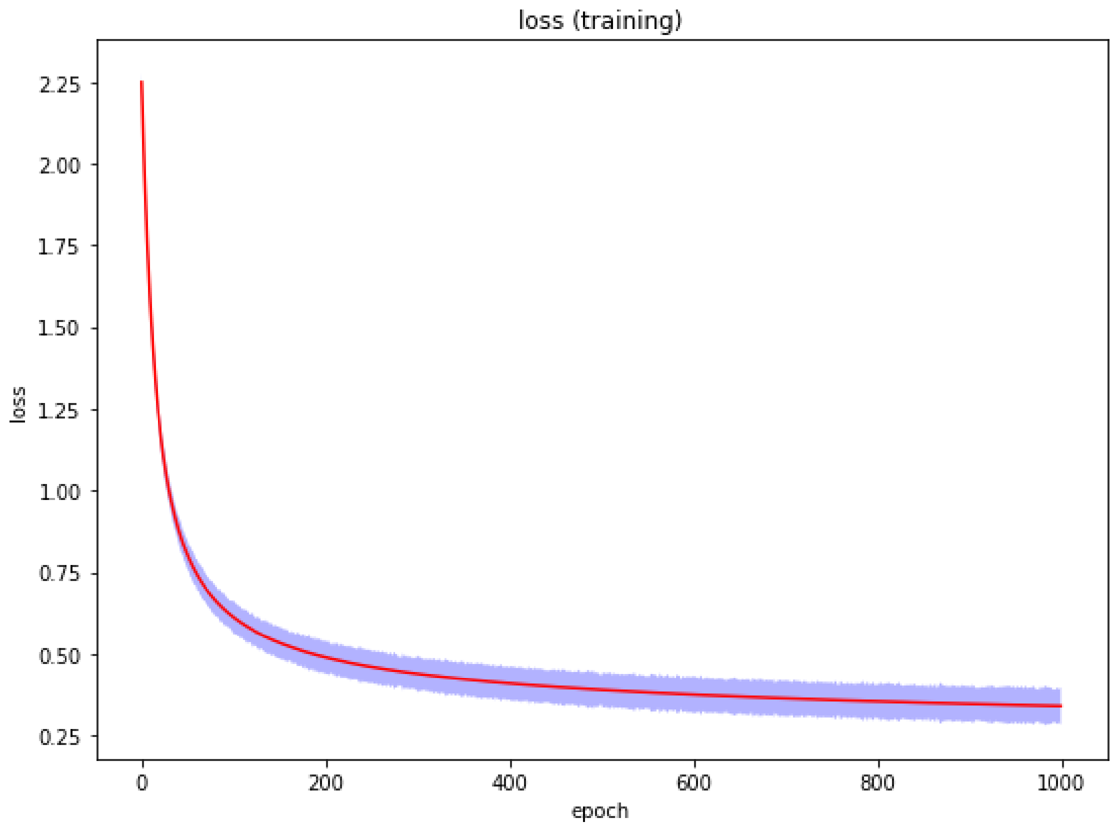
1.26 loss (training) curve : mini-batch size = 100, weight decay = 0

[ ]:
```
plot_curve_error(train_loss_mean_minibatch[1], train_loss_std_minibatch[1],␣
↪'epoch', 'loss', 'loss (training)')
```



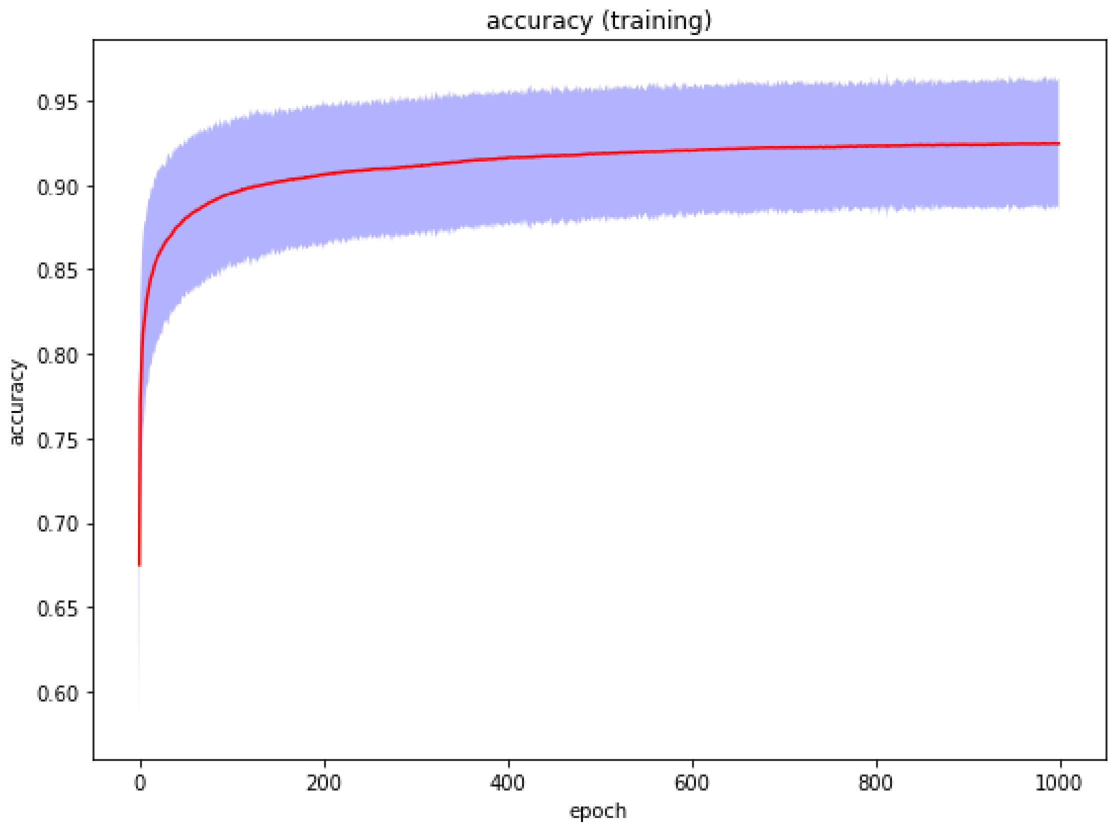
1.27 loss (training) curve : mini-batch size = 200, weight decay = 0

```
[ ]: plot_curve_error(train_loss_mean_minibatch[2], train_loss_std_minibatch[2],␣
      ↪'epoch', 'loss', 'loss (training)')
```



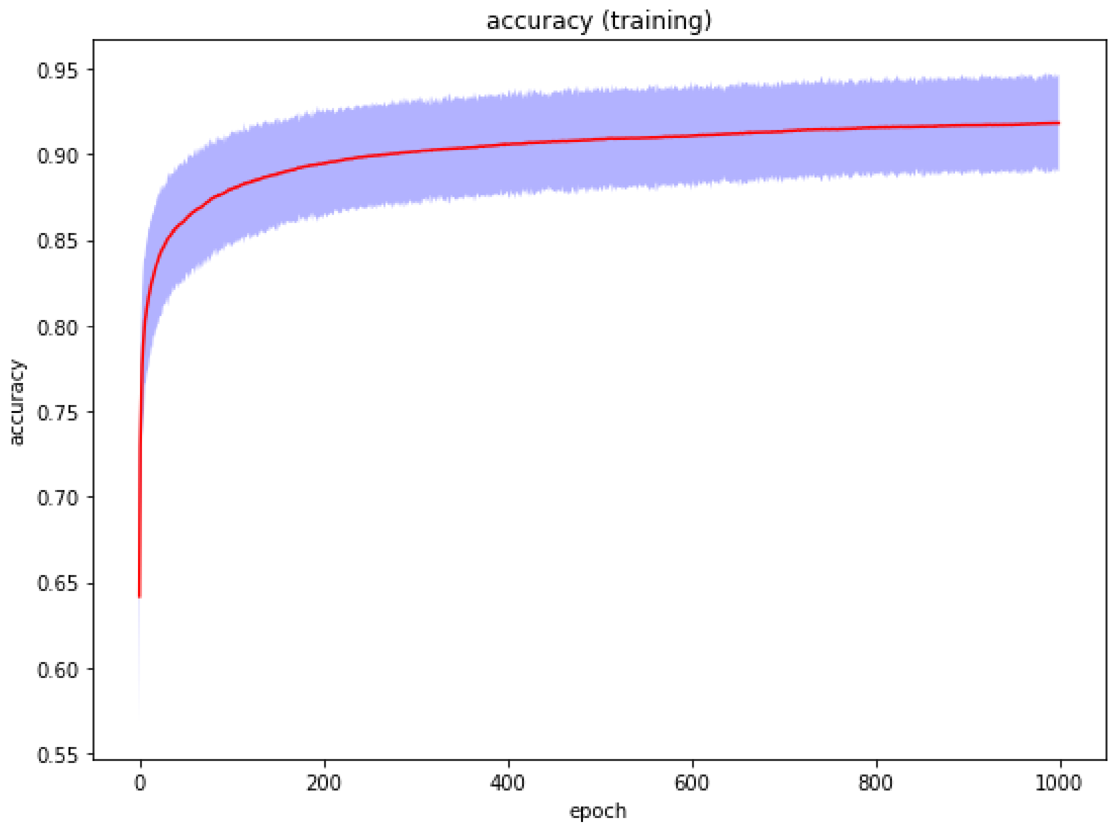
1.28 accuracy (training) curve : mini-batch size = 50, weight decay = 0

```
[ ]: plot_curve_error(train_accuracy_mean_minibatch[0],␣
     ↪train_accuracy_std_minibatch[0], 'epoch', 'accuracy', 'accuracy (training)')
```



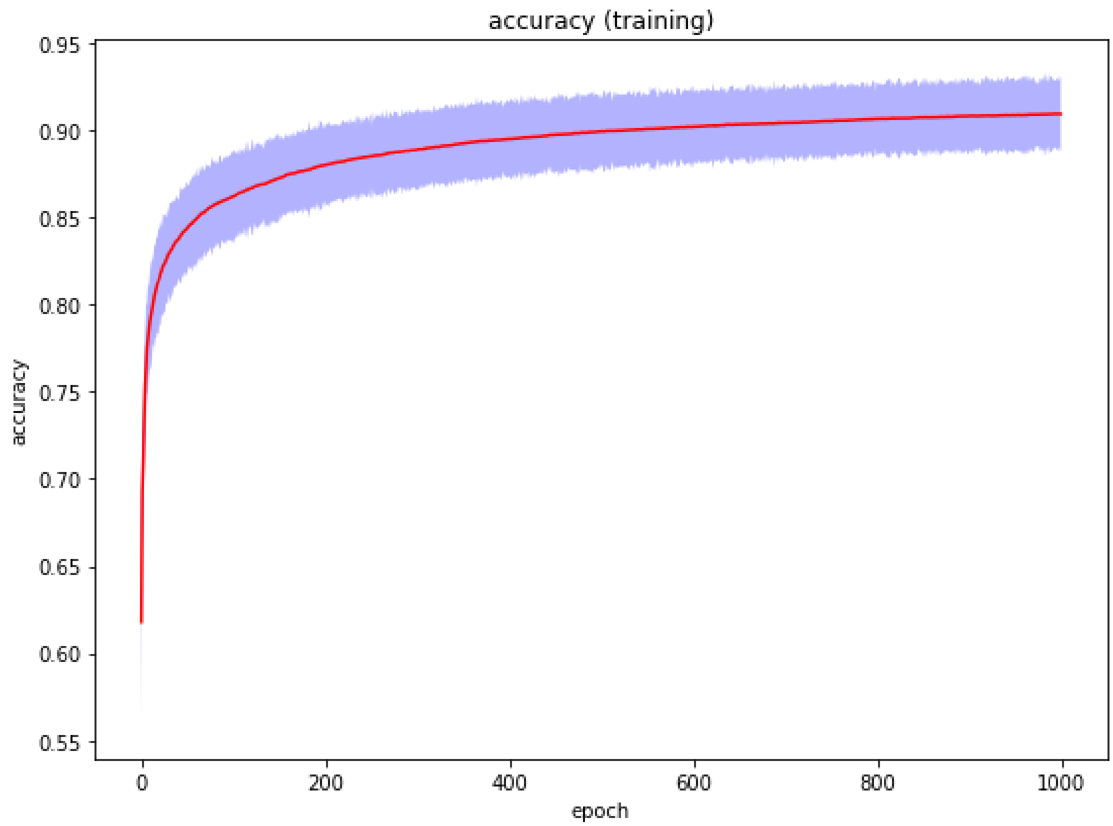
1.29 accuracy (training) curve : mini-batch size = 100, weight decay = 0

[ ]:
```
plot_curve_error(train_accuracy_mean_minibatch[1],␣
→train_accuracy_std_minibatch[1], 'epoch', 'accuracy', 'accuracy (training)')
```



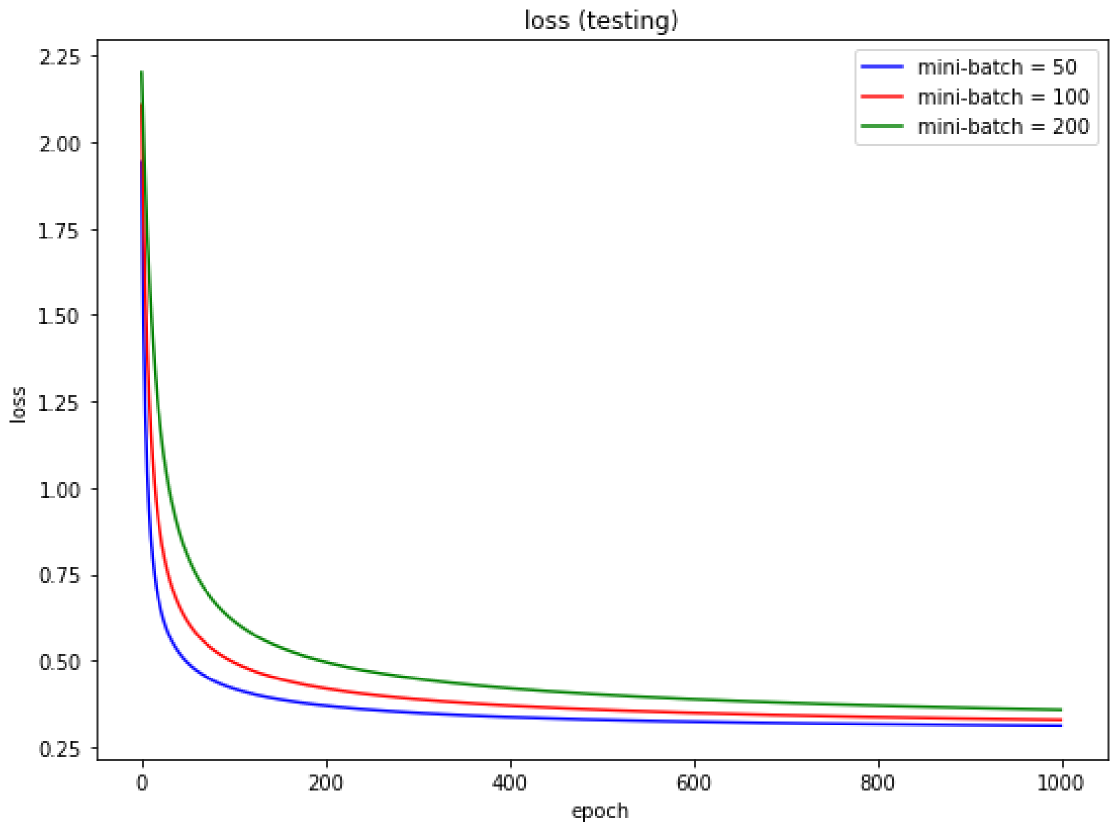
1.30 accuracy (training) curve : mini-batch size = 200, weight decay = 0

[ ]:
```
plot_curve_error(train_accuracy_mean_minibatch[2],␣
↪train_accuracy_std_minibatch[2], 'epoch', 'accuracy', 'accuracy (training)')
```



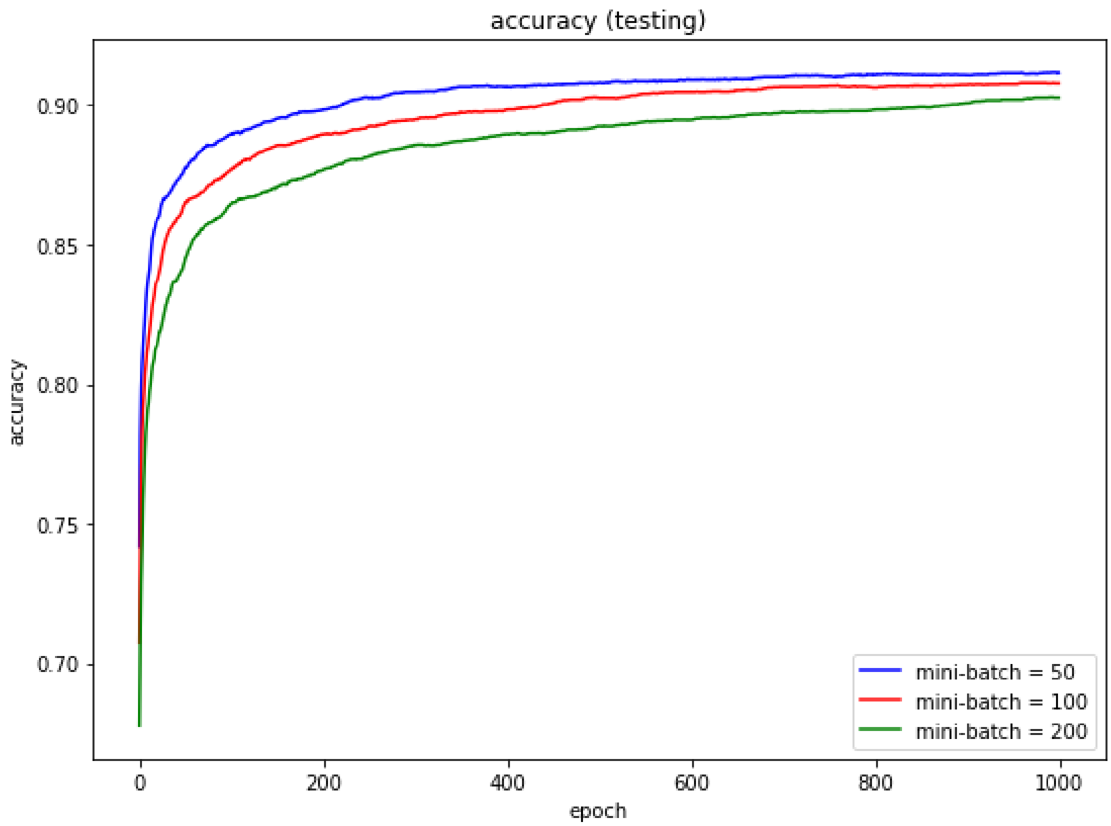
1.31 loss (testing) curve with different mini-batch

[ ]:
```
plot_curve3(test_loss_minibatch[0], 'mini-batch = 50', test_loss_minibatch[1],␣
→'mini-batch = 100', test_loss_minibatch[2], 'mini-batch = 200', 'epoch',␣
→'loss', 'loss (testing)')
```



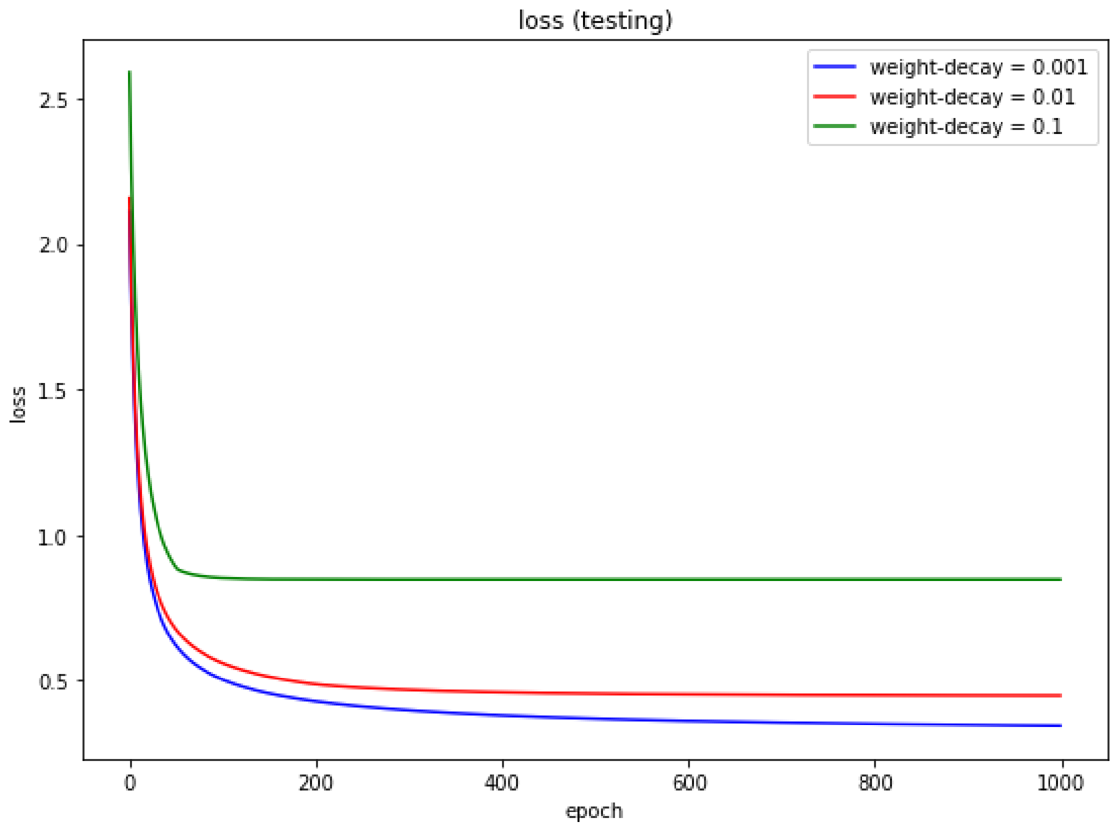
1.32 accuracy (testing) curve with different mini-batch

[ ]:
```
plot_curve3(test_accuracy_minibatch[0], 'mini-batch = 50',␣
↪test_accuracy_minibatch[1], 'mini-batch = 100', test_accuracy_minibatch[2],␣
↪'mini-batch = 200', 'epoch', 'accuracy', 'accuracy (testing)')
```



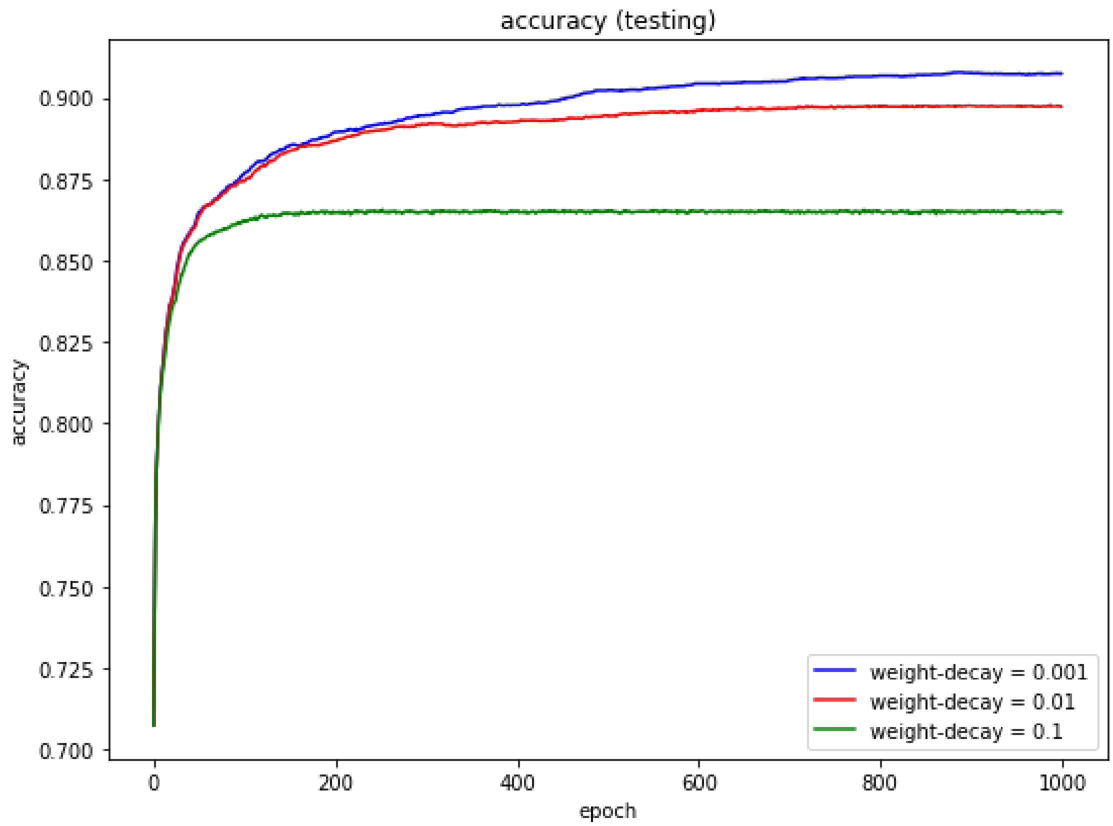
1.33 loss (testing) curve with different weight-decay

[ ]:
```
plot_curve3(test_loss_weight_decay[0], 'weight-decay = 0.001',␣
↪test_loss_weight_decay[1], 'weight-decay = 0.01', test_loss_weight_decay[2],␣
↪'weight-decay = 0.1', 'epoch', 'loss', 'loss (testing)')
```



1.34 accuracy (testing) curve with different weight-decay

[ ]:
```
plot_curve3(test_accuracy_weight_decay[0], 'weight-decay = 0.001',␣
↪test_accuracy_weight_decay[1], 'weight-decay = 0.01',␣
↪test_accuracy_weight_decay[2], 'weight-decay = 0.1', 'epoch', 'accuracy',␣
↪'accuracy (testing)')
```



1.35 print values

[ ]:
```
def print_curve(data, index):

    for i in range(len(index)):

        idx = index[i]
        val = data[idx]

        print('index = %4d, value = %12.10f' % (idx, val))
```

1.36 given iterations at which the values are presented

[ ]:
```
index = np.array([0, 100, 200, 300, 400, 500, 600, 700, 800, 900])
```

1.37 training loss (mean) : mini-batch = 50, weight-decay = 0

[ ]:
```
print_curve(train_loss_mean_minibatch[0], index)
```

```
index =    0, value = 2.1095174326
index =  100, value = 0.4083112742
index =  200, value = 0.3536532803
index =  300, value = 0.3289345117
index =  400, value = 0.3136814192
index =  500, value = 0.3029182430
index =  600, value = 0.2947188323
index =  700, value = 0.2881619450
index =  800, value = 0.2827342002
index =  900, value = 0.2781186702
```

1.38 training loss (mean) : mini-batch = 100, weight-decay = 0

[ ]:
```
print_curve(train_loss_mean_minibatch[1], index)
```

```
index =    0, value = 2.2003866772
index =  100, value = 0.4884452279
index =  200, value = 0.4086303762
index =  300, value = 0.3743102039
index =  400, value = 0.3538169113
index =  500, value = 0.3396656612
index =  600, value = 0.3290494042
index =  700, value = 0.3206579243
index =  800, value = 0.3137801756
index =  900, value = 0.3079845796
```

1.39 training loss (mean) : mini-batch = 200, weight-decay = 0

[ ]:
```
print_curve(train_loss_mean_minibatch[2], index)
```

```
index =    0, value = 2.2496536286
index =  100, value = 0.6111343709
index =  200, value = 0.4888445621
index =  300, value = 0.4381704814
index =  400, value = 0.4087920721
index =  500, value = 0.3889821374
index =  600, value = 0.3744117757
index =  700, value = 0.3630728201
index =  800, value = 0.3538961488
index =  900, value = 0.3462466136
```

1.40 training accuracy (mean) : mini-batch = 50, weight-decay = 0

[ ]:
```
print_curve(train_accuracy_mean_minibatch[0], index)
```

```
index =    0, value = 0.6751000000
index =  100, value = 0.8951500000
index =  200, value = 0.9062000000
```

```
index =  300, value = 0.9112000000
index =  400, value = 0.9160000000
index =  500, value = 0.9184000000
index =  600, value = 0.9204000000
index =  700, value = 0.9224000000
index =  800, value = 0.9230000000
index =  900, value = 0.9238500000
```

1.41 training accuracy (mean) : mini-batch = 100, weight-decay = 0

[ ]:
```
print_curve(train_accuracy_mean_minibatch[1], index)
```

```
index =    0, value = 0.6415000000
index =  100, value = 0.8798500000
index =  200, value = 0.8948000000
index =  300, value = 0.9018000000
index =  400, value = 0.9062000000
index =  500, value = 0.9090500000
index =  600, value = 0.9109500000
index =  700, value = 0.9137000000
index =  800, value = 0.9158000000
index =  900, value = 0.9171500000
```

1.42 training accuracy (mean) : mini-batch = 200, weight-decay = 0

[ ]:
```
print_curve(train_accuracy_mean_minibatch[2], index)
```

```
index =    0, value = 0.6180500000
index =  100, value = 0.8617500000
index =  200, value = 0.8798500000
index =  300, value = 0.8886000000
index =  400, value = 0.8947000000
index =  500, value = 0.8989500000
index =  600, value = 0.9015500000
index =  700, value = 0.9039000000
index =  800, value = 0.9062000000
index =  900, value = 0.9078000000
```

1.43 training loss (mean) : mini-batch = 100, weight-decay = 0.001

[ ]:
```
print_curve(train_loss_mean_weight_decay[0], index)
```

```
index =    0, value = 2.2053959737
index =  100, value = 0.4962629800
index =  200, value = 0.4175434109
index =  300, value = 0.3840709465
index =  400, value = 0.3643988464
index =  500, value = 0.3514216500
```

```
index =  600, value = 0.3419763652
index =  700, value = 0.3346759415
index =  800, value = 0.3288218803
index =  900, value = 0.3239931453
```

1.44 training loss (mean) : mini-batch = 100, weight-decay = 0.01

[ ]:
```
print_curve(train_loss_mean_weight_decay[1], index)
```

```
index =    0, value = 2.2503892272
index =  100, value = 0.5527154423
index =  200, value = 0.4768299350
index =  300, value = 0.4555187148
index =  400, value = 0.4458227005
index =  500, value = 0.4405324514
index =  600, value = 0.4373688185
index =  700, value = 0.4353625905
index =  800, value = 0.4340460613
index =  900, value = 0.4331493839
```

1.45 training loss (mean) : mini-batch = 100, weight-decay = 0.1

[ ]:
```
print_curve(train_loss_mean_weight_decay[2], index)
```

```
index =    0, value = 2.6914887181
index =  100, value = 0.8456924353
index =  200, value = 0.8404500365
index =  300, value = 0.8401991509
index =  400, value = 0.8401929995
index =  500, value = 0.8402133683
index =  600, value = 0.8401931172
index =  700, value = 0.8401618275
index =  800, value = 0.8401630632
index =  900, value = 0.8402009377
```

1.46 training accuracy (mean) : mini-batch = 100, weight-decay = 0.001

[ ]:
```
print_curve(train_accuracy_mean_weight_decay[0], index)
```

```
index =    0, value = 0.6415000000
index =  100, value = 0.8794000000
index =  200, value = 0.8944000000
index =  300, value = 0.9014500000
index =  400, value = 0.9053000000
index =  500, value = 0.9083000000
index =  600, value = 0.9104000000
index =  700, value = 0.9124000000
```

```
index =   800, value = 0.9144500000
index =   900, value = 0.9157000000
```

1.47 training accuracy (mean) : mini-batch = 100, weight-decay = 0.01

```
[ ]: print_curve(train_accuracy_mean_weight_decay[1], index)
```

```
index =     0, value = 0.6414500000
index =   100, value = 0.8781500000
index =   200, value = 0.8913000000
index =   300, value = 0.8969500000
index =   400, value = 0.8999500000
index =   500, value = 0.9016500000
index =   600, value = 0.9028000000
index =   700, value = 0.9032000000
index =   800, value = 0.9034500000
index =   900, value = 0.9040000000
```

1.48 training accuracy (mean) : mini-batch = 100, weight-decay = 0.1

```
[ ]: print_curve(train_accuracy_mean_weight_decay[2], index)
```

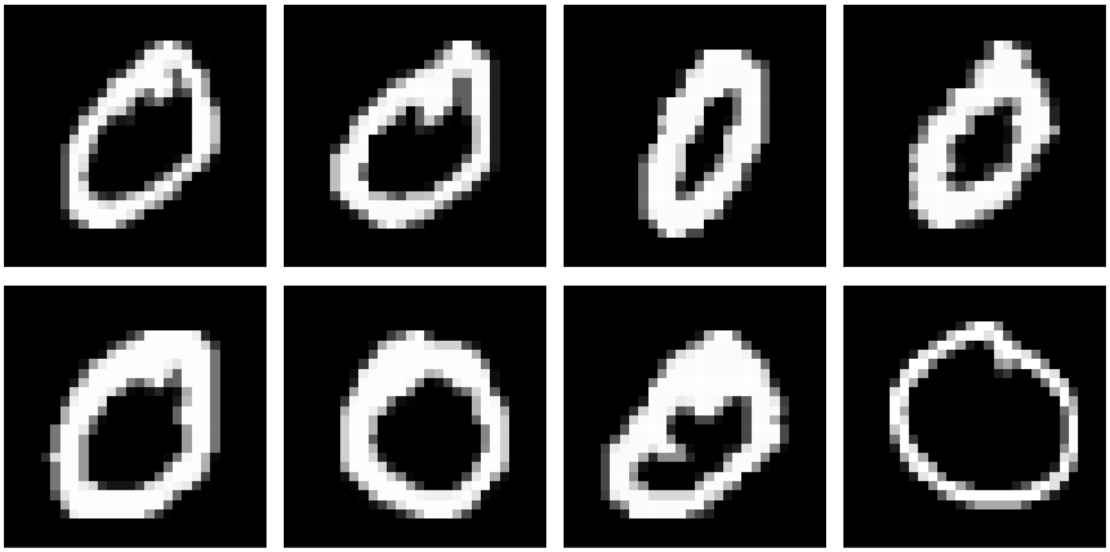
```
index =     0, value = 0.6413000000
index =   100, value = 0.8620000000
index =   200, value = 0.8649000000
index =   300, value = 0.8649500000
index =   400, value = 0.8648000000
index =   500, value = 0.8650000000
index =   600, value = 0.8651500000
index =   700, value = 0.8648000000
index =   800, value = 0.8651500000
index =   900, value = 0.8653500000
```

1.49 function definition for presenting the results

```
[ ]: def function_results_01(data,nRow,nCol):
         fig, axes = plt.subplots(nRow, nCol, constrained_layout=True, figsize=(nCol␣
      ↪* 3, nRow * 3))
         nPlot      = nRow * nCol

         index_data = index_train[0][0][0:nPlot]

         for i in range(nRow):
             for j in range(nCol):

                 k       = i * nCol + j
                 index   = index_data[k]
```

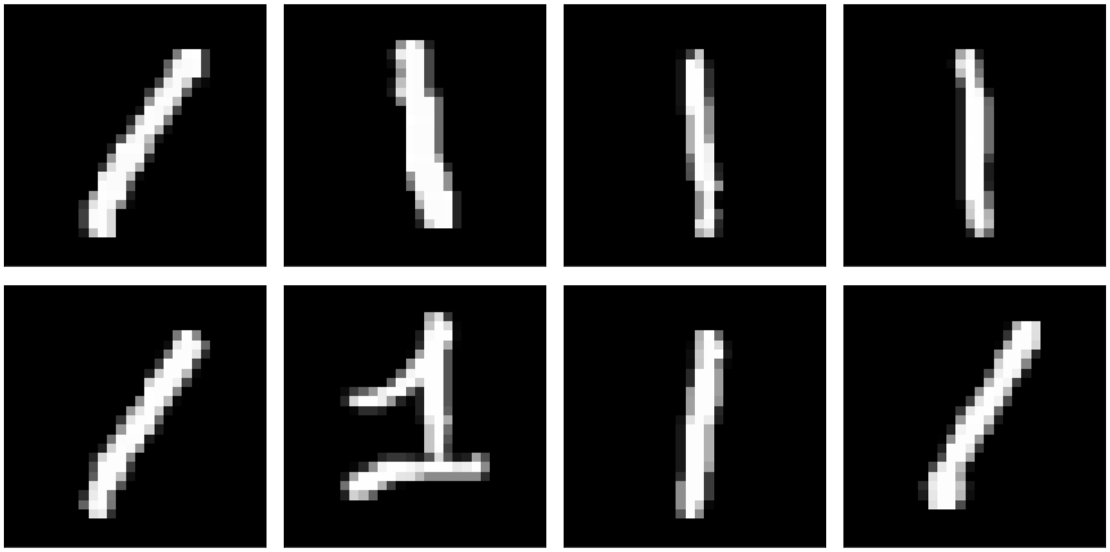
```
            axes[i, j].imshow(data[index], cmap='gray', vmin=0, vmax=1)
            axes[i, j].xaxis.set_visible(False)
            axes[i, j].yaxis.set_visible(False)

    plt.show()
```

[ ]:
```
function_results_01(x_train,2,4)
```



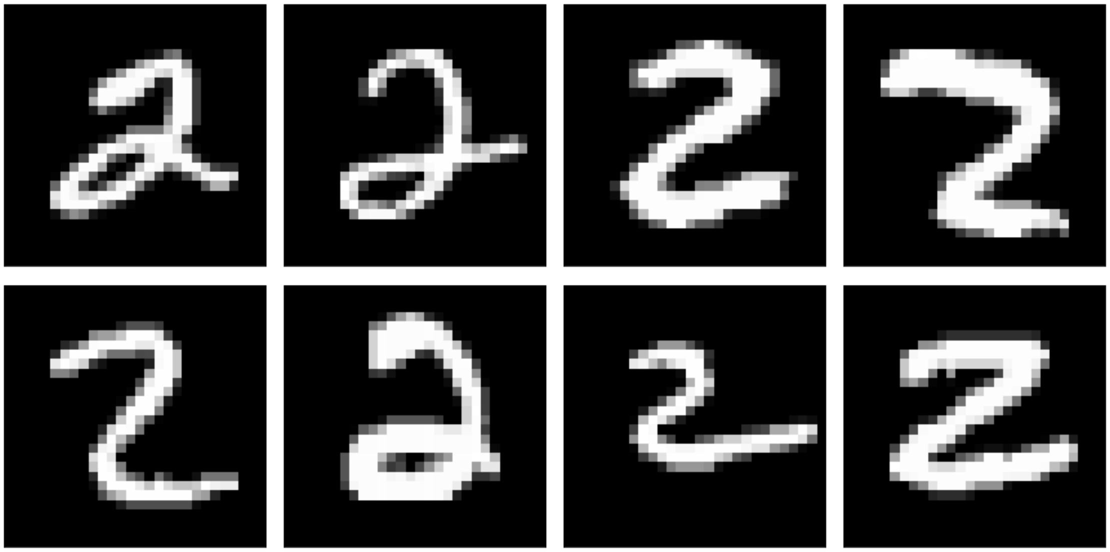
[ ]:
```
def function_results_02(data,nRow,nCol):
    fig, axes = plt.subplots(nRow, nCol, constrained_layout=True, figsize=(nCol␣
 ↪* 3, nRow * 3))
    nPlot     = nRow * nCol

    index_data = index_train[1][0][0:nPlot]

    for i in range(nRow):
        for j in range(nCol):

            k       = i * nCol + j
            index   = index_data[k]

            axes[i, j].imshow(data[index], cmap='gray', vmin=0, vmax=1)
            axes[i, j].xaxis.set_visible(False)
            axes[i, j].yaxis.set_visible(False)

    plt.show()
```

[ ]:
```
function_results_02(x_train,2,4)
```



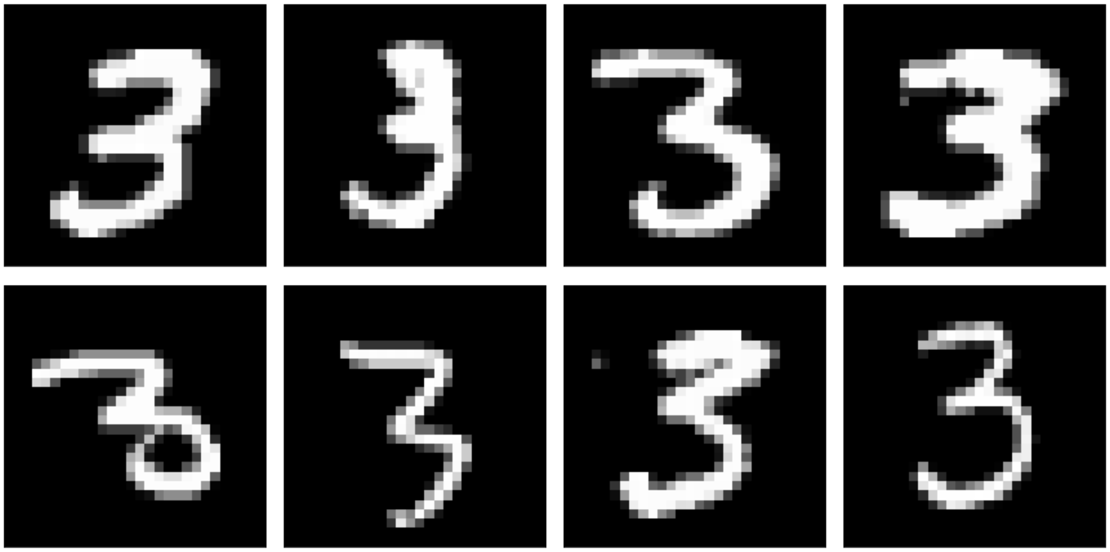
[ ]:
```
def function_results_03(data,nRow,nCol):
    fig, axes = plt.subplots(nRow, nCol, constrained_layout=True, figsize=(nCol␣
↪* 3, nRow * 3))
    nPlot      = nRow * nCol

    index_data = index_train[2][0][0:nPlot]

    for i in range(nRow):
        for j in range(nCol):

            k        = i * nCol + j
            index    = index_data[k]

            axes[i, j].imshow(data[index], cmap='gray', vmin=0, vmax=1)
            axes[i, j].xaxis.set_visible(False)
            axes[i, j].yaxis.set_visible(False)

    plt.show()
```

[ ]:
```
function_results_03(x_train,2,4)
```



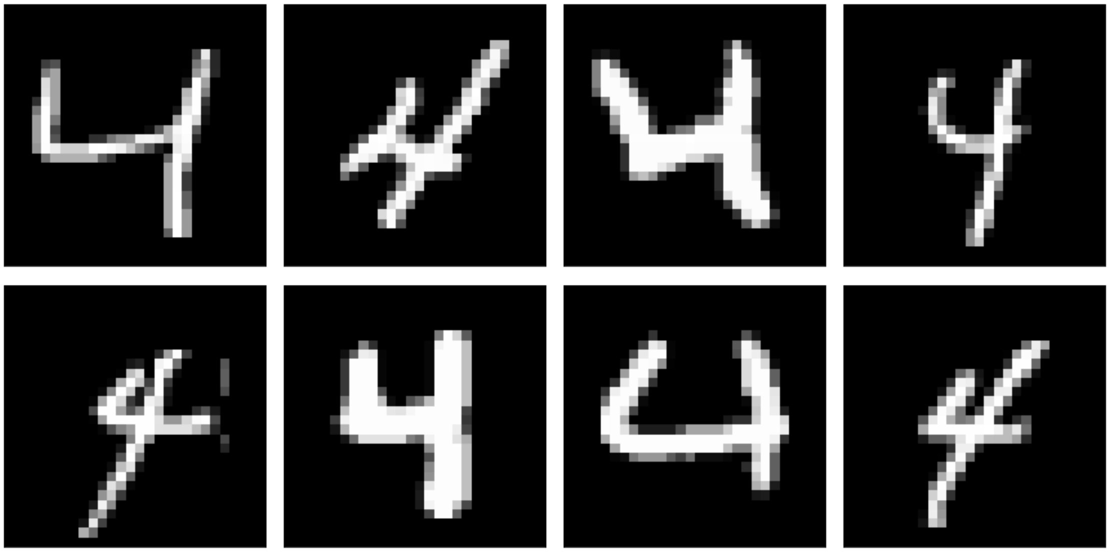
```
[ ]: def function_results_04(data,nRow,nCol):
         fig, axes = plt.subplots(nRow, nCol, constrained_layout=True, figsize=(nCol␣
      ↪* 3, nRow * 3))
         nPlot     = nRow * nCol

         index_data = index_train[3][0][0:nPlot]

         for i in range(nRow):
             for j in range(nCol):

                 k       = i * nCol + j
                 index   = index_data[k]

                 axes[i, j].imshow(data[index], cmap='gray', vmin=0, vmax=1)
                 axes[i, j].xaxis.set_visible(False)
                 axes[i, j].yaxis.set_visible(False)

         plt.show()
```

```
[ ]: function_results_04(x_train,2,4)
```



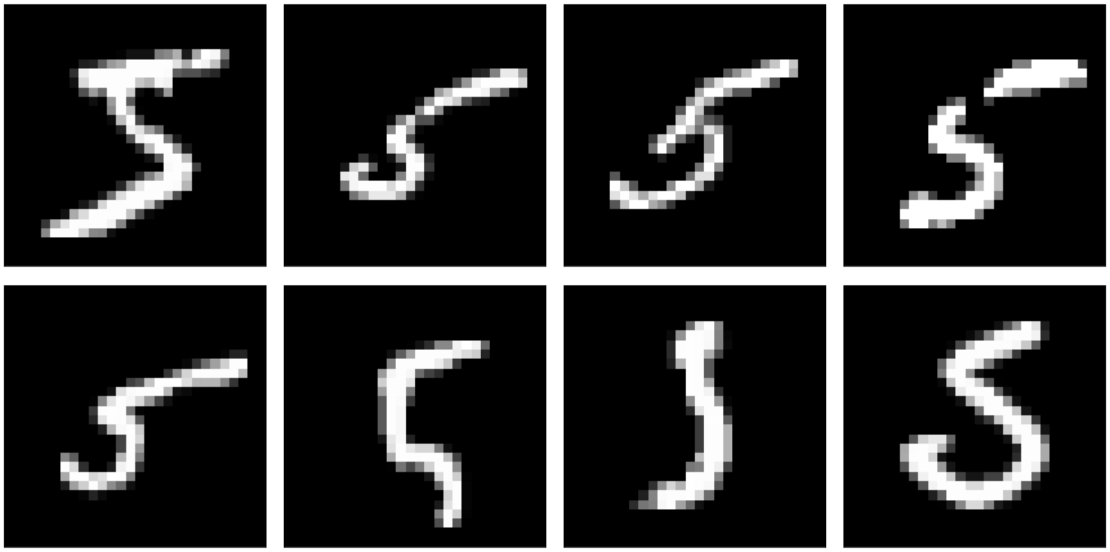
```
[ ]: def function_results_05(data,nRow,nCol):
         fig, axes = plt.subplots(nRow, nCol, constrained_layout=True, figsize=(nCol␣
      ↪* 3, nRow * 3))
         nPlot      = nRow * nCol

         index_data = index_train[4][0][0:nPlot]

         for i in range(nRow):
             for j in range(nCol):

                 k        = i * nCol + j
                 index    = index_data[k]

                 axes[i, j].imshow(data[index], cmap='gray', vmin=0, vmax=1)
                 axes[i, j].xaxis.set_visible(False)
                 axes[i, j].yaxis.set_visible(False)

         plt.show()
```

```
[ ]: function_results_05(x_train,2,4)
```



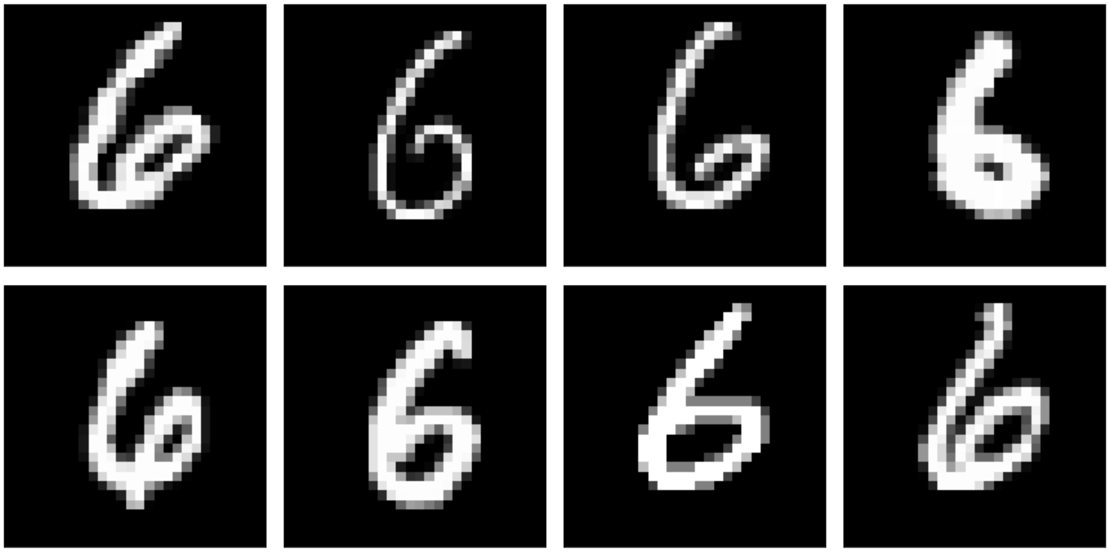
[ ]:
```python
def function_results_06(data,nRow,nCol):
    fig, axes = plt.subplots(nRow, nCol, constrained_layout=True, figsize=(nCol␣
↪* 3, nRow * 3))
    nPlot     = nRow * nCol

    index_data = index_train[5][0][0:nPlot]

    for i in range(nRow):
        for j in range(nCol):

            k       = i * nCol + j
            index   = index_data[k]

            axes[i, j].imshow(data[index], cmap='gray', vmin=0, vmax=1)
            axes[i, j].xaxis.set_visible(False)
            axes[i, j].yaxis.set_visible(False)

    plt.show()
```

[ ]:
```python
function_results_06(x_train,2,4)
```



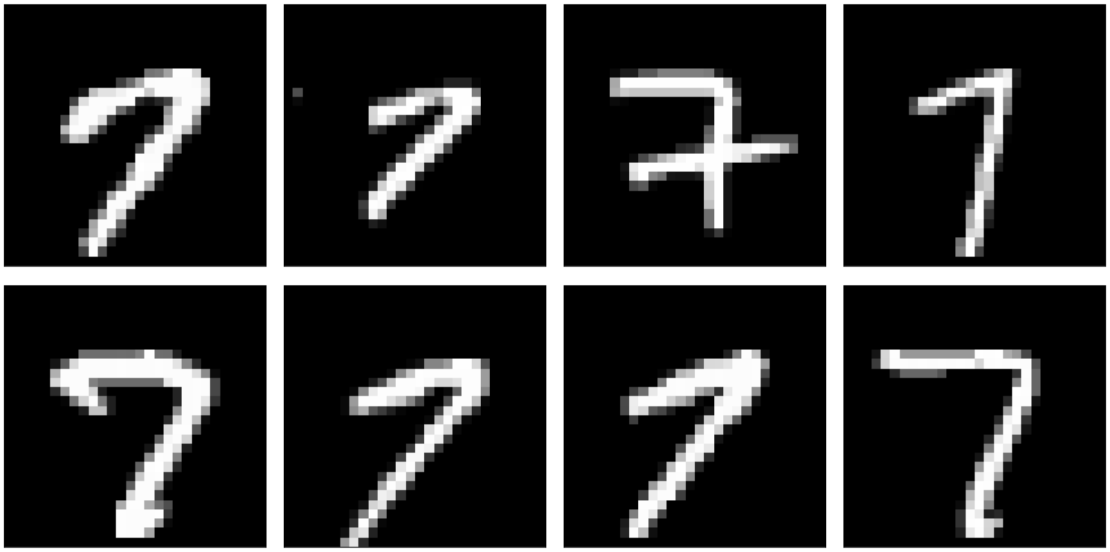
```
[ ]: def function_results_07(data,nRow,nCol):
         fig, axes = plt.subplots(nRow, nCol, constrained_layout=True, figsize=(nCol␣
      ↪* 3, nRow * 3))
         nPlot     = nRow * nCol

         index_data = index_train[6][0][0:nPlot]

         for i in range(nRow):
             for j in range(nCol):

                 k       = i * nCol + j
                 index   = index_data[k]

                 axes[i, j].imshow(data[index], cmap='gray', vmin=0, vmax=1)
                 axes[i, j].xaxis.set_visible(False)
                 axes[i, j].yaxis.set_visible(False)

         plt.show()
```

```
[ ]: function_results_07(x_train,2,4)
```



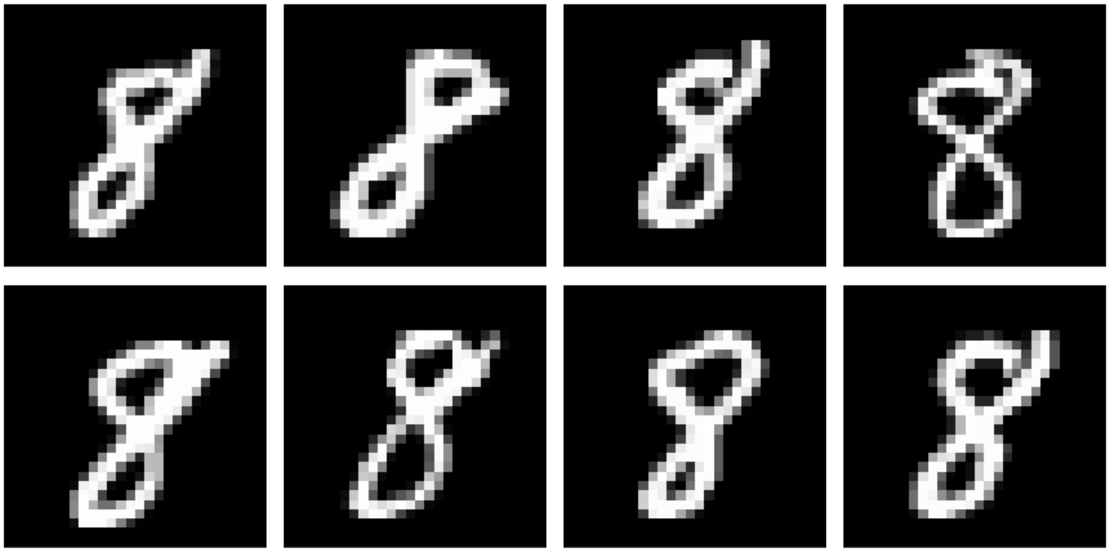
```
[ ]: def function_results_08(data,nRow,nCol):
         fig, axes = plt.subplots(nRow, nCol, constrained_layout=True, figsize=(nCol␣
     ↪* 3, nRow * 3))
         nPlot      = nRow * nCol

         index_data = index_train[7][0][0:nPlot]

         for i in range(nRow):
             for j in range(nCol):

                 k        = i * nCol + j
                 index    = index_data[k]

                 axes[i, j].imshow(data[index], cmap='gray', vmin=0, vmax=1)
                 axes[i, j].xaxis.set_visible(False)
                 axes[i, j].yaxis.set_visible(False)

         plt.show()
```

```
[ ]: function_results_08(x_train,2,4)
```



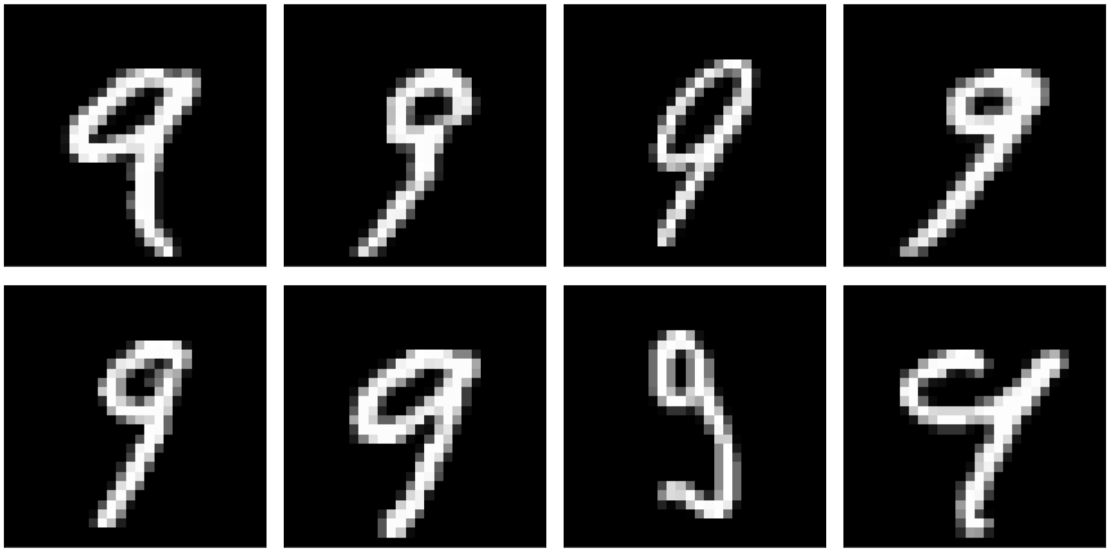
```
[ ]: def function_results_09(data,nRow,nCol):
         fig, axes = plt.subplots(nRow, nCol, constrained_layout=True, figsize=(nCol␣
     ↪* 3, nRow * 3))
         nPlot     = nRow * nCol

         index_data = index_train[8][0][0:nPlot]

         for i in range(nRow):
             for j in range(nCol):

                 k       = i * nCol + j
                 index   = index_data[k]

                 axes[i, j].imshow(data[index], cmap='gray', vmin=0, vmax=1)
                 axes[i, j].xaxis.set_visible(False)
                 axes[i, j].yaxis.set_visible(False)

         plt.show()
```

```
[ ]: function_results_09(x_train,2,4)
```



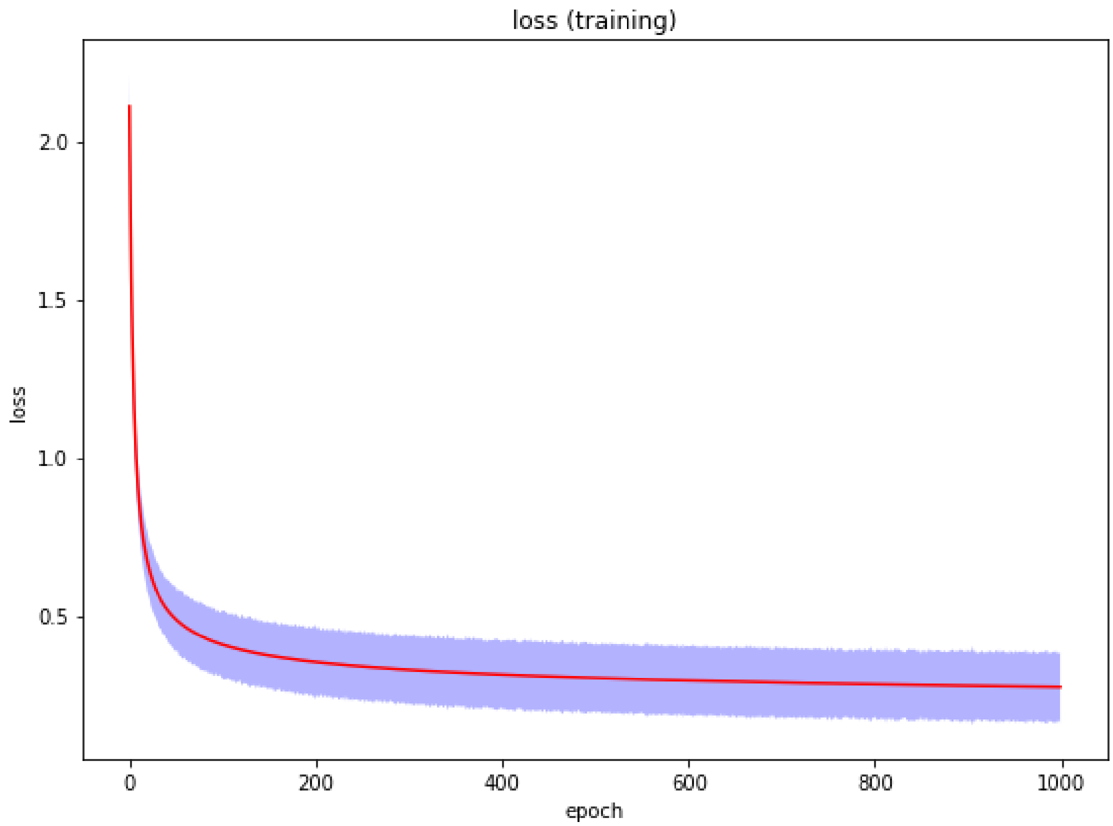
```
[ ]: def function_results_10(data,nRow,nCol):
         fig, axes = plt.subplots(nRow, nCol, constrained_layout=True, figsize=(nCol␣
      ↪* 3, nRow * 3))
         nPlot      = nRow * nCol

         index_data = index_train[9][0][0:nPlot]

         for i in range(nRow):
             for j in range(nCol):

                 k        = i * nCol + j
                 index    = index_data[k]

                 axes[i, j].imshow(data[index], cmap='gray', vmin=0, vmax=1)
                 axes[i, j].xaxis.set_visible(False)
                 axes[i, j].yaxis.set_visible(False)

         plt.show()
```

```
[ ]: function_results_10(x_train,2,4)
```



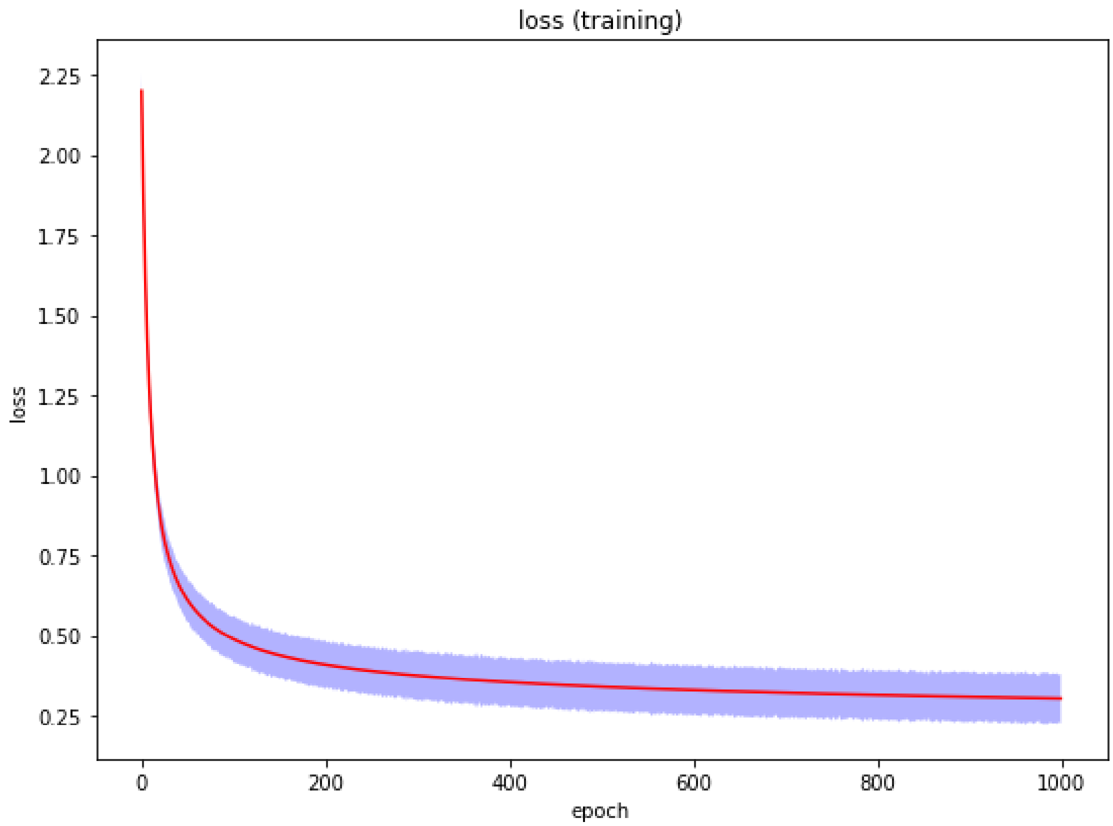
[ ]:
```
def function_results_11():
    plot_curve_error(train_loss_mean_minibatch[0], train_loss_std_minibatch[0],␣
↪'epoch', 'loss', 'loss (training)')
```

[ ]:
```
function_results_11()
```



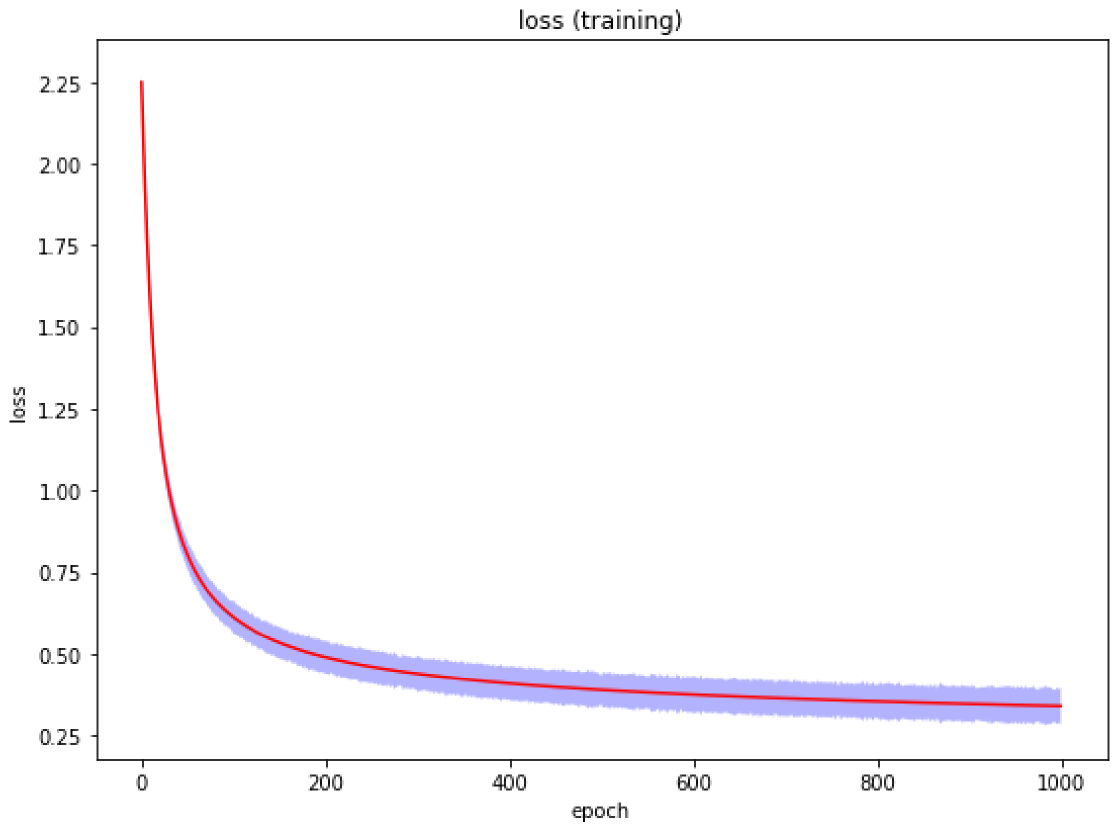
[ ]:
```
def function_results_12():
    plot_curve_error(train_loss_mean_minibatch[1], train_loss_std_minibatch[1],␣
↪'epoch', 'loss', 'loss (training)')
```

[ ]:
```
function_results_12()
```



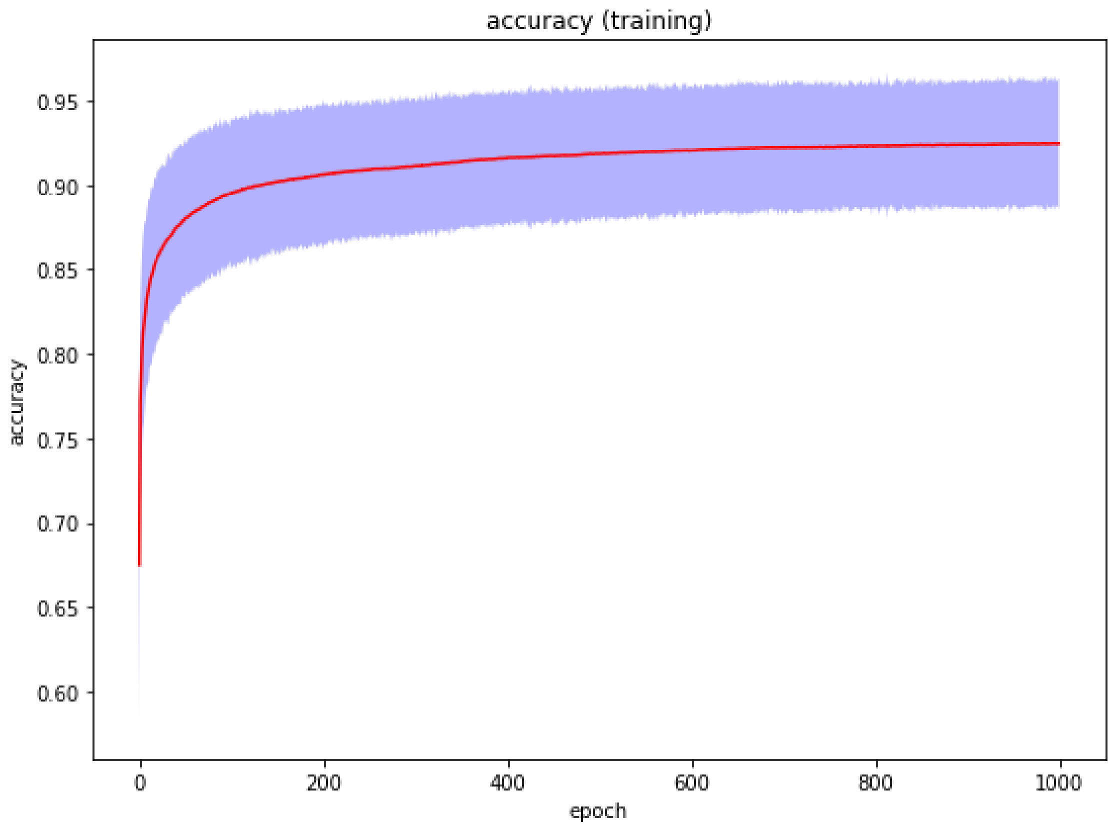

```
[ ]: def function_results_13():
         plot_curve_error(train_loss_mean_minibatch[2], train_loss_std_minibatch[2],␣
      ↪'epoch', 'loss', 'loss (training)')
```

```
[ ]: function_results_13()
```



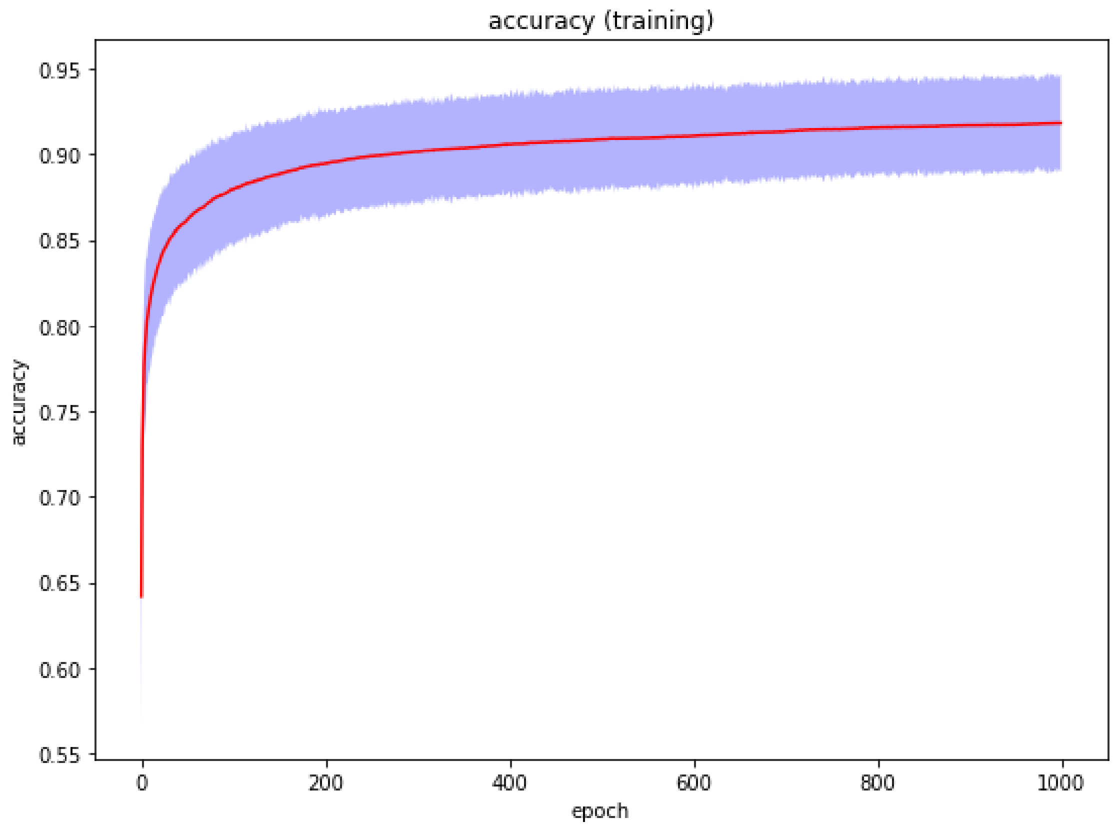

```
[ ]: def function_results_14():
         plot_curve_error(train_accuracy_mean_minibatch[0],␣
      ↪train_accuracy_std_minibatch[0], 'epoch', 'accuracy', 'accuracy (training)')
```

```
[ ]: function_results_14()
```



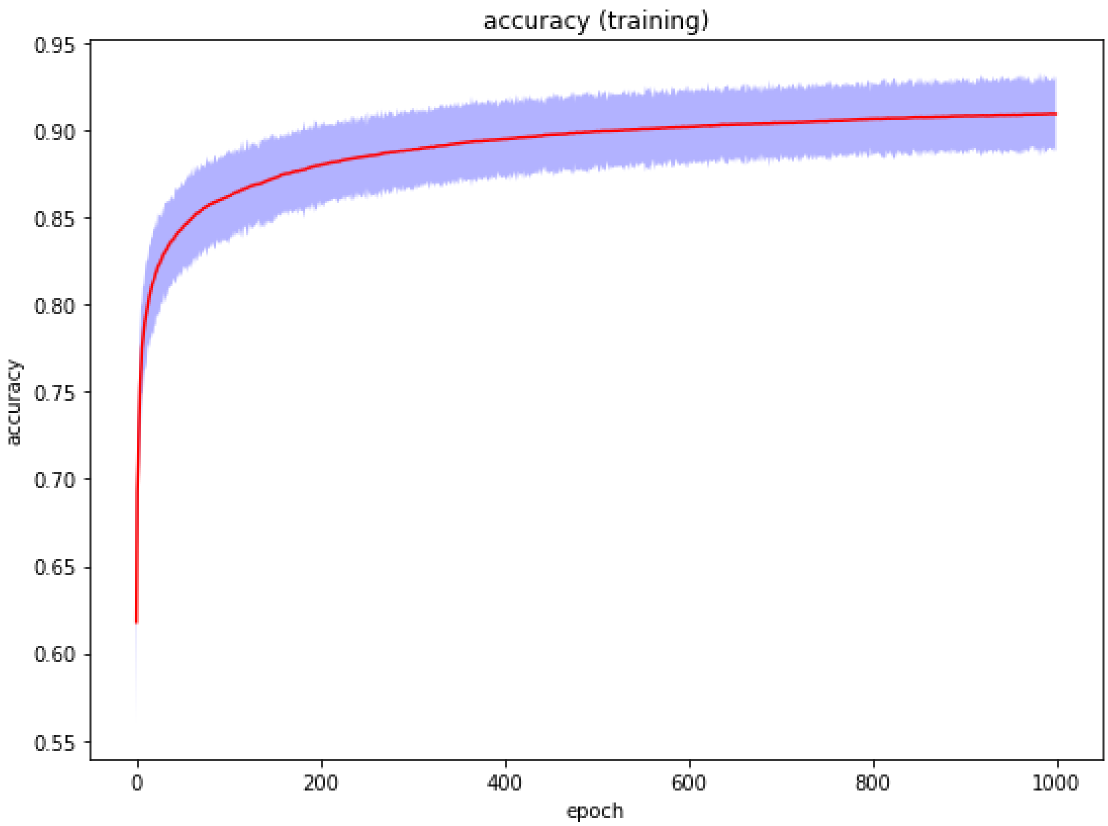

```
[ ]: def function_results_15():
         plot_curve_error(train_accuracy_mean_minibatch[1],␣
      ↪train_accuracy_std_minibatch[1], 'epoch', 'accuracy', 'accuracy (training)')
```

```
[ ]: function_results_15()
```



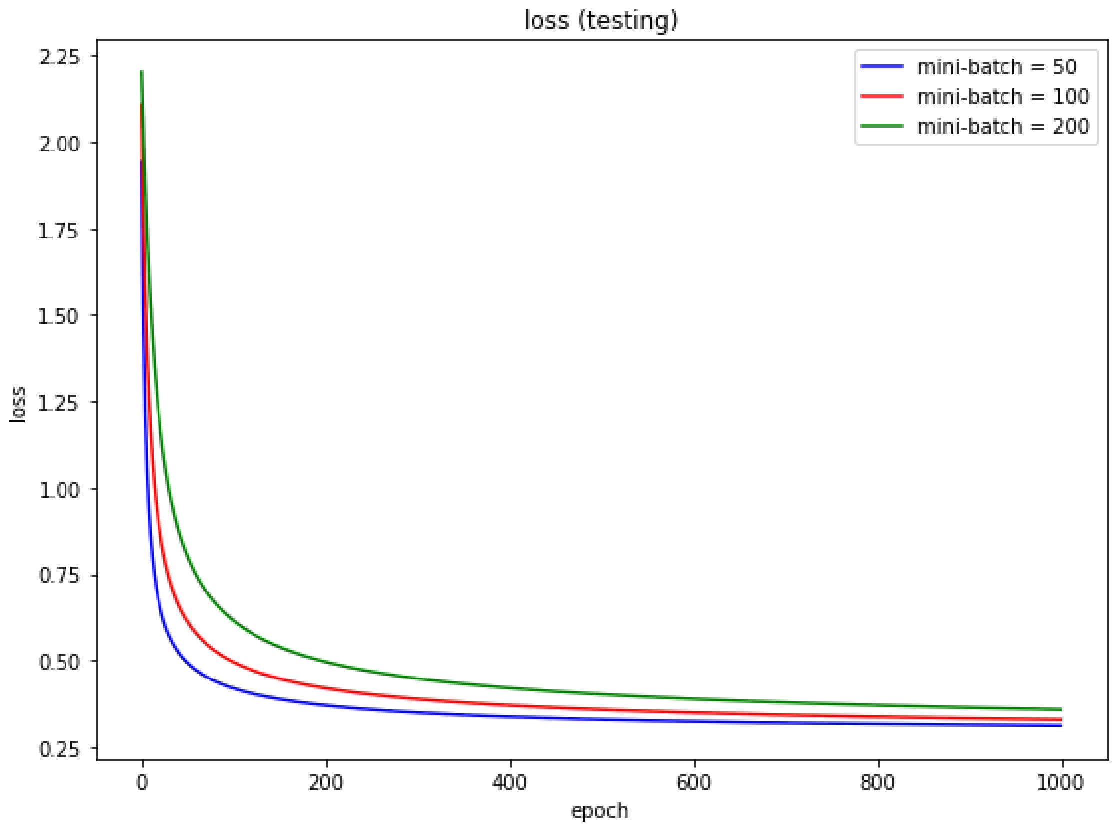
[ ]:
```
def function_results_16():
    plot_curve_error(train_accuracy_mean_minibatch[2],␣
 ↪train_accuracy_std_minibatch[2], 'epoch', 'accuracy', 'accuracy (training)')
```

[ ]:
```
function_results_16()
```



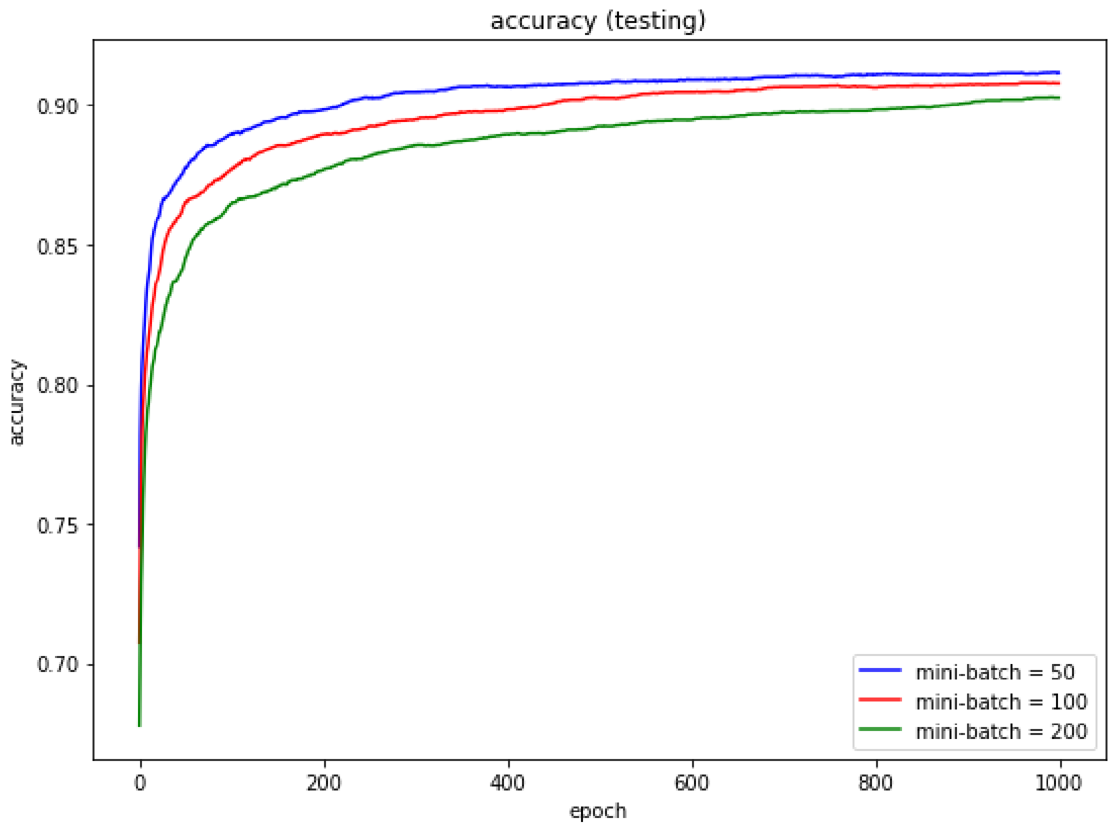

```
[ ]: def function_results_17():
         plot_curve3(test_loss_minibatch[0], 'mini-batch = 50',␣
      ↪test_loss_minibatch[1], 'mini-batch = 100', test_loss_minibatch[2],␣
      ↪'mini-batch = 200', 'epoch', 'loss', 'loss (testing)')
```

```
[ ]: function_results_17()
```



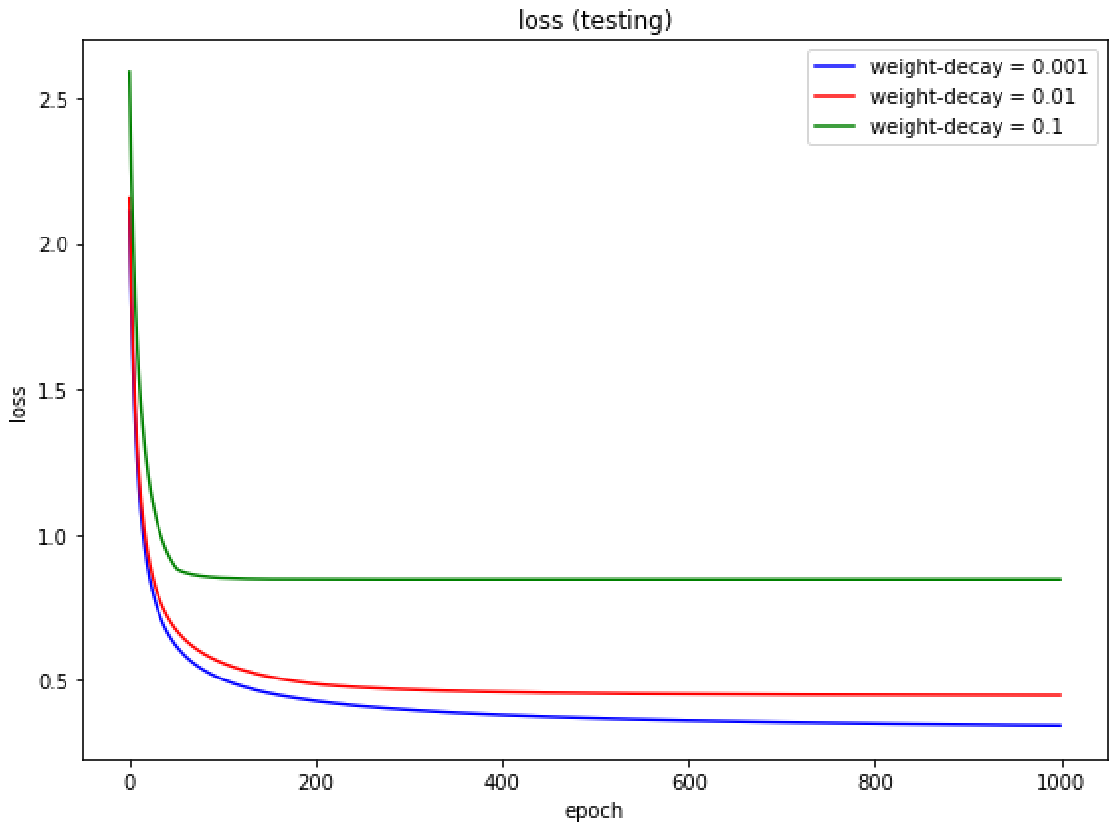
[ ]:
```
def function_results_18():
    plot_curve3(test_accuracy_minibatch[0], 'mini-batch = 50',
 ↪test_accuracy_minibatch[1], 'mini-batch = 100', test_accuracy_minibatch[2],
 ↪'mini-batch = 200', 'epoch', 'accuracy', 'accuracy (testing)')
```

[ ]:
```
function_results_18()
```



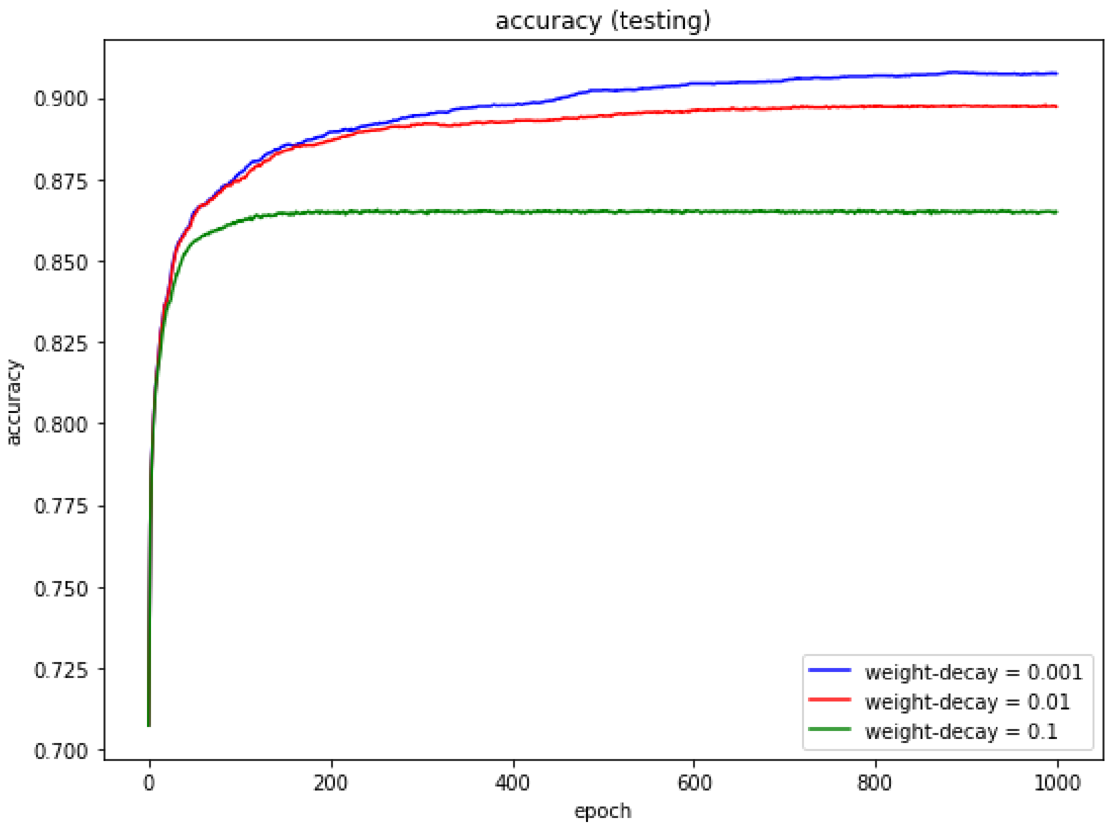
[ ]:
```
def function_results_19():
    plot_curve3(test_loss_weight_decay[0], 'weight-decay = 0.001',␣
 ↪test_loss_weight_decay[1], 'weight-decay = 0.01', test_loss_weight_decay[2],␣
 ↪'weight-decay = 0.1', 'epoch', 'loss', 'loss (testing)')
```

[ ]:
```
function_results_19()
```




```
[ ]: def function_results_20():
         plot_curve3(test_accuracy_weight_decay[0], 'weight-decay = 0.001',␣
      ↪test_accuracy_weight_decay[1], 'weight-decay = 0.01',␣
      ↪test_accuracy_weight_decay[2], 'weight-decay = 0.1', 'epoch', 'accuracy',␣
      ↪'accuracy (testing)')
```

```
[ ]: function_results_20()
```



[ ]:
```python
def function_results_21(data):

    index = np.array([0, 100, 200, 300, 400, 500, 600, 700, 800, 900])

    for i in range(len(index)):

        idx = index[i]
        val = data[idx]

        print('index = %4d, value = %12.10f' % (idx, val))
```

[ ]:
```python
function_results_21(train_loss_mean_minibatch[1])
```

```
index =    0, value = 2.2003866772
index =  100, value = 0.4884452279
index =  200, value = 0.4086303762
index =  300, value = 0.3743102039
index =  400, value = 0.3538169113
index =  500, value = 0.3396656612
index =  600, value = 0.3290494042
index =  700, value = 0.3206579243
```

```
index =  800, value = 0.3137801756
index =  900, value = 0.3079845796
```

[ ]:
```python
def function_results_22(data):
    index = np.array([0, 100, 200, 300, 400, 500, 600, 700, 800, 900])

    for i in range(len(index)):

        idx = index[i]
        val = data[idx]

        print('index = %4d, value = %12.10f' % (idx, val))
```

[ ]:
```python
function_results_22(train_accuracy_mean_minibatch[1])
```

```
index =    0, value = 0.6415000000
index =  100, value = 0.8798500000
index =  200, value = 0.8948000000
index =  300, value = 0.9018000000
index =  400, value = 0.9062000000
index =  500, value = 0.9090500000
index =  600, value = 0.9109500000
index =  700, value = 0.9137000000
index =  800, value = 0.9158000000
index =  900, value = 0.9171500000
```

[ ]:
```python
def function_results_23(data):
    index = np.array([0, 100, 200, 300, 400, 500, 600, 700, 800, 900])

    for i in range(len(index)):

        idx = index[i]
        val = data[idx]

        print('index = %4d, value = %12.10f' % (idx, val))
```

[ ]:
```python
function_results_23(train_loss_mean_weight_decay[0])
```

```
index =    0, value = 2.2053959737
index =  100, value = 0.4962629800
index =  200, value = 0.4175434109
index =  300, value = 0.3840709465
index =  400, value = 0.3643988464
index =  500, value = 0.3514216500
index =  600, value = 0.3419763652
index =  700, value = 0.3346759415
index =  800, value = 0.3288218803
index =  900, value = 0.3239931453
```

[ ]:
```
def function_results_24(data):
    index = np.array([0, 100, 200, 300, 400, 500, 600, 700, 800, 900])

    for i in range(len(index)):

        idx = index[i]
        val = data[idx]

        print('index = %4d, value = %12.10f' % (idx, val))
```

[ ]:
```
function_results_24(train_accuracy_mean_weight_decay[0])
```

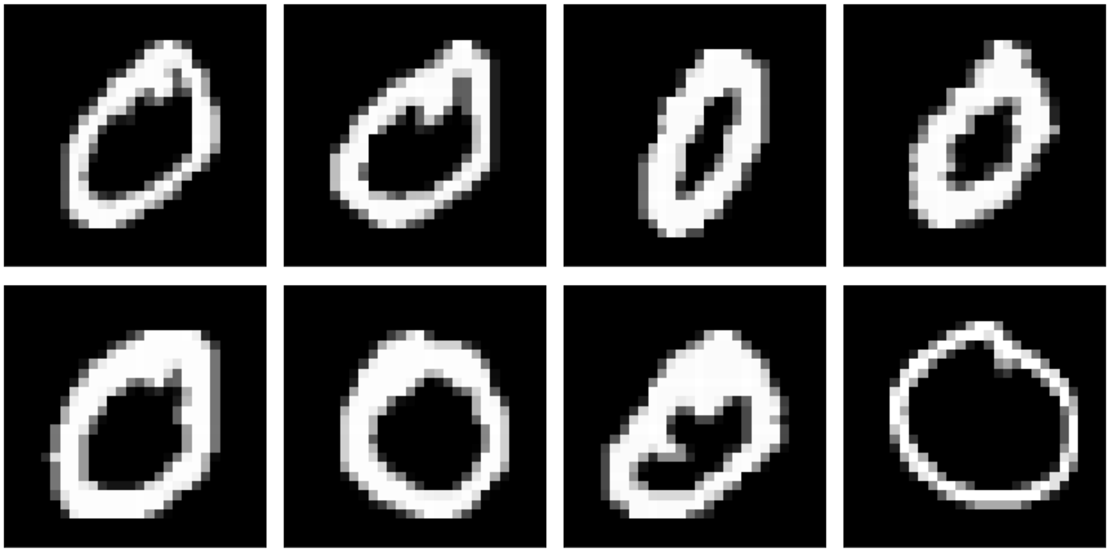
```
index =    0, value = 0.6415000000
index =  100, value = 0.8794000000
index =  200, value = 0.8944000000
index =  300, value = 0.9014500000
index =  400, value = 0.9053000000
index =  500, value = 0.9083000000
index =  600, value = 0.9104000000
index =  700, value = 0.9124000000
index =  800, value = 0.9144500000
index =  900, value = 0.9157000000
```

---

2 RESULTS

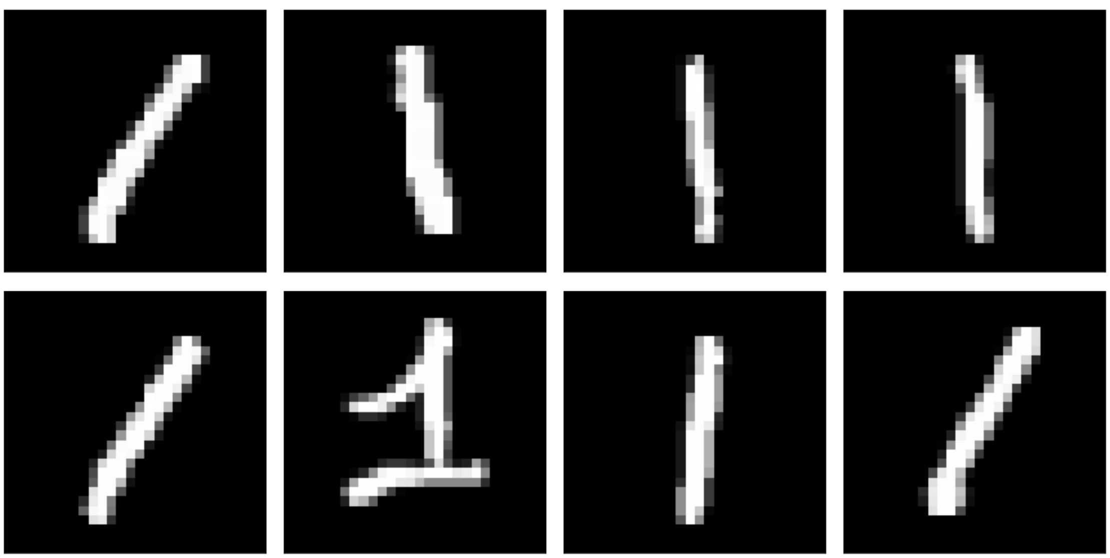
2.1 # 01. plot the input images for '0'

[ ]:
```
function_results_01(x_train,2,4)
```



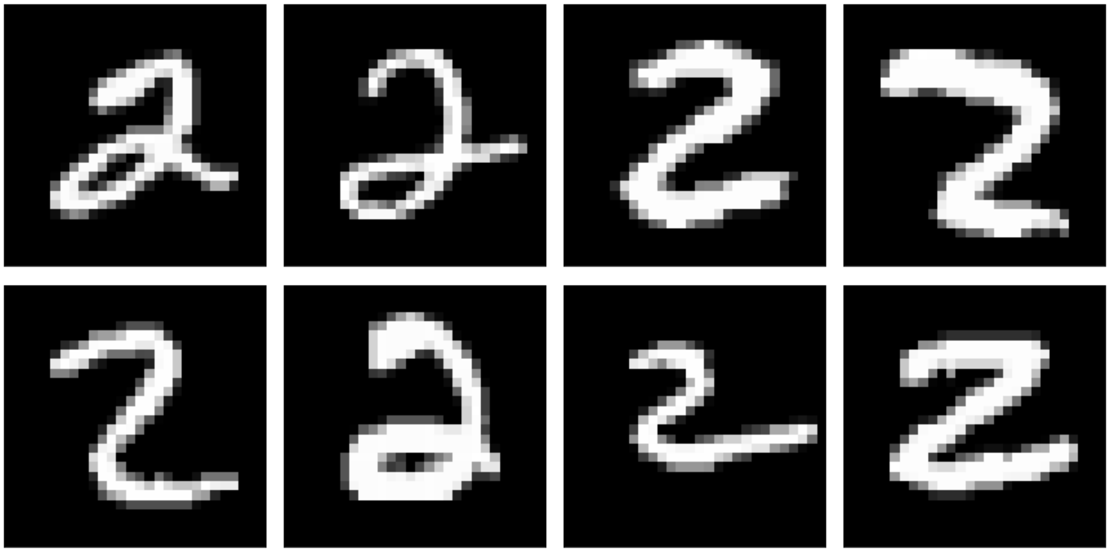
2.2 # 02. plot the input images for '1'

```
[ ]: function_results_02(x_train,2,4)
```



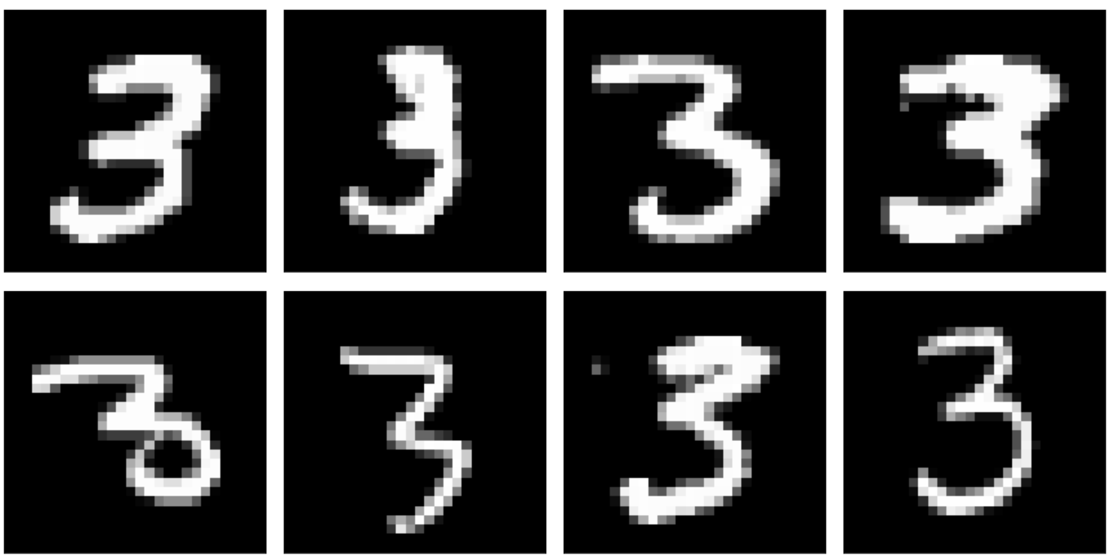
2.3 # 03. plot the input images for '2'

```
[ ]: function_results_03(x_train,2,4)
```



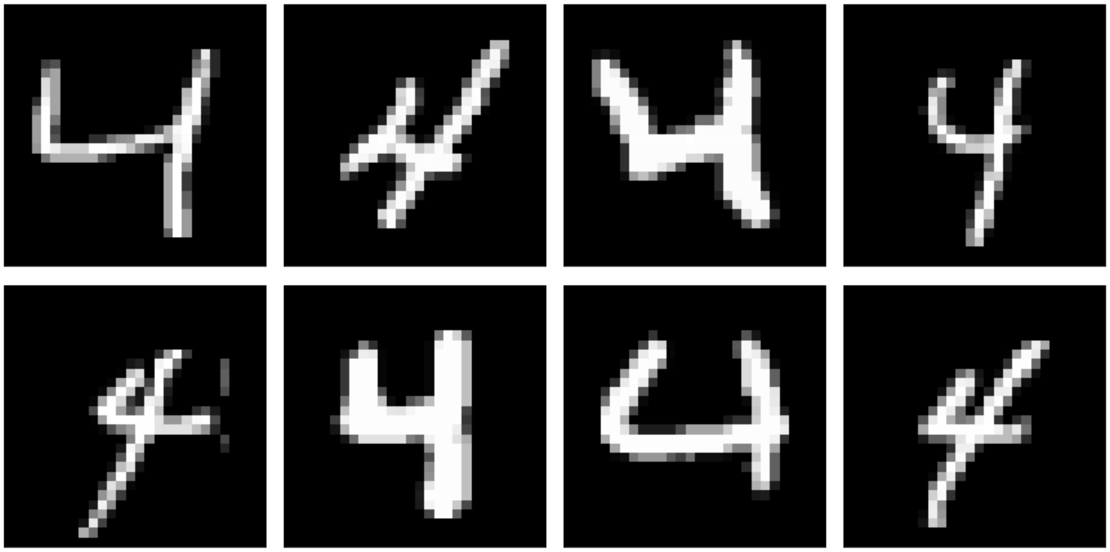
2.4 # 04. plot the input images for '3'

[ ]:
```
function_results_04(x_train,2,4)
```



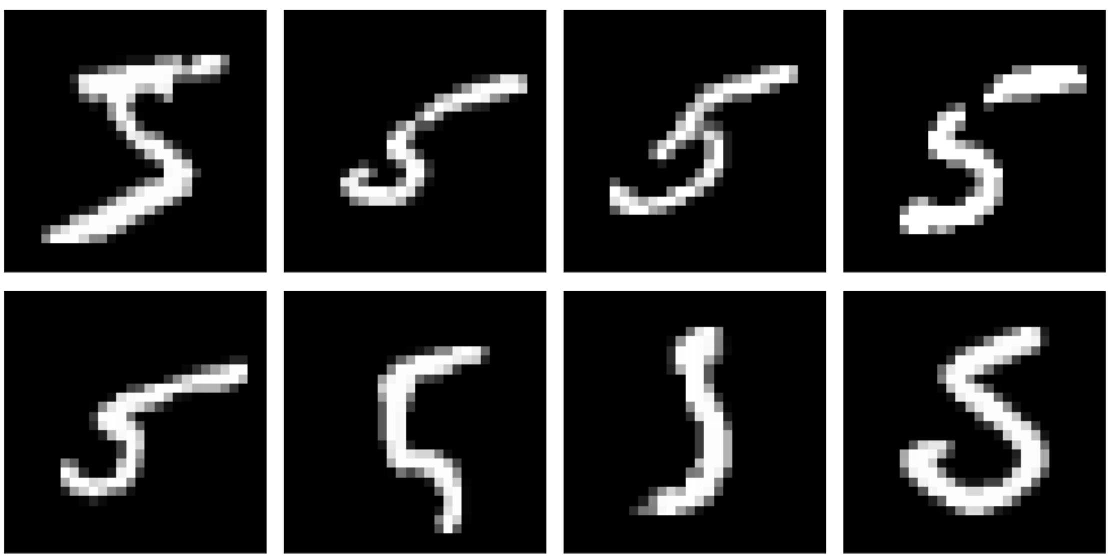
2.5 # 05. plot the input images for '4'

[ ]:
```
function_results_05(x_train,2,4)
```



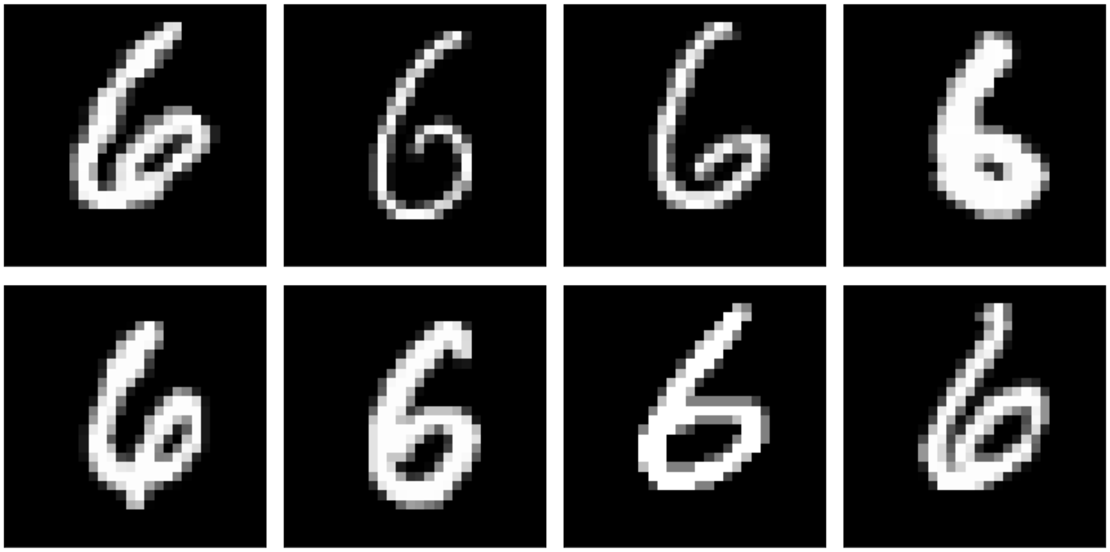
2.6 # 06. plot the input images for '5'

[ ]:
```
function_results_06(x_train,2,4)
```



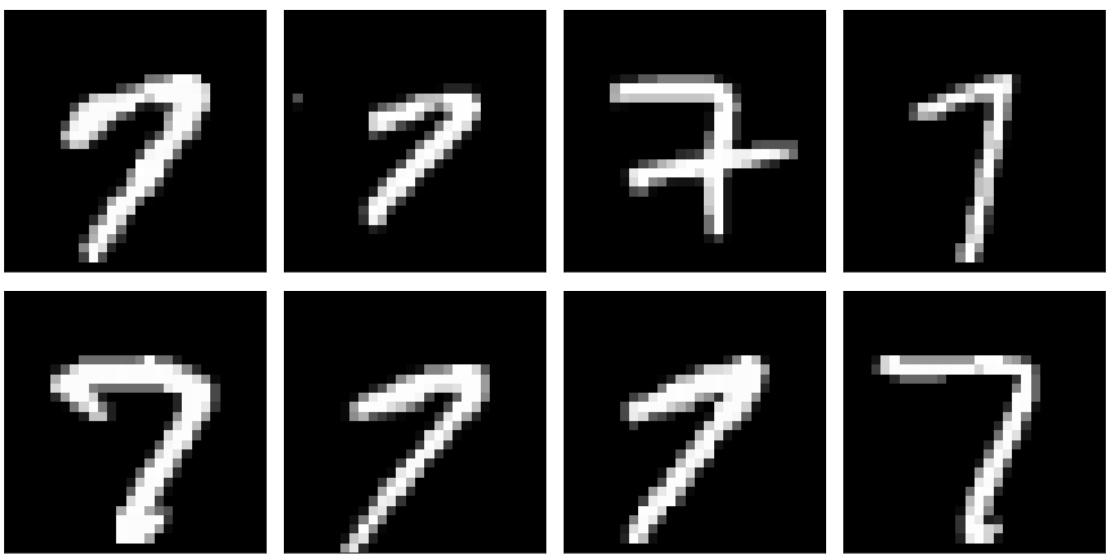
2.7 # 07. plot the input images for '6'

[ ]:
```
function_results_07(x_train,2,4)
```



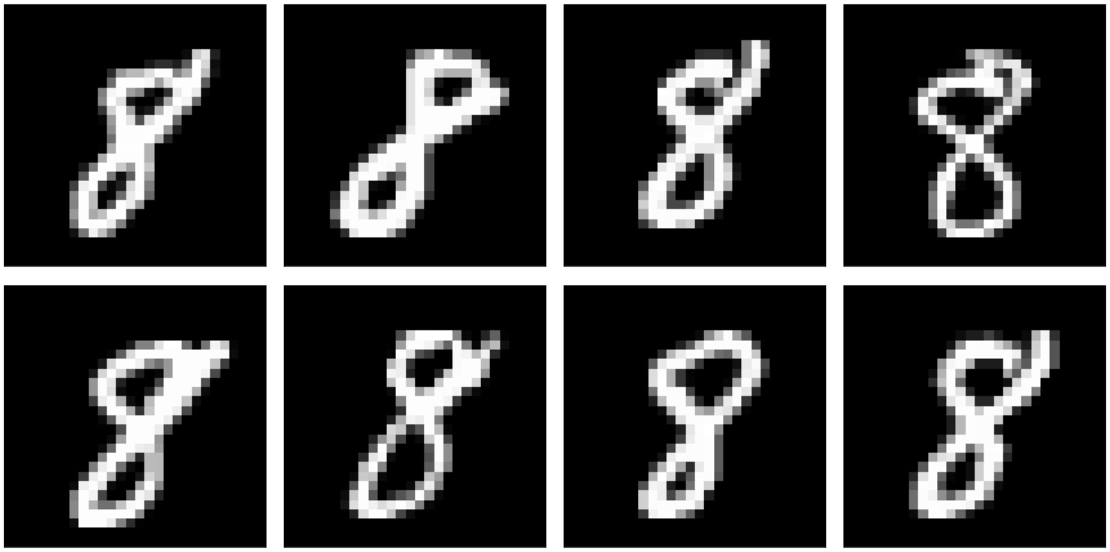
2.8 # 08. plot the input images for '7'

[ ]:
```
function_results_08(x_train,2,4)
```



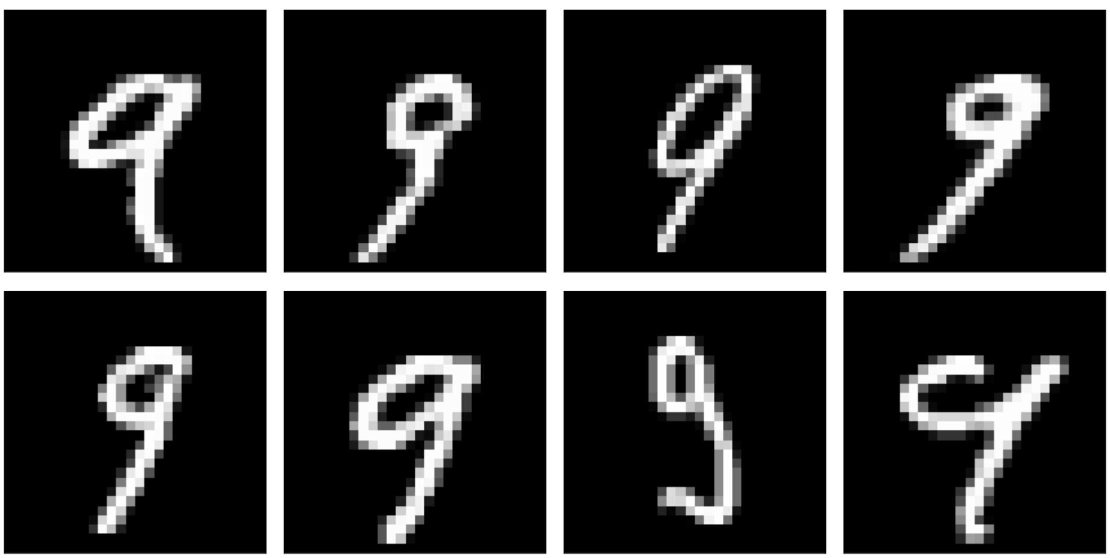
2.9 # 09. plot the input images for '8'

[ ]:
```
function_results_09(x_train,2,4)
```



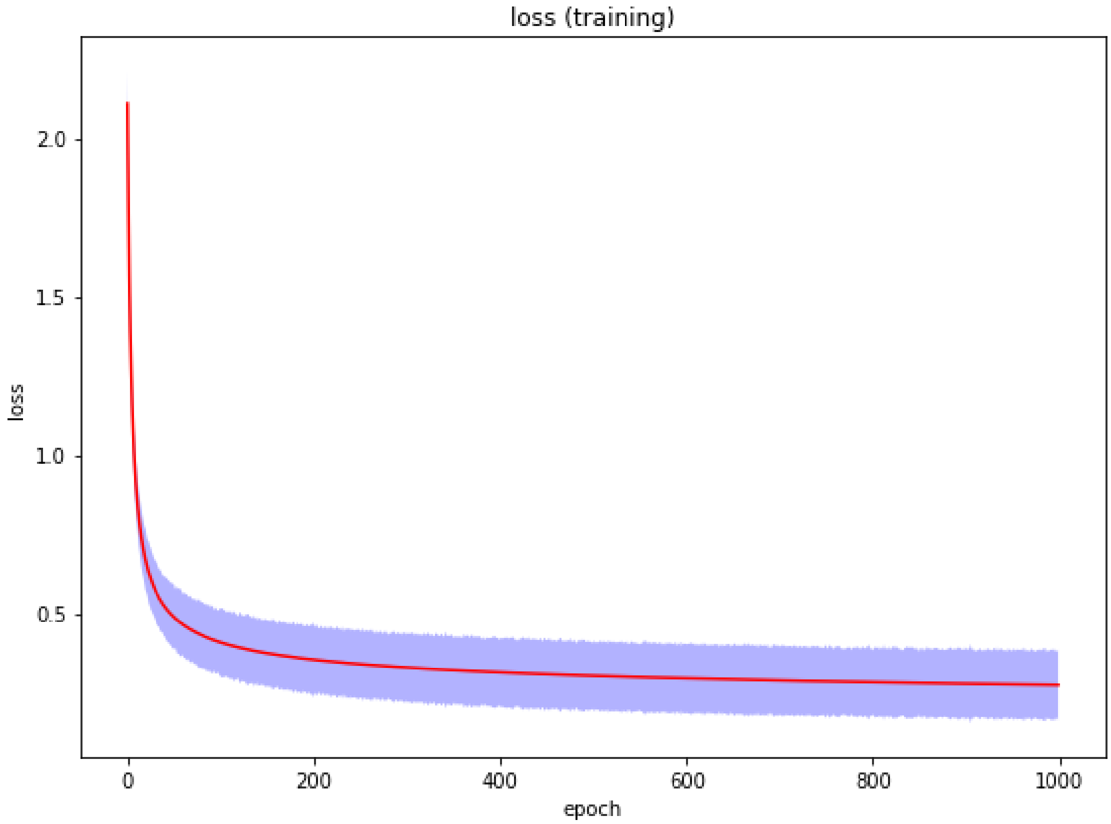
2.10 # 10. plot the input images for '9'

[ ]:
```
function_results_10(x_train,2,4)
```



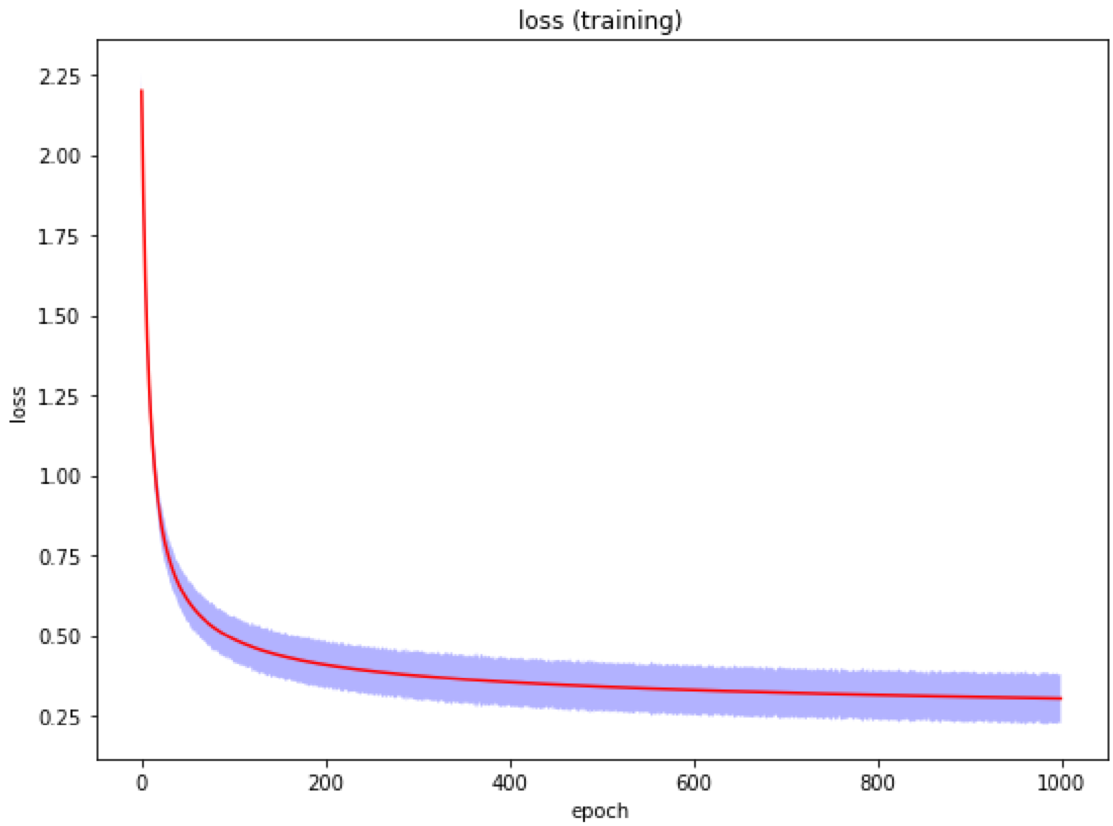
2.11 # 11. plot loss (training) curve (mean, std) : mini-batch size = 50, weight decay = 0

[ ]:
```
function_results_11()
```



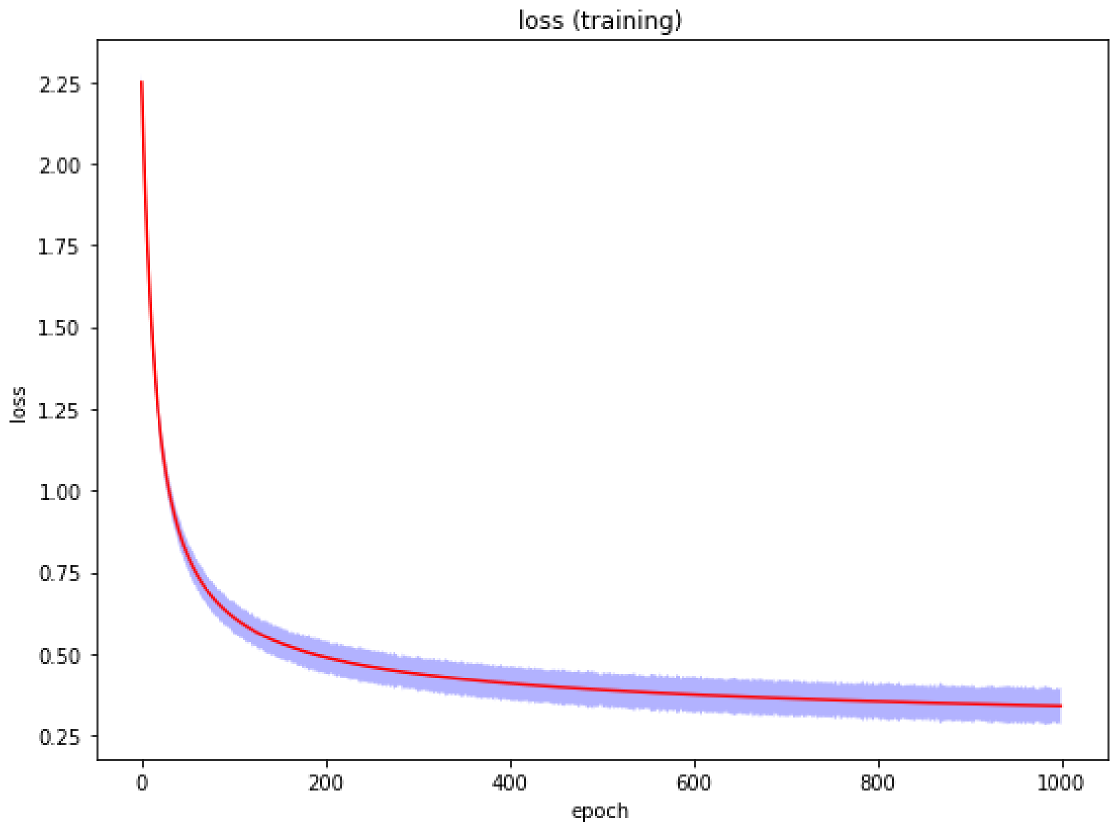
2.12 # 12. plot loss (training) curve (mean, std) : mini-batch size = 100, weight decay = 0

[ ]:
```
function_results_12()
```



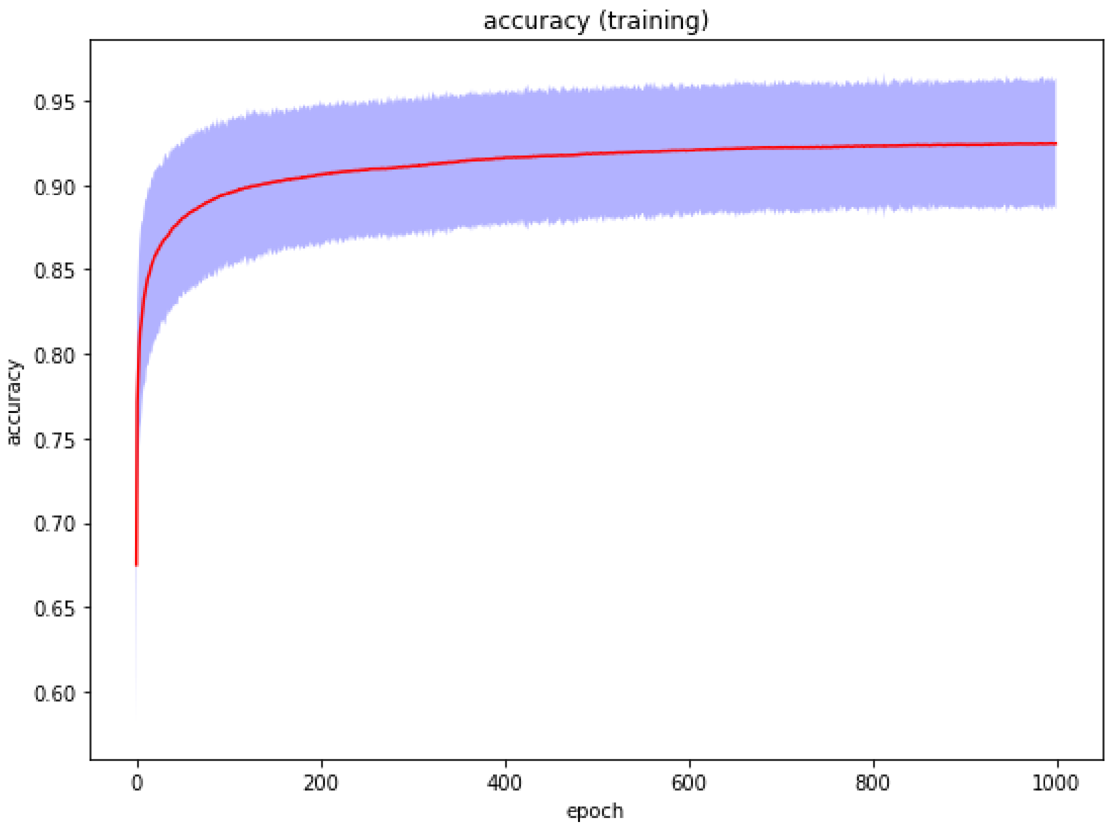
2.13 # 13. plot loss (training) curve (mean, std) : mini-batch size = 200, weight decay = 0

[ ]:
```
function_results_13()
```



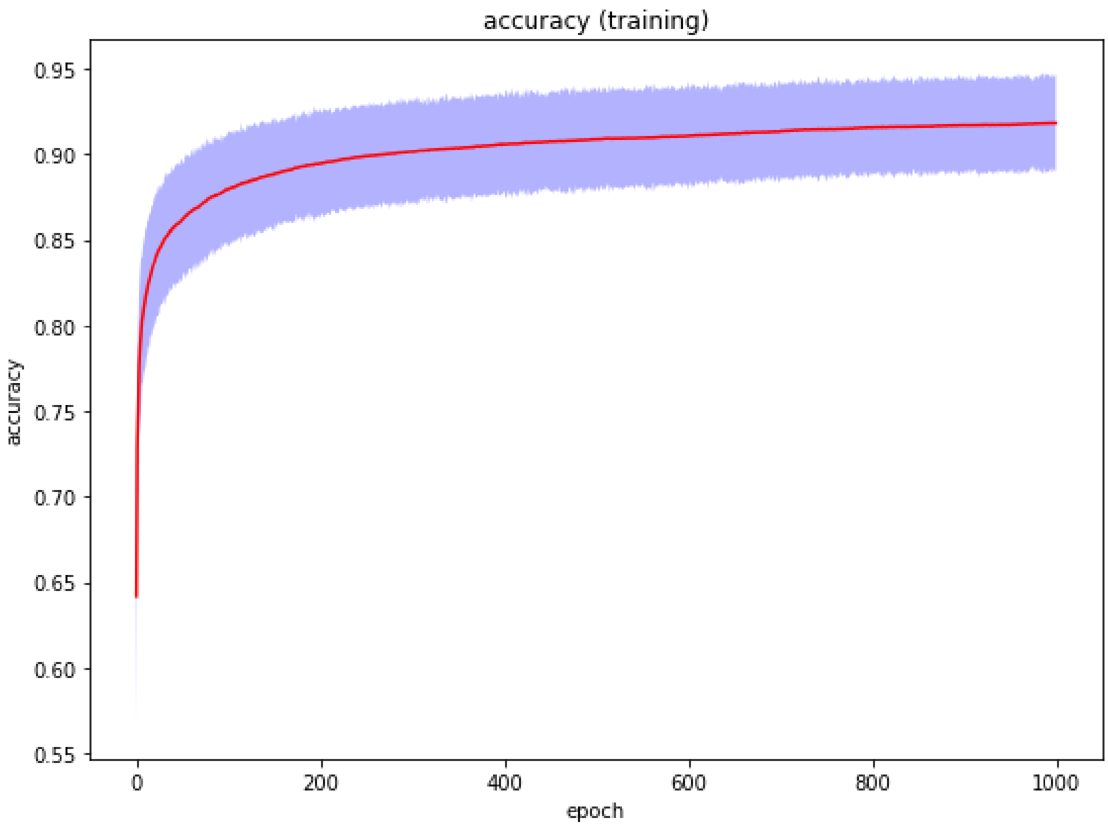
2.14 # 14. plot accuracy (training) curve (mean, std) : mini-batch size = 50, weight decay = 0

[ ]:
```
function_results_14()
```



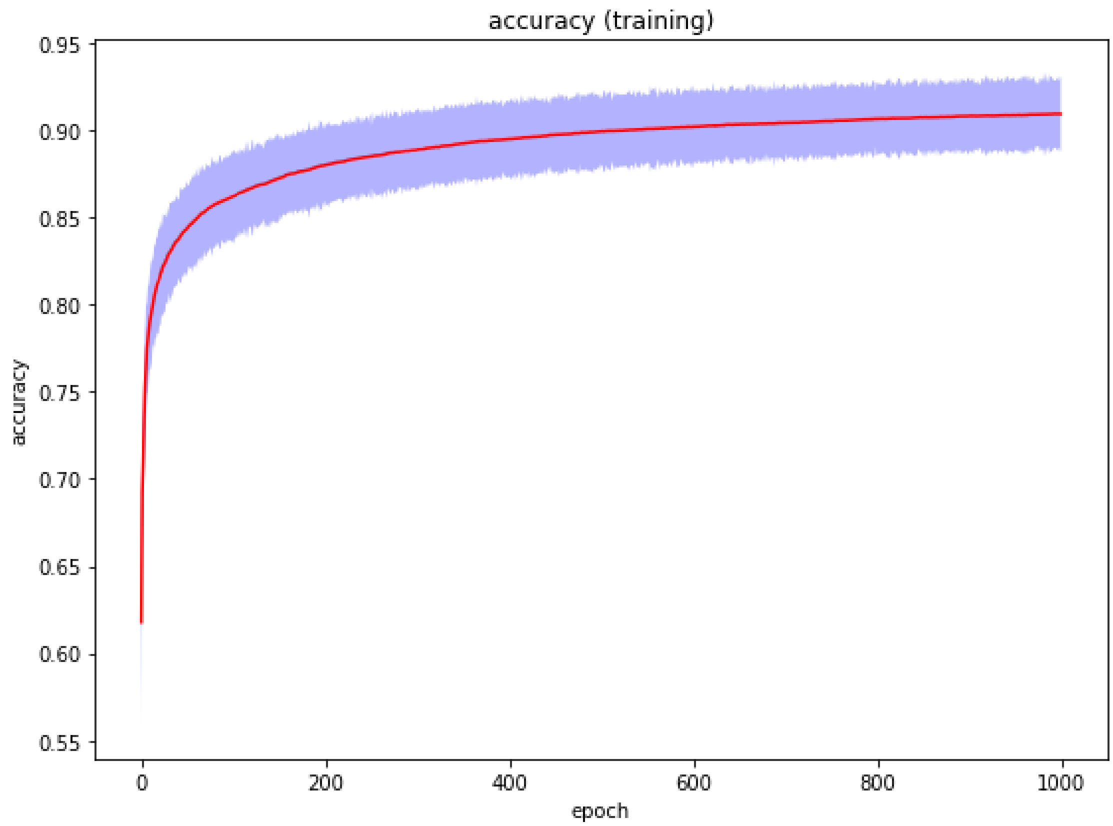
2.15 # 15. plot accuracy (training) curve (mean, std) : mini-batch size = 100, weight decay = 0

[ ]:
```
function_results_15()
```



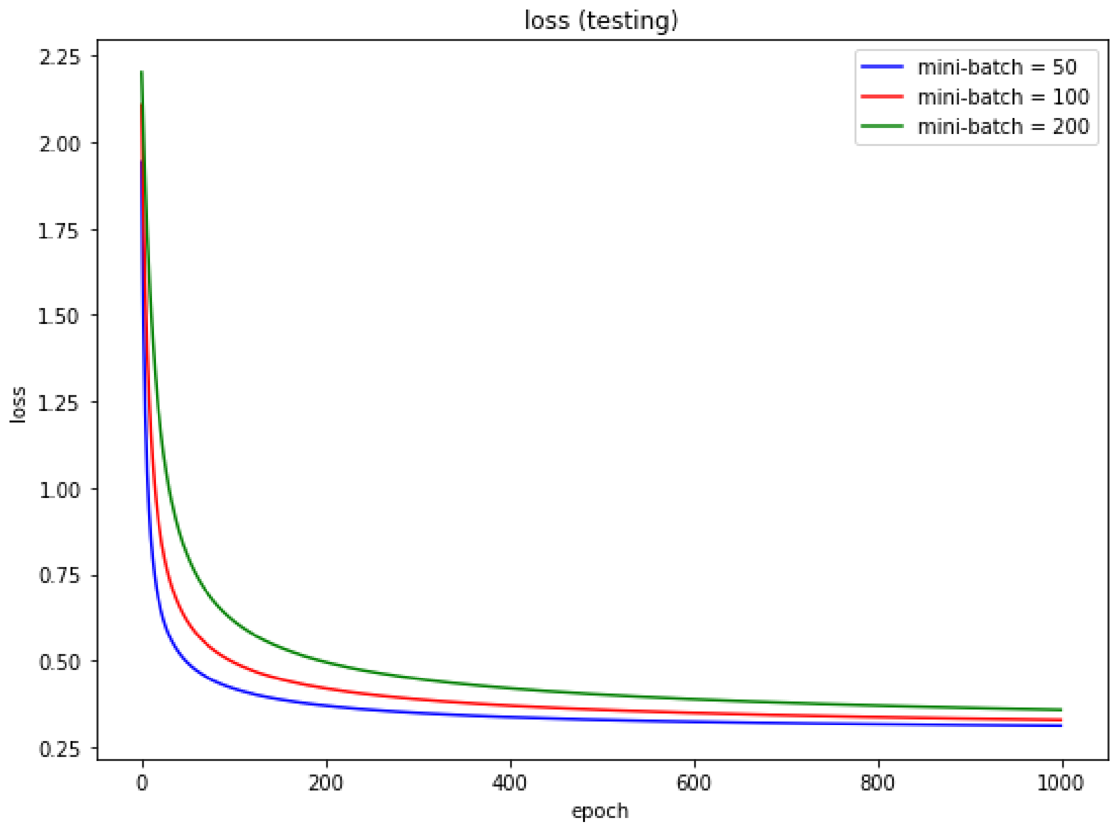
2.16 # 16. plot accuracy (training) curve (mean, std) : mini-batch size = 200, weight decay = 0

[ ]:
```
function_results_16()
```



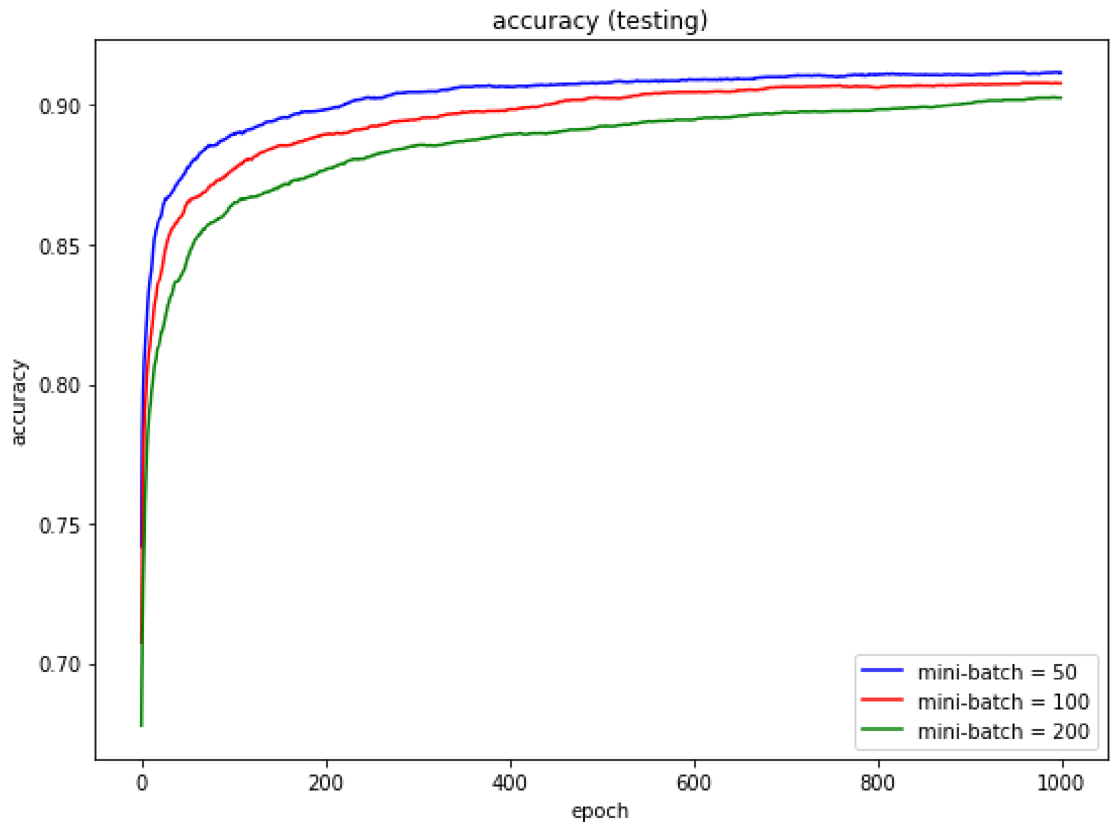
2.17 # 17. plot loss (testing) curve (mean) with different mini-batch

[ ]:
```
function_results_17()
```



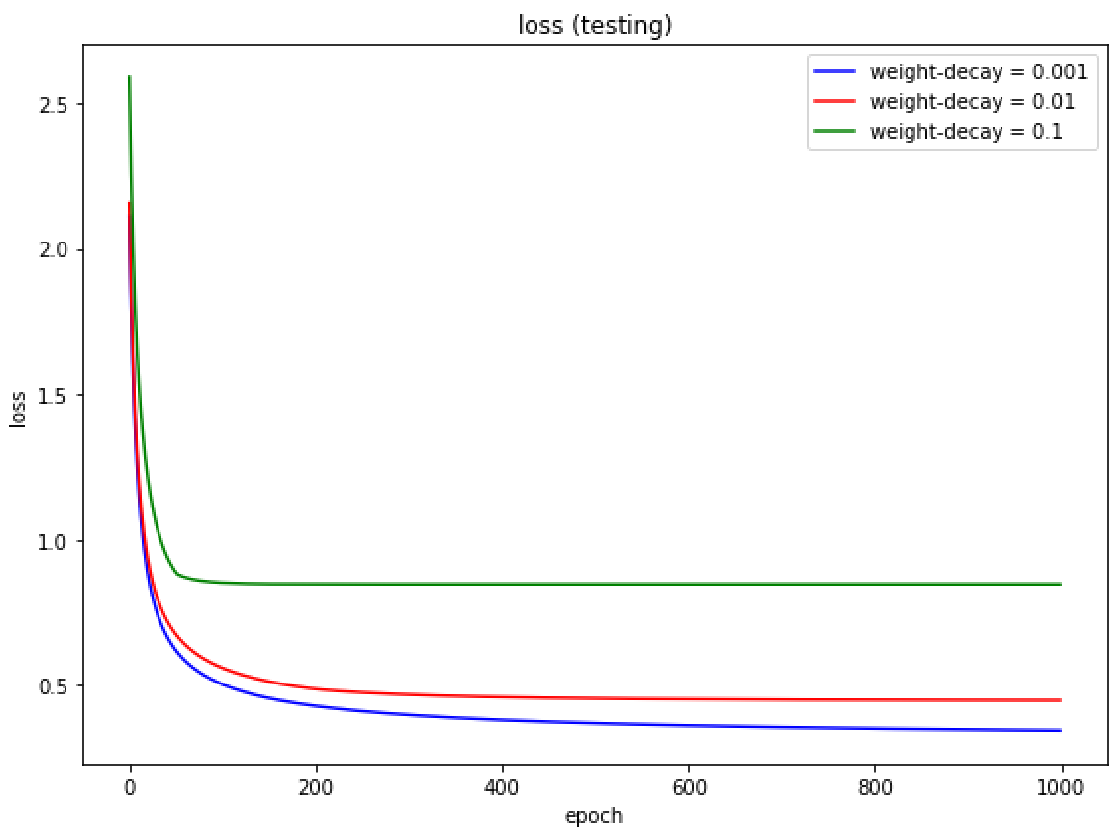
2.18 # 18. plot accuracy (testing) curve (mean) with different mini-batch

[ ]:
```
function_results_18()
```



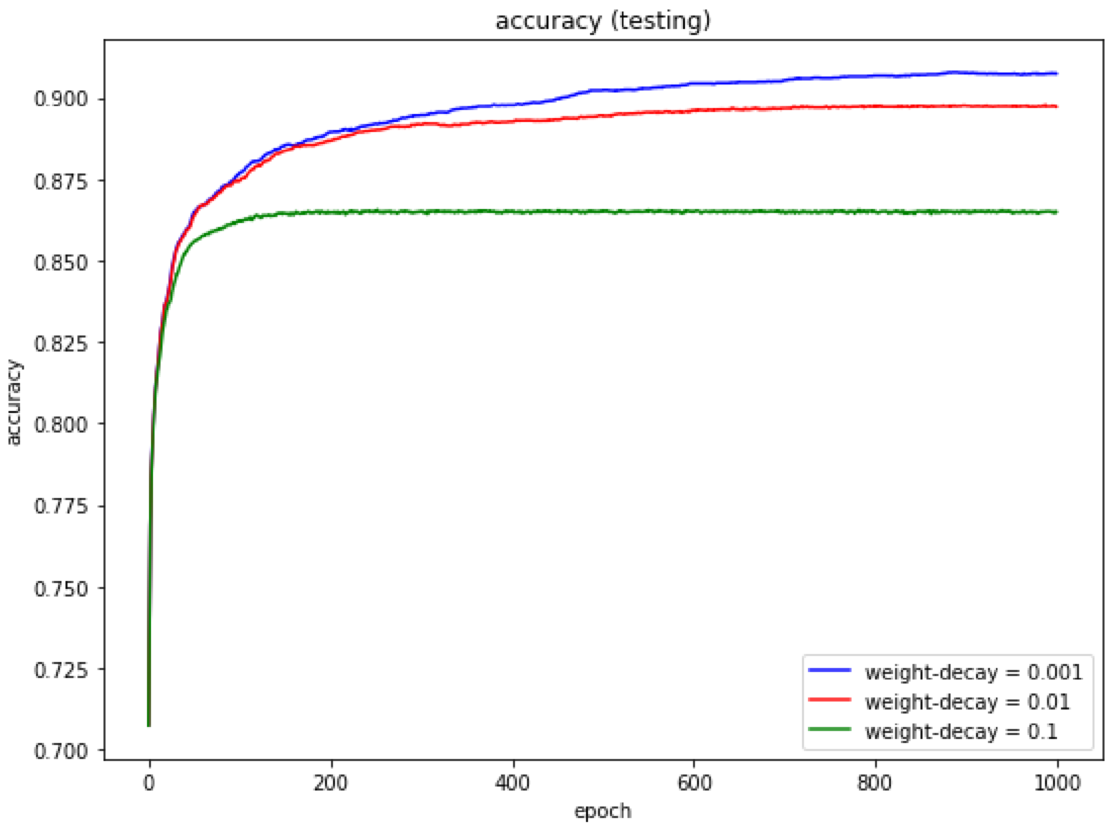
2.19 # 19. plot loss (testing) curve (mean) with different weight-decay

[ ]:
```
function_results_19()
```



2.20 # 20. plot accuracy (testing) curve (mean) with different weight-decay

[ ]:
```
function_results_20()
```



2.21 # 21. print training loss (mean) at iterations 0, 100, 200, 300, 400, 500, 600, 700, 800, 900 : mini-batch = 100, weight-decay = 0

[ ]:
```
function_results_21(train_loss_mean_minibatch[1])
```

```
index =    0, value = 2.2003866772
index =  100, value = 0.4884452279
index =  200, value = 0.4086303762
index =  300, value = 0.3743102039
index =  400, value = 0.3538169113
index =  500, value = 0.3396656612
index =  600, value = 0.3290494042
index =  700, value = 0.3206579243
index =  800, value = 0.3137801756
index =  900, value = 0.3079845796
```

2.22 # 22. print training accuracy (mean) at iterations 0, 100, 200, 300, 400, 500, 600, 700, 800, 900 : mini-batch = 100, weight-decay = 0

[ ]:
```
function_results_22(train_accuracy_mean_minibatch[1])
```

```
index =    0, value = 0.6415000000
index =  100, value = 0.8798500000
index =  200, value = 0.8948000000
index =  300, value = 0.9018000000
index =  400, value = 0.9062000000
index =  500, value = 0.9090500000
index =  600, value = 0.9109500000
index =  700, value = 0.9137000000
index =  800, value = 0.9158000000
index =  900, value = 0.9171500000
```

2.23 # 23. print training loss (mean) at iterations 0, 100, 200, 300, 400, 500, 600, 700, 800, 900 : mini-batch = 100, weight-decay = 0.001

[ ]:
```
function_results_23(train_loss_mean_weight_decay[0])
```

```
index =    0, value = 2.2053959737
index =  100, value = 0.4962629800
index =  200, value = 0.4175434109
index =  300, value = 0.3840709465
index =  400, value = 0.3643988464
index =  500, value = 0.3514216500
index =  600, value = 0.3419763652
index =  700, value = 0.3346759415
index =  800, value = 0.3288218803
index =  900, value = 0.3239931453
```

2.24 # 24. print training accuracy (mean) at iterations 0, 100, 200, 300, 400, 500, 600, 700, 800, 900 : mini-batch = 100, weight-decay = 0.001

[ ]:
```
function_results_24(train_accuracy_mean_weight_decay[0])
```

```
index =    0, value = 0.6415000000
index =  100, value = 0.8794000000
index =  200, value = 0.8944000000
index =  300, value = 0.9014500000
index =  400, value = 0.9053000000
index =  500, value = 0.9083000000
index =  600, value = 0.9104000000
index =  700, value = 0.9124000000
index =  800, value = 0.9144500000
index =  900, value = 0.9157000000
```