MFE 230I: Problem Set 1

Due: Monday, June 14, 2021 by 9:30 a.m.

1. You take out a 30-year, fixed-rate mortgage for $1,000,000. The interest rate on the mortgage loan is 5.25% (APR, compounded monthly), and it requires you to make equal payments at the end of each of the next 360 months (i.e., the first payment is 1 month from today and the last payment is 360 months from today).

   (a) What is the amount of each monthly payment?

   (b) What is the remaining balance on the loan immediately after the 100th loan payment?

   (c) How much interest in total will you pay during year 10 of the mortgage (i.e., what is the total amount of interest included in the 12 payments ending with the payment 120 months from today)?

2. The annually compounded one-, two- and three-year spot rates ($r_1(0,1)$, $r_1(0,2)$ and $r_1(0,3)$) are 5%, 6% and 7% respectively.

   (a) What are the first three annually compounded (one-year) forward rates for years 1, 2 and 3 ($f_1(0,0,1)$, $f_1(0,1,2)$ and $f_1(0,2,3)$)?

   (b) If no explicit forward-rate agreements existed, how would you use a combination of spot lending and borrowing to effectively borrow $1,000 a year from today, and pay it back 3 years from today (plus interest)?

3. The annually compounded one-, two- and three-year forward rates ($f_1(0,0,1)$, $f_1(0,1,2)$ and $f_1(0,2,3)$) are 11%, 13% and 17% respectively. What are the annually compounded one, two and three-year spot rates ($r_1(0,1)$, $r_1(0,2)$ and $r_1(0,3)$)?

4. A Treasury Bill has 32 days to maturity. If the quoted discount is 5.2%, what is the price of one Bill (assume a $10,000 face value)?

5. Prove that for a discount bond, YTM > current yield > coupon rate.

6. Here is the start of the Bloomberg "23" yield curve for trade date May 29, 2018 that we studied in class:

Date	Zero Rate	Forward Rate
05/31/2018	—	2.31813000000000
08/31/2018	2.37666301585850	2.33346095337569
11/30/2018	2.37150478655406	2.46702480555569
02/28/2019	2.42393753171055	2.55633514771236

   (a) Calculate the discount factor for February 28, 2019 from the *zero rates*.

   (b) Calculate the discount factor for February 28, 2019 from the *forward rates*.

The next three problems involve fitting yield curves. To obtain the input data for your analysis, start by using Bloomberg to download LIBOR/swap rates from Feb. 28, 2019 (see Appendix A for detailed instructions). **In your analysis, make sure to use *mid-quotes*, i.e., (Bid + Ask) / 2.**

7. Fit a yield curve to the input data you just downloaded, assuming that

   - The *simple interest rate* r_s is a continuous function of maturity, $r_s(t)$.
   - $r_s(t)$ is piecewise linear, with kinks at the maturity dates of the instruments in the input data set.
   - $r_s(t)$ is constant outside the range of maturities in the input data. I.e., if $t_{\min}$ and $t_{\max}$ are, respectively, the shortest and longest maturities in the input sample,

$$r_s(t) = \begin{cases} r_s(t_{\min}) & \text{if } t \le t_{\min}, \\ r_s(t_{\max}) & \text{if } t \ge t_{\max}. \end{cases}$$

   [This is Bloomberg's default "method 1" (see Bloomberg, 2012, pp. 7–8).]

   Your goal is to match the zero rates and forward rates calculated by Bloomberg, which you can obtain by selecting `Curve Analysis` → `Forward Analysis` and then exporting to Excel [make sure that `Interpolation` is set to `Piecewise Linear (Simple)` and that `Curve Side` is set to `mid`].

   Using your fitted yield curve, generate a table showing, at 3-month intervals from 3/4/2019 to 12/4/2068 (use the dates in the output spreadsheet you just downloaded from Bloomberg):

   (a) Your fitted discount factor.
   (b) Your fitted zero rate.
   (c) Bloomberg's fitted zero rate
   (d) Your fitted 3-month forward rate
   (e) Bloomberg's fitted 3-month forward rate
   (f) The difference between your fitted zero rate and Bloomberg's
   (g) The difference between your fitted forward rate and Bloomberg's

   Report all numbers to 10 decimal places, and report the numbers in columns (b)–(g) as percentages (e.g., 2.6151300000%). **Your numbers should be identical to Bloomberg's to at least this degree of precision. In other words, the numbers in columns (f) and (g) should all be ±0.0000000000%.** If you manage to match only to, say, 7 decimal places, e.g., a difference of 0.000000026%, your calculations are *not* 100% right.

   Note: Bloomberg uses the following quoting conventions:

Rate	Compounding	Day count
Zero rates	Semi-annual	30I/360
Forward rates	Quarterly	Actual/360
Simple rates	Simple	Actual/360

Note: this is not easy! Nobody got all the numbers to match exactly last year. In answering this question, make sure to read — and reread — the description in Bloomberg (2012) in detail.

8. Fit the 5-parameter model of Nelson and Siegel (1987) to your input data. Generate the same table as in Question 7.

9. Repeat Question 8, this time using the model of Svensson (1994).

References

Bloomberg, 2012, Building the Bloomberg interest rate curve: Definitions and methodology, Technical report.

Nelson, C. R., and A. F. Siegel, 1987, Parsimonious modeling of yield curves, *Journal of Business* 60, 473–489.

Svensson, L. E. O., 1994, Estimating and interpreting forward rates: Sweden 1992–4, Working Paper 4871, NBER.

A Downloading data from Bloomberg

LIBOR curves are typically estimated from quotes on three different types of instrument:

1. LIBOR deposit rates for short maturities.
2. Rates calculated from Eurodollar futures for intermediate maturities.
3. Swap rates for longer maturities.

For this problem set, you need to download rates for February 28, 2019 (corresponding settlement date: March 4, 2019) by using Bloomberg's swap-curve-builder command, `ICVS`:[1]

- Run the command `ICVS` to display the curve-selection screen (see Figure 1).
- Pick curve number 23, `USD (30/360, S/A vs. 3M LIBOR)`, to display the curve-construction screen (see Figure 2), showing yields for today's date plus a graph of Bloomberg's fitted yield curve (see Bloomberg, 2012, pp. 3–6).
- Change the trade date (at the top-right of the screen) to 2/28/19 and press `Enter` to change to the desired date.
- Download the displayed data to Excel (at higher precision than shown on the screen) by selecting `Actions` → `Export to Excel`. The columns you care about are:
  - `Term`: Number of months/years or actual date.
  - `Unit`: Months (MO), years (YR) or maturity date (ACTDATE)
  - `Bid` and `Ask`: Bid and ask yields on the security.
  - `Rate Type`: The type of interest rate:
    - `Cash Rates`: Deposit rate
    - `Contiguous Futures`: Rate calculated from Eurodollar futures contract.
    - `Swap Rates`: LIBOR swap rate.
  - `Daycount`: The day-count convention being used.
  - `Freq`: Compounding/payment frequency.

[1] Further information about `ICVS`, and about the methods used by Bloomberg for interpolating yield curves, can be found in Bloomberg (2012). While Bloomberg allows the user to customize exactly which quotes to use at each maturity, we shall just use the default settings.

Swap Curve Builder

	Currency	Country	Curve Number	Curve
	SEK	Sweden	20	SEK (vs. 3M STIBOR)
	SEK	Sweden	137	SEK vs. USD Basis
	SEK	Sweden	185	SEK OIS
	SEK	Sweden	295	SEK vs. EUR Basis
	SEK	Sweden	347	SEK (vs. 1M STIBOR)
	SEK	Sweden	348	SEK (vs. 6M STIBOR)
	SEK	Sweden	417	SEK Cashflow CSA Curve(s)
	SEK	Sweden	393	SEK EIOPA UFR Curve
	SEK	Sweden	Inflation	Sweden CPI SWCPI
	USD	United States of Ame	23	USD (30/360, S/A vs. 3M LIBOR)
	USD	United States of Ame	42	USD OIS
	USD	United States of Ame	47	USD (ACT360, Ann vs. 3M LIBOR)
	USD	United States of Ame	50	USD (vs. 1M LIBOR)
	USD	United States of Ame	51	USD (vs. 6M LIBOR)
	USD	United States of Ame	52	USD (vs. T-BILL)
	USD	United States of Ame	85	USD (vs. FED FUNDS)
	USD	United States of Ame	86	USD (vs. PRIME)
	USD	United States of Ame	87	USD (vs. Comm Paper)
	USD	United States of Ame	349	USD (vs. 12M LIBOR)
	USD	United States of Ame	357	USD MUNI (vs. % LIBOR)
	USD	United States of Ame	400	USD Cashflow CSA Curve(s)

Figure 1: Bloomberg ICVS curve-selection screen

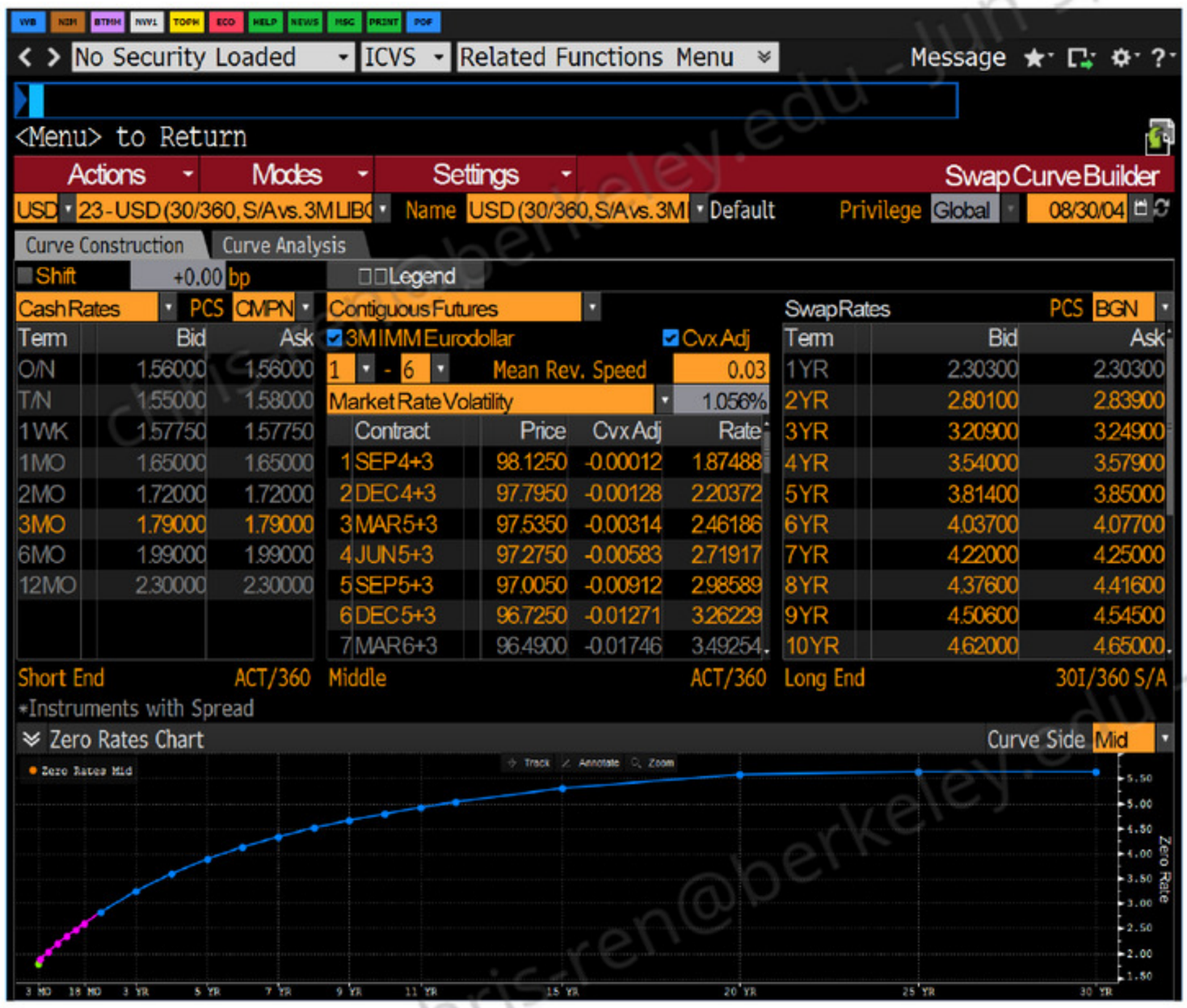

Figure 2: Bloomberg ICVS curve-construction screen