

MFE230E Problem Set 6

Due May 5 6:00pm via bCourses

You may use the `arch` package (<https://pypi.org/project/arch>). It is not included in Anaconda and you need to install it yourself.

1. Download daily prices of the S&P500 index and GE from starting in 1/1/1970 and construct daily, monthly, and annual log returns from prices.
 - (a) Compute the autocorrelations of daily, monthly, and annual log returns up to lag 5.
 - (b) Compute the average volatility over the sample using the following volatility measures (all expressed in annual units):
 - i. Average annualized volatility of *daily* returns.
 - ii. Average annualized volatility of *monthly* returns.
 - iii. Average volatility of *annual* returns.

Why are the sample averages different?

- (c) Next, compare the time series of
 - i. Volatility of *monthly* returns.
 - ii. Monthly volatility based on daily returns in each month.

Plot these monthly series and compute their means, variances and correlation.

- (d) Estimate an AR(1) model for realized monthly volatility. What features of the data and model are noteworthy?
- (e) Using the AR(1) model, compute one-month-ahead forecasts of the realized volatility. Plot realized volatility and forecasted realized volatility. (For simplicity, use the forecasts from the in-sample AR(1) regression instead of true out-of-sample forecasts.)
- (f) Compute the Mean-Square-Error (MSE) of the AR(1) forecasts $E[(\sigma_{t+1} - \hat{\sigma}_{t+1|t})^2]$, where σ_{t+1} is the realized value of the volatility at $t + 1$ and $\hat{\sigma}_{t+1|t}$ is its forecast given information at time t .

2.
 - (a) Estimate a GARCH(1,1) model for monthly returns. What features of the data and model are noteworthy?
 - (b) Regress realized monthly volatility on conditional GARCH(1,1) volatility. Plot realized volatility and conditional volatility.
 - (c) Compute the RMSE of the GARCH(1,1) forecasts.
 - (d) Find the “best” GARCH(p, q) model.

3. Repeat the above exercise for the GJR-GARCH(1,1) model

$$\sigma_t^2 = \omega + \delta \sigma_{t-1}^2 + \alpha \epsilon_{t-1}^2 + \phi \epsilon_{t-1}^2 I_{\epsilon_{t-1} < 0}.$$

4. Compare the RMSE and MAE of the AR(1), GARCH(1,1) and GJR-GARCH(1,1) models.

Optional: The `arch` package includes other xGARCH specifications. Find the “best” model among all applicable specifications that are part of the `arch` package.