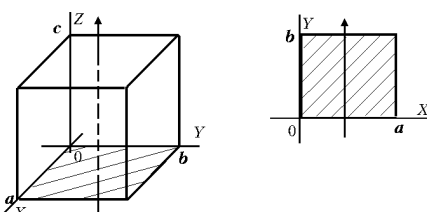
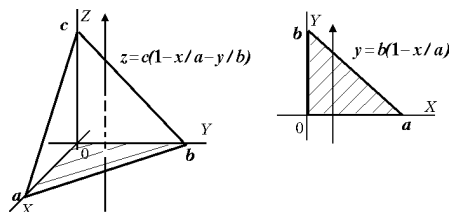
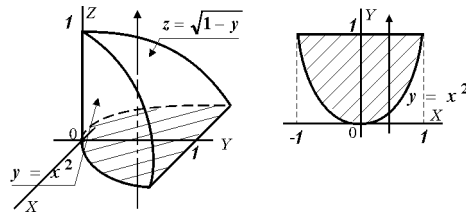
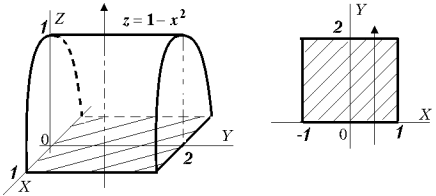
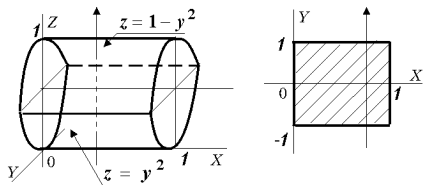
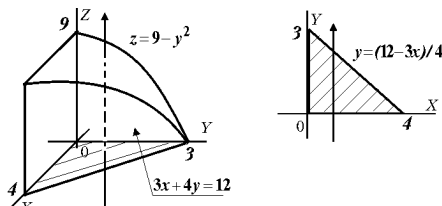
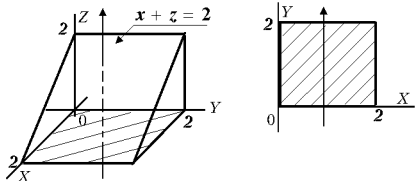
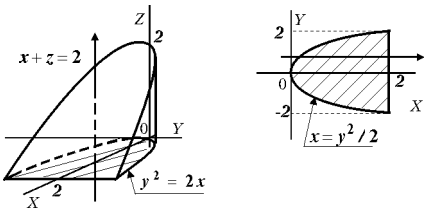
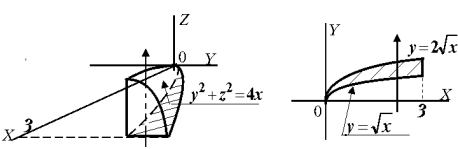
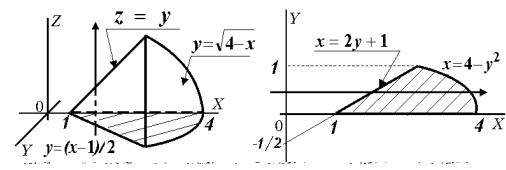
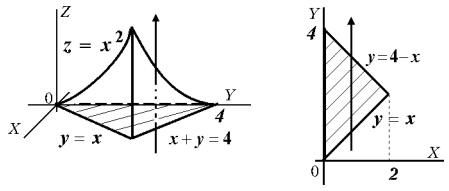


### Приложение 3.

#### Тройной интеграл в прямоугольных координатах

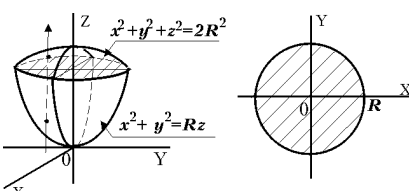
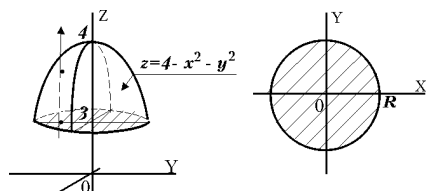
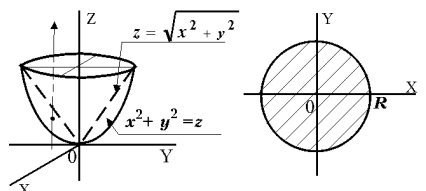
|    | Область интегрирования  | Рисунок<br>Повторный интеграл   |
|----|---|---|
| 1. | $\begin{cases} x = 0, & x = a, \\ y = 0, & y = b, \\ z = 0, & z = c \end{cases}$                      |  $\int_0^a dx \int_0^b dy \int_0^c f(x, y, z) dz$                         |
| 2. | $\begin{cases} \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1, \\ x = 0, \\ y = 0, \\ z = 0 \end{cases}$ |  $\int_0^a dx \int_0^{b(1-x/a)} dy \int_0^{c(1-x/a-y/b)} f(x, y, z) dz$ |
| 3. | $\begin{cases} z = \sqrt{1-y}, \\ y = x^2, \\ z \geq 0 \end{cases}$                                   |  $\int_{-1}^1 dx \int_{x^2}^1 dy \int_0^{\sqrt{1-y}} f(x, y, z) dz$     |

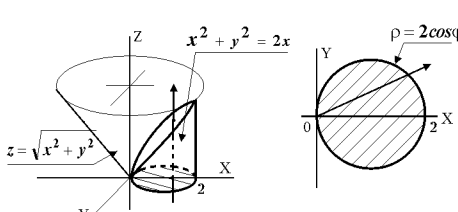
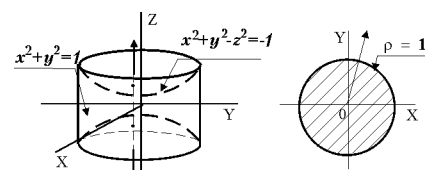
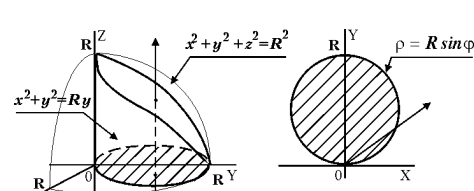
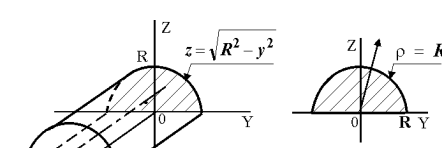
|    |  |  |
|----|--|--|
| 4. | $\begin{cases} z = 1 - x^2, \\ y = 0, \quad y = 2, \\ z \geq 0 \end{cases}$                        |  $\int_{-1}^1 dx \int_0^2 dy \int_0^{1-x^2} f(x, y, z) dz$       |
| 5. | $\begin{cases} z = 1 - y^2, \\ z = y^2, \\ x = 0, \quad x = 1 \end{cases}$                         |  $\int_0^1 dx \int_{-1}^1 dy \int_{y^2}^{1-y^2} f(x, y, z) dz$   |
| 6. | $\begin{cases} z = 9 - y^2, \\ 3x + 4y = 12, \\ x \geq 0, \quad y \geq 0, \\ z \geq 0 \end{cases}$ |  $\int_0^4 dx \int_0^{3-3/4x} dy \int_0^{9-y^2} f(x, y, z) dz$ |
| 7. | $\begin{cases} x + z = 2, \\ y = 0, \quad y = 2, \\ x \geq 0, \quad z \geq 0 \end{cases}$          |  $\int_0^2 dx \int_0^2 dy \int_0^{2-x} f(x, y, z) dz$          |

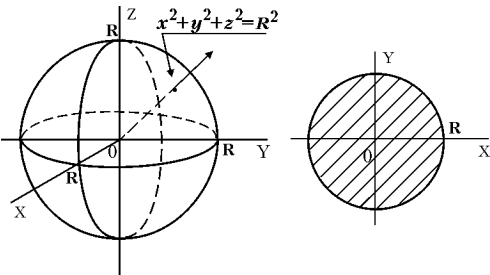
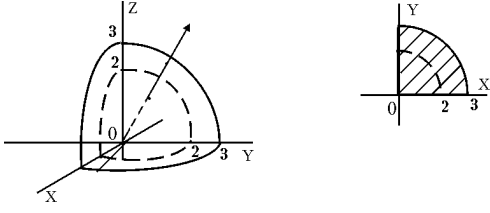
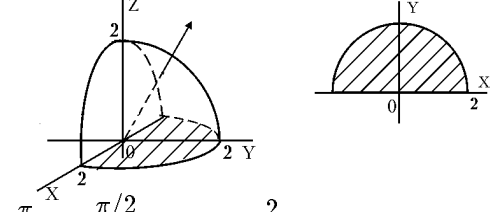
|     |   |  |
|-----|---|--|
| 8.  | $\begin{cases} x + z = 2, \\ y^2 = 2x, \\ z \geq 0 \end{cases}$   |  $\int_{-2}^2 dy \int_{y^2/2}^2 dx \int_0^{2-x} f(x, y, z) dz$                     |
| 9.  | $\begin{cases} y^2 + z^2 = 4x, \\ y^2 = x, \quad y^2 = 4x, \\ x = 3, \\ z \geq 0, \quad y \geq 0 \end{cases}$ |  $\int_0^3 dx \int_{\sqrt{x}}^{2\sqrt{x}} dy \int_0^{\sqrt{4x-y^2}} f(x, y, z) dz$ |
| 10. | $\begin{cases} z = y, \quad z \geq 0, \\ y = \sqrt{4-x}, \\ x - 2y = 1 \end{cases}$                           |  $\int_0^1 dy \int_{2y+1}^{4-y^2} dx \int_0^y f(x, y, z) dz$                     |
| 11. | $\begin{cases} z = x^2, \\ x + y = 4, \\ y = x, \quad z \geq 0 \end{cases}$                                   |  $\int_0^2 dx \int_x^{4-x} dy \int_0^{x^2} f(x, y, z) dz$                        |

## Приложение 4.

### Тройной интеграл в цилиндрических и сферических координатах

|    | Область<br>интегрирования  | Рисунок<br>Повторный интеграл   |
|----|--|---|
| 1. | $\begin{cases} x^2 + y^2 + z^2 = 2R^2, \\ x^2 + y^2 = Rz, \\ z \geq 0 \end{cases}$ $\begin{cases} z = \sqrt{2R^2 - \rho^2}, \\ z = \rho^2/R \end{cases}$ |  $\int_0^{2\pi} d\varphi \int_0^R \rho d\rho \int_{\rho^2/R}^{\sqrt{2R^2 - \rho^2}} f(\rho, \varphi, z) dz$ |
| 2. | $\begin{cases} z = 4 - x^2 - y^2, \\ z = 3 \end{cases}$ $\begin{cases} z = 4 - \rho^2, \\ z = 3 \end{cases}$   |  $\int_0^{2\pi} d\varphi \int_0^1 \rho d\rho \int_3^{4 - \rho^2} f(\rho, \varphi, z) dz$                  |
| 3. | $\begin{cases} z = \sqrt{x^2 + y^2}, \\ z = x^2 + y^2 \end{cases}$ $\begin{cases} z = \rho, \\ z = \rho^2 \end{cases}$                                   |  $\int_0^{2\pi} d\varphi \int_0^1 \rho d\rho \int_{\rho^2}^{\rho} f(\rho, \varphi, z) dz$                 |

|    |   |   |
|----|---|---|
| 4. | $\begin{cases} z = \sqrt{x^2 + y^2}, \\ x^2 + y^2 = 2x, \\ z = 0 \end{cases}$ $\begin{cases} z = 0, & z = \rho, \\ \rho = 2 \cos \varphi \end{cases}$                     |  $\int_{-\pi/2}^{\pi/2} d\varphi \int_0^{2 \cos \varphi} \rho d\rho \int_0^{\rho} f(\rho, \varphi, z) dz$         |
| 5. | $\begin{cases} x^2 + y^2 - z^2 = -1, \\ x^2 + y^2 = 1 \end{cases}$ $\begin{cases} z = \pm \sqrt{1 + \rho^2}, \\ \rho = 1 \end{cases}$                                     |  $\int_0^{2\pi} d\varphi \int_0^1 \rho d\rho \int_{-\sqrt{1+\rho^2}}^{\sqrt{1+\rho^2}} f(\rho, \varphi, z) dz$    |
| 6. | $\begin{cases} x^2 + y^2 + z^2 = R^2, \\ x^2 + y^2 = Ry, \\ z \geq 0 \end{cases}$ $\begin{cases} z = \sqrt{R^2 - \rho^2}, \\ \rho = R \sin \varphi, \\ z = 0 \end{cases}$ |  $\int_0^{\pi} d\varphi \int_0^{R \sin \varphi} \rho d\rho \int_0^{\sqrt{R^2 - \rho^2}} f(\rho, \varphi, z) dz$ |
| 7. | $\begin{cases} z \leq \sqrt{R^2 - y^2}, \\ x = 0, & x = 1, \\ z \geq 0 \end{cases}$ $\begin{cases} x = 0, & x = 1, \\ \rho = R \end{cases}$                               |  $\int_0^1 dx \int_0^{\pi} d\varphi \int_0^R \rho d\rho f(\rho, \varphi, z)$                                    |

|     | Область<br>интегрирования  | Рисунок<br>Повторный интеграл   |
|-----|--|---|
| 8.  | $x^2 + y^2 + z^2 = R^2$ $\rho = R$   |  $\int_0^{2\pi} d\varphi \int_0^{\pi} \sin \theta d\theta \int_0^R f(\rho, \varphi, \theta) \rho^2 d\rho$     |
| 9.  | $\begin{cases} x^2 + y^2 + z^2 \geq 4, \\ x^2 + y^2 + z^2 \leq 9, \\ x \geq 0, \quad y \geq 0, \\ z \geq 0 \end{cases}$ $2 \leq \rho \leq 3$ |  $\int_0^{\pi/2} d\varphi \int_0^{\pi/2} \sin \theta d\theta \int_2^3 f(\rho, \theta, \varphi) \rho^2 d\rho$ |
| 10. | $\begin{cases} x^2 + y^2 + z^2 = 4, \\ y \geq 0, \\ z \geq 0 \end{cases}$ $\rho = 2.$  |  $\int_0^{\pi} d\varphi \int_0^{\pi/2} \sin \theta d\theta \int_0^2 f(\rho, \theta, \varphi) \rho^2 d\rho$  |