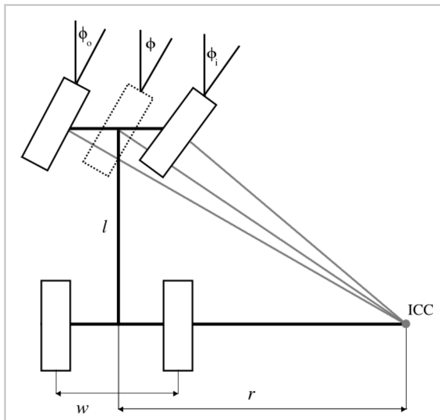
Ackerman Steering

The ackerman steering is used in car-like vehicles. The basic idea consists of rotating the inner wheel slightly sharper than the outer wheel to reduce tire slippage.



With the track width w (the lateral wheel separation), the wheel base l (the longitudinal wheel separation), ϕ_i the relative steering angle of the inner wheel, ϕ_o the relative steering angle of the outer wheel and r the distance between ICC (instantaneous center of curvature) and the center of the car.

The ackerman steering equation can then be derivated quite easily by considering the three triangles formed by the vertical side l and the side r plus or minus $\frac{w}{2}$. So we get

$$\begin{aligned} \tan\phi &= \frac{l}{r} \\ \tan\phi_i &= \frac{l}{r-w/2} \\ \tan\phi_o &= \frac{l}{r+w/2} \end{aligned}$$

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By subtracting the reciprocal of the latter two equations, we arrive at the ackerman steering equation:

$$\frac{1}{\tan\phi_o} - \frac{1}{\tan\phi_i} = \cot\phi_o - \cot\phi_i = \frac{r+w/2}{l} - \frac{r-w/2}{l} = \frac{w}{l}$$

Equivalently, the two cotangents can be expressed with base angle ϕ as follows:

$$\begin{aligned}\cot\phi_i - \cot\phi &= \tfrac{r-w/2}{l} - \tfrac{r}{l} = -\tfrac{w}{2l} \Leftrightarrow \cot\phi_i = \cot\phi - \tfrac{w}{2l}\\ \cot\phi_o - \cot\phi &= \tfrac{r+w/2}{l} - \tfrac{r}{l} = +\tfrac{w}{2l} \Leftrightarrow \cot\phi_o = \cot\phi + \tfrac{w}{2l}\end{aligned}$$

These equations have a problem for the case $\phi = 0$, since $\cot(0)$ is not defined. However, when considering the fact that $\cot\alpha = \frac{\cos\alpha}{\sin\alpha}$, we can reformulate the equations as

$$\begin{aligned}\phi_i &= \tan^{-1}\left(\tfrac{2l\sin\phi}{2l\cos\phi - w\sin\phi}\right)\\ \phi_o &= \tan^{-1}\left(\tfrac{2l\sin\phi}{2l\cos\phi + w\sin\phi}\right)\end{aligned}$$

Forward Kinematic for Car-Like vehicles

The forward kinematic for a car-like vehicle, which is the prediction of the future state, given the configuration can be formulated as follows. Considering the state of the vehicle is represented as a quadruple (x, y, θ, ϕ), with θ being the heading and (x, y) the position in the world. The most important thing about the ackerman steering is that the rotational center is not in the middle of the car but between the back wheels. As the car is non-holonomic, we can derive the kinmatics only for the derivatives. Given the speed s of the vehicle we get

$$\begin{aligned}\dot{x} &= s\cos\theta\\ \dot{y} &= s\sin\theta\\ \dot{\theta} &= \frac{s}{l}\tan\phi \approx \frac{s}{l}\phi\end{aligned}$$

(Max taylor approximation error of the tangent simplification is about 3° at 30° steering lock)

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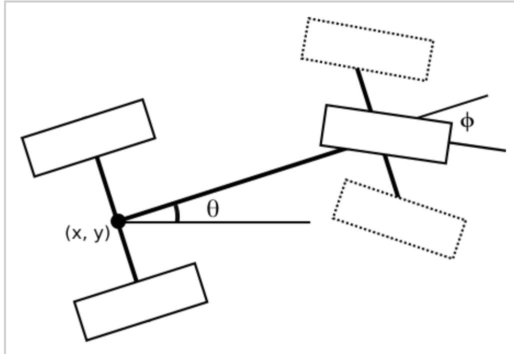

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