

## DEMORGAN'S THEOREMS

DeMorgan, a mathematician who knew Boole, proposed two theorems that are an important part of Boolean algebra. In practical terms. DeMorgan's theorems provide mathematical verification of the equivalency of the NAND and negative-OR gates and the equivalency of the NOR and negative-AND gates, which were discussed in part 3.

One of DeMorgan's theorems is stated as follows:

***The complement of a product of variables is equal to the sum of the complements of the variables,***

Stated another way,

***The complement of two or more ANDed variables is equivalent to the OR of the complements of the individual variables.***

The formula for expressing this theorem for two variables is

$$XY = \overline{X} + \overline{Y}$$

DeMorgan's second theorem is stated as follows:

***The complement of a sum of variables is equal to the product of the complements of the variables.***

Stated another way,

***The complement of two or more ORed variables is equivalent to the AND of the complements of the individual variables,***

The formula for expressing this theorem for two variables is

$$\overline{X + Y} = \overline{X} \overline{Y}$$

Fig.(4-15) shows the gate equivalencies and truth tables for the two equations above.

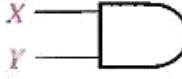
 NAND	 Negative-OR	<table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th colspan="2">Inputs</th><th colspan="2">Output</th></tr> <tr> <th>X</th><th>Y</th><th><math>\overline{XY}</math></th><th><math>\overline{X} + \overline{Y}</math></th></tr> </thead> <tbody> <tr> <td>0</td><td>0</td><td>1</td><td>1</td></tr> <tr> <td>0</td><td>1</td><td>1</td><td>1</td></tr> <tr> <td>1</td><td>0</td><td>1</td><td>1</td></tr> <tr> <td>1</td><td>1</td><td>0</td><td>0</td></tr> </tbody> </table>	Inputs		Output		X	Y	$\overline{XY}$	$\overline{X} + \overline{Y}$	0	0	1	1	0	1	1	1	1	0	1	1	1	1	0	0
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Fig.(4-15) Gate equivalencies and the corresponding truth tables that illustrate DeMorgan's theorems.

As stated, DeMorgan's theorems also apply to expressions in which there are more than two variables. The following examples illustrate the application of DeMorgan's theorems to 3-variable and 4-variable expressions.

### Example

Apply DeMorgan's theorems to the expressions  $XYZ$  and  $X + Y + z$ .

$$XYZ = \overline{\overline{X}} + \overline{\overline{Y}} + \overline{\overline{Z}}$$

$$\overline{X + Y + Z} = \overline{\overline{X}} \overline{\overline{Y}} \overline{\overline{Z}}$$

### Example

Apply DeMorgan's theorems to the expressions  $WXYZ$  and  $W + X + y + z$ .

$$WXYZ = \overline{\overline{W}} + \overline{\overline{X}} + \overline{\overline{y}} + \overline{\overline{Z}}$$

$$\overline{W + X + y + Z} = \overline{\overline{W}} \overline{\overline{X}} \overline{\overline{y}} \overline{\overline{Z}}$$

### Applying DeMorgan's Theorems

The following procedure illustrates the application of DeMorgan's theorems and Boolean algebra to the specific expression

$$\overline{\overline{A + \overline{BC}} + D(\overline{E + \overline{F}})}$$

Step 1. Identify the terms to which you can apply DeMorgan's theorems, and think of each term as a single variable. Let  $A + \overline{BC} = X$  and  $D(E + \overline{F}) = Y$ .

Step 2. Since  $\overline{X + Y} = \overline{X}\overline{Y}$ ,

$$\overline{\overline{A + \overline{BC}} + D(\overline{E + \overline{F}})} = (A + \overline{BC})(D(E + \overline{F}))$$

Step 3. Use rule 9 ( $A = \overline{\overline{A}}$ ) to cancel the double bars over the left term (this is not part of DeMorgan's theorem).

$$(A + \overline{BC})(D(E + \overline{F})) = (A + \overline{BC})(D(E + \overline{F}))$$

Step 4. Applying DeMorgan's theorem to the second term,

$$(A + \overline{BC})(D(E + \overline{F})) = (A + \overline{BC})(\overline{D} + (E + \overline{F}))$$

Step 5. Use rule 9 ( $A = \overline{\overline{A}}$ ) to cancel the double bars over the  $E + \overline{F}$  part of the term.

$$(A + \overline{BC})(\overline{D} + \overline{E + \overline{F}}) = (A + \overline{BC})(\overline{D} + E + \overline{F})$$

### Example

Apply DeMorgan's theorems to each of the following expressions:

(a)  $(A + B + C)D$       (b)  $ABC + DEF$       (c)  $A\overline{B} + C\overline{D} + EF$