Example

The Boolean expression for an exclusive-OR gate is $A\overline{B} + \overline{A}B$. With this as a starting point, use DeMorgan's theorems and any other rules or laws that are applicable to develop an expression for the exclusive-NOR gate.

BOOLEAN ANALYSIS OF LOGIC CIRCUITS

Boolean algebra provides a concise way to express the operation of a logic circuit formed by a combination of logic gates so that the output can be determined for various combinations of input values.

Boolean Expression for a Logic Circuit

To derive the Boolean expression for a given logic circuit, begin at the left-most inputs and work toward the final output, writing the expression for each gate. For the example circuit in Fig.(4-16), the Boolean expression is determined as follows:

The expression for the left-most AND gate with inputs C and D is CD. The output of the left-most AND gate is one of the inputs to the OR gate and B is the other input. Therefore, the expression for the OR gate is B + CD.

The output of the OR gate is one of the inputs to the right-most AND gate and A is the other input. Therefore, the expression for this AND gate is A(B + CD), which is the final output expression for the entire circuit.

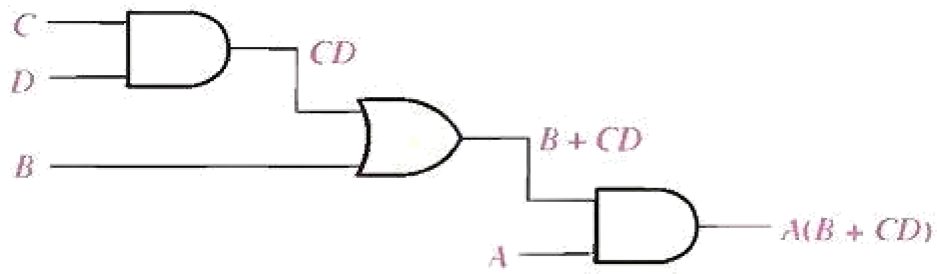

Fig.(4-16) A logic circuit showing the development of the Boolean expression for the output.

<u>Constructing a Truth Table for a Logic Circuit</u>

Once the Boolean expression for a given logic circuit has been determined, a truth table that shows the output for all possible values of the input variables can be developed. The procedure requires that you evaluate the Boolean expression for all possible combinations of values for the input variables. In the case of the circuit in Fig.(4-16), there are four input variables (A, B, C, and D) and therefore sixteen ($2^4 = 16$) combinations of values are possible.

<u>Putting the Results in Truth Table format</u>

The first step is to list the sixteen input variable combinations of 1s and 0s in a binary sequence as shown in Table 4-5. Next, place a 1 in the output column for each combination of input variables that was determined in the evaluation. Finally, place a 0 in the output column for all other combinations of input variables. These results are shown in the truth table in Table 4-5.

Table 4-5

INPUTS				OUTPUT
A	B	C	D	A(B + CD)
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

SIMPLIFICATION USING BOOLEAN ALGEBRA

A simplified Boolean expression uses the fewest gates possible to implement a given expression.

Example

Using Boolean algebra techniques, simplify this expression:

$$AB + A(B + C) + B(B + C)$$

Solution

Step 1: Apply the distributive law to the second and third terms in the expression, as follows:

$$AB + AB + AC + BB + BC$$

Step 2: Apply rule 7 (BB = B) to the fourth term.

$$AB + AB + AC + B + BC$$

Step 3: Apply rule 5 (AB + AB = AB) to the first two terms.

$$AB + AC + B + BC$$

Step 4: Apply rule 10 (B + BC = B) to the last two terms.

$AB + AC + B$

Step 5: Apply rule 10 ($AB + B = B$) to the first and third terms.

$B+AC$

At this point the expression is simplified as much as possible.

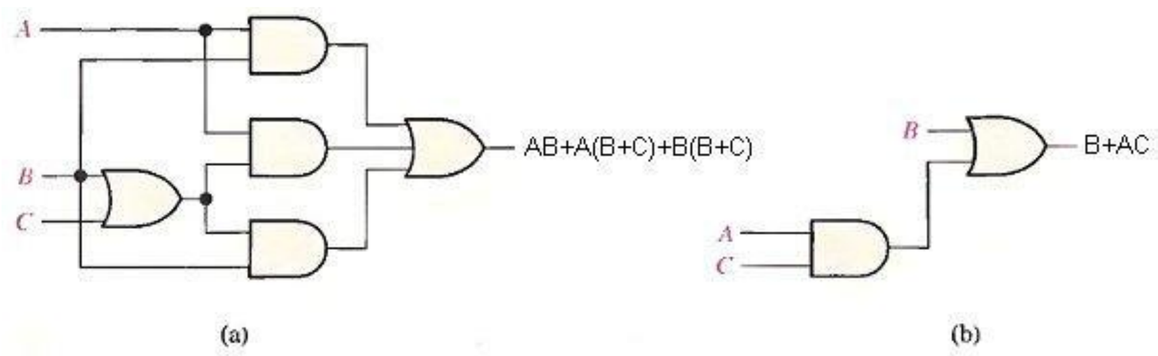

Fig.(4-17) Gate circuits for example above.

<u>Example</u>

Simplify the Boolean expressions:

1- $A\overline{B} + A(B + C) + B(B + C)$.

2- $[A\overline{B}(C + BD) + \overline{A}\,\overline{B}]C$

3- $\overline{A}BC + A\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C} + A\overline{B}C + ABC$