## ■ *Rules of Boolean Algebra*

Table 4-1 lists 12 basic rules that are useful in manipulating and simplifying Boolean expressions. Rules 1 through 9 will be viewed in terms of their application to logic gates. Rules 10 through 12 will be derived in terms of the simpler rules and the laws previously discussed.

Table 4-1 Basic rules of Boolean algebra.

| | |
|---|---|
| 1. $A + 0 = A$ | 7. $A \cdot A = A$ |
| 2. $A + 1 = 1$ | 8. $A \cdot \overline{A} = 0$ |
| 3. $A \cdot 0 = 0$ | 9. $\overline{\overline{A}} = A$ |
| 4. $A \cdot 1 = A$ | 10. $A + AB = A$ |
| 5. $A + A = A$ | 11. $A + \overline{A}B = A + B$ |
| 6. $A + \overline{A} = 1$ | 12. $(A + B)(A + C) = A + BC$ |

$A$, $B$, or $C$ can represent a single variable or a combination of variables.

Rule 1.      $A + 0 = A$

A variable ORed with 0 is always equal to the variable. If the input variable A is 1, the output variable X is 1, which is equal to A. If A is 0, the output is 0, which is also equal to A. This rule is illustrated in Fig.(4-6), where the lower input is fixed at 0.
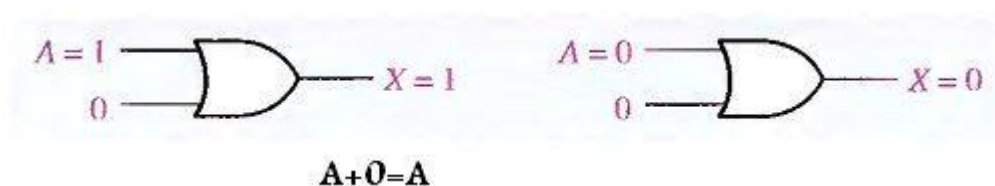


$A+0=A$

Fig.(4-6)

Rule 2.       A + 1 = 1

A variable ORed with 1 is always equal to 1. A 1 on an input to an OR gate produces a 1 on the output, regardless of the value of the variable on the other input. This rule is illustrated in Fig.(4-7), where the lower input is fixed at 1.
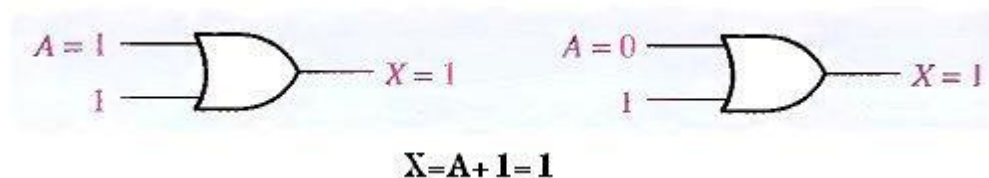


$$X = A + 1 = 1$$

Fig.(4-7)

Rule 3.       A . 0 = 0

A variable ANDed with 0 is always equal to 0. Any time one input to an AND gate is 0, the output is 0, regardless of the value of the variable on the other input. This rule is illustrated in Fig.(4-8), where the lower input is fixed at 0.
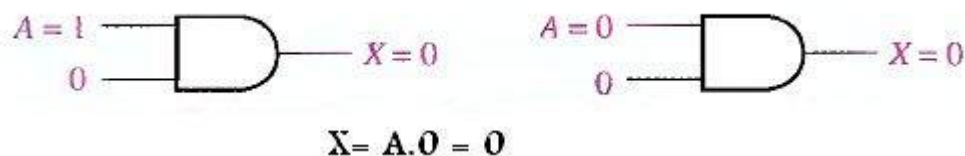


$$X = A.0 = 0$$

Fig.(4-8)

Rule 4.       A . 1 = A

A variable ANDed with 1 is always equal to the variable. If A is 0 the output of the AND gate is 0. If A is 1, the output of the AND gate is 1 because both inputs are now 1s. This rule is shown in Fig.(4-9), where the lower input is fixed at 1.
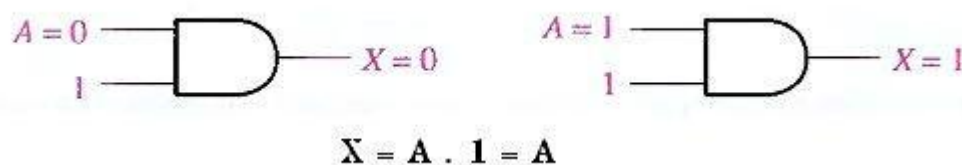


$$X = A . 1 = A$$

Fig.(4-9)

**DSD UNIT 2 NOTES**

Rule 5.        $A + A = A$

A variable ORed with itself is always equal to the variable. If A is 0, then 0 + 0 = 0; and if A is 1, then 1 + 1 = 1. This is shown in Fig.(4-10), where both inputs are the same variable.
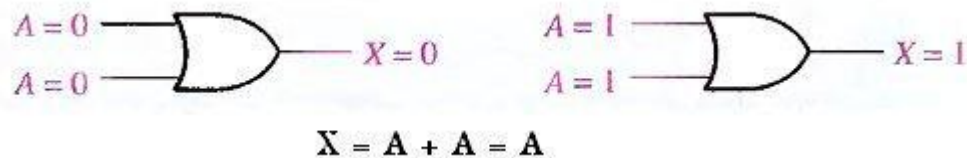


$$X = A + A = A$$

Fig.(4-10)

Rule 6.        $A + \overline{A} = 1$

A variable ORed with its complement is always equal to 1. If A is 0, then 0 + $\overline{0}$ = 0 + 1 = 1. If A is 1, then 1 + $\overline{1}$ = 1+ 0 = 1. See Fig.(4-11), where one input is the complement of the other.
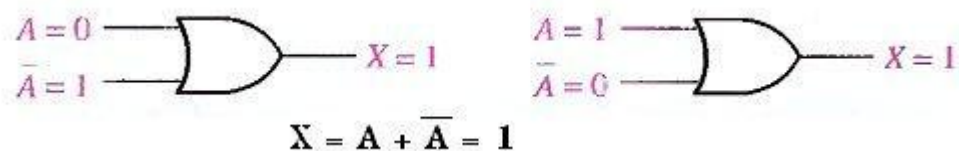


$$X = A + \overline{A} = 1$$

Fig.(4-11)

Rule 7.        $A . A = A$

A variable ANDed with itself is always equal to the variable. If A = 0, then 0.0 = 0; and if A = 1. then 1.1 = 1. Fig.(4-12) illustrates this rule.



$$A . A = A$$
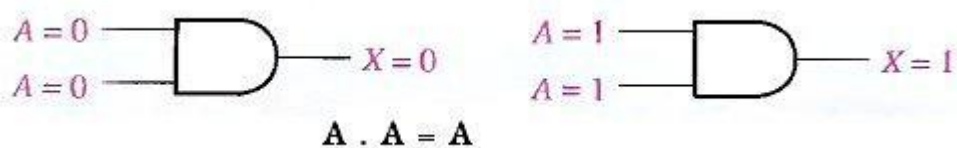
Fig.(4-12)

**DSD UNIT 2 NOTES**

Rule 8.                 $A \cdot \overline{A} = 0$

A variable ANDed with its complement is always equal to 0. Either A or $\overline{A}$
will always be 0: and when a 0 is applied to the input of an AND gate. the
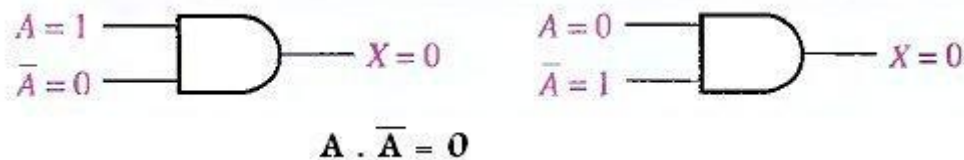output will be 0 also. Fig.(4-13) illustrates this rule.



Fig.(4-13)

Rule 9        $A = \overline{\overline{A}}$

The double complement of a variable is always equal to the variable. If you
start with the variable A and complement (invert) it once, you get $\overline{A}$. If you
then take $\overline{A}$ and complement (invert) it, you get A, which is the original
variable. This rule is shown in Fig.(4-14) using inverters.



Fig.(4-14)

Rule 10.       $A + AB = A$

This rule can be proved by applying the distributive law, rule 2, and rule 4
as follows:

$$A + AB = A(1 + B) \qquad \text{Factoring (distributive law)}$$
$$= A \cdot 1 \qquad \qquad \text{Rule 2: } (1 + B) = 1$$
$$= A \qquad \qquad \text{Rule 4: } A \cdot 1 = A$$

The proof is shown in Table 4-2, which shows the truth table and the
resulting logic circuit simplification.

**DSD UNIT 2 NOTES**

Table 4-2



| A | B | AB | A + AB |
|---|---|---|--------|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

equal

A
B
A   straight connection

Rule 11.   $A + A\overline{B} = A + B$

This rule can be proved as follows:

$$A + \overline{A}B = (A + AB) + \overline{A}B \qquad \text{Rule 10: } A = A + AB$$
$$= (AA + AB) + \overline{A}B \qquad \text{Rule 7: } A = AA$$
$$= AA + AB + A\overline{A} + \overline{A}B \qquad \text{Rule 8: adding } A\overline{A} = 0$$
$$= (A + \overline{A})(A + B) \qquad \text{Factoring}$$
$$= 1.(A + B) \qquad \text{Rule 6: } A + \overline{A} = 1$$
$$= A + B \qquad \text{Rule 4: drop the 1}$$

The proof is shown in Table 4-3, which shows the truth table and the resulting logic circuit simplification.

Table 4-3



| A | B | $\overline{A}B$ | $A + \overline{A}B$ | A + B |
|---|---|-----------------|---------------------|-------|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 | 1 |

equal

A
B

A
B

**DSD UNIT 2 NOTES**

Rule 12.   $(A + B)(A + C) = A + BC$

This rule can be proved as follows:

$(A + B)(A + C) = AA + AC + AB + BC$   Distributive law

$\qquad\qquad\quad = A + AC + AB + BC$        Rule 7: AA = A

$\qquad\qquad\quad = A( 1 + C) + AB + BC$   Rule 2: 1 + C = 1

$\qquad\qquad\quad = A. 1 + AB + BC$            Factoring (distributive law)

$\qquad\qquad\quad = A(1 + B) + BC$            Rule 2: 1 + B = 1

$\qquad\qquad\quad = A. 1 + BC$                  Rule 4: A . 1 = A

$\qquad\qquad\quad = A + BC$

The proof is shown in Table 4-4, which shows the truth table and the
resulting logic circuit
simplification.

Table 4-4

| A | B | C | A + B | A + C | (A + B)(A + C) | BC | A + BC |
|---|---|---|-------|-------|----------------|----|--------|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 0 | 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |