Discrete stream function method
for the incompressible Navier-Stokes equations
with simple boundary conditions

Rauan

February 14, 2024

Abstract

The goal of these notes is to present the detailed overview of discrete stream function method for solving incompressible Navier-Stokes equations with simple boundary conditions. We will discuss in detail the scheme formulation, transient and spatial discretizations. Special attention will be paid to the change of unknown variables. After studying these notes one must get a coherent picture of the application of discrete stream function method to incompressible flows with simple BCs and be able to implement the scheme in code.

Contents

1 Introduction

In these notes we will be concerned with the discretization of the Navier-Stokes equations describing the flow of an incompressible fluid:

$$\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v} \cdot \nabla \boldsymbol{v} = -\nabla p + \epsilon \nabla \cdot \nabla \boldsymbol{v} \tag{1.1a}$$

$$\nabla \cdot \boldsymbol{v} = 0, \tag{1.1b}$$

which are written here in the non-dimensional form, i.e. $\epsilon \equiv Re^{-1}$ for brevity; also $\boldsymbol{v}$ is the velocity and p pressure fields. The boundary conditions are assumed to be simple, i.e. boundary conditions to be Dirichlet and Neumann for the normal and tangential velocities, respectively.

2 Summary

References

A Appendix

A.1 Transient schemes