Вычислительные схемы для Higher Order модели

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Дискретная сетка

В пространстве $\{x, y, z\} \in \mathbb{R}^3$ строится сетка

Перед введением схемы по z производится координатное отображение.

$$\xi|_{x,y} = \frac{z|_{x,y}}{s(x,y) - b(x,y)}$$

- равномерная сетка по x: $\{x_i\}:\quad x_i = i \cdot \Delta x$
- равномерная сетка по y: $\{y_j\}:\quad y_j = j \cdot \Delta y$
- неравномерная сетка по ξ : $\{\xi_k\}$

Также для получения схем, включающих z нужны следующие соотношения.

$$\begin{aligned}
&\Delta\xi_{k-\frac{1}{2}} = \xi_k - \xi_{k-1}\\
&\Delta\xi_{k+\frac{1}{2}} = \xi_{k+1} - \xi_k\\
&\Delta\xi_k = \frac{\Delta\xi_{k+\frac{1}{2}} - \Delta\xi_{k-\frac{1}{2}}}{2}\\
&f_{k-\frac{1}{2}} = \frac{f_k - f_{k-1}}{2}\\
&f_{k+\frac{1}{2}} = \frac{f_{k+1} - f_k}{2}\\
&f_k = \frac{\Delta\xi_{k-\frac{1}{2}}}{2\Delta\xi_k} f_{k+\frac{1}{2}} + \frac{\Delta\xi_{k+\frac{1}{2}}}{2\Delta\xi_k} f_{k-\frac{1}{2}}\\
&\left(\frac{\partial f}{\partial \xi}\right)_k = \frac{\Delta\xi_{k-\frac{1}{2}}}{\Delta\xi_k\Delta\xi_{k+\frac{1}{2}}} f_{k+\frac{1}{2}} - \frac{\Delta\xi_{k+\frac{1}{2}}}{\Delta\xi_k\Delta\xi_{k-\frac{1}{2}}} f_{k-\frac{1}{2}} + 2\frac{\Delta\xi_{k+\frac{1}{2}} - \Delta\xi_{k-\frac{1}{2}}}{\Delta\xi_{k+\frac{1}{2}}\Delta\xi_{k-\frac{1}{2}}} f_k
\end{aligned}$$

Схемы основных операторов

В данном разделе приведены дискретные схемы основных дифференциальных операторов.

2D операторы

$$\frac{\partial}{\partial x}\left(g\frac{\partial f}{\partial x}\right)_{i,j} = \frac{1}{\Delta x^2}\left\{f_{i-1,j}\left[g_{i-\frac{1}{2},j}\right] + f_{i,j}\left[-g_{i-\frac{1}{2},j} - g_{i+\frac{1}{2},j}\right] + f_{i+1,j}\left[g_{i+\frac{1}{2},j}\right]\right\}$$

$$\frac{\partial}{\partial x}\left(g\frac{\partial f}{\partial y}\right)_{i,j}=\frac{1}{4\Delta x\Delta y}\left\{f_{i-1,j-1}\left[g_{i-\frac{1}{2},j-\frac{1}{2}}\right]+f_{i-1,j}\left[-g_{i-\frac{1}{2},j-\frac{1}{2}}+g_{i-\frac{1}{2},j+\frac{1}{2}}\right]+\right.$$
$$+f_{i-1,j+1}\left[-g_{i-\frac{1}{2},j+\frac{1}{2}}\right]+f_{i,j-1}\left[g_{i-\frac{1}{2},j-\frac{1}{2}}-g_{i+\frac{1}{2},j-\frac{1}{2}}\right]+$$
$$+f_{i,j}\left[-g_{i-\frac{1}{2},j-\frac{1}{2}}+g_{i-\frac{1}{2},j+\frac{1}{2}}+g_{i+\frac{1}{2},j-\frac{1}{2}}-g_{i+\frac{1}{2},j+\frac{1}{2}}\right]+$$
$$+f_{i,j+1}\left[-g_{i-\frac{1}{2},j+\frac{1}{2}}+g_{i+\frac{1}{2},j+\frac{1}{2}}\right]+f_{i+1,j-1}\left[-g_{i+\frac{1}{2},j-\frac{1}{2}}\right]+$$
$$\left.+f_{i+1,j}\left[g_{i+\frac{1}{2},j-\frac{1}{2}}-g_{i+\frac{1}{2},j+\frac{1}{2}}\right]+f_{i+1,j+1}\left[g_{i+\frac{1}{2},j+\frac{1}{2}}\right]\right\}$$

$$\frac{\partial}{\partial y}\left(g\frac{\partial f}{\partial x}\right)_{i,j}=\frac{1}{4\Delta x\Delta y}\left\{f_{i-1,j-1}\left[g_{i-\frac{1}{2},j-\frac{1}{2}}\right]+f_{i-1,j}\left[g_{i-\frac{1}{2},j-\frac{1}{2}}-g_{i-\frac{1}{2},j+\frac{1}{2}}\right]+\right.$$
$$+f_{i-1,j+1}\left[-g_{i-\frac{1}{2},j+\frac{1}{2}}\right]+f_{i,j-1}\left[-g_{i-\frac{1}{2},j-\frac{1}{2}}+g_{i+\frac{1}{2},j-\frac{1}{2}}\right]+$$
$$+f_{i,j}\left[-g_{i-\frac{1}{2},j-\frac{1}{2}}+g_{i-\frac{1}{2},j+\frac{1}{2}}+g_{i+\frac{1}{2},j-\frac{1}{2}}-g_{i+\frac{1}{2},j+\frac{1}{2}}\right]+$$
$$+f_{i,j+1}\left[g_{i-\frac{1}{2},j+\frac{1}{2}}-g_{i+\frac{1}{2},j+\frac{1}{2}}\right]+f_{i+1,j-1}\left[-g_{i+\frac{1}{2},j-\frac{1}{2}}\right]+$$
$$\left.+f_{i+1,j}\left[-g_{i+\frac{1}{2},j-\frac{1}{2}}+g_{i+\frac{1}{2},j+\frac{1}{2}}\right]+f_{i+1,j+1}\left[g_{i+\frac{1}{2},j+\frac{1}{2}}\right]\right\}$$

$$\frac{\partial}{\partial y}\left(g\frac{\partial f}{\partial y}\right)_{i,j}=\frac{1}{\Delta y^2}\left\{f_{i,j-1}\left[g_{i,j-\frac{1}{2}}\right]+f_{i,j}\left[-g_{i,j-\frac{1}{2}}-g_{i,j+\frac{1}{2}}\right]+f_{i,j+1}\left[g_{i,j+\frac{1}{2}}\right]\right\}$$

3D операторы

$$\frac{\partial}{\partial x}\left(g\frac{\partial f}{\partial x}\right)_{i,j,k}=\frac{1}{\Delta x^2}\left\{f_{i-1,j,k}\left[g_{i-\frac{1}{2},j,k}\right]+f_{i,j,k}\left[-g_{i-\frac{1}{2},j,k}-g_{i+\frac{1}{2},j,k}\right]+f_{i+1,j,k}\left[g_{i+\frac{1}{2},j,k}\right]\right\}$$

$$\frac{\partial}{\partial x}\left(g\frac{\partial f}{\partial y}\right)_{i,j,k}=\frac{1}{4\Delta x\Delta y}\left\{f_{i-1,j-1,k}\left[g_{i-\frac{1}{2},j-\frac{1}{2},k}\right]+f_{i-1,j,k}\left[-g_{i-\frac{1}{2},j-\frac{1}{2},k}+g_{i-\frac{1}{2},j+\frac{1}{2},k}\right]+\right.$$
$$+f_{i-1,j+1,k}\left[-g_{i-\frac{1}{2},j+\frac{1}{2},k}\right]+f_{i,j-1,k}\left[g_{i-\frac{1}{2},j-\frac{1}{2},k}-g_{i+\frac{1}{2},j-\frac{1}{2},k}\right]+$$
$$+f_{i,j,k}\left\{-g_{i-\frac{1}{2},j-\frac{1}{2},k}+g_{i-\frac{1}{2},j+\frac{1}{2},k}+g_{i+\frac{1}{2},j-\frac{1}{2},k}-g_{i+\frac{1}{2},j+\frac{1}{2},k}\right\}+$$
$$+f_{i,j+1,k}\left[-g_{i-\frac{1}{2},j+\frac{1}{2},k}+g_{i+\frac{1}{2},j+\frac{1}{2},k}\right]+f_{i+1,j-1,k}\left[-g_{i+\frac{1}{2},j-\frac{1}{2},k}\right]+$$
$$\left.+f_{i+1,j,k}\left[g_{i+\frac{1}{2},j-\frac{1}{2},k}-g_{i+\frac{1}{2},j+\frac{1}{2},k}\right]+f_{i+1,j+1,k}\left[g_{i+\frac{1}{2},j+\frac{1}{2},k}\right]\right\}$$

$$\frac{\partial}{\partial x}\left(g\frac{\partial f}{\partial \xi}\right)_{i,j,k}=\frac{1}{4\Delta x\Delta\xi_k\Delta\xi_{k-\frac{1}{2}}\Delta\xi_{k+\frac{1}{2}}}\times$$
$$\times\left\{f_{i-1,j,k-1}\left[\Delta\xi^2_{k+\frac{1}{2}}g_{i-\frac{1}{2},j,k-\frac{1}{2}}\right]+f_{i-1,j,k}\left[-\Delta\xi^2_{k+\frac{1}{2}}g_{i-\frac{1}{2},j,k-\frac{1}{2}}+\Delta\xi^2_{k-\frac{1}{2}}g_{i-\frac{1}{2},j,k+\frac{1}{2}}\right]+\right.$$
$$+f_{i-1,j,k+1}\left[-\Delta\xi^2_{k-\frac{1}{2}}g_{i-\frac{1}{2},j,k+\frac{1}{2}}\right]+f_{i,j,k-1}\left[\Delta\xi^2_{k+\frac{1}{2}}g_{i-\frac{1}{2},j,k-\frac{1}{2}}-\Delta\xi^2_{k+\frac{1}{2}}g_{i+\frac{1}{2},j,k-\frac{1}{2}}\right]+$$
$$+f_{i,j,k}\left[-\Delta\xi^2_{k+\frac{1}{2}}g_{i-\frac{1}{2},j,k-\frac{1}{2}}+\Delta\xi^2_{k-\frac{1}{2}}g_{i-\frac{1}{2},j,k+\frac{1}{2}}+\Delta\xi^2_{k+\frac{1}{2}}g_{i+\frac{1}{2},j,k-\frac{1}{2}}-\Delta\xi^2_{k-\frac{1}{2}}g_{i+\frac{1}{2},j,k+\frac{1}{2}}\right]+$$
$$+f_{i,j,k+1}\left[-\Delta\xi^2_{k-\frac{1}{2}}g_{i-\frac{1}{2},j,k+\frac{1}{2}}+\Delta\xi^2_{k-\frac{1}{2}}g_{i+\frac{1}{2},j,k+\frac{1}{2}}\right]+f_{i+1,j,k-1}\left[-\Delta\xi^2_{k+\frac{1}{2}}g_{i+\frac{1}{2},j,k-\frac{1}{2}}\right]+$$
$$\left.+f_{i+1,j,k}\left[\Delta\xi^2_{k+\frac{1}{2}}g_{i+\frac{1}{2},j,k-\frac{1}{2}}-\Delta\xi^2_{k-\frac{1}{2}}g_{i+\frac{1}{2},j,k+\frac{1}{2}}\right]+f_{i+1,j,k+1}\left[\Delta\xi^2_{k-\frac{1}{2}}g_{i+\frac{1}{2},j,k+\frac{1}{2}}\right]\right\}$$

$$\begin{aligned}\frac{\partial}{\partial y}\left(g\frac{\partial f}{\partial x}\right)_{i,j,k} =& \frac{1}{4\Delta x\Delta y}\left\{f_{i-1,j-1,k}\left[g_{i-\frac{1}{2},j-\frac{1}{2},k}\right] + f_{i-1,j,k}\left[g_{i-\frac{1}{2},j-\frac{1}{2},k} - g_{i-\frac{1}{2},j+\frac{1}{2},k}\right] + \right.\\ &+ f_{i-1,j+1,k}\left[-g_{i-\frac{1}{2},j+\frac{1}{2},k}\right] + f_{i,j-1,k}\left[-g_{i-\frac{1}{2},j-\frac{1}{2},k} + g_{i+\frac{1}{2},j-\frac{1}{2},k}\right] +\\ &+ f_{i,j,k}\left\{-g_{i-\frac{1}{2},j-\frac{1}{2},k} + g_{i-\frac{1}{2},j+\frac{1}{2},k} + g_{i+\frac{1}{2},j-\frac{1}{2},k} - g_{i+\frac{1}{2},j+\frac{1}{2},k}\right\} +\\ &+ f_{i,j+1,k}\left[g_{i-\frac{1}{2},j+\frac{1}{2},k} - g_{i+\frac{1}{2},j+\frac{1}{2},k}\right] + f_{i+1,j-1,k}\left[-g_{i+\frac{1}{2},j-\frac{1}{2},k}\right] +\\ &\left.+ f_{i+1,j,k}\left[-g_{i+\frac{1}{2},j-\frac{1}{2},k} + g_{i+\frac{1}{2},j+\frac{1}{2},k}\right] + f_{i+1,j+1,k}\left[g_{i+\frac{1}{2},j+\frac{1}{2},k}\right]\right\}\end{aligned}$$

$$\frac{\partial}{\partial y}\left(g\frac{\partial f}{\partial y}\right)_{i,j,k} = \frac{1}{\Delta y^2}\left\{f_{i,j-1,k}\left[g_{i,j-\frac{1}{2},k}\right] + f_{i,j,k}\left[-g_{i,j-\frac{1}{2},k} - g_{i,j+\frac{1}{2},k}\right] + f_{i,j+1,k}\left[g_{i,j+\frac{1}{2},k}\right]\right\}$$

$$\begin{aligned}\frac{\partial}{\partial y}\left(g\frac{\partial f}{\partial \xi}\right)_{i,j,k} =& \frac{1}{4\Delta y\Delta\xi_k\Delta\xi_{k-\frac{1}{2}}\Delta\xi_{k+\frac{1}{2}}}\times\\ &\times\left\{f_{i,j-1,k-1}\left[\Delta\xi^2_{k+\frac{1}{2}}g_{i,j-\frac{1}{2},k-\frac{1}{2}}\right] + f_{i,j-1,k}\left[-\Delta\xi^2_{k+\frac{1}{2}}g_{i,j-\frac{1}{2},k-\frac{1}{2}} + \Delta\xi^2_{k-\frac{1}{2}}g_{i,j-\frac{1}{2},k+\frac{1}{2}}\right] +\right.\\ &+ f_{i,j-1,k+1}\left[-\Delta\xi^2_{k-\frac{1}{2}}g_{i,j-\frac{1}{2},k+\frac{1}{2}}\right] + f_{i,j,k-1}\left[\Delta\xi^2_{k+\frac{1}{2}}g_{i,j-\frac{1}{2},k-\frac{1}{2}} - \Delta\xi^2_{k+\frac{1}{2}}g_{i,j+\frac{1}{2},k-\frac{1}{2}}\right] +\\ &+ f_{i,j,k}\left[-\Delta\xi^2_{k+\frac{1}{2}}g_{i,j-\frac{1}{2},k-\frac{1}{2}} + \Delta\xi^2_{k-\frac{1}{2}}g_{i,j-\frac{1}{2},k+\frac{1}{2}} + \Delta\xi^2_{k+\frac{1}{2}}g_{i,j+\frac{1}{2},k-\frac{1}{2}} - \Delta\xi^2_{k-\frac{1}{2}}g_{i,j+\frac{1}{2},k+\frac{1}{2}}\right] +\\ &+ f_{i,j,k+1}\left[-\Delta\xi^2_{k-\frac{1}{2}}g_{i,j-\frac{1}{2},k+\frac{1}{2}} + \Delta\xi^2_{k-\frac{1}{2}}g_{i,j+\frac{1}{2},k+\frac{1}{2}}\right] + f_{i,j+1,k-1}\left[-\Delta\xi^2_{k+\frac{1}{2}}g_{i,j+\frac{1}{2},k-\frac{1}{2}}\right] +\\ &\left.+ f_{i,j+1,k}\left[\Delta\xi^2_{k+\frac{1}{2}}g_{i,j+\frac{1}{2},k-\frac{1}{2}} - \Delta\xi^2_{k-\frac{1}{2}}g_{i,j+\frac{1}{2},k+\frac{1}{2}}\right] + f_{i,j+1,k+1}\left[\Delta\xi^2_{k-\frac{1}{2}}g_{i,j+\frac{1}{2},k+\frac{1}{2}}\right]\right\}\end{aligned}$$

$$\begin{aligned}\frac{\partial}{\partial \xi}\left(g\frac{\partial f}{\partial x}\right)_{i,j,k} =& \frac{1}{4\Delta x\Delta\xi_k\Delta\xi_{k-\frac{1}{2}}\Delta\xi_{k+\frac{1}{2}}} \times \left\{f_{i-1,j,k-1}\left[\Delta\xi^2_{k+\frac{1}{2}}g_{i-\frac{1}{2},j,k-\frac{1}{2}}\right] +\right.\\ &+ f_{i-1,j,k}\left[-4\Delta\xi_k\left(\Delta\xi_{k+\frac{1}{2}} - \Delta\xi_{k-\frac{1}{2}}\right)g_{i-\frac{1}{2},j,k} + \Delta\xi_{k+\frac{1}{2}}g_{i-\frac{1}{2},j,k-\frac{1}{2}} - \Delta\xi_{k-\frac{1}{2}}g_{i-\frac{1}{2},j,k+\frac{1}{2}}\right] +\\ &+ f_{i-1,j,k+1}\left[-\Delta\xi^2_{k-\frac{1}{2}}g_{i-\frac{1}{2},j,k+\frac{1}{2}}\right] + f_{i,j,k-1}\left[-\Delta\xi^2_{k+\frac{1}{2}}g_{i-\frac{1}{2},j,k-\frac{1}{2}} + \Delta\xi^2_{k+\frac{1}{2}}g_{i+\frac{1}{2},j,k-\frac{1}{2}}\right] +\\ &+ f_{i,j,k}\left[4\Delta\xi_k\left(\Delta\xi_{k+\frac{1}{2}} - \Delta\xi_{k-\frac{1}{2}}\right)\left(g_{i-\frac{1}{2},j,k} - g_{i+\frac{1}{2},j,k}\right) -\right.\\ &\qquad\left.-\Delta\xi^2_{k+\frac{1}{2}}g_{i-\frac{1}{2},j,k-\frac{1}{2}} + \Delta\xi^2_{k+\frac{1}{2}}g_{i-\frac{1}{2},j,k+\frac{1}{2}} + \Delta\xi^2_{k-\frac{1}{2}}g_{i+\frac{1}{2},j,k-\frac{1}{2}} - \Delta\xi^2_{k+\frac{1}{2}}g_{i+\frac{1}{2},j,k+\frac{1}{2}}\right] +\\ &+ f_{i,j,k+1}\left[\Delta\xi^2_{k-\frac{1}{2}}g_{i-\frac{1}{2},j,k+\frac{1}{2}} - \Delta\xi^2_{k-\frac{1}{2}}g_{i+\frac{1}{2},j,k+\frac{1}{2}}\right] + f_{i+1,j,k-1}\left[-\Delta\xi^2_{k+\frac{1}{2}}g_{i+\frac{1}{2},j,k-\frac{1}{2}}\right] +\\ &+ f_{i+1,j,k}\left[4\Delta\xi_k\left(\Delta\xi^2_{k+\frac{1}{2}} - \Delta\xi^2_{k-\frac{1}{2}}\right)g_{i+\frac{1}{2},j,k} - \Delta\xi^2_{k+\frac{1}{2}}g_{i+\frac{1}{2},j,k-\frac{1}{2}} + \Delta\xi^2_{k-\frac{1}{2}}g_{i+\frac{1}{2},j,k+\frac{1}{2}}\right] +\\ &\left.+ f_{i+1,j,k+1}\left[\Delta\xi^2_{k-\frac{1}{2}}g_{i+\frac{1}{2},j,k+\frac{1}{2}}\right]\right\}\end{aligned}$$

$$
\begin{aligned}
\frac{\partial}{\partial\xi}\left(g\frac{\partial f}{\partial y}\right)_{i,j,k} =& \frac{1}{4\Delta y\Delta\xi_k\Delta\xi_{k-\frac{1}{2}}\Delta\xi_{k+\frac{1}{2}}} \times \Big\{ f_{i,j-1,k-1}\left[\Delta\xi^2_{k+\frac{1}{2}} g_{i,j-\frac{1}{2},k-\frac{1}{2}}\right] + \\
&+ f_{i,j-1,k}\left[-4\Delta\xi_k\left(\Delta\xi_{k+\frac{1}{2}} - \Delta\xi_{k-\frac{1}{2}}\right) g_{i,j-\frac{1}{2},k} + \Delta\xi_{k+\frac{1}{2}} g_{i,j-\frac{1}{2},k-\frac{1}{2}} - \Delta\xi_{k-\frac{1}{2}} g_{i,j-\frac{1}{2},k+\frac{1}{2}}\right] + \\
&+ f_{i,j-1,k+1}\left[-\Delta\xi^2_{k-\frac{1}{2}} g_{i,j-\frac{1}{2},k+\frac{1}{2}}\right] + f_{i,j,k-1}\left[-\Delta\xi^2_{k+\frac{1}{2}} g_{i,j-\frac{1}{2},k-\frac{1}{2}} + \Delta\xi^2_{k+\frac{1}{2}} g_{i,j+\frac{1}{2},k-\frac{1}{2}}\right] + \\
&+ f_{i,j,k}\Big[4\Delta\xi_k\left(\Delta\xi_{k+\frac{1}{2}} - \Delta\xi_{k-\frac{1}{2}}\right)\left(g_{i,j-\frac{1}{2},k} - g_{i,j+\frac{1}{2},k}\right) - \\
&\qquad -\Delta\xi^2_{k+\frac{1}{2}} g_{i,j-\frac{1}{2},k-\frac{1}{2}} + \Delta\xi^2_{k+\frac{1}{2}} g_{i,j-\frac{1}{2},k+\frac{1}{2}} + \Delta\xi^2_{k-\frac{1}{2}} g_{i,j+\frac{1}{2},k-\frac{1}{2}} - \Delta\xi^2_{k+\frac{1}{2}} g_{i,j+\frac{1}{2},k+\frac{1}{2}}\Big] + \\
&+ f_{i,j,k+1}\left[\Delta\xi^2_{k-\frac{1}{2}} g_{i,j-\frac{1}{2},k+\frac{1}{2}} - \Delta\xi^2_{k-\frac{1}{2}} g_{i,j+\frac{1}{2},k+\frac{1}{2}}\right] + f_{i,j+1,k-1}\left[-\Delta\xi^2_{k+\frac{1}{2}} g_{i,j+\frac{1}{2},k-\frac{1}{2}}\right] + \\
&+ f_{i,j+1,k}\left[4\Delta\xi_k\left(\Delta\xi^2_{k+\frac{1}{2}} - \Delta\xi^2_{k-\frac{1}{2}}\right) g_{i,j+\frac{1}{2},k} - \Delta\xi^2_{k+\frac{1}{2}} g_{i,j+\frac{1}{2},k-\frac{1}{2}} + \Delta\xi^2_{k-\frac{1}{2}} g_{i,j+\frac{1}{2},k+\frac{1}{2}}\right] + \\
&+ f_{i,j+1,k+1}\left[\Delta\xi^2_{k-\frac{1}{2}} g_{i,j+\frac{1}{2},k+\frac{1}{2}}\right]\Big\}
\end{aligned}
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\begin{aligned}
\frac{\partial}{\partial\xi}\left(g\frac{\partial f}{\partial\xi}\right)_{i,j,k} =& \frac{1}{\Delta\xi_k\Delta\xi_{k-\frac{1}{2}}\Delta\xi_{k+\frac{1}{2}}} \left\{ f_{i,j,k-1}\left[-\frac{\left(\Delta\xi_{k+\frac{1}{2}} - \Delta\xi_{k-\frac{1}{2}}\right)\Delta\xi_{k-\frac{1}{2}}}{\Delta\xi_{k+\frac{1}{2}}} g_{i,j,k} + \frac{\Delta\xi^2_{k-\frac{1}{2}}}{\Delta\xi_{k+\frac{1}{2}}} g_{i,j,k-\frac{1}{2}}\right] + \right. \\
&+ f_{i,j,k}\left[2\frac{\Delta\xi_k\left(\Delta\xi_{k+\frac{1}{2}} - \Delta\xi_{k-\frac{1}{2}}\right)^2}{\Delta\xi_{k+\frac{1}{2}}\Delta\xi_{k-\frac{1}{2}}} g_{i,j,k} - \frac{\Delta\xi^2_{k+\frac{1}{2}}}{\Delta\xi_{k-\frac{1}{2}}} g_{i,j,k-\frac{1}{2}} - \frac{\Delta\xi^2_{k-\frac{1}{2}}}{\Delta\xi_{k+\frac{1}{2}}} g_{i,j,k+\frac{1}{2}}\right] + \\
&\left. + f_{i,j,k+1}\left[\frac{\left(\Delta\xi_{k+\frac{1}{2}} - \Delta\xi_{k-\frac{1}{2}}\right)\Delta\xi_{k-\frac{1}{2}}}{\Delta\xi_{k+\frac{1}{2}}} g_{i,j,k} + \frac{\Delta\xi^2_{k-\frac{1}{2}}}{\Delta\xi_{k+\frac{1}{2}}} g_{i,j,k+\frac{1}{2}}\right]\right\}
\end{aligned}
$$