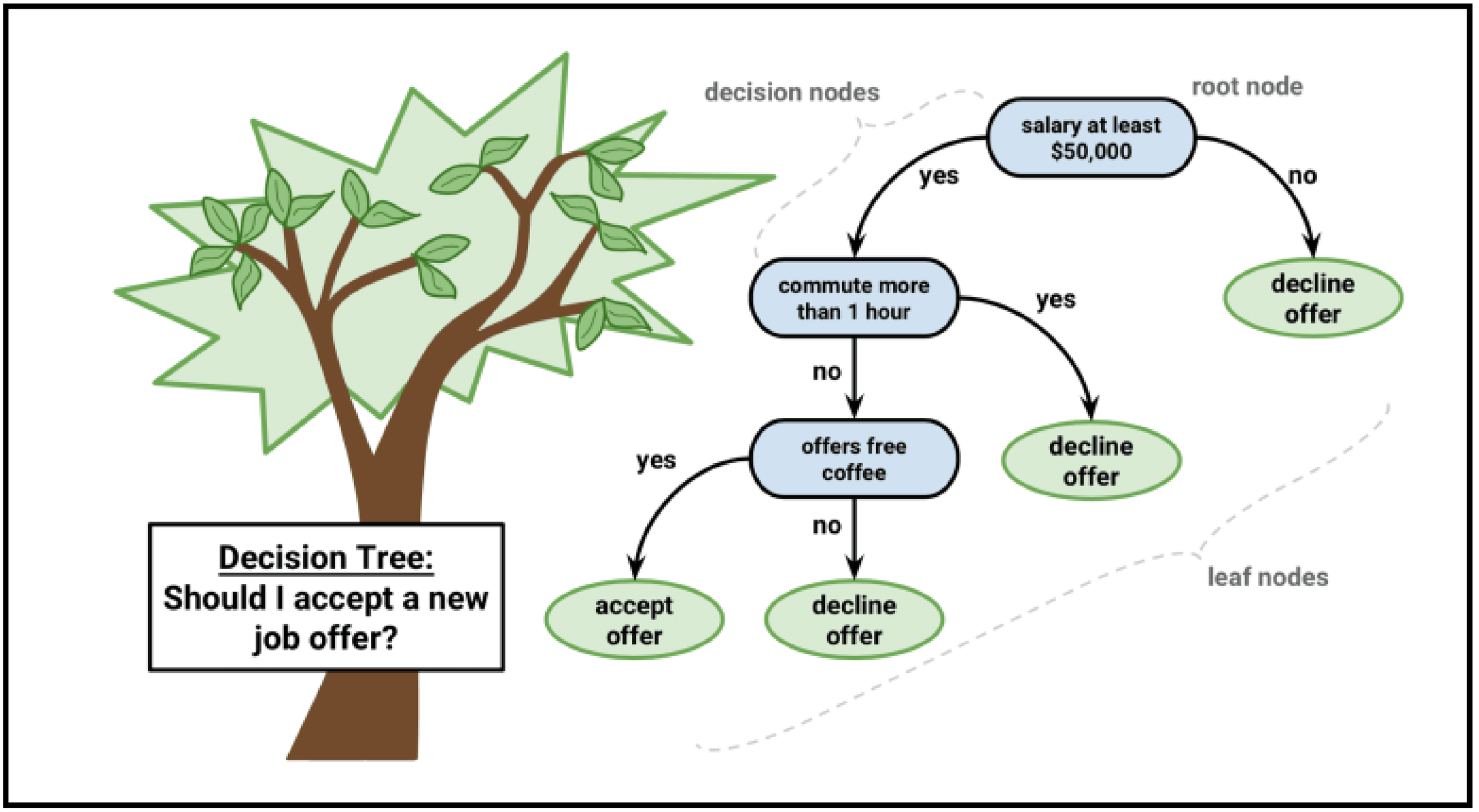
Decision Tree (ID3 Algorithm)

(Numerical)

Decision Tree and ID3 Algorithm

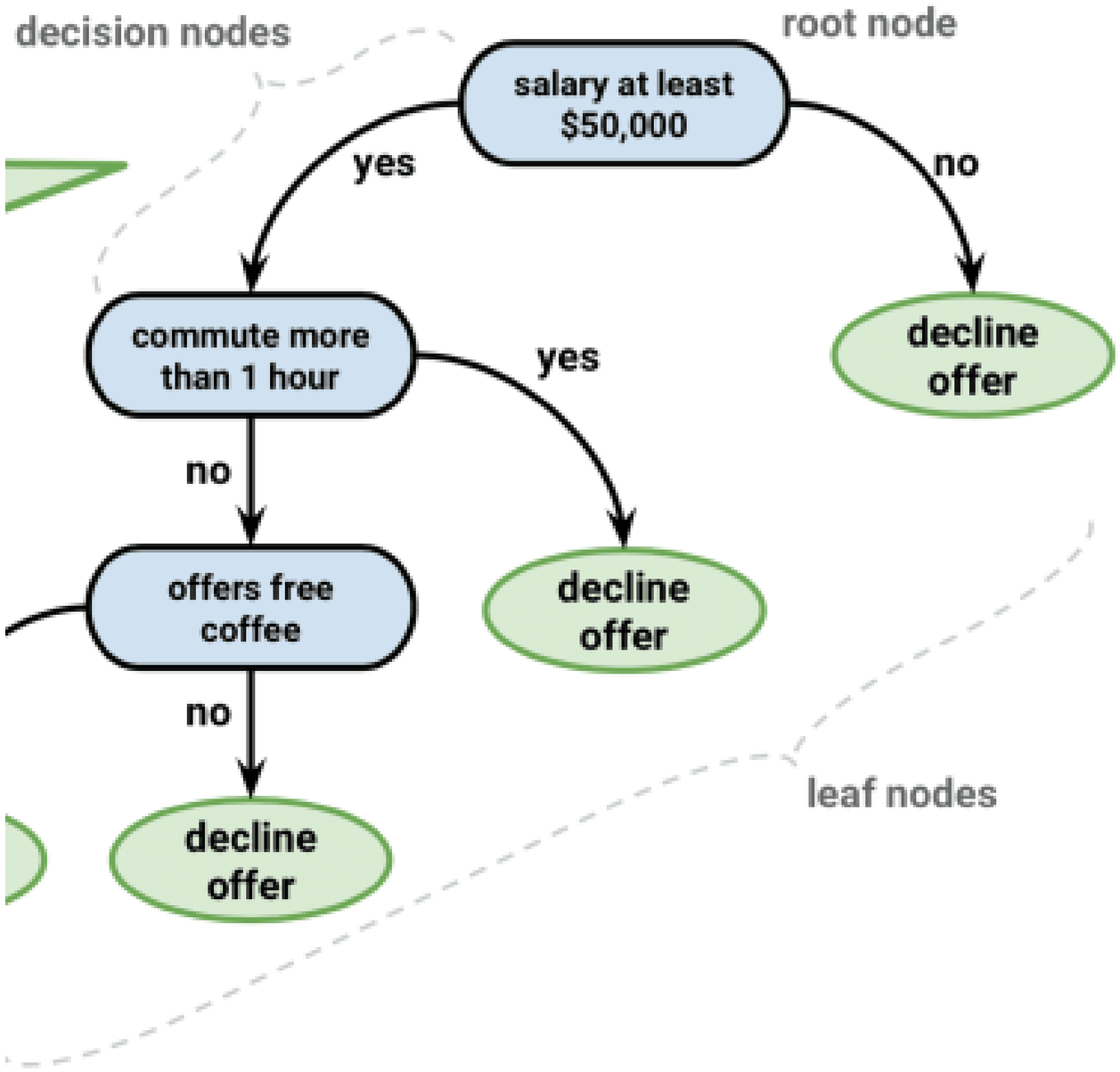
S. No.	Outlook	Temperature	Humidity	Windy	PlayTennis
1	Sunny	Hot	High	Weak	No
2	Sunny	Hot	High	Strong	No
3	Overcast	Hot	High	Weak	Yes
4	Rainy	Mild	High	Weak	Yes
5	Rainy	Cool	Normal	Weak	Yes
6	Rainy	Cool	Normal	Strong	No
7	Overcast	Cool	Normal	Strong	Yes
8	Sunny	Mild	High	Weak	No
9	Sunny	Cool	Normal	Weak	Yes
10	Rainy	Mild	Normal	Weak	Yes
11	Sunny	Mild	Normal	Strong	Yes
12	Overcast	Mild	High	Strong	Yes
13	Overcast	Hot	Normal	Weak	Yes
14	Rainy	Mild	High	Strong	No

Make a decision tree that predicts whether tennis will be played on the day?

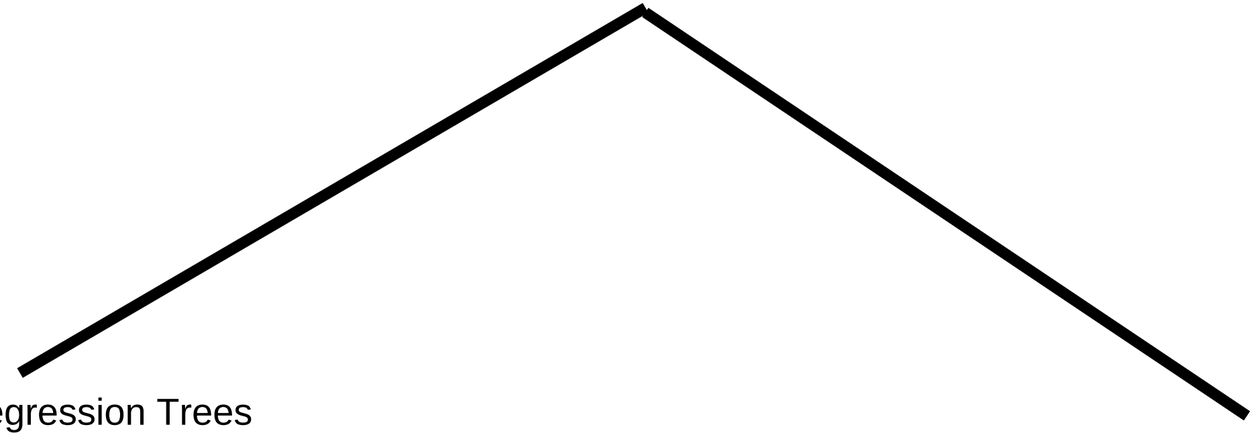

Decision Tree:
Should I accept a new job offer?
decision nodes
root node
salary at least $50,000
yes
no
commute more than 1 hour
yes
decline offer
no
offers free coffee
yes
decline offer
yes
no
accept offer
decline offer
leaf nodes

WHAT IS DECISON TREE?

A ***Decision Tree*** is a tree where *each node* represents a **Feature (Attribute)**, each link (branch) represents a **decision (rule)** and each leaf represents an **outcome.**



Algorithms



Classification And Regression Trees

CART

- Gini Index

ID3

- Entropy Function
- Information Gain

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Step 1: Create a root node

- How to choose the root node?

  The attribute that best classifies the training data, use this attribute at the root of the tree.

Step 1: Create a root node

- How to choose the root node?

  The attribute that best classifies the training data, use this attribute at the root of the tree.

- How to choose the best attribute?

  So from here, *ID3 algorithm* begins

- Calculate **Entropy** (Amount of uncertainity in dataset):

$$Entropy = \frac{-p}{p+n} log_2(\frac{p}{p+n}) - \frac{n}{p+n} log_2(\frac{n}{p+n})$$

- Calculate **Average Information**:

$$I(Attribute) = \sum \frac{p_i + n_i}{p_T + n_T} Entropy(A)$$

- Calculate **Information Gain**: (Difference in Entropy before and after splitting dataset on attribute A)

$$Gain = Entropy(S) - I(Attribute)$$

1. COMPUTE THE **ENTROPY** FOR DATA-SET **ENTROPY(S)**

2. FOR EVERY ATTRIBUTE/FEATURE:

    1. CALCULATE ENTROPY FOR ALL OTHER VALUES **ENTROPY(A)**

    2. TAKE **AVERAGE INFORMATION ENTROPY** FOR THE CURRENT ATTRIBUTE

    3. CALCULATE **GAIN** FOR THE CURRENT ATTRIBUTE

3. PICK THE **HIGHEST GAIN ATTRIBUTE**.

4. **REPEAT** UNTIL WE GET THE TREE WE DESIRED.

1.

S. No.	Outlook	Temperature	Humidity	Windy	PlayTennis
1	Sunny	Hot	High	Weak	No •
2	Sunny	Hot	High	Strong	No •
3	Overcast	Hot	High	Weak	Yes
4	Rainy	Mild	High	Weak	Yes
5	Rainy	Cool	Normal	Weak	Yes
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13	Overcast	Hot	Normal	Weak	Yes
14	Rainy	Mild	High	Strong	No •

P = 9

N = 5

Total = 14

- Calculate **Entropy(S):**

$$Entropy = \frac{-p}{p+n} log_2(\frac{p}{p+n}) - \frac{n}{p+n} log_2(\frac{n}{p+n})$$

$$Entropy(S) = \frac{-9}{9+5} log_2(\frac{9}{9+5}) - \frac{5}{9+5} log_2(\frac{5}{9+5})$$

$$Entropy(S) = \frac{-9}{14} log_2(\frac{9}{14}) - \frac{5}{14} log_2(\frac{5}{14}) \quad = 0.940$$

- For each Attribute: (let say **Outlook**)
  - Calculate Entropy for each Values, i.e for 'Sunny', 'Rainy','Overcast'

Outlook	PlayTennis
Sunny	No
Sunny	No
Sunny	No
Sunny	Yes
Sunny	Yes

Outlook	PlayTennis
Rainy	Yes
Rainy	Yes
Rainy	No
Rainy	Yes
Rainy	No

Outlook	PlayTennis
Overcast	Yes
Overcast	Yes
Overcast	Yes
Overcast	Yes

Outlook	p	n	Entropy
Sunny	2	3	0.971
Rainy	3	2	0.971
Overcast	4	0	0

Calculate **Entropy(Outlook='Value'):**

$$Entropy = \frac{-p}{p+n} log_2(\frac{p}{p+n}) - \frac{n}{p+n} log_2(\frac{n}{p+n})$$

$$\text{E (Outlook=sunny)} = -\frac{2}{5}\log\left(\frac{2}{5}\right) - \frac{3}{5}\log\left(\frac{3}{5}\right) = 0.971$$

$$\text{E (Outlook=overcast)} = -1\log(1) - 0\log(0) = 0$$

$$\text{E (Outlook=rainy)} = -\frac{3}{5}\log\left(\frac{3}{5}\right) - \frac{2}{5}\log\left(\frac{2}{5}\right) = 0.971$$

- Calculate **Average Information Entropy**:

$$I(Outlook) = \frac{p_{sunny} + n_{sunny}}{p+n} Entropy(Outlook = Sunny) +$$

$$\frac{p_{rainy} + n_{rainy}}{p+n} Entropy(Outlook = Rainy) +$$

$$\frac{p_{Overcast} + n_{Overcast}}{p+n} Entropy(Outlook = Overcast)$$

$$I(Outlook) = \frac{3+2}{9+5} * 0.971 + \frac{2+3}{9+5} * 0.971 + \frac{4+0}{9+5} * 0 \quad = 0.693$$

- Calculate **Gain**: attribute is Outlook

$$Gain = Entropy(S) - I(Attribute)$$

$$Entropy(S) = 0.940$$

$$Gain(Outlook) = 0.940 - 0.693 = 0.247$$

- For each Attribute: (let say **Temperature**)
  - Calculate Entropy for each Temp, i.e for 'Hot', 'Mild' and 'Cool'

Temperature	PlayTennis
Hot	No
Hot	No
Hot	Yes
Hot	Yes

Temperature	PlayTennis
Mild	Yes
Mild	No
Mild	Yes
Mild	Yes
Mild	Yes
Mild	No

Temperature	PlayTennis
Cool	Yes
Cool	No
Cool	Yes
Cool	Yes

Temperature	p	n	Entropy
Hot	2	2	1
Mild	4	2	0.918
Cool	3	1	0.811

- Calculate **Average Information Entropy**:

$$I(Temperature) = \frac{p_{hot} + n_{hot}}{p+n} Entropy(Temperature = Hot) +$$

$$\frac{p_{mild} + n_{mild}}{p+n} Entropy(Temperature = Mild) +$$

$$\frac{p_{Cool} + n_{Cool}}{p+n} Entropy(Temperature = Cool)$$

$$I(Temperature) = \frac{2+2}{9+5} * 1 + \frac{4+2}{9+5} * 0.918 + \frac{3+1}{9+5} * 0.811 => 0.911$$

- Calculate **Gain**: attribute is Temperature

$$Gain = Entropy(S) - I(Attribute)$$

$$Entropy(S) = 0.940$$

$$Gain(Temperature) = 0.940 - 0.911 = 0.029$$

- For each Attribute: (let say **Humidity**)
  - Calculate Entropy for each Humidity, i.e for 'High', 'Normal'

Humidity	PlayTennis
Normal	Yes
Normal	No
Normal	Yes
Normal	Yes
Normal	Yes
Normal	Yes
Normal	Yes

Humidity	PlayTennis
High	No
High	No
High	Yes
High	Yes
High	No
High	Yes
High	No

Humidity	p	n	Entropy
High	3	4	0.985
Normal	6	1	0.591

- Calculate **Average Information Entropy**:

$$I(Humidity) = \frac{p_{High} + n_{High}}{p+n} Entropy(Humidity = High) +$$

$$\frac{p_{Normal} + n_{Normal}}{p+n} Entropy(Humidity = Normal)$$

$$I(Humidity) = \frac{3+4}{9+5} * 0.985 + \frac{6+1}{9+5} * 0.591 => 0.788$$

- Calculate **Gain**: attribute is Humidity

$$Gain = Entropy(S) - I(Attribute)$$

$$Entropy(S) = 0.940$$

$$Gain(Humidity) = 0.940 - 0.788 = 0.152$$

- For each Attribute: (let say **Windy**)
  - Calculate Entropy for each Windy, i.e for 'Strong' and 'Weak'

Windy	PlayTennis
Weak	No
Weak	Yes
Weak	Yes
Weak	Yes
Weak	No
Weak	Yes
Weak	Yes
Weak	Yes

Windy	PlayTennis
Strong	No
Strong	No
Strong	Yes
Strong	Yes
Strong	Yes
Strong	No

Windy	p	n	Entropy
Strong	3	3	1
Weak	6	2	0.811

- Calculate **Average Information Entropy**:

$$I(Windy) = \frac{p_{Strong} + n_{Strong}}{p + n} Entropy(Windy = Strong) +$$

$$\frac{p_{Weak} + n_{Weak}}{p + n} Entropy(Windy = Weak)$$

$$I(Windy) = \frac{3+3}{9+5} * 1 + \frac{6+2}{9+5} * 0.811 => 0.892$$

- Calculate **Gain**: attribute is Windy

$$Gain = Entropy(S) - I(Attribute)$$

$$Entropy(S) = 0.940$$

$$Gain(Windy) = 0.940 - 0.892 = 0.048$$

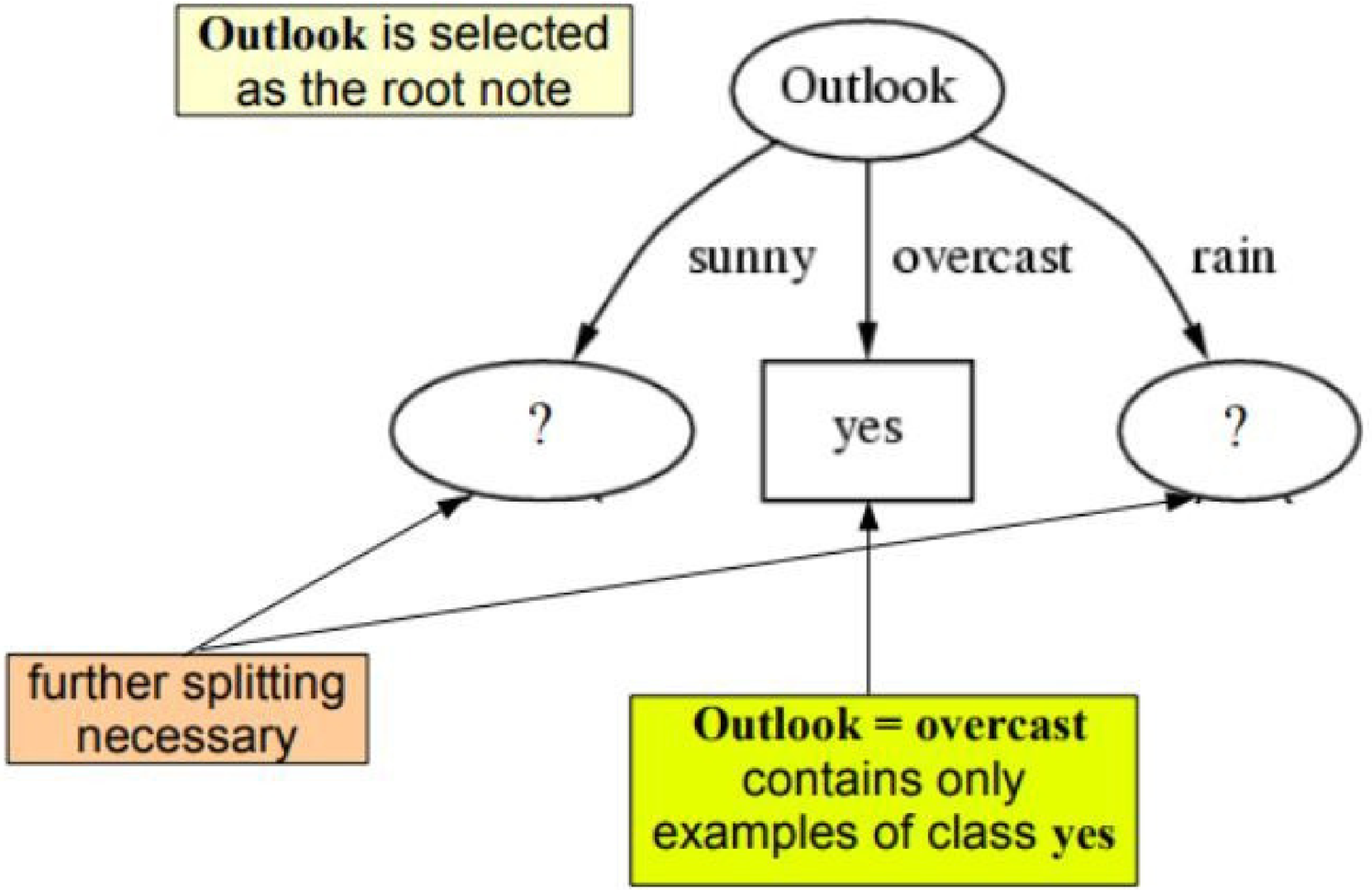
- Pick the highest gain attribute.

Attributes	Gain
Outlook	0.247
Temperature	0.029
Humidity	0.152
Windy	0.048

Root Node:

OUTLOOK

Outlook	Temperature	Humidity	Windy	PlayTennis
Overcast	Hot	High	Weak	Yes
Overcast	Cool	Normal	Strong	Yes
Overcast	Mild	High	Strong	Yes
Overcast	Hot	Normal	Weak	Yes



- Repeat the same thing for sub-trees till we get the tree.

Outlook	Temperature	Humidity	Windy	PlayTennis
Sunny	Hot	High	Weak	No
Sunny	Hot	High	Strong	No
Sunny	Mild	High	Weak	No
Sunny	Cool	Normal	Weak	Yes
Sunny	Mild	Normal	Strong	Yes

Outlook = "Sunny"

Outlook	Temperature	Humidity	Windy	PlayTennis
Rainy	Mild	High	Weak	Yes
Rainy	Cool	Normal	Weak	Yes
Rainy	Cool	Normal	Strong	No
Rainy	Mild	Normal	Weak	Yes
Rainy	Mild	High	Strong	No

Outlook = "Rainy"

Outlook	Temperature	Humidity	Windy	PlayTennis
Sunny	Hot	High	Weak	No
Sunny	Hot	High	Strong	No
Sunny	Mild	High	Weak	No
Sunny	Cool	Normal	Weak	Yes
Sunny	Mild	Normal	Strong	Yes

P= 2

N= 3

Total= 5

- ENTROPY:

$$Entropy = \frac{-p}{p+n} log_2(\frac{p}{p+n}) - \frac{n}{p+n} log_2(\frac{n}{p+n})$$

$$Entropy(S_{sunny}) = \frac{-2}{2+3} log_2(\frac{2}{2+3}) - \frac{3}{2+3} log_2(\frac{3}{2+3})$$

=>0.971

- For each Attribute: (let say **Humidity**):
  - Calculate Entropy for each Humidity, i.e for 'High' and 'Normal'

Outlook	Humidity	PlayTennis
Sunny	High	No
Sunny	High	No
Sunny	High	No
Sunny	Normal	Yes
Sunny	Normal	Yes

Humidity	p	n	Entropy
high	0	3	0
normal	2	0	0

- Calculate **Average Information Entropy**: I(Humidity) = 0
- Calculate **Gain:** Gain = 0.971

- For each Attribute: (let say **Windy**):
  - Calculate Entropy for each Windy, i.e for 'Strong' and 'Weak'

Outlook	Windy	PlayTennis
Sunny	Strong	No
Sunny	Strong	Yes
Sunny	Weak	No
Sunny	Weak	No
Sunny	Weak	Yes

Windy	p	n	Entropy
Strong	1	1	1
Weak	1	2	0.918

- Calculate **Average Information Entropy**: I(Windy) = 0.951
- Calculate **Gain:** Gain = 0.020

- For each Attribute: (let say **Temperature**):
  - Calculate Entropy for each Windy, i.e for 'Cool', 'Hot' and 'Mild'

Outlook	Temperature	PlayTennis
Sunny	Cool	Yes
Sunny	Hot	No
Sunny	Hot	No
Sunny	Mild	No
Sunny	Mild	Yes

Temperature	p	n	Entropy
Cool	1	0	0
Hot	0	2	0
Mild	1	1	1

- Calculate **Average Information Entropy**: I(Temp) = 0.4
- Calculate **Gain:** Gain = 0.571

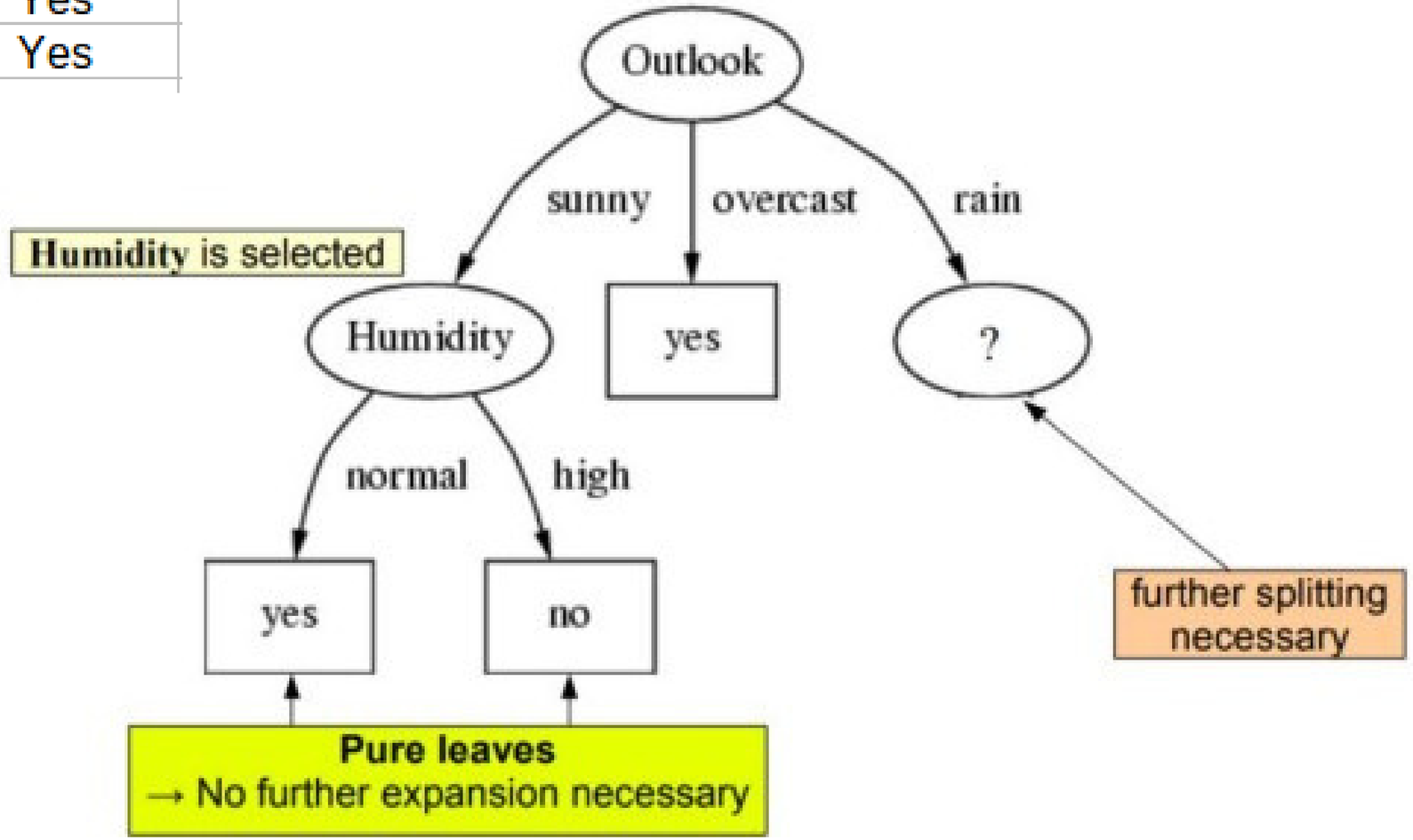
- Pick the highest gain attribute.

Attributes	Gain
Temperature	0.571
Humidity	0.971
Windy	0.02

Next Node in sunny:

Humidity

Outlook	Humidity	PlayTennis
Sunny	High	No
Sunny	High	No
Sunny	High	No
Sunny	Normal	Yes
Sunny	Normal	Yes



Outlook	Temperature	Humidity	Windy	PlayTennis
Rainy	Mild	High	Weak	Yes
Rainy	Cool	Normal	Weak	Yes
Rainy	Cool	Normal	Strong	No
Rainy	Mild	Normal	Weak	Yes
Rainy	Mild	High	Strong	No

P= 3 N= 2

Total = 5

- ENTROPY:

$$Entropy = \frac{-p}{p+n} log_2(\frac{p}{p+n}) - \frac{n}{p+n} log_2(\frac{n}{p+n})$$

$$Entropy(S_{Rainy}) = \frac{-3}{3+2} log_2(\frac{3}{3+2}) - \frac{2}{3+2} log_2(\frac{2}{2+3})$$

=>0.971

- For each Attribute: (let say **Humidity**):
  - Calculate Entropy for each Humidity, i.e for 'High' and 'Normal'

Outlook	Humidity	PlayTennis
Rainy	High	Yes
Rainy	High	No
Rainy	Normal	Yes
Rainy	Normal	No
Rainy	Normal	Yes

Attribute	p	n	Entropy
High	1	1	1
Normal	2	1	0.918

  - Calculate **Average Information Entropy**: I(Humidity) = 0.951
  - Calculate **Gain:** Gain = 0.020

- For each Attribute: (let say **Windy**):
  - Calculate Entropy for each Windy, i.e for 'Strong' and 'Weak'

Outlook	Windy	PlayTennis
Rainy	Strong	No
Rainy	Strong	No
Rainy	Weak	Yes
Rainy	Weak	Yes
Rainy	Weak	Yes

Attribute	p	n	Entropy
Strong	0	2	0
Weak	3	0	0

- Calculate **Average Information Entropy**: I(Windy) = 0
- Calculate **Gain:** Gain = 0.971

- For each Attribute: (let say **Temperature**):
  - Calculate Entropy for each Windy, i.e for 'Cool', 'Hot' and 'Mild'

Outlook	Temperature	PlayTennis
Rainy	Mild	Yes
Rainy	Cool	Yes
Rainy	Cool	No
Rainy	Mild	Yes
Rainy	Mild	No

Attribute	p	n	Entropy
Cool	1	1	1
Mild	2	1	0.918

- Calculate **Average Information Entropy**: I(Temp) = 0.951
- Calculate **Gain:** Gain = 0.020

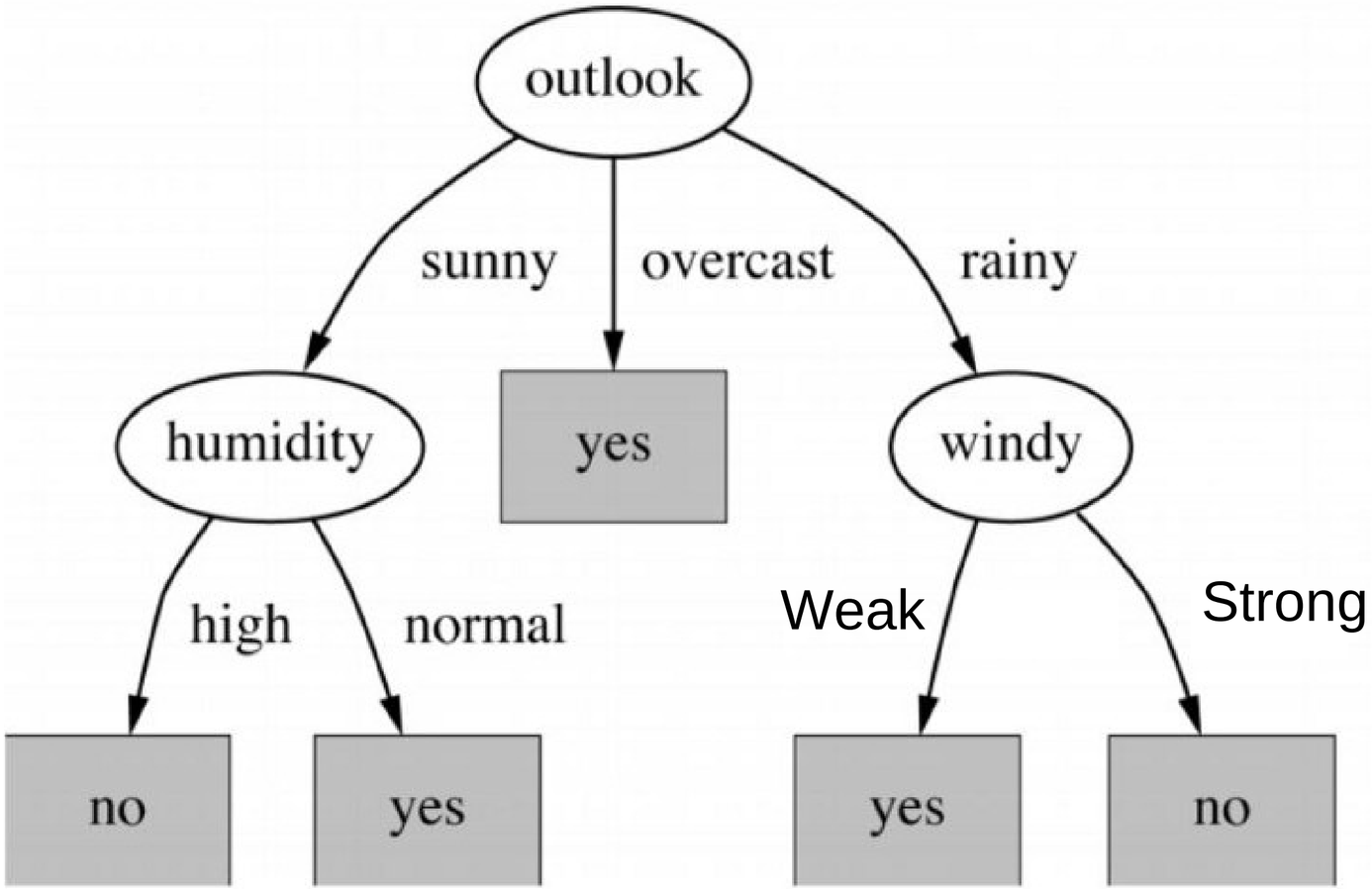
- Pick the highest gain attribute.

Attributes	Gain
Humidity	0.02
Windy	0.971
Temperature	0.02

Next Node in Rainy:

Windy

Final decision tree



Thank You